

# A computational study of a decomposition approach for the dynamic two-level uncapacitated facility location problem with single and multiple allocation

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## Abstract

This work presents a computational study for two variants of a dynamic or multi-period two-level uncapacitated facility location problem. In this problem, first-level plants serve different demand patterns of scattered clients over a planning horizon via second-level facilities. In the first variant, second-level facilities can be supplied by only one of the plants (single assignment); whereas, in the second, they can be served by more than one of the first-level plants (multiple allocation). As the demands vary over time, the different operating settings for plants and facilities, and client assignments need to be sought in each period to serve demands at minimal installation and transportation costs. Since both problem variants arise naturally in the context of logistics systems, it is of interest to have solution methods at hand for practitioners and researchers. To provide such a tool, this work presents an efficient decomposition approach to solve the two problem variants. It relies on Benders decomposition reformulations combined with a greedy randomized adaptive search procedure and different Benders cut separation procedures. The devised solution framework outperformed CPLEX and its Benders built-in algorithm on solving two different challenging large-scale instance sets.

*Keywords:* discrete location, dynamic two-level facility location, Benders decomposition method, Benders optimality cuts, GRASP.

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## 1. Tables referring to the complete and detailed computational results

Table 1: Computational running times in seconds for the multiple assignment case.

	Instance	UB	CPU (s) M					CPU (s) M <sup>2</sup>					GRASP gap <sup>a</sup> (%)		
			CPX	I	CF	MW	MW <sub>r</sub>	P	CPX	I	CF	MW		MW <sub>r</sub>	P
Ro and Tcha 100×200	R-10-100-200-5-a	11,044,461.79	14.81	9.23	8.81	24.95	10.20	8.68	19.41	8.44	18.00	8.82	13.31	9.34	1.23
	R-10-100-200-5-b	11,090,359.93	22.37	14.87	9.93	27.09	15.08	18.27	19.69	14.46	24.78	24.41	10.01	17.40	0.64
	R-10-100-200-5-c	11,043,872.61	17.66	7.18	13.90	20.34	14.70	13.39	18.56	10.58	13.11	10.39	11.84	14.82	1.03
	R-10-100-200-5-d	11,081,448.06	8.61	15.93	19.15	23.94	28.29	11.53	10.55	11.28	20.78	26.12	27.72	12.26	0.50
	R-10-100-200-10-a	22,083,067.87	35.72	125.35	89.68	94.65	118.15	56.37	34.42	164.48	72.72	97.42	151.24	65.78	0.87
	R-10-100-200-10-b	21,149,324.01	34.04	78.15	66.60	50.82	103.18	40.58	34.49	91.05	101.27	65.59	91.38	47.02	2.21
	R-10-100-200-10-c	20,965,204.59	36.68	83.21	84.34	83.90	79.52	79.98	24.19	83.05	88.75	100.71	79.88	83.62	1.22
	R-10-100-200-10-d	21,064,035.66	31.20	86.29	100.77	84.13	89.92	74.46	26.64	76.25	123.89	83.22	120.45	77.52	1.45
	I-10-100-200-5-a	10,930,717.93	42.67	16.13	14.76	16.70	12.00	11.14	35.11	16.36	22.47	28.34	28.42	15.41	1.05
	I-10-100-200-5-b	10,690,447.41	36.74	24.39	22.01	16.42	15.78	14.78	30.92	26.35	34.67	23.43	34.00	21.25	0.93
	I-10-100-200-5-c	11,078,305.04	32.32	12.74	13.65	11.62	25.58	15.17	38.53	29.91	29.66	23.75	44.30	22.78	0.80
	I-10-100-200-5-d	11,075,044.96	29.45	27.71	23.18	28.28	22.55	12.68	25.79	21.30	30.16	35.47	81.44	23.43	0.91
	I-10-100-200-10-a	21,781,896.60	692.95	250.90	203.74	229.73	298.74	121.85	1285.04	602.33	460.61	893.37	492.57	253.81	2.14
	I-10-100-200-10-b	21,042,819.94	194.31	105.87	111.76	111.40	90.27	70.60	329.28	293.86	206.57	351.31	128.56	111.91	0.00
	I-10-100-200-10-c	20,890,629.12	366.62	138.14	156.62	157.65	159.74	118.22	364.53	267.15	334.80	258.25	197.20	205.70	0.47
	I-10-100-200-10-d	21,080,876.67	3167.58	322.54	307.97	433.79	351.58	247.46	2137.50	1985.35	1379.13	813.72	1462.87	1356.47	0.58
	D-10-100-200-5-a	10,933,566.69	27.85	14.11	12.76	30.58	13.20	8.50	32.45	14.96	13.06	19.29	19.10	8.38	2.27
	D-10-100-200-5-b	10,783,782.89	7.05	16.45	11.79	15.09	16.41	10.34	6.75	13.03	21.20	14.21	15.18	11.74	1.59
	D-10-100-200-5-c	11,085,731.24	21.69	13.57	11.82	12.44	14.27	14.58	11.76	12.43	18.28	12.09	18.08	14.50	1.21
	D-10-100-200-5-d	11,086,101.70	6.73	22.41	22.15	22.45	19.02	12.31	7.73	25.13	24.51	23.08	27.47	11.66	0.68
D-10-100-200-10-a	21,828,372.18	200.66	168.35	174.51	189.90	168.63	98.99	508.68	275.30	280.57	306.14	450.66	167.46	2.17	
D-10-100-200-10-b	21,090,585.34	165.82	113.70	115.10	146.65	107.95	69.01	354.39	185.84	192.70	406.88	302.91	133.28	1.37	
D-10-100-200-10-c	20,967,669.47	95.74	153.00	142.33	139.02	111.99	107.40	255.55	248.71	227.25	278.98	184.39	138.75	1.15	
D-10-100-200-10-d	21,155,643.74	700.74	215.71	199.59	256.24	304.44	164.66	8896.24	686.52	574.63	512.37	776.48	929.27	0.78	
Torres-Soto and Uster 50×50	S-50-50-50-5-a	2,937,408.00	3.54	6.42	5.97	7.39	3.36	5.91	2.64	3.51	3.76	4.93	2.27	5.39	6.48
	S-50-50-50-5-b	2,896,559.00	3.07	5.70	5.95	6.48	2.66	6.28	2.44	5.03	4.69	5.76	4.15	5.47	4.10
	S-50-50-50-5-c	2,825,342.00	3.32	5.20	5.85	9.06	2.53	7.15	2.47	6.91	8.80	6.93	4.10	6.91	4.90
	S-50-50-50-5-d	2,724,222.50	3.19	3.44	3.35	4.71	2.37	5.44	2.16	3.46	3.32	3.18	2.10	5.48	2.21
	S-50-50-50-10-a	5,288,873.00	8.14	18.50	20.90	19.87	21.06	16.73	8.73	15.71	13.10	20.82	17.02	9.27	0.00
	S-50-50-50-10-b	4,937,556.00	16.84	14.92	16.96	13.56	10.14	11.80	14.70	15.58	23.88	18.27	8.70	15.29	0.00
	S-50-50-50-10-c	5,231,665.00	6.47	16.68	10.01	10.08	3.19	13.28	6.68	18.09	16.74	15.61	3.29	12.76	0.00
	S-50-50-50-10-d	4,868,281.00	6.54	3.99	9.36	7.15	5.52	12.56	6.58	7.62	6.57	10.80	5.14	9.62	2.56
	I-50-50-50-5-a	1,941,445.00	4.46	5.05	4.90	6.84	4.28	5.91	3.42	6.74	5.64	6.07	2.19	6.59	3.77
	I-50-50-50-5-b	1,980,111.50	4.13	3.87	4.98	5.59	4.17	6.21	3.16	3.83	4.23	4.55	3.31	5.22	0.00
	I-50-50-50-5-c	1,950,775.50	4.31	4.07	4.41	4.88	4.84	4.90	3.10	6.10	6.02	4.54	3.67	4.79	1.17
	I-50-50-50-5-d	1,949,023.00	2.68	3.40	2.52	4.39	1.60	5.14	2.25	3.09	2.36	3.37	1.97	4.76	1.73
	I-50-50-50-10-a	3,218,694.00	12.97	15.94	16.43	24.49	8.91	20.08	11.95	34.47	43.92	30.19	16.81	26.94	6.08
	I-50-50-50-10-b	2,745,488.00	12.56	15.86	20.78	21.77	9.91	18.49	12.53	30.11	36.06	24.70	13.85	20.57	4.13
	I-50-50-50-10-c	3,728,704.00	11.02	13.10	14.30	18.87	13.12	12.37	9.27	18.27	15.39	23.07	19.62	17.56	7.99
	I-50-50-50-10-d	3,350,786.00	11.74	19.49	25.52	19.18	11.77	12.95	9.37	15.28	14.62	28.32	16.55	13.91	8.74
	D-50-50-50-5-a	1,841,128.00	4.11	4.09	4.40	4.99	2.56	7.22	4.33	7.78	7.82	9.92	4.69	6.72	7.34
	D-50-50-50-5-b	1,995,085.50	3.44	4.81	4.07	4.95	4.03	6.51	3.30	4.19	4.25	4.78	3.40	5.11	3.29
	D-50-50-50-5-c	1,875,603.50	3.52	2.82	2.14	6.56	2.00	5.49	3.30	2.95	2.38	5.38	1.87	5.23	1.17
	D-50-50-50-5-d	1,856,735.00	2.66	3.67	3.51	2.94	2.01	6.38	2.27	2.36	2.36	3.12	2.88	5.31	5.72
D-50-50-50-10-a	3,037,055.00	9.34	12.86	10.90	19.84	5.55	15.83	10.47	24.93	26.18	40.24	18.84	18.24	7.13	
D-50-50-50-10-b	2,760,789.00	8.22	10.44	14.97	12.41	6.71	13.62	10.61	28.74	31.75	31.65	7.94	15.76	3.35	
D-50-50-50-10-c	3,139,103.00	9.14	24.74	27.18	22.76	18.89	17.21	9.94	39.52	42.93	44.65	19.44	14.73	1.74	
D-50-50-50-10-d	3,167,577.00	8.53	5.39	5.63	9.33	4.78	15.03	10.25	15.35	15.38	34.63	5.37	14.60	5.62	

<sup>a</sup> GRASP with time limit set to 1 second or 10 iterations.

Table 2: Number of explored branch-and-bound nodes for the multiple assignment case.

	Instance	UB	M						M <sup>2</sup>					
			CPX	I	CF	MW	MW <sub>t</sub>	P	CPX	I	CF	MW	MW <sub>t</sub>	P
Re and Tcha 100×200	R-10-100-200-5-a	11,044,461.79	8	25	40	58	35	26	17	25	33	42	63	41
	R-10-100-200-5-b	11,090,359.93	18	35	32	50	23	50	9	33	48	28	21	40
	R-10-100-200-5-c	11,043,872.61	14	33	36	42	23	18	20	34	70	39	33	43
	R-10-100-200-5-d	11,081,448.06	3	25	68	55	49	47	3	42	49	41	35	42
	R-10-100-200-10-a	22,083,067.87	10	36	35	56	86	31	12	69	80	194	171	98
	R-10-100-200-10-b	21,149,324.01	10	46	40	47	103	31	10	42	40	62	39	57
	R-10-100-200-10-c	20,965,204.59	18	23	22	30	8	28	13	38	38	63	15	69
	R-10-100-200-10-d	21,064,035.66	16	27	48	53	52	37	8	49	49	89	52	32
	I-10-100-200-5-a	10,930,717.93	20	98	86	102	41	58	20	90	141	135	157	83
	I-10-100-200-5-b	10,690,447.41	22	79	42	72	82	61	12	95	105	104	49	66
	I-10-100-200-5-c	11,078,305.04	20	53	65	70	98	44	90	211	166	215	108	102
	I-10-100-200-5-d	11,075,044.96	21	65	67	73	44	31	22	65	72	122	122	149
	I-10-100-200-10-a	21,781,844.65	544	510	447	478	366	304	468	923	1035	2208	719	1022
	I-10-100-200-10-b	21,042,819.94	69	188	188	171	136	160	69	278	277	345	116	224
	I-10-100-200-10-c	20,890,360.66	317	273	376	370	209	215	144	369	528	433	225	950
	I-10-100-200-10-d	21,080,876.67	3039	973	1162	1260	881	1042	1221	4345	3626	2417	2052	6576
	D-10-100-200-5-a	10,933,566.69	12	42	72	77	41	34	12	42	60	84	35	18
	D-10-100-200-5-b	10,783,782.89		25	29	46	34	36		53	27	37	28	33
	D-10-100-200-5-c	11,085,731.24	22	82	61	86	68	35	14	36	45	65	32	48
	D-10-100-200-5-d	11,086,101.70		30	54	71	57	41		36	49	53	54	32
	D-10-100-200-10-a	21,828,372.18	86	257	291	264	165	212	344	532	534	679	465	550
	D-10-100-200-10-b	21,090,570.13	108	121	170	197	129	133	315	591	522	750	626	543
	D-10-100-200-10-c	20,967,615.88	48	220	249	197	159	208	227	330	343	604	203	442
	D-10-100-200-10-d	21,155,643.74	749	580	578	579	434	536	6704	3522	3066	2004	1487	7573
Torres-Soto and Uster 50×50	S-50-50-50-5-a	2,937,408.00		70	82	118	40	31		37	43	71	27	26
	S-50-50-50-5-b	2,896,559.00		96	104	156	35	34		101	120	140	90	40
	S-50-50-50-5-c	2,825,342.00		52	51	204	27	39		182	219	188	36	33
	S-50-50-50-5-d	2,724,222.50		55	57	115	29	35		53	75	46	33	38
	S-50-50-50-10-a	5,288,873.00		55	83	61	109	64		113	63	72	124	30
	S-50-50-50-10-b	4,937,556.00	20	62	105	80	78	49	6	129	334	100	48	90
	S-50-50-50-10-c	5,231,665.00		96	58	54	16	29		118	140	80	17	33
	S-50-50-50-10-d	4,868,281.00		23	45	39	31	17		38	32	44	21	8
	I-50-50-50-5-a	1,941,445.00		84	84	129	58	58		221	123	165	17	36
	I-50-50-50-5-b	1,980,111.50		56	85	97	70	33		84	80	141	55	37
	I-50-50-50-5-c	1,950,775.50		75	76	109	65	28		226	213	126	151	49
	I-50-50-50-5-d	1,949,023.00		45	35	64	12	28		58	51	102	25	12
	I-50-50-50-10-a	3,218,694.00		127	74	189	43	66		225	360	327	98	55
	I-50-50-50-10-b	2,745,488.00		63	137	105	43	45		349	519	196	59	71
	I-50-50-50-10-c	3,728,704.00		69	96	118	78	54		89	101	196	151	39
	I-50-50-50-10-d	3,350,786.00		80	245	122	73	20		138	110	367	112	45
	D-50-50-50-5-a	1,841,128.00		44	46	77	21	37		199	194	251	104	38
	D-50-50-50-5-b	1,995,085.50		70	63	78	80	29		52	53	92	57	45
	D-50-50-50-5-c	1,875,603.50		30	30	135	23	25		31	28	154	15	21
	D-50-50-50-5-d	1,856,735.00		76	58	42	32	66		48	48	64	43	41
	D-50-50-50-10-a	3,037,055.00		57	69	120	33	35		90	97	339	105	41
	D-50-50-50-10-b	2,760,789.00		71	87	68	34	26		165	242	208	26	24
	D-50-50-50-10-c	3,139,103.00		153	166	126	97	55		241	278	312	118	42
	D-50-50-50-10-d	3,167,577.00		30	30	42	31	27		88	78	326	19	19

Blank results refer to a zero value.

Table 3: Computational running times for the single assignment case.

	Instance	UB	S						S <sup>#</sup>						S <sup>##</sup>						GRASP						
			CPX	I	CF	MW	MW <sub>t</sub>	P	CPX	I	CF	MW	MW <sub>t</sub>	P	CPX	I	CF	MW	MW <sub>t</sub>	P	CPX	I	CF	MW	MW <sub>t</sub>	P	gap <sup>b</sup> (%)
Ro and Tcha 100×200	R-10-100-200-5-a	11,044,461.79	144.54	15.99	16.31	16.11	16.85	16.30	133.56	16.23	14.84	16.35	20.28	20.01	78.52	13.04	16.16	13.14	16.00	15.46	121.66	17.28	18.15	17.41	14.56	15.13	1.23
	R-10-100-200-5-b	11,090,359.93	70.94	14.03	12.79	14.13	6.45	13.51	64.58	13.77	12.96	13.71	15.75	13.59	52.34	12.28	11.57	12.42	19.25	13.37	46.70	11.22	11.97	11.23	21.84	13.30	0.64
	R-10-100-200-5-c	11,043,872.61	44.03	11.92	12.32	12.01	15.34	17.30	60.11	10.90	11.00	11.08	15.50	13.09	38.01	14.31	11.64	12.85	13.43	13.85	64.49	12.14	11.94	12.18	14.74	14.30	1.03
	R-10-100-200-5-d	11,081,448.06	31.40	12.49	12.61	12.56	14.16	13.33	29.56	16.70	15.22	16.37	12.44	12.07	30.59	12.12	11.23	12.09	13.86	12.75	26.80	11.75	11.53	11.90	13.48	14.07	0.50
	R-10-100-200-10-a	22,083,067.87	162.02	37.75	50.30	37.09	50.45	35.93	144.34	42.41	54.31	43.60	43.40	37.26	116.21	32.15	30.47	32.26	46.00	31.58	112.19	33.81	40.37	33.64	41.24	38.23	2.14
	R-10-100-200-10-b	21,149,324.01	78.62	52.39	63.75	80.07	46.82	45.54	99.03	65.69	51.93	46.83	45.28	37.90	63.88	50.46	174.40	50.47	47.93	34.16	144.32	132.70	80.19	133.09	45.78	32.64	0.00
	R-10-100-200-10-c	20,965,204.59	71.65	43.78	52.26	43.65	59.61	31.89	122.10	53.23	45.16	53.93	74.38	32.55	53.71	51.59	42.17	51.30	128.08	28.64	76.20	43.79	36.98	44.29	152.33	36.77	0.47
	R-10-100-200-10-d	21,064,035.66	192.62	92.58	123.68	93.81	49.16	30.51	163.48	64.54	234.14	63.86	50.07	32.26	184.45	136.33	200.77	136.20	45.94	32.64	221.60	122.48	203.59	122.27	46.49	31.92	0.58
	I-10-100-200-5-a	10,930,717.93	165.42	18.20	18.34	18.14	21.53	16.68	252.23	21.81	21.98	22.39	29.77	20.24	270.39	24.95	20.65	24.77	28.01	24.37	331.66	21.55	19.88	21.84	27.19	22.52	0.87
	I-10-100-200-5-b	10,690,447.41	100.80	14.78	14.05	14.84	22.36	16.66	108.55	14.26	13.82	14.32	15.02	16.69	90.65	17.25	19.18	17.40	16.44	15.77	102.02	15.90	16.93	17.23	16.35	19.03	2.21
	I-10-100-200-5-c	11,078,305.04	127.62	15.92	17.00	16.08	27.22	17.76	79.28	17.06	16.85	17.30	25.67	19.62	135.61	18.95	19.53	19.04	32.61	21.79	163.88	17.86	18.26	17.94	25.52	18.21	1.22
	I-10-100-200-5-d	11,075,044.96	59.11	16.44	13.74	16.44	11.76	15.70	46.71	14.80	15.23	14.74	19.57	15.24	51.16	16.76	13.92	16.54	14.45	15.27	57.30	16.37	14.70	16.45	15.16	14.46	1.45
	I-10-100-200-10-a	21,781,844.65	2671.56	171.03	119.90	163.10	169.66	130.77	2660.45	145.45	118.39	143.66	102.63	126.76	3998.62	178.20	181.85	158.73	454.73	154.05	7089.30	197.43	183.16	197.92	175.47	123.30	2.27
	I-10-100-200-10-b	21,042,819.94	813.97	87.04	80.96	85.90	88.85	64.60	929.90	94.56	76.81	98.32	91.41	71.71	1472.43	230.32	127.08	142.04	73.19	82.77	1107.87	118.17	77.14	112.17	70.96	112.30	1.43
	I-10-100-200-10-c	20,890,360.66	1247.49	90.04	88.38	89.92	134.38	81.66	1401.66	111.82	115.94	116.79	55.74	102.21	1031.72	122.71	108.53	120.66	177.21	89.22	1691.26	117.40	122.77	87.63	440.73	107.45	1.21
	I-10-100-200-10-d	21,080,876.67	10020.83	278.13	244.92	282.92	317.50	152.37	14160.98	313.46	359.67	316.47	256.23	248.65	9439.16	510.51	441.17	477.08	340.65	432.40	14590.83	433.19	715.43	430.47	303.18	449.65	0.68
	D-10-100-200-5-a	10,933,566.69	129.58	16.97	16.62	16.96	18.08	19.02	118.32	17.44	17.62	17.65	18.03	16.75	145.12	15.18	13.95	15.19	16.38	14.60	112.98	12.79	16.87	12.80	14.13	13.78	1.05
	D-10-100-200-5-b	10,783,782.89	51.52	15.36	17.93	15.56	16.76	16.67	38.86	13.73	14.24	14.03	18.62	17.11	47.90	13.62	15.38	13.60	15.18	16.23	33.15	14.19	13.65	14.21	17.98	16.89	0.93
	D-10-100-200-5-c	11,085,731.24	73.15	17.80	18.52	17.89	17.08	18.00	43.88	21.54	17.54	21.99	16.66	16.19	90.88	16.65	15.61	16.78	27.98	17.88	81.96	18.21	17.69	18.23	18.47	17.03	0.80
	D-10-100-200-5-d	11,086,101.70	22.65	14.91	14.09	14.82	12.80	14.68	23.57	12.96	12.90	13.12	13.07	15.99	29.11	15.03	13.69	14.98	22.29	14.48	28.14	12.61	15.07	12.96	12.59	14.51	0.92
	D-10-100-200-10-a	21,828,372.18	873.83	93.85	93.31	94.07	113.26	69.60	1176.48	86.55	99.42	86.28	82.09	66.63	2617.52	119.03	95.58	136.51	153.84	100.60	2227.13	109.87	117.43	113.26	87.03	89.35	2.17
	D-10-100-200-10-b	21,090,570.13	804.08	94.39	82.77	96.08	86.52	59.60	833.04	121.22	82.29	126.38	79.31	64.24	826.74	124.03	174.30	130.42	212.14	104.78	1107.54	79.73	151.51	84.04	106.15	73.25	1.37
	D-10-100-200-10-c	20,967,615.88	564.71	91.13	74.54	64.18	138.13	66.97	635.24	80.95	77.95	87.38	119.12	68.60	821.56	76.08	79.12	72.21	169.00	69.40	611.75	62.11	81.01	59.19	117.60	77.72	1.15
	D-10-100-200-10-d	21,155,643.74	2429.24	143.06	154.63	172.09	186.84	108.26	4007.28	185.94	168.25	186.47	152.96	125.91	18746.38	408.24	238.20	531.25	390.56	257.62	11224.57	427.24	312.52	324.79	450.13	233.80	0.78
Torres-Soto and Uster 50×50	S-50-50-50-5-a	2,937,408.00	7.11	87.10	108.31	83.74	101.30	80.59	7.49	91.37	100.36	87.80	89.43	97.22	6.47	76.80	75.80	77.23	99.31	108.46	6.50	79.74	80.13	82.10	91.36	100.43	6.48
	S-50-50-50-5-b	2,896,559.00	7.44	99.92	110.18	106.32	114.34	137.43	6.88	79.88	91.80	81.45	152.89	80.75	6.33	67.62	72.20	70.08	82.67	119.66	5.99	100.90	101.06	100.50	140.96	110.86	4.10
	S-50-50-50-5-c	2,825,342.00	7.57	91.29	79.84	97.10	98.67	73.44	7.85	73.93	78.97	72.04	73.37	79.25	7.07	77.46	68.16	76.80	90.25	84.18	6.89	84.13	87.14	81.93	73.14	82.71	4.90
	S-50-50-50-5-d	2,724,222.50	6.34	84.22	81.99	83.38	144.38	110.46	6.43	122.56	101.91	118.99	118.14	137.55	5.55	88.16	68.65	84.34	100.41	123.66	5.71	79.77	92.24	83.98	104.24	133.56	2.21
	S-50-50-50-10-a	5,288,873.00	17.43	487.24	657.13	524.34	496.71	437.77	18.05	583.54	620.55	625.25	476.26	635.84	23.74	1098.08	845.64	1167.70	760.46	819.20	19.97	784.36	932.38	866.35	660.83	672.45	6.08
	S-50-50-50-10-b	4,937,556.00	51.52	196.87	338.91	205.40	608.46	212.81	51.56	571.40	383.74	594.62	510.23	260.05	50.18	862.70	674.49	868.76	714.67	1027.82	40.84	407.14	530.74	391.42	1197.08	1027.92	4.13
	S-50-50-50-10-c	5,231,665.00	14.27	353.14	398.59	409.35	263.66	331.26	13.32	255.44	462.14	245.58	568.98	288.37	17.23	467.76	452.36	466.62	340.09	860.08	14.15	694.58	653.22	757.89	833.52	533.38	7.99
	S-50-50-50-10-d	4,868,281.00	12.54	282.62	304.90	295.35	251.43	305.05	12.64	251.17	331.76	259.53	321.09	295.28	17.96	628.05	252.38	689.47	318.00	494.18	14.91	643.29	569.97	535.47	684.04	708.15	8.74
	I-50-50-50-5-a	1,941,445.00	9.94	71.06	78.77	71.00	83.81	94.54	9.23	89.29	78.73	88.16	97.43	107.89	9.49	79.76	77.18	79.85	79.10	132.85	9.70	74.62	79.66	76.14	89.42	101.22	0.00
	I-50-50-50-5-b	1,980,111.50	9.08	94.49	96.72	93.28	126.73	170.47	10.74	95.18	96.52	94.83	119.88	141.26	8.25	77.45	73.91	77.63	88.73	147.37	9.46	73.56	88.75	74.20	102.87	133.77	0.00
	I-50-50-50-5-c	1,950,775.50	8.91	57.65	63.34	58.31	69.45	106.60	9.02	56.78	57.08	56.60	67.33	99.71	8.04	50.82	63.80	50.73	72.21	51.45	8.43	50.05	46.23	49.95	58.97	92.36	0.00
	I-50-50-50-5-d	1,949,023.00	6.12	50.99	51.50	50.62	66.52	168.34	6.18	90.59	90.06	90.97	85.46	111.23	5.83	59.35	67.55	60.29	62.82	87.48	6.06	51.39	50.53	51.95	77.97	123.45	2.56
I-50-50-50-10-a	3,218,694.00	39.69	1040.53	1101.09	988.43	497.05	863.22	33.86	872.79	807.82	975.91	759.61	828.78	4.41	1175.31	1379.91	1283.57	969.86	1628.54	51.06	1589.12	1366.66	1611.45	970.76	2778.96	7.34	
I-50-50-50-10-b	3,205,448.00	33.04	610.43	610.43	610.43	497.05	863.22	33.66	864.54	807.82	975.91	757															

Table 4: Number of explored branch-and-bound nodes for the single assignment case.

	Instance	UB	S					S <sup>g</sup>					S <sup>g2</sup>					S <sup>g2</sup>								
			CPX	I	CF	MW	MW <sub>f</sub>	P	CPX	I	CF	MW	MW <sub>f</sub>	P	CPX	I	CF	MW	MW <sub>f</sub>	P	CPX	I	CF	MW	MW <sub>f</sub>	P
Ro and Tcha 100×200	R-10-100-200-5-a	11,044,461.79	18	42	76	42	36	34	13	61	41	61	63	59	10	41	40	41	71	46	18	76	74	76	36	37
	R-10-100-200-5-b	11,090,359.93	14	37	32	37	34	30	25	38	34	38	45	40	12	43	29	43	63	42	14	28	32	28	37	37
	R-10-100-200-5-c	11,043,872.61	12	42	34	42	52	33	20	41	50	41	49	30	8	65	38	50	83	33	19	58	29	58	40	36
	R-10-100-200-5-d	11,081,448.06		35	27	35	32	33		117	55	117	39	32		34	38	34	60	43		32	35	32	47	40
	R-10-100-200-10-a	22,083,067.87	18	34	117	34	8	40	20	65	95	64	12	43	13	34	33	34	9	29	24	66	54	79	8	33
	R-10-100-200-10-b	21,149,324.01		48	46	90		69		69	51	69		59		38	91	38		42		92	187	92		41
	R-10-100-200-10-c	20,965,204.59	8	45	44	45	80	59	21	85	40	85	15	45	6	75	48	75	42	29	8	42	53	42	51	41
	R-10-100-200-10-d	21,064,035.66	15	88	80	89	15	69	19	50	141	50	16	48	26	78	111	78	10	42	21	80	203	80	15	45
	L-10-100-200-5-a	10,930,717.93	6	50	57	50	44	40	14	89	76	89	92	55	12	96	86	99	105	62	24	57	94	57	86	87
	L-10-100-200-5-b	10,690,447.41	17	39	46	38	37	41	17	47	34	47	66	54	16	75	89	77	69	33	17	61	65	60	63	74
	L-10-100-200-5-c	11,078,305.04	22	61	51	61	59	56	14	60	63	60	58	57	21	137	111	137	91	101	29	100	78	101	94	68
	L-10-100-200-5-d	11,075,044.96		48	28	48	37	33		58	33	57	39	39		40	47	49	37	31		81	44	81	58	39
	L-10-100-200-10-a	21,781,844.65	474	367	305	343	245	412	376	395	320	396	179	402	186	627	810	532	496	670	520	688	614	690	484	455
	L-10-100-200-10-b	21,042,819.94	35	131	93	131	50	133	32	159	115	157	37	128	70	245	187	120	27	175	30	192	136	202	28	159
	L-10-100-200-10-c	20,890,360.66	214	146	168	147	125	221	178	314	234	315	73	277	60	278	285	268	224	237	125	220	324	186	178	225
	L-10-100-200-10-d	21,080,876.67	2513	708	578	718	640	506	1778	718	660	725	590	903	538	1448	1657	1492	1152	1693	1079	1356	2262	1375	1030	2099
	D-10-100-200-5-a	10,933,566.69	17	39	48	39	59	83	15	44	39	44	37	52	24	40	34	40	47	37	14	32	51	32	40	34
	D-10-100-200-5-b	10,783,782.89		41	57	41	53	42		47	32	47	57	35		35	65	35	33	36		33	49	33	41	46
	D-10-100-200-5-c	11,085,731.24	23	67	69	67	56	53	11	84	46	84	48	48	27	58	39	58	85	51	21	59	44	60	85	49
	D-10-100-200-5-d	11,086,101.70		44	35	44	33	39		35	33	35	38	35		69	41	69	47	33		38	45	40	35	38
	D-10-100-200-10-a	21,828,372.18	85	172	209	172	102	157	112	183	176	164	95	117	339	206	308	350	209	233	262	217	180	211	162	226
	D-10-100-200-10-b	21,090,570.13	58	152	109	152	37	113	37	172	106	171	46	118	113	239	310	209	425	365	101	178	299	172	82	220
	D-10-100-200-10-c	20,967,615.88	33	119	92	128	150	126	39	144	140	147	101	157	172	218	212	192	280	159	64	121	260	114	101	191
	D-10-100-200-10-d	21,155,643.74	506	386	353	421	253	311	703	409	431	409	223	390	3198	1761	1039	3148	1533	1081	1213	2054	1393	1570	1408	1128
Torres-Soto and Uster 50×50	S-50-50-50-5-a	2,937,408.00		531	514	531	420	355		495	530	495	458	471		547	491	547	576	451		578	529	578	518	472
	S-50-50-50-5-b	2,896,559.00		547	494	547	556	639		485	564	484	823	327		509	484	509	572	636		479	613	477	526	633
	S-50-50-50-5-c	2,825,342.00		539	431	536	404	250		358	469	358	463	288		711	440	711	484	391		538	601	538	488	437
	S-50-50-50-5-d	2,724,222.50		357	616	357	700	800		762	445	762	665	789		614	330	614	684	658		480	443	480	619	858
	S-50-50-50-10-a	5,288,873.00		974	947	974	972	526		821	779	821	463	953		1729	1110	1729	977	1363		1014	2011	1014	1352	679
	S-50-50-50-10-b	4,937,556.00	11	99	184	101	786	90	9	1173	738	1171	680	179	5	884	1025	869	856	1566	5	494	1020	494	1400	1119
	S-50-50-50-10-c	5,231,665.00		563	736	566	278	363		275	1020	275	804	229		979	715	979	841	1134		956	1153	956	917	888
	S-50-50-50-10-d	4,868,281.00		416	410	416	514	356		560	312	560	464	341		1212	413	1208	849	1213		689	1052	699	1382	1381
	L-50-50-50-5-a	1,941,445.00		305	342	305	432	404		396	310	396	495	480		397	521	397	382	750		429	307	429	419	346
	L-50-50-50-5-b	1,980,111.50		375	353	375	371	502		397	374	397	358	645		363	316	363	385	639		410	380	410	490	662
	L-50-50-50-5-c	1,950,775.50		273	255	273	280	472		233	193	233	278	491		266	341	266	421	144		237	204	237	253	476
	L-50-50-50-5-d	1,949,023.00		259	259	259	377	725		370	370	370	437	567		310	353	310	298	344		278	291	278	402	553
	L-50-50-50-10-a	3,218,694.00	1037	1089	1036	483	881		1204	854	1204	866	934		1509	2278	1539	1142	1674		1851	1453	1842	1094	1979	
	L-50-50-50-10-b	2,745,488.00		531	631	531	549	823		602	721	602	749	1037		1160	1405	1174	1249	1316		1054	1084	1054	1421	2111
	L-50-50-50-10-c	3,728,704.00		934	1050	934	546	1152		819	737	819	1173	1087		978	1165	994	440	1373		1414	1116	1415	1262	1431
	L-50-50-50-10-d	3,350,786.00		316	925	316	712	958		755	714	753	1191	1123		1686	1265	1686	1243	1409		1003	1223	1002	1108	1742
	D-50-50-50-5-a	1,841,128.00		414	277	414	359	369		378	496	378	397	307		443	352	443	272	352		298	228	298	438	375
	D-50-50-50-5-b	1,995,085.50		499	428	499	562	704		280	368	280	388	717		374	295	374	327	563		317	310	317	312	685
	D-50-50-50-5-c	1,875,603.50		331	217	331	159	291		187	256	187	232	263		236	257	236	191	697		246	270	246	295	566
	D-50-50-50-5-d	1,856,735.00		411	254	411	273	562		303	196	303	376	672		245	224	245	315	413		241	164	241	269	380
	D-50-50-50-10-a	3,037,055.00		876	618	875	389	739		1005	948	1005	528	1030		1085	1467	1082	1255	1836		1295	1061	1292	1232	1890
	D-50-50-50-10-b	2,760,789.00		599	173	599	796	713		802	467	802	880	657		656	621	656	865	1197		1330	1676	1330	919	1189
	D-50-50-50-10-c	3,139,103.00		543	730	535	887	767		708	684	696	596	596		1156	1197	1154	1079	1197		812	978	827	963	1371
	D-50-50-50-10-d	3,167,577.00		469	686	469	818	929		534	566	532	688	1116		656	961	656	1005	1520		1105	880	1105	635	1425

Blank results refer to a zero value.

Table 5: Computational running times in seconds and number of branch-and-bound nodes for the multiple case (larger instances).

	Instance	UB	LR-CPX		CPX		BD-CPX		MW <sub>t</sub>		P		GRASP		
			gap <sup>tr</sup> (%)	CPU (s)	Nodes	CPU (s)	Nodes	CPU (s)	Nodes	CPU (s)	Nodes	CPU (s)	gap <sup>h</sup> (%)	CPU (s)	
Ro and Tcha 100×200	R-20-100-200-5-a	10,590,175.22	0.79	17.89	24	87.52	50	67.91	53	50.61	75	57.34	3.56	1.43	
	R-20-100-200-5-b	10,481,887.22	1.77	22.83	209	651.18	1122	140.11	287	204.86	342	173.16	1.81	1.38	
	R-20-100-200-5-c	10,923,773.43	2.14	20.49	524	1210.58	7905	540.56	866	932.15	1554	484.91	1.74	1.49	
	R-20-100-200-5-d	10,387,605.05	0.46	10.53	12	37.12	32	54.67	23	29.60	22	31.21	1.39	1.41	
	R-20-100-200-10-a	20,615,632.65	1.46	87.40	946	5743.54	11164	1883.43	2970	5857.70	2712	2038.74	0.75	1.62	
	R-20-100-200-10-b	19,511,892.27	0.48	41.87	45	236.85	191	186.56	30	309.38	35	282.55	1.84	1.47	
	R-20-100-200-10-c	20,009,438.04	1.51	58.11	483	2162.83	8748	973.89	752	582.44	813	541.90	1.46	1.80	
	R-20-100-200-10-d	20,647,763.14	0.63	72.29	225	1302.10	668	260.23	209	396.21	388	440.40	0.53	1.34	
	I-20-100-200-5-a	10,238,315.28	0.89	35.10	122	480.34	446	112.82	144	179.07	188	111.62	4.14	1.39	
	I-20-100-200-5-b	10,411,596.33	1.98	45.01	816	3629.64	10826	1219.86	1069	951.86	751	461.46	1.56	1.53	
	I-20-100-200-5-c	10,901,065.11	2.21	42.67	7725	29135.53	13026	1420.03	1517	2819.08	1157	564.54	3.55	1.64	
	I-20-100-200-5-d	10,427,267.63	0.60	23.57	26	119.81	169	78.02	65	87.15	34	58.75	2.02	1.43	
	I-20-100-200-10-a	20,389,681.88	1.65	230.62	0.61	3943	86400.00	*	*	14705	32402.91	12071	22379.00	2.01	1.79
	I-20-100-200-10-b	19,651,376.26	0.55	143.66	641	3647.71	5510	808.84	413	790.53	428	537.98	0.85	1.66	
	I-20-100-200-10-c	19,745,108.18	1.45	181.75	4608	47582.30	85283	12284.90	1810	2528.73	2522	3270.86	2.01	1.97	
	I-20-100-200-10-d	20,364,175.13	0.55	213.02	702	11856.25	4981	2674.22	369	869.32	528	1145.15	1.59	1.38	
	D-20-100-200-5-a	10,218,221.55	0.76	21.69	25	152.91	106	76.49	53	69.19	57	66.04	3.79	1.52	
	D-20-100-200-5-b	10,425,793.69	2.02	31.93	270	1282.36	9886	799.50	574	856.07	680	357.16	1.89	1.49	
	D-20-100-200-5-c	10,960,055.47	2.27	26.34	558	2227.30	16004	1166.92	1043	986.91	763	405.77	2.11	1.56	
	D-20-100-200-5-d	10,445,131.19	0.66	13.57	16	51.06	147	73.27	35	53.36	43	48.24	2.26	1.50	
D-20-100-200-10-a	20,414,334.84	1.68	136.13	0.16	5636	86400.00	25471	11989.75	9277	26832.87	9335	17960.42	1.95	1.55	
D-20-100-200-10-b	19,730,466.19	0.53	84.99	177	1130.30	814	330.06	162	535.62	110	405.50	0.75	1.68		
D-20-100-200-10-c	19,773,648.61	1.41	106.94	709	5559.54	18312	3353.52	737	4063.54	1825	1988.47	1.90	1.91		
D-20-100-200-10-d	20,440,933.21	0.64	141.77	162	2834.58	1036	475.96	267	2790.44	352	847.35	1.10	1.44		
Torres-Soto and Uster 100×100	S-100-100-100-5-a	4,422,960.50		11.70		34.76		77.81	37	30.11	16	46.25	6.69	8.97	
	S-100-100-100-5-b	4,478,869.50		14.62		39.45		77.88	53	39.29	21	39.87	5.87	7.17	
	S-100-100-100-5-c	4,383,907.00		13.69		34.48		77.63	44	34.93	25	45.58	6.50	7.70	
	S-100-100-100-5-d	4,573,928.00		12.34		36.14		78.26	78	42.70	11	38.03	3.08	7.49	
	S-100-100-100-10-a	8,225,841.00		26.87		76.86	*	*	11	105.08	16	302.33	0.58	18.08	
	S-100-100-100-10-b	8,157,227.00		30.92		82.70	*	*	45	172.19	9	99.39	6.95	18.02	
	S-100-100-100-10-c	8,239,255.00		40.48		100.93	*	*	21	108.05	5	328.72	4.60	17.50	
	S-100-100-100-10-d	8,326,957.00	0.01	29.21		80.72	*	*	20	94.63	1	110.21	4.63	15.72	
	I-100-100-100-5-a	2,891,966.50		17.29		46.04		77.54	9	18.32	10	48.89	7.38	9.24	
	I-100-100-100-5-b	2,861,982.00		26.85		52.07		79.11	24	28.30	3	47.67	4.40	8.38	
	I-100-100-100-5-c	2,904,623.50		18.37		45.42		76.46	22	37.88	1	61.78	6.14	11.23	
	I-100-100-100-5-d	3,087,996.00		24.72		52.51		79.39	38	55.18	29	67.03	5.90	11.35	
	I-100-100-100-10-a	4,675,005.00		83.22		147.44	*	*	67	171.83	13	132.14	7.68	22.62	
	I-100-100-100-10-b	4,641,771.00		70.12		143.09	*	*	37	189.31		96.84	11.10	22.19	
	I-100-100-100-10-c	4,954,900.00		181.48		252.15	*	*	22	329.24	19	186.03	2.81	19.24	
	I-100-100-100-10-d	5,231,316.00		83.44		144.08	*	*	33	143.63	6	131.04	2.51	14.71	
	D-100-100-100-5-a	2,940,466.50		13.32		38.39		77.44	38	26.10	24	44.69	7.53	9.25	
	D-100-100-100-5-b	3,004,935.00		24.21		49.86		78.73	16	27.32	7	53.08	4.46	9.77	
	D-100-100-100-5-c	2,937,341.50		19.21		42.72		76.87	40	41.95	1	59.26	5.27	11.26	
	D-100-100-100-5-d	3,060,773.00		20.46		43.94		77.27	22	37.67		46.43	5.42	10.14	
D-100-100-100-10-a	4,546,091.00		57.14		121.98	*	*	18	115.75	2	254.08	7.73	22.62		
D-100-100-100-10-b	4,730,662.00		42.34		83.67	*	*	6	66.70		118.14	11.16	22.16		
D-100-100-100-10-c	4,784,380.00		103.57		144.29	*	*	22	190.05	7	211.80	3.15	24.10		
D-100-100-100-10-d	5,154,930.00		60.94		110.92	*	*	68	150.11	14	145.91	2.00	14.72		

Blank results refer to a zero value.

\* Out of memory.

<sup>tr</sup> Linear Relaxation gap (%).

<sup>c</sup> CPLEX optimality gap (%).

<sup>h</sup> GRASP with time limit set to 1 second or 10 iterations.

Table 6: Computational running times in seconds and number of branch-and-bound nodes for the single assignment case (larger instances).

	Instance	UB	LR-CPX		CPX		BD-CPX		I		CF		MW		MW <sub>t</sub>		P		GRASP		
			gap <sup>r</sup> (%)	CPU (s)	gap <sup>r</sup> (%)	Nodes	CPU (s)	Nodes	CPU (s)	Nodes	CPU (s)	Nodes	CPU (s)	Nodes	CPU (s)	Nodes	CPU (s)	Nodes	CPU (s)	gap <sup>h</sup> (%)	CPU (s)
Ro and Tcha 100×200	R-20-100-200-5-a	10,590,175.22	0.71	248.54		20	1374.57	49	354.49	35	50.61	36	52.91	35	53.29	29	53.52	37	56.72	3.56	1.49
	R-20-100-200-5-b	10,481,887.22	1.57	298.11		107	6256.95	180	326.28	169	115.03	194	115.59	165	112.11	137	103.86	274	148.55	1.81	1.43
	R-20-100-200-5-c	10,923,773.43	1.98	273.15		165	10989.00	752	449.28	791	247.29	741	242.60	728	246.99	445	202.18	1001	350.71	1.74	1.52
	R-20-100-200-5-d	10,387,605.05	0.36	101.06		12	282.27	26	248.63	23	41.28	17	40.23	23	40.24	19	44.21	21	43.25	1.39	1.41
	R-20-100-200-10-a	20,615,632.65	1.34	1299.74	0.33	523	86400.00	*	*	2741	1980.90	2548	1758.86	2741	1971.38	2013	1629.32	2357	1587.00	0.75	1.68
	R-20-100-200-10-b	19,511,892.27	0.32	623.11		18	2683.35	*	*	44	149.26	44	147.85	46	151.37	26	143.01	28	140.01	1.84	1.51
	R-20-100-200-10-c	20,009,438.04	1.46	706.11	*	*	*	*	*	1158	707.57	895	569.66	1157	720.47	868	597.37	772	544.34	1.46	1.86
	R-20-100-200-10-d	20,647,763.14	0.57	889.93		82	8597.01	*	*	206	264.73	135	236.93	206	266.28	294	308.83	222	331.86	0.53	1.30
	I-20-100-200-5-a	10,238,315.28	0.79	413.22		70	5980.55	457	363.92	63	68.27	75	66.14	63	69.59	85	96.11	66	70.49	4.14	1.43
	I-20-100-200-5-b	10,411,596.33	1.79	556.43		672	61124.21	3918	960.46	5044	1694.23	2416	979.99	5044	1696.37	1894	759.85	910	402.14	1.56	1.53
	I-20-100-200-5-c	10,901,065.11	2.03	505.05	0.82	870	86400.00	4982	1018.56	5238	1869.49	3191	1122.10	5238	1881.20	4193	1644.76	1489	576.10	3.55	1.65
	I-20-100-200-5-d	10,427,267.63	0.45	231.18		16	865.45	66	305.78	28	48.22	26	51.10	28	49.97	27	53.67	29	59.74	2.02	1.48
	I-20-100-200-10-a	20,528,863.39	2.14	4207.56	1.90	198	86345.69	*	*	10105	14073.30	14952	19090.29	12664	22405.69	10002	32210.17	7548	8822.70	2.01	1.74
	I-20-100-200-10-b	19,651,376.26	0.47	2270.40		471	45072.43	*	*	422	383.59	346	423.26	458	416.06	405	549.42	318	405.68	0.85	1.64
	I-20-100-200-10-c	19,790,998.81	1.61	2645.53	1.10	262	86400.00	*	*	1162	1384.08	998	1310.51	1144	1428.29	1197	1480.25	1208	1313.13	2.01	1.89
	I-20-100-200-10-d	20,364,175.13	0.50	2569.21	0.11	218	86400.00	*	*	746	964.33	397	575.35	945	1145.96	332	848.72	251	536.51	1.59	1.38
	D-20-100-200-5-a	10,218,221.55	0.67	397.04		14	2210.22	57	329.14	40	56.62	45	63.00	30	53.00	28	57.84	29	64.75	3.79	1.53
	D-20-100-200-5-b	10,425,793.69	1.82	427.37		218	23216.33	694	467.38	280	154.24	250	169.23	306	162.44	370	300.78	294	175.60	1.89	1.53
	D-20-100-200-5-c	10,960,055.47	2.11	395.82		300	29816.88	1675	647.96	1691	627.77	1675	648.63	1706	648.05	678	461.32	996	461.50	2.11	1.57
	D-20-100-200-5-d	10,445,131.19	0.52	169.48		12	557.37	55	298.09	31	50.04	26	48.69	31	48.42	32	61.69	23	56.81	2.26	1.49
	D-20-100-200-10-a	20,481,513.41	1.86	2817.20	1.30	152	86400.00	*	*	4128	5182.01	4229	4835.94	4137	5538.64	4023	19304.68	4496	4724.45	1.95	1.56
	D-20-100-200-10-b	19,730,466.19	0.46	1746.18		154	13953.70	*	*	181	275.66	178	263.20	177	283.83	156	312.47	126	274.39	0.75	1.64
	D-20-100-200-10-c	19,773,648.61	1.35	1631.52		381	64017.49	*	*	784	786.68	546	648.08	790	839.58	684	1403.97	856	922.40	1.90	1.89
	D-20-100-200-10-d	20,440,933.21	0.59	2269.21		86	43096.93	*	*	283	496.96	246	458.99	290	558.06	279	809.99	254	567.46	1.10	1.53
Torres-Soto and Üster 100×100	S-100-100-100-5-a	4,422,960.50		34.61			83.28	*	*	704	1513.71	587	900.17	705	1425.96	820	1966.60	824	1354.83	6.69	9.04
	S-100-100-100-5-b	4,478,869.50		33.16			101.00	*	*	849	1585.19	1371	2132.31	849	1683.60	1287	2004.35	908	1865.59	5.87	7.17
	S-100-100-100-5-c	4,383,907.00		22.29			72.69	*	*	1559	2111.01	1387	2228.33	1558	2590.91	640	942.12	1587	3174.34	6.50	7.71
	S-100-100-100-5-d	4,573,928.00		33.10			94.90	*	*	1242	1641.12	707	1229.68	1242	1972.99	879	1646.23	419	1096.27	3.08	7.55
	S-100-100-100-10-a	8,225,841.00		85.24	*	*	*	*	*	1005	10902.89	1209	6898.11	1005	10909.45	1309	21805.60	1606	16607.71	0.58	18.16
	S-100-100-100-10-b	8,157,227.00		105.00	*	*	*	*	*	445	3624.17	217	2522.28	445	3698.95	1176	5973.02	1657	13262.38	6.95	18.08
	S-100-100-100-10-c	8,239,255.00		198.74	*	*	*	*	*	1408	7285.91	1692	18441.28	1408	7465.99	1512	8985.24	721	8296.41	4.60	17.61
	S-100-100-100-10-d	8,326,957.00	0.01	81.15	*	*	*	*	*	515	3093.38	752	5272.26	512	3165.24	829	6524.65	1288	7414.89	4.63	15.88
	I-100-100-100-5-a	2,891,966.50		34.13			137.64	*	*	958	1722.12	740	1516.40	952	1673.14	1521	2327.48	941	2736.15	7.38	9.23
	I-100-100-100-5-b	2,861,982.00		47.16			158.18	*	*	1292	2573.25	1272	2899.71	1290	2945.94	1654	3594.19	1738	4104.34	4.40	8.39
	I-100-100-100-5-c	2,904,623.50		32.66			138.20	*	*	1735	3580.05	1654	2868.58	1735	3075.24	1540	2489.76	1557	5163.99	6.14	11.44
	I-100-100-100-5-d	3,087,996.00		63.06			155.98	*	*	1745	3392.87	1272	2427.12	1723	3755.49	1118	3861.93	1277	2574.12	5.90	11.28
	I-100-100-100-10-a	4,675,005.00		234.96	*	*	*	*	*	1700	18668.81	1157	19283.19	1581	16268.54	1617	18031.33	1437	15960.22	7.68	22.95
	I-100-100-100-10-b	4,641,771.00		221.83	*	*	*	*	*	1030	11950.45	1030	9540.51	1030	10239.97	1531	16138.87	1747	49347.31	11.10	22.30
	I-100-100-100-10-c	4,641,771.00		646.55	*	*	*	*	*	2153	30559.67	1903	32201.02	2096	31916.28	2019	43359.38	2116	41026.47	2.81	19.24
	I-100-100-100-10-d	5,231,316.00		309.80	*	*	*	*	*	706	6593.89	590	5084.15	706	7377.37	1176	10242.47	1098	10456.60	2.51	14.77
	D-100-100-100-5-a	2,940,466.50		27.40			93.79	*	*	994	1655.57	1154	1802.33	994	1653.06	1182	2662.76	1167	2740.99	7.53	9.32
	D-100-100-100-5-b	3,004,935.00		40.84			167.91	*	*	1009	2598.75	671	2373.57	1004	2966.04	997	3322.22	1287	2919.24	4.46	9.79
	D-100-100-100-5-c	2,937,341.50		46.92			118.09	*	*	1349	4697.10	2017	6107.26	1310	3575.72	1923	4687.13	1360	3153.38	5.27	11.32
	D-100-100-100-5-d	3,060,773.00		44.56			93.30	*	*	1023	2315.27	662	2258.74	1007	2329.78	917	1473.94	511	2148.15	5.42	10.12
	D-100-100-100-10-a	4,546,091.00		180.16	*	*	*	*	*	1382	14538.75	1341	10420.21	1382	14101.21	1473	11844.00	1182	17753.04	7.73	22.67
	D-100-100-100-10-b	4,730,662.00		83.91	*	*	*	*	*	2508	23493.62	894	17471.50	2506	25440.94	1332	12386.63	1684	29599.90	11.16	22.18
	D-100-100-100-10-c	4,784,380.00		333.10	*	*	*	*	*	1436	19924.23	1065	14742.03	1436	24758.95	629	7270.27	1815	26816.86	3.15	24.04
	D-100-100-100-10-d	5,154,930.00		135.23	*	*	*	*	*	1411	16193.80	1293	7975.11	1002	8848.30	1485	22075.66	1270	11310.15	2.00	14.83

Blank results refer to a zero value.

<sup>r</sup> Out of memory.<sup>c</sup> Linear Relaxation gap (%).<sup>e</sup> CPLEX optimality gap (%).<sup>h</sup> GRASP with time limit set to 1 second or 10 iterations.

## References

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