



Vertical Stiffness

Canopy Equation:

To a first approximation the tyres behave like a linear spring in the vertical direction, however some additional features require modelling. The equation governing the loaded radius is as follows:

$$R = R_0 - \frac{|F_z|}{k_0 - k_1|a_c|} + (k_2 + k_6|F_z|)n^2 + k_3p_{\text{inf}} + k_4|F_z| + k_5|F_z|^2 + k_7|F_z|p_{\text{inf}}$$

in which R_0 is the unloaded radius, F_z the vertical force on the tyre, a_c the camber angle of the tyre, n the wheel rotational speed and p_{inf} the inflation pressure. The coefficients k_0, \dots, k_4 are set as the user parameter vector **krLoaded**. This equation attempts to capture the effects of:

1. Vertical stiffness.
2. Reduced vertical stiffness at non-zero camber angles.
3. Growth of the tyre due to rotational speed.
4. Increase in stiffness due to increased inflation pressure.

We are aware that many tyre manufacturers supply their own loaded radius equations and parameters. If you would like these added to the model, Canopy will be happy to oblige. The Pirelli and Pacejka loaded radius and rolling radius equations are already implemented, and can be selected from the drop-down menu next to *RadiusEquations*.

Pacejka Equation:

The Pacejka loaded radius equation is given below for reference:

$$F_z = - \left(1 + q_{v2} \left(\frac{nR_0}{V_0} \right) - \left(q_{fcx1} \frac{F_x}{F_{z0}} \right)^2 - \left(q_{fcy1} \frac{F_y}{F_{z0}} \right)^2 + q_{fcg1} aCamber^2 \right) \left(q_{fz1} \frac{\rho}{R_0} + q_{fz2} \left(\frac{\rho}{R_0} \right)^2 \right) F_{z0}$$

Where:

$$\rho = R_0 + q_{v1} R_0 \left(\frac{nR_0}{V_0} \right)^2 - r_{\text{Loaded}}$$

Pacejka with pInflation Equation:

The "Pacejka with pInflation" loaded radius equation is an adapted version of the Pacejka loaded radius equation, with an added dependency on pInflation and the removal of the dependency on aCamber, among with a couple of other small changes. This equation is given: -

$$F_z = - \left(1 + q_{v2} \left(\frac{nR_0}{V_0} \right) - \left(q_{fcx1} \frac{F_x}{F_{z0}} \right)^2 - \left(q_{fcy1} \frac{F_y}{F_{z0}} \right)^2 \right) \left(q_{fz1} \frac{\rho}{R_0} + q_{fz2} \left(\frac{\rho}{R_0} \right)^2 \right) (1 + p_{fz1} dp_i) F_{z0}$$

Where:

$$\rho = q_{re0} R_0 + q_{v1} R_0 \left(\frac{nR_0}{V_0} \right)^2 - r_{\text{Loaded}}$$

Rolling Radius

Canopy Equation:

The rolling radius of the tyre (in the case of using the Canopy radius equations) is governed by a subset of the **krLoaded** terms via the following relation:

$$R_{\text{effroll}} = R_0 + (k_2 + k_6|F_z|)n^2 + k_3p_{\text{inf}} + k_4|F_z| + k_5|F_z|^2$$

This captures the inflation and growth-with-speed effects but effectively keeps the belt radius almost constant.

Pacejka Equation:

The Pacejka rolling radius equation is given below for reference:

$$R_{\text{effroll}} = R_0 \left(1 + q_{v1} \left(\frac{nR_0}{V_0} \right)^2 \right) - \frac{F_{z0}}{k_{\text{Vert}}} (D_{\text{Reff}} \arctan(B_{\text{Reff}} \rho^d) + F_{\text{Reff}} \rho^d)$$

