Planck Parameter Definition Document

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Abstract: *This document describes general parameter definitions to be used in the framework of Planck Data reduction and analysis. It is presently maintained by Karim Benabed (benabed@iap.fr).*

Document Change History

Title: Planck Parameter Definition Document			
Issue	**Rev.**	**Date**	**Reason for Change**
Draft 0.4	0	2003-07-07	HFI-L2 PDD becomes draft of common document
Draft 0.5	0	2003-10-20	updated to include LFI remarks
Draft 0.5	1	2003-10-23	General clean-up and additional definitions
Draft 0.6	1	2004-7-23	Evolution of the document. Some Clean up of definitions.
Draft 0.7	0	2004-12-15	**Modification of the structure.** Some sections where shuffled and reworked to achieve (hopefully) a greater ease of use. **Introduction of quaternions.** Quaternions can be used to describe rotations and frame changes. **Definitions of pointing, angles, reference frames** as a set of values tagged by a keyword describing a convention. **Introduction of a keyword** to describe polarization convention. **Introduction of a beam aligned reference frame** Bxx.
	1	2005-1-10	Correction of orthograph and grammar.
	2	2005-1-17	Update history chart from version 0.7.0.
	3	2005-1-20	Corrected an error in euler angles definition.
	4	2005-6-22	Added definition of phase for binned rings.
Draft 0.8	0	2007-03-12	Some clarifications proposed by F. van Leeuwen Reworked section on Beams (this still need some work). Changed sec. 3.1.1 and fig. 1 section to uniformize ϕ and θ definition in the document. Some typo corrections by M. Reineke SHORTQUATERNION flagged for possible removal in future versions
Draft 0.8	1	2007-03-12	better definition of Dxx in sec. 3.2.9 added a section describing the notation for reference frame exchanges and rotations

Document Approval

NAME	Organization	Signature	Date
N. Mandolesi	CNR - Bologna		
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1 Introduction

The following document defines parameters, notations and conventions used in the Planck data processing. It starts with general definitions : reference frames, geometry, angles, time reference frames, polarisation parameters, spectral responses, brightness, etc.

This document originates from the HFI DPC L2 Parameter Definition Document, which was originally designed at the Planck HFI L2 data processing level only for the same convention definition purpose.

This document has been accepted as the standard parameter definition document for HFI at large in autumn 2003, HFI-L2 remaining the custodian. The present version, which includes comments received from LFI so far and merges some of the LFI convention, is a draft of a common HFI-LFI parameter definition document.

For questions, suggestions and updates please contact K. Benabed (benabed@iap.fr), F. Pasian (pasian@ts.astro.it), and L. Vibert (laurent.vibert@ias.u-psud.fr).

1.1 Definition of acronyms

AOCS	Attitude and Orbit Control System
ATT	Attitude
DPC	Data Processing Centres
ECL	Ecliptic
EQU	Equatorial
GAL	Galactic
HFI	High Frequency Instrument
LFI	Low Frequency Instrument
LOS	Line Of Sight
MOC	Mission Operations Centre
N/A	Not Applicable
OBT	On-Board Time
PLM	Payload Module
RDP	Reference Detector Plane
S/C	Spacecraft
SA	Spin Axis
SC	Spacecraft
SS	Spacecraft Spinning
SVM	Service Module
TAI	Temps atomique international
TBC	To Be Confirmed
TBD	To Be Determined
TBN	To Be Nominated
TBW	To Be Written

1.2 Applicable documents

1.3 Reference documents

RD-1. Planck Telescope Design Specification, SCI-PT-RS-07024, (August 31 2000)

RD-2. ESA-ESTEC Tech. Note ”Conventions and coordinate systems for the polarized radiation patterns simulated at ESTEC using GRASP8”, P. Fosalba & A.M. Polegre, 2000

2 Rationale

The multiplication of conventions and notations used in the Planck project at large being a serious potential source of future trouble, this document aims at defining in the best possible consensual way conventions to be used by the Planck consortia at large.

Existing documents have been reviewed and compared. The present document tries at best to summarize the consensus, or the most commonly used conventions. Discrepancies or incompatibilities have been sorted out by discussions leading either to an agreement or compromise, or to a formal decision at DPC management level.

This ongoing process is likely to continue for the duration of the project, with updates and definitions being added as needed.

3 Reference frames, positions, angles

3.1 Definition of pointing, angle and reference frame.

We need to define the position on the sky of each detector as a function of time, for any time of the mission during which science data is collected, and with sufficient accuracy so as not to deteriorate in any way the accumulated science data. We define the position of each detector through a number of transformations between reference frames.

The first reference frame is some "fixed" celestial reference frame on the sky like the galactic and ecliptic reference frames.

The observations of the satellite are linked to this fixed reference frame through the star mapper observations. This link is usually referred to as the satellite's attitude.

The second reference frame defines the orientation of the payload with respect to the the orientation of the star mapper. The payload is defined here by means of a Fiducial Reference Point in the focal plane and a nominal orientation of the focal plane geometry.

A third transformation describes the position of each detector in the focal plane as projected on the sky, relative to the fiducial reference point and a nominal orientation of the focal plane. Within this reference frame are also defined the beam profiles.

This document will define all aspects of the transformations described above, to provide a unique definition of the position of each detector on the sky as a function of time.

3.1.1 Pointing

We will call pointing a position on the celestial sphere, in any reference frame. Of course, we will mainly refer as the *pointing of a bolometer* as the position it observes at a given time t, expressed in a known reference frame.

A position on the sphere can be represented in many different ways. We will use three conventions for representing position on the sphere in Planck.

CARTESIAN This is the simplest and recommended solution. Any point on the unit sphere can be represented by its three cartesian coordinates (x, y, z). This convention is of course redundant, as the coordinates have to satisfy the relation $x^2 + y^2 + z^2 = 1$. The range of values for x, y and z is $[-1, +1]$. The x, y, z are unitless. Assuming that IO operations are close to optimal, this choice of coordinates, even if redundant, should the most effective for all tasks involving rotation, projections of coordinates and frame change.

SPHERICAL This corresponds to the mathematical convention or at least the "mathematical-for-physicist" convention. A point on the sphere is described by its spherical coordinates ϕ and θ. The first coordinate spans the range $[0, 2\pi]$; the second one $[0, \pi]$. The units are radians. The point $M = (\phi, \theta)$ is such that the angle $(zOM) = \theta$ and lies in a plane orthogonal to the xy plane and forming an angle ϕ with the xz plane (see fig. 1). The ϕ and θ coordinates are also usually called *longitude* and *colatitude*. By convention, the poles ($(0, 0, 1)$ and $(0, 0 - 1)$ in cartesian coordinates) are represented by $(0, 0)$ and $(0, \pi)$. Any couple of values of the form$(\phi, 0)$ or (ϕ, π) with $\theta \neq 0$ is considered meaningless and should not be used.

GEOGRAPHICAL This convention relates to the previous one by the fact that the second coordinate, usually called δ is given by $\delta = \pi/2 - \theta$, with θ defined above. This coordinates spans the range $[-\pi/2, \pi/2]$, unit is also radian. The coordinate labelled δ is also named *latitude*. The above remark concerning the poles holds here.

It is strongly recommended that each pointing data available to the Planck community is tagged with the right convention keyword (*CARTESIAN*, *SPHERICAL* or *GEOGRAPHICAL*), and with the name of the reference frame into which it is defined.

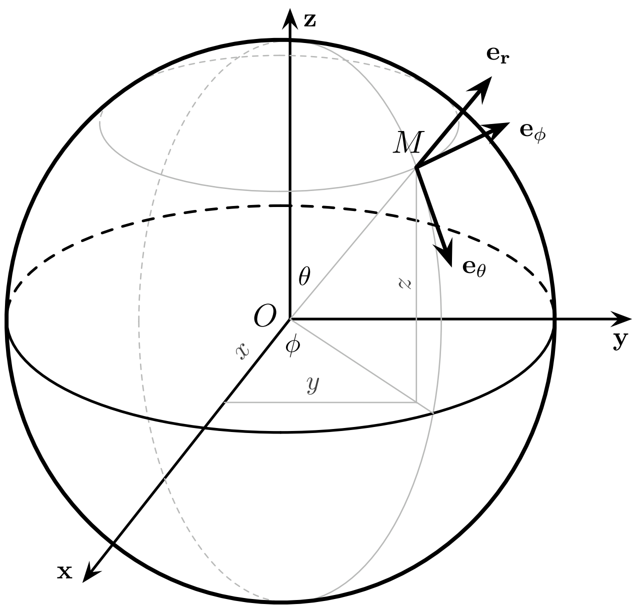

Figure 1: Definition of the SPHERICAL coordinates convention.

3.1.2 Reference Frame

A reference frame is a set of three orthogonal vectors $(\mathbf{e_1}, \mathbf{e_2}, \mathbf{e_3})$ such that $\mathbf{e_1} \times \mathbf{e_2} = \mathbf{e_3}$. We are only interested in right-handed reference frames. In general one should also pick an origin for the reference frame, but in our case, since we are observing the sky on large scales, we will be able to assume that all our reference frames will have the same origin.

We will use many different reference frames in the processing of the Planck mission. Some of them will have to be attached to the sky. However, as the spacecraft rotates, it will be much easier to define the fiducial pointings of its detector in reference frames attached to it.

In reference frames attached to the sky, diffuse emission such as CMB anisotropies are static, but the beams and sidelobes (i.e. optical responses) of the Planck detectors change with time because of the scanning – the instantaneous rotation of the instrument with respect to the sky. Sky coordinates serve to define the location of fixed celestial bodies and features (CMB fluctuations, galaxy clusters).

In reference frames attached to the spacecraft, the beams and sidelobes (i.e. optical responses) of the Planck detectors are fixed (at least to first order), and do not change with the scanning. The sky rotates around Planck, presenting at each moment a different configuration to the spacecraft. Reference frames attached to the spacecraft serve for defining the location and relative orientations of spacecraft elements, and for defining instrumental beams – responses as a function of direction as measured relative to the instrument.

One very common task of the data processing will be to transform the pointing of the bolometers from one reference frame to the other. Since all our reference frames have the same origin, we can relate them to one another by rotations. Some of these rotation will be time dependent, like for example the one linking the star mapper reference frame to the celestial sphere (also called the attitude of the satellite).

In the following, we will define the different reference frames in use in the Planck data processing. However, for data reduction purposes, the only interesting information is the rotation linking one reference frame to the other. Those rotations will be numerically represented by coordinates, i.e. sets of numerical values, tagged by convention keywords. Here follows the list of Planck aknowledged conventions

QUATERNION A set of four unitless values $(q_i)_{i=0,3}$, spanning the range $[-1,1]$, such that

$$\bar{q}q = \sum_i q_i^2 = 1. \tag{1}$$

Those four values represent a unit-normalized quaternion. It is well known that the group of quaternion of norm unity for the multiplication operator provides a representation of the rotation group on the sphere. The non-commutative multiplication of two quaternions q and p is given by

$$\begin{aligned} q\,p &\equiv (q_i)_{i=0,3} \times (p_i)_{i=0,3} \\ &\equiv \begin{pmatrix} q_0p_0 - \sum_{\nu=1}^{3} q_\nu p_\nu \\ q_0p_1 + q_1p_0 + q_2p_3 - q_3p_2 \\ q_0p_2 + q_2p_0 + q_3p_1 - q_1p_3 \\ q_0p_3 + q_3p_0 + q_1p_2 - q_2p_1 \end{pmatrix}. \end{aligned} \tag{2}$$

The conjugate of a quaternion q is defined by

$$\bar{q} \equiv (q_0, -q_\nu)_{\nu=1,3}. \tag{3}$$

The action of the rotation represented by a quaternion q, on a vector $\mathbf{M} = (x_M, y_M, z_M)$ (in cartesian coordinates) is given by

$$M' = qM\bar{q}, \tag{4}$$

where

$$M = (0, x_M, y_M, z_M) \tag{5}$$

is the quaternion representation of vector $\mathbf{M}$. The combinaison of two rotations q_1, q_2 is simply given by

$$q = q_2 q_1. \tag{6}$$

Finally, there is a simple relation between the quaternion and the rotation axis ((x, y, z)) and angle (α) as

$$q = (\cos\frac{\alpha}{2}, x\ \sin\frac{\alpha}{2}, y\ \sin\frac{\alpha}{2}, z\ \sin\frac{\alpha}{2}). \tag{7}$$

More information on the quaternion algebra can be found at http://mathworld.wolfram.com/Quaternio

SHORTQUATERNION Only unit-norm quaternions represent a valid rotation so that only three values and a sign are needed. This sign can be ignored by fixing a convention. Indeed, the rotation of axis $\mathbf{n}$ and angle α, and the rotation $(-\mathbf{n}, -\alpha)$ are identical, i.e. unit norm quaternions q and $-q$ represent the same rotation. Any quaternion $(q_i)_{i=0,3}$ representing a rotation can then be represented as a set of three values, $(q'_\nu)_{\nu=1,3}$, in the range $[-1, 1]$, such that $q'_\nu = \mathrm{sign}(q_0)q_\nu$ for $\nu = 1, 3$. This set of three values will be called a SHORTQUATERNION. From the three values, one can retrieve the quaternion

$$q'' = \left(\sqrt{\left(\sum_{\mu=1}^{3} {q'_\mu}^2\right) - 1}, q'_\nu\right)_{\nu=1,3}, \tag{8}$$

that is identical, up to a sign, to the original quaternion. The rotations of any vector $\mathbf{M}$ by q and q'' lead to the same result.
N.B. The SHORTQUATERNION was added to the document in order to be able to follow the notation of the AHF. Since then, the AHF have changed, and now contains quaternions. If the SHORTQUATERNION is not used, elsewhere it can be removed from the document.

EULER($a - b - c$) A set of three numbers (θ, ϕ, ψ) spanning the range $[0, 2\pi]$. The described rotation is the result of the combination of the three rotation $(\mathbf{e}_a, \theta)$, $(\mathbf{e}_b, \phi)$ and $(\mathbf{e}_c, \psi)$. Among all the possible values of a, b and c, two sets are the most common **EULER($3 - 2 - 3$)** and **EULER($3 - 1 - 3$).**

It is expected that any reference frame transformation is tagged with the appropriate keyword, the intial reference frame and the final reference frame. The QUATERNION convention should to be favored as the easiest and most optimal way of sharing rotation information among the Planck data processing community. Using only quaternions for all reference frame rotations, and cartesian coordinates for pointing definitions allows to avoid most trigonometric operations and to use only additions and multiplications.

3.1.3 Orientations on the sky

Very often it will be necessary not only to know the pointing direction in a given reference frame, but also the orientation on the sky of a given observation. This orientation is only relevant when dealing with observations of extended asymmetrical objects, or with polarisation. In other words, orientation will almost be always needed when dealing with non ideal bolometers with asymmetric beams.

An orientation on the sky will be given by the angle made between a fiducial vector and a vector describing the orientation. Both vectors have to be tangential to the sky at the pointing direction. The angle is always measured counterclockwise around the z direction of the reference frame attached to the pointing. There are two ways of defining the reference frame attached to the pointing.

IAU At each pointing on the sky is attached a single reference frame defined as follows: $\mathbf{e}_1$ points toward the north pole and is tangential to the sphere, $\mathbf{e}_2$ points toward the east and $\mathbf{e}_3$ is equal to the inverse of the pointing direction (as defined for example in the Dxx reference frame, see below). This corresponds to the infinitesimal reference frame attached to a pointing position in the GEOGRAPHICAL convention. It is the reference frame preferred by the IAU/IEEE convention. As such, it is the most spread convention in the astronomy/astrophysics community.

COSMO At each pointing on the sky is attached a single reference frame defined as follows: $\mathbf{e}_1$ points toward the south pole and is tangential to the sphere, $\mathbf{e}_2$ points toward the east and $\mathbf{e}_3$ is equal to the pointing direction. This corresponds to the infinitesimal reference frame attached to a pointing position in the SPHERICAL convention (see fig. 1). This convention is widely used in the cosmology/CMB community. As such, it is the one used in most reference scientific software dealing with the computation of the anisotropies of CMB and reduction of CMB data. Most notably, this is the convention used to describe polarization in the widely used HEALPix pixelization scheme.

The choice of the fiducial vector in the plane tangent to the sphere at the pointing direction is difficult. In most cases, a good candidate is either one of the two other vectors of the reference frame in the IAU or COSMO convention. However, this choice is not well defined when pointing at the poles. We will thus assume that the IAU and COSMO frames are defined, at the north and south poles, by the rotation of $\pi/2$ around the y axis of the frame attached to repectively (1,0,0) and (-1,0,0) in the CARTESIAN conventions.

Depending on the use of the orientation angle, it can be more efficient to store it not as an angle in radian, but as its cosine, or both sine and cosine. Three keyword are defined to describe this choices

ANGLE($i = 1, 2$) A single numerical value in the range $[0, 2\pi]$ describing the angle between $\mathbf{e}_i$ and the interesting orientation on the sky. Unit is radian.

TRIGO($i = 1, 2$) Two values giving the cosine and sine of the angle defined above, and spanning the range $[-1, 1]$, unitless. Of course, the couple of values (c, s) must verify $c^2 + s^2 = 1$ This convention is mainly present to allow for optimisation of the data processing.

COS($i = 1, 2$) A single unitless value spanning the interval $[-1, 1]$ and corresponding to the cosine of the angle defined above. This convention is present to allow for optimisation when orientation are only relevant up to π, with is the case for polarization and symmetric beams.

An orientation should always be described, by its convention keyword (ANGLE, TRIGO or COS), by the measurement convention (IAU or COSMO), by a position on the sky where the orientation is measured, and by the reference frame into which this position on the sky is defined. Note also that defining all this information is completely equivalent to giving the frame transformation between the reference frame into which the pointing is defined, and the reference frame with, for example, $\mathbf{e}_3$ along the pointing direction and $\mathbf{e}_1$ along the direction represented by the orientation.

3.2 Reference frame list

The processing of Planck data requires using several different reference frames. Each frame is labelled using a character code for easy reference. The list of useful frames is:

ECL Ecliptic reference frame

GAL Galactic reference frame

EQU(xxxx.xx) Equatorial reference frame, epoch xxxx.xx

L2 L2 reference frame

SC Spacecraft reference frame

ATT Attitude reference frame

LOS Line-Of-Sight reference frame

Dxx Detector reference frame for detector xx

Bxx Detector reference frame for detector xx aligned with a fiducial reference direction for the definition of the beam

Pxx Polarimeter reference frame for polarimeter xx

RDP Reference Detector Plane reference frame

SA(t) Spin-Axis reference frame

SS(t_1, t_2) Spacecraft Spinning reference frame between $t1$ and $t2$

In this list, frames ECL, GAL, EQU(xxxx.xx), SA(t), SS(t_1, t_2) are not “attached” to the spacecraft. SC, ATT, LOS, Dxx, Pxx, RDP are fixed with respect to the S/C frame, and are connected to each other through time–independent transformations. The L2 reference frame is attached to the solar system.

In the following definitions, the coordinates follows the CARTESIAN convention, they are labelled x, y and z with the reference frame in wich the coordinates are expressed as an upperscript e.q. for a point $\mathbf{M}$ on the unit sphere, x^{L2} is its x coordinate expressed in the L2 reference frame. Unit vectors $(\mathbf{e}_i)$ are labelled as capital letters $\mathbf{X}$, $\mathbf{Y}$ and $\mathbf{Z}$ with their reference frame as subscript e.g. $\mathbf{X}_{\mathrm{yyy}}^{\mathrm{yyy}} \equiv (1, 0, 0)$ is the first reference vector of the yyy reference frame, expressed in the yyy reference frame, whereas $\mathbf{X}_{\mathrm{yyy}}^{\mathrm{xxx}}$ is the first reference vector of the yyy reference frame expressed in the xxx coordinates, it can be different from $(1, 0, 0)$. We will use the upperscript notation only when it is relevant to specify the reference frame into which the coordinates of a vector are described.

3.2.1 ECL, GAL and EQU(xxxx.xx) sky reference frames

The Ecliptic, Galactic and Equatorial reference frames are the usual astronomical reference frames.

3.2.2 L2 reference frame

The L2 reference frame (fig. 2 has its origin on the L2 point, with $\mathbf{X}_{\mathrm{L2}}$ parallel to the ecliptic plane along the Sun-Earth axis, $\mathbf{Z}_{\mathrm{L2}}$ along the perperdicular to the ecliptic plane towards the North Ecliptic Pole, and $\mathbf{Y}_{\mathrm{L2}}$ such that $\mathbf{X}_{\mathrm{L2}}$, $\mathbf{Y}_{\mathrm{L2}}$ and $\mathbf{Z}_{\mathrm{L2}}$ form a direct system.

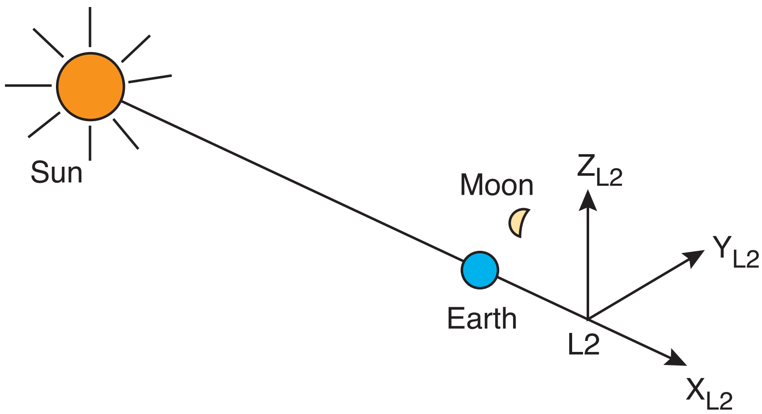

Figure 2: L2 reference frame.

3.2.3 SC reference frame

The SC reference frame is defined in RD-1 (and is consistent with the definitions used by the industrial contractor). The origin O_{SC} of this reference system is located in the telescope symmetry plane at the center of the S/C to launcher interface. The set of axes is fixed with respect to the mechanical structure of the telescope, $\mathbf{X}_{\mathrm{SC}}$ axis along the nominal spin axis (the direction being from SVM to PLM) in the telescope symmetry plane, $\mathbf{Z}_{\mathrm{SC}}$ is orthogonal to $\mathbf{X}_{\mathrm{SC}}$ in the telescope symmetry plane, and oriented so that the field of view is on positive Z sides, and $\mathbf{Y}_{\mathrm{SC}}$ such that $\mathbf{X}_{\mathrm{SC}}$, $\mathbf{Y}_{\mathrm{SC}}$ and $\mathbf{Z}_{\mathrm{SC}}$ form a direct system (Fig. 4).

Note: Here the SC reference frame is fixed with respect to the optics (i.e. attached to the telescope). The exact connection between a frame attached mechanically to the spacecraft, and a frame attached to the telescope can not be calibrated in flight, nor can it be known to accuracies required for the final pointing reconstruction (especially for polarization).

3.2.4 LOS reference frame

The optical axis (or Line Of Sight: **LOS**) is defined as the direction obtained by imaging a reference point in the focal plane (called the Focal Plane Center or FPC, uniquely defined in agreement by the consortia of the two instruments[1]) onto the sky through the optics. The optical axis corresponds to the $\mathbf{Z}_{\rm LOS}$ axis of the LOS reference frame. $\mathbf{Y}_{\rm LOS}$ is along $\mathbf{Z}_{\rm LOS} \times \mathbf{X}_{\rm SC}$ (it thus points opposite to the scanning direction on the sky corresponding to positive angular velocity $\Omega_{\rm spin}$). $\mathbf{X}_{\rm LOS}$ is such that $\mathbf{X}_{\rm LOS}$, $\mathbf{Y}_{\rm LOS}$ and $\mathbf{Z}_{\rm LOS}$ form a direct system. $\mathbf{X}_{\rm LOS}$ and $\mathbf{Y}_{\rm LOS}$ are in the plane tangent to the sky at the intersection of the LOS with the celestial sphere.

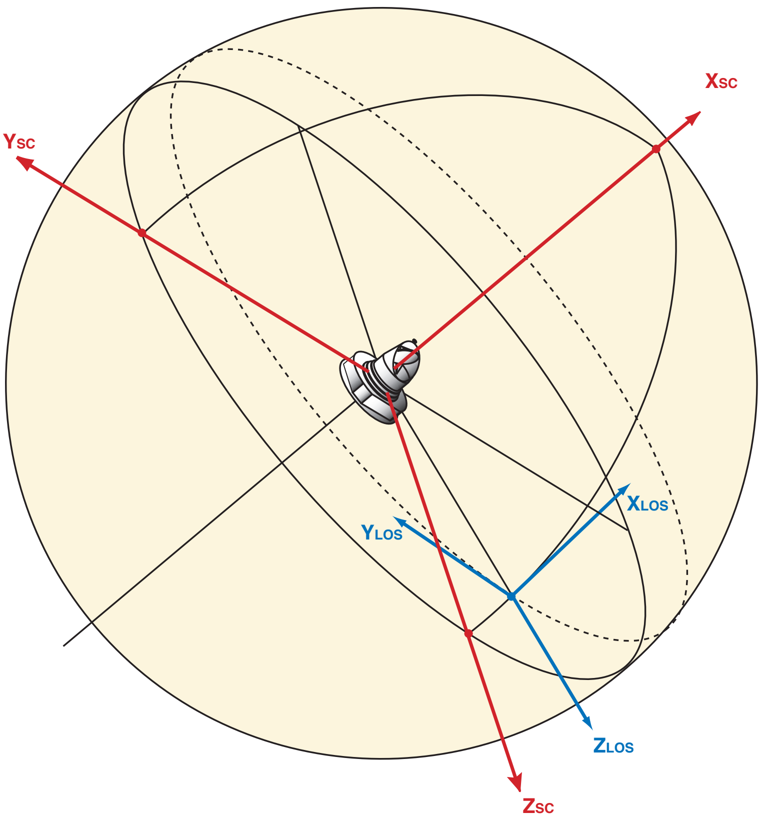

Figure 3: Spacecraft (SC) and Line of Sight (LOS) reference frames. The SC reference frame is obtained from the LOS reference frame by a rotation of angle $\theta_b - 90^\circ$ around $\mathbf{Y}_{\rm SC}$.

[1]Note that the word *Center* here does not refer to any special geometrical property. This *Center* is merely a fiducial point common to HFI and LFI.

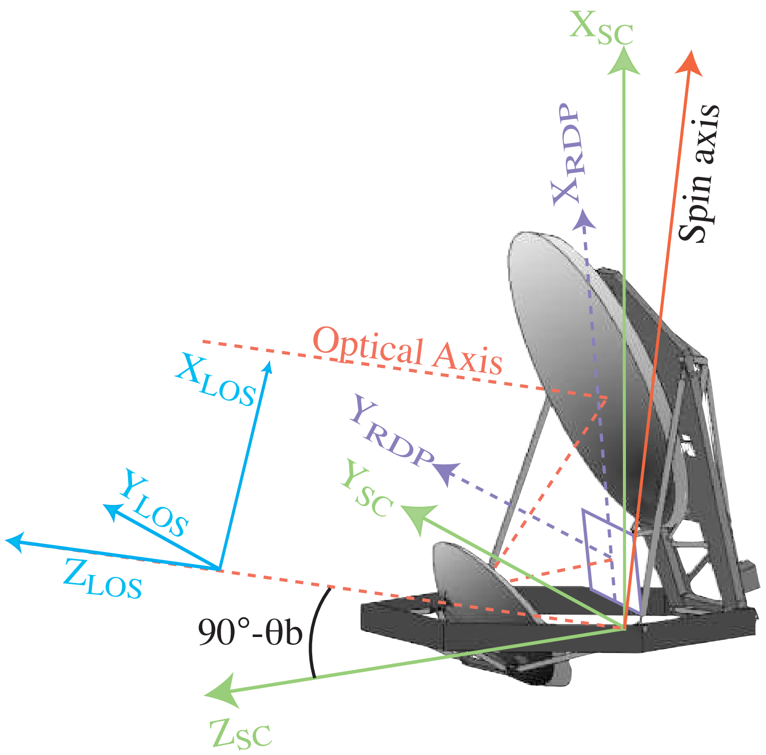

Figure 4: Spacecraft (SC), Line of Sight (LOS) and Reference Detector Plane (RDP) reference frames.

3.2.5 RDP reference frame

We adopt the focal plane axis system defined in RD-1, which will be useful when relating quantities to the focal plane geometry. The origin O_{RDP} is at the center of the RDP and also defines the optical axis. $\mathbf{X}_{\mathrm{RDP}}$ is the intersection of the RDP and the $\mathbf{X}_{\mathrm{SC}}$-$\mathbf{Z}_{\mathrm{SC}}$ plane of the telescope with positive direction towards the primary mirror. $\mathbf{Z}_{\mathrm{RDP}}$ is normal to the RDP with positive direction toward the secondary mirror. $\mathbf{Y}_{\mathrm{RDP}}$ complements the right handed triad $\mathbf{X}_{\mathrm{RDP}}, \mathbf{Y}_{\mathrm{RDP}}, \mathbf{Z}_{\mathrm{RDP}}$.

Note that the optics of Planck image $\mathbf{X}_{\mathrm{RDP}}$ and $\mathbf{Y}_{\mathrm{RDP}}$ upward and to the right respectively, so that nominally, $\mathbf{X}_{\mathrm{RDP}}$ and $\mathbf{Y}_{\mathrm{RDP}}$ project to $\mathbf{X}_{\mathrm{LOS}}$ and $\mathbf{Y}_{\mathrm{LOS}}$ respectively through the optics.

3.2.6 ATT reference frame

This reference frame attached to the Star Mapper. The attitude information provided by the MOC will be the reference frame transfomation exchanging the ATT reference frame and the ECL reference frame (see ICD006). Nominally, the ATT and SC frame should be identicals, and the ATT and LOS frames should be linked by a fixed reference frame transformation. However, relative positioning and alignment uncertainties make the prior adjustment on the SC frame of both the LOS reference frame (defined by the payload) and the pointing reference frame (defined by the pointing instrument, e.g. stellar sensor) impossible. Moreover, it is possible, although not with a high probability, that the relation between LOS and ATT will change slowly during the mission, as the mechanical stress adds on the satellite structure. For all this reason, the relative orientation of ATT and LOS will have to be monitored during the course of the mission. Note that modification of the ATT to LOS transformation are indicernible with errors in the attitude determination.

3.2.7 SA reference frame

The SA reference frame is defined at each time t as the frame for which $\mathbf{Z}_{\mathrm{SA}}$ is along the instantaneous vector of rotation Ω_{spin} of the spacecraft (with respect to the sky), $\mathbf{Y}_{\mathrm{SA}}$ along $\mathbf{Z}_{\mathrm{SA}} \times \mathbf{Z}_{\mathrm{LOS}}$ (and points along the positive scanning direction on the sky), and $\mathbf{X}_{\mathrm{SA}}$ is such that $\mathbf{X}_{\mathrm{SA}}$, $\mathbf{Y}_{\mathrm{SA}}$ and $\mathbf{Z}_{\mathrm{SA}}$ form a direct system.

Nominally, the SA reference frame is such that $\mathbf{Z}_{\mathrm{SA}} = \mathbf{X}_{\mathrm{SC}}$. Deviations from this nominal direction define the wobble angle (see below).

The SA reference frame is obtained by a rotation of the SC reference frame which puts $\mathbf{X}_{\mathrm{SC}}$ along the spin axis, followed by a relabelling (permutation of axes $X \to Z$, $Y \to X$, $Z \to Y$), followed by a rotation which brings the $\mathbf{Y}_{SA}$ axis along the instantaneous positive scanning direction.

3.2.8 SS reference frame

The spacecraft spinning reference frame is defined, for a period of nominal stable spinning, as the frame for which the Z axis is the average of all spin-axes directions on the sky during that period, the Y axis is along $\mathbf{Z}_{\mathrm{SS}} \times \mathbf{Z}_{\mathrm{ECL}}$, and $\mathbf{X}_{\mathrm{SS}}$ is such that $\mathbf{X}_{\mathrm{SS}}$, $\mathbf{Y}_{\mathrm{SS}}$ and $\mathbf{Z}_{\mathrm{SS}}$ form a direct system. $\mathbf{X}_{\mathrm{SS}}$ is in the plane defined by the Ecliptic North Pole and the time-averaged spin-axis. The spherical coordinate angle ϕ_{SS} of a detector beam on the sky is the phase along the corresponding Planck circle or ring for that detector.

We define the "spin axis" plane as the plane containing the ecliptic polar axis and the average spin axis direction. For perfectly stable spinning, the average spin axis direction is the direction of the stable spin axis.

3.2.9 Dxx reference frame

The detector reference frame for detector xx is defined from the LOS frame by parallel transport of the LOS reference frame to the detector reference point. In other words, if $\mathbf{Z}_{\mathrm{Dxx}}^{\mathrm{LOS}} = (z_i)_{i=0,2}$ is the pointing direction of the xx bolometer expressed in the LOS, then $\mathbf{X}_{\mathrm{Dxx}}^{\mathrm{LOS}}$ and $\mathbf{Y}_{\mathrm{Dxx}}^{\mathrm{LOS}}$, the x and y axis of the Dxx frame expressed in the LOS frame are given by

$$\mathbf{Y}_{\mathrm{Dxx}}^{\mathrm{LOS}} = \frac{1}{\sqrt{z_1^2 + z_2^2}} \begin{pmatrix} 0 \\ z_2 \\ -z_1 \end{pmatrix} \tag{9}$$

and

$$\mathbf{X}_{\mathrm{Dxx}}^{\mathrm{LOS}} = \mathbf{Y}_{\mathrm{Dxx}}^{\mathrm{LOS}} \times \mathbf{Z}_{\mathrm{Dxx}}^{\mathrm{LOS}}. \tag{10}$$

The formula for $\mathbf{Y}_{\mathrm{Dxx}}$ simply express that we want to define the Y axis of the Dxx reference frame not to gain any x component in the LOS frame, and that this axis have to be identical to $\mathbf{Y}_{\mathrm{LOS}}$ when the detector is situated in the center of the focal plane.

Each Dxx reference frame is defined with respect to the ”beam center” (see below).

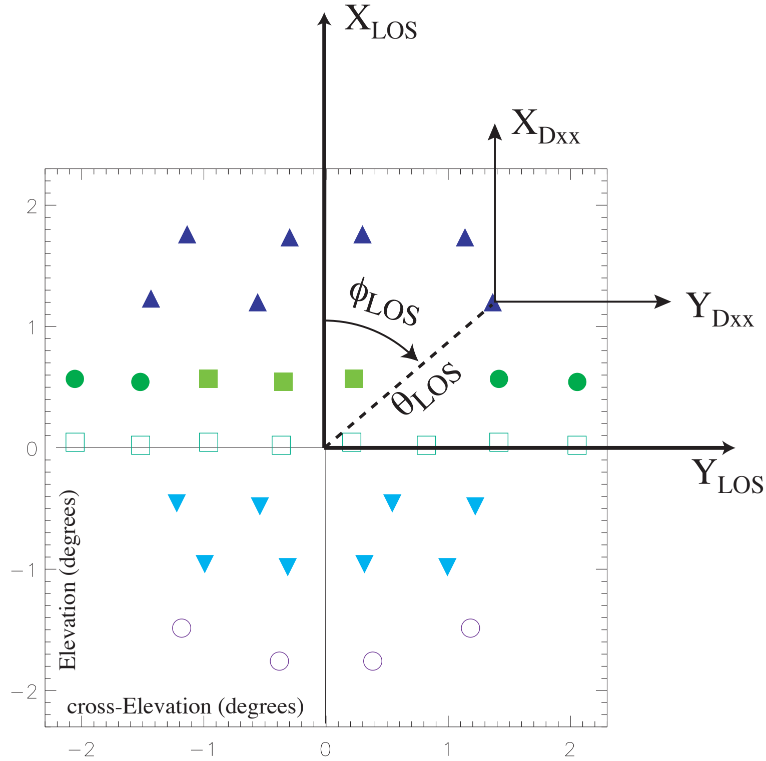

Figure 5: LOS and Detector (Dxx) reference frames on the image of the focal plane on the sky. The Dxx reference frame is obtained from the LOS reference frame by the rotation bringing along a great circle O_{LOS} onto the beam center of detector xx. Note that the LOS subscript indicates, as everywhere else, that θ and ϕ are quantities measured in the LOS reference frame (here polar angles of the location of the beam center of a detector).

3.2.10 Bxx reference frame

This reference frame is related to the former by a rotation around $\mathbf{Z}_{Dxx}$in order to ensure that $\mathbf{X}_{Bxx}$ is aligned to some fiducial direction relevant for the beam description. For example, when describing the beams by elliptical gaussian functions, it can be convinient to define the Bxx reference frame so that $\mathbf{X}_{Bxx}$ is along the major axis of the ellipse.

3.2.11 Pxx reference frame

This is essentially the same as the Bxx frame but now, we are aligning $\mathbf{X}_{Pxx}$ with the polarization main axis. This frame is convenient for defining polarized beam patterns as discussed in section 5.5. This convention is the same as the main beam reference frame adopted in RD-2. Of course, the Z_{pxx} direction points toward the nominal unpolarised beam center for polarimeter xx. One can define a Pxx reference frame for an unpolarized detector; in this case, Pxx = Dxx.

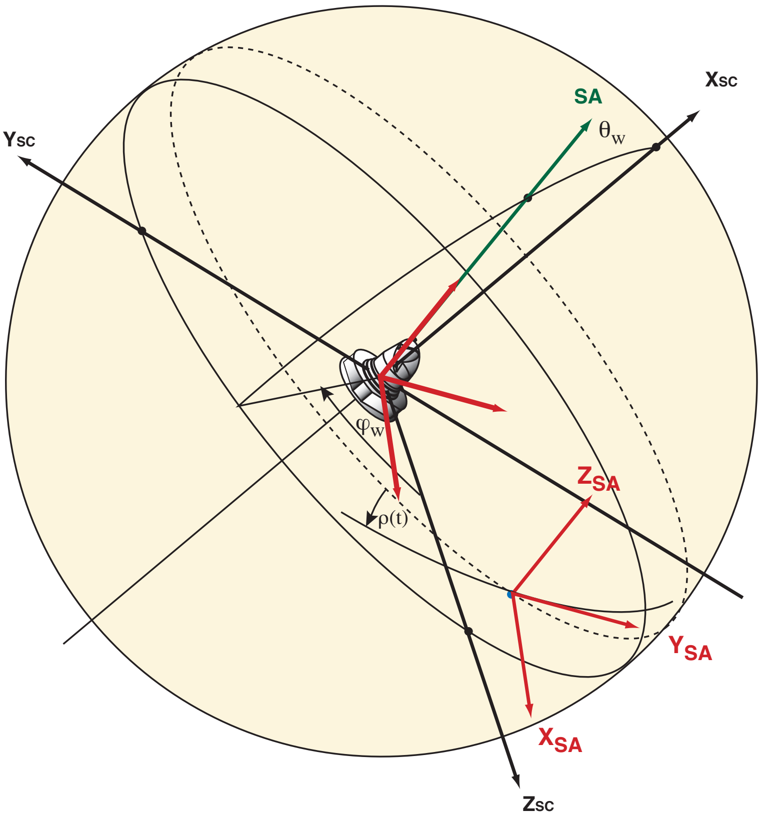

Figure 6: SC and SA frames. The roll angle $\rho(t)$ is the angle between the nominal and actual trajectories of the LOS on the sky. In other words, it is the angle between Y_{LOS} and $-Y_{\mathrm{SA}}$ in the plane tangent to the sky at the LOS center.

3.3 Pointing and attitude

At each time t, any spacecraft reference frame is connected to a sky reference frame by a single rotation. The knowledge of the spacecraft attitude amounts to knowing, for each time, the rotation connecting one single spacecraft reference frame with one single sky reference frame.

It will be assumed that such knowledge will be available from the MOC. We assume that (possibly after some reprocessing) the attitude of a specific reference frame (the attitude reference frame – ATT) with respect to the ecliptic reference frame (ECL) will be available. The ATT reference frame can be, for instance, attached to optical telecopes serving as stellar sensors. No strong requirements are put on the connection of the ATT reference frame and other frames attached to the spacecraft (including those attached to the telescope), except that this connection is assumed to be time–independent.

The rationale for this approach to pointing requirements is that a large fraction of the difficulty of the pointing is the problem of the knowledge to sub–arcminute accuracy of the relative orientation of the sensors serving to reconstruct the spacecraft attitude (stellar sensors, gyroscopes, to which the ATT reference frame is attached) and of the telescope (to which the Line Of Sight (LOS) reference frame and instrumental beams are attached). The relative orientation of the ATT and the LOS reference frame has to be calibrated in flight, using science data.

3.4 Combining rotation and reference frame exchanges

We will briefly remind you a few known results on rotation and reference frame exchanges. In the following, we will denote $R(\mathbf{v}, \phi)$ the rotation around the axis $\mathbf{v}$ and of angle ϕ. We will also denote $\times$ the application of a rotation to a vector, and $\circ$ combination of two rotation such that

$$R = R_2 \circ R_1 \tag{11}$$

is the rotation R_1 followed by rotation R_2.

If R is a rotation that brings the axis of the xxx reference frame to the yyy reference frame, then the coordinates of any vector $\mathbf{M}^{\text{xxx}}$ expressed in the xxx reference frame can be expressed in the yyy reference frame by

$$\mathbf{M}^{\text{yyy}} = R^{-1} \times \mathbf{M}^{\text{xxx}}. \tag{12}$$

This is the usual results that the coordinate exchange is the inverse of the rotation matrix that brings one reference frame to the other. This coordinate exchange is also sometime named a *passive rotation*. To avoid further confusion we will define $R_{\text{xxx}}^{\text{yyy}}$ the reference frame exchange that allows to compute in the reference frame yyy the coordinate of a vector defined in the reference frame xxx, and we will pretend that we forgot that this coordinate frame exchange is in fact a rotation. In this case, we have

$$\mathbf{M}^{\text{yyy}} = R_{\text{xxx}}^{\text{yyy}} \times \mathbf{M}^{\text{xxx}}, \tag{13}$$

and we recover

$$R_{\text{xxx}}^{\text{yyy}} = R^{-1}. \tag{14}$$

As an example, let's compute the reference frame exchange linking the ATT reference frame and the ECL reference frame at two different times. t_0 and t_1. Let's assume that we know this reference frame exchange at time t_0 ; $R_{\text{ATT}(t_0)}^{\text{ECL}}$ is such that

$$\mathbf{Z}^{\text{ECL}}(t_0) = R_{\text{ATT}(t_0)}^{\text{ECL}} \times (0, 0, 1) \tag{15}$$

is the direction seen by the star tracker at t_0, expressed in the ECL reference frame. Let's assume that between t_0 and t_1 the satelite turned of ϕ radian around some axis $\mathbf{v}$, that is represented by $\mathbf{v}^{\text{ATT}(t_0)}$ in the ATT reference frame. at t_0 Then, expressed in the ATT reference frame at t_0, the new Z axis is given by

$$\mathbf{Z}_{\text{ATT}(t_1)}^{\text{ATT}(t_0)} = R(\mathbf{v}^{\text{ATT}(t_0)}, \phi) \times (0, 0, 1). \tag{16}$$

The same holds for the other coordinates, such that the reference frame exchange between ECL and ATT at t_1 is

$$R^{\mathrm{ECL}}_{\mathrm{ATT}(t_1)} = R^{\mathrm{ECL}}_{\mathrm{ATT}(t_0)} \circ R(\mathbf{v}^{\mathrm{ATT}(t_0)}, \phi). \tag{17}$$

If we now decide to describe the rotation axis in the ECL coordinates, we got

$$R^{\mathrm{ECL}}_{\mathrm{ATT}(t_1)} = R(\mathbf{v}^{\mathrm{ECL}}, \phi) \circ R^{\mathrm{ECL}}_{\mathrm{ATT}(t_0)} \tag{18}$$

and

$$\mathbf{v}^{\mathrm{ECL}} = R^{\mathrm{ECL}}_{\mathrm{ATT}(t_0)} \times \mathbf{v}^{\mathrm{ATT}(t_0)}. \tag{19}$$

This last result can be also obtained by noting that

$$\mathbf{Z}^{\mathrm{ATT}(t_0)}_{\mathrm{ATT}(t_1)} = R^{\mathrm{ATT}(t_0)}_{\mathrm{ECL}} \circ R(\mathbf{v}^{\mathrm{ECL}}, \phi) \circ R^{\mathrm{ECL}}_{\mathrm{ATT}(t_0)} \times (0, 0, 1), \tag{20}$$

so that

$$R^{\mathrm{ECL}}_{\mathrm{ATT}(t_1)} = R^{\mathrm{ECL}}_{\mathrm{ATT}(t_0)} \circ R^{\mathrm{ATT}(t_0)}_{\mathrm{ECL}} \circ R(\mathbf{v}^{\mathrm{ECL}}, \phi) \circ R^{\mathrm{ECL}}_{\mathrm{ATT}(t_0)} = R(\mathbf{v}^{\mathrm{ECL}}, \phi) \circ R^{\mathrm{ECL}}_{\mathrm{ATT}(t_0)}. \tag{21}$$

Note that the above discussion is valid regardless of the convetion used to describe rotations and vectors.

4 Some definitions relevant to the Instrument Model

4.1 Focal Plane Image Geometry

The knowledge of the set of LOS-Dxx, and LOS-Pxx rotations is enough to define the focal plane image. The use of quaternion is strongly recommended to provide this information. Optionally, the LOS-Bxx rotations can be provided for special use.

4.2 Beams

4.2.1 Optical beam

The optical beam of a detector is the radiation pattern produced by the optics alone.

4.2.2 Effective beam

The effective beam of a detector is the effective beam due to a combination of the optical beam and the smearing effect due to the time response of the detector chain (time constants, integration windows for sampling, filters).

4.2.3 Beam center

The location of the beam center for each detector is defined in the instrument model by the appropriate instrumental team. This definition may be revised after in-flight calibration of detector relative pointings, under the responsibility of the instrument scientists.

Nominally, the beam center is defined as the center of the ideal gaussian beam shape. For distorted beams, it is defined as the center of the gaussian best fit.

4.2.4 Beam Description

Beams have to be described in the Dxx reference frames. Optionally, it is possible to define a Bxx reference frame suitable for an easier description of the beam. This is not recommended.

Different approximations of the beam are possible. As of now, the Instrument Model should contain circular and elliptical gaussian approximations of the beam. It has also been proposed to store a three circular gaussian (same size and amplitude) approximation of the beam. Finally, a pixelized approximation of the beams, on a projection plane or on the sphere, in real space or Fourier/Spherical harmonics will be stored.

4.3 Attitude and nominal Attitude

4.3.1 Bore–sight angle

The angle between the optical axis (LOS) and the $\mathbf{X}_{\mathrm{SC}}$ axis defines the bore–sight angle θ_b, nominally equal to $85°$.

4.3.2 Positive scanning direction

The positive scanning direction is defined as the instantaneous direction of scanning of the image on the sky of the FPC. The positive scanning direction is thus given by $\mathbf{Z}_{\mathrm{SA}} \times \mathbf{Z}_{\mathrm{LOS}}$ (and thus equals $-\mathbf{Y}_{\mathrm{LOS}}$). The negative scanning direction is the opposite, and points along $\mathbf{Y}_{\mathrm{LOS}}$.

4.3.3 Wobble angle

The nominal spin axis of the SC is along $\mathbf{X}_{\mathrm{SC}}$. The angle between $\mathbf{X}_{\mathrm{SC}}$ and the instantaneous spin axis is called wobble angle, and can be decomposed into a set of 2 polar angles θ_w (angular distance between the spin axis and $\mathbf{X}_{\mathrm{SC}}$) and ϕ_w, longitude angle of the real instantaneous spin axis in the nominal spin axis reference frame (fig. 6).

4.3.4 Roll angle

The angle, in the LOS XY plane, between $\mathbf{Y}_{\mathrm{LOS}}$ and $-\mathbf{Y}_{\mathrm{SA}}$ is defined as the roll angle $\rho(t)$. Note that $\rho(t) = 0$ when $\phi_w = 0$.

4.3.5 Satellite position

In order to be able to position solar-system objects with respect to the satellite, the Spacecraft position has to be known all along the mission. Its coordinates will be given either in the Ecliptic reference frame, or in the L2 frame.

4.4 Detector pointings on the sky, in a ring

4.4.1 Detector pointings

The detector pointing of a detector xx at time t (or at sample t) is defined as the corresponding location on the sky of the detector beam center. That is to say, the direction in which Z_{Dxx} points.

This location has to be expressed in either GEOGRAPHICAL, SPHERICAL or CARTESIAN coordinates in a ”fixed” (i.e. not attached to the spacecraft) reference frame YYY. The use of CARTESIAN coordinates is recommended.

Alternatively, the rotation that exchanges the Dxx and YYY frame can also be considered as a detector pointing. The knowledge of the full rotation is needed to compute the orientation of the detector.

It is recommended that detector pointings are not stored during the data processing. Indeed, this information can be simply reconstructed, at a very low cost, from the knowledge of the rotation between the ATT and some fixed reference frame YYY (given by the MOC) and the knowledge of the fixed ATT-LOS and LOS-Dxx rotations.

4.4.2 Polarization angle / Orientation of beams

At each time, the detector xx observes the sky, forming an angle α with some fiducial orientation. The knowledge of this angle is needed to project the polarization vector onto the detector sensitivity directions, or to convolve the beam with the real sky. As described in section 3.1.3, this angle has to be measured using one of the two conventions IAU or COSMO.

For polarization and beams, the fiducial direction will be the $\mathbf{e}_1$ vector of the frame attached with the pointing direction (see above). This corresponds of course to the angle between the orientation and the local meridian.

Note that the cosine and sine of this angle can be computed very simply and at low cost from the knowledge of the ATT-YYY rotation at time t, along with the fixed ATT-LOS and LOS-Pxx rotations. It is thus recommended to avoid propagating angles and to compute them each time they are needed.

4.4.3 Phase along circles

The DPC will need to manipulate the phase information along a given observed circle (for instance, in order to average data in phase into phase–binned–rings).

Each circle crosses the plane defined by $\mathbf{Z}_{\mathrm{ECL}}$ and $\mathbf{Z}_{\mathrm{SS}}$ in two points A and B (Fig.7). The LOS phase angle $(\phi(t))$ is defined as the angle between the current direction of the optical axis $(\lambda(t), \beta(t))$ and the Northernmost intersection point (A), and is counted positive for an anti-clockwise (positive trigonometric) rotation around $\mathbf{Z}_{\mathrm{SS}}$ and negative for a clockwise rotation around $\mathbf{Z}_{\mathrm{SS}}$.

In the case of binned rings, the phase corresponds to the phase of the middle of the bin.

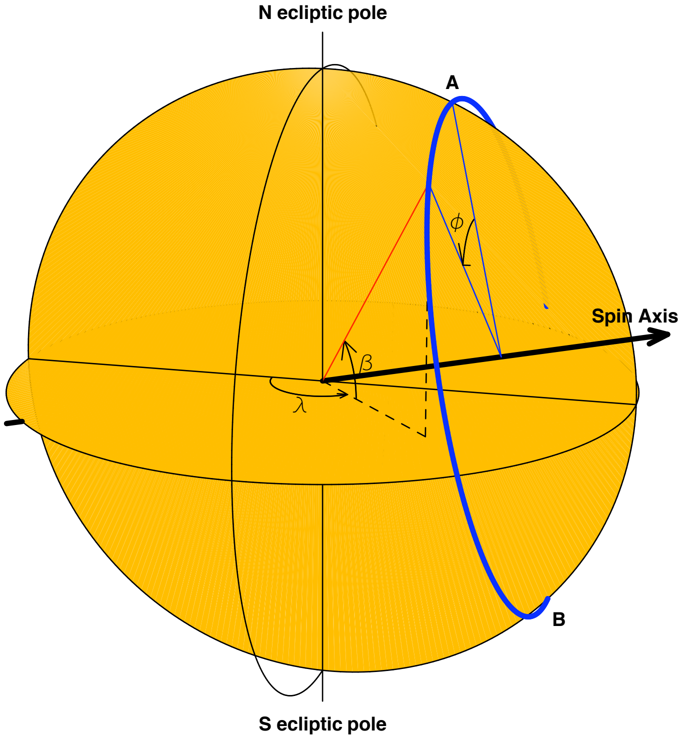

Figure 7: Ecliptic coordinates of the spin axis direction, defining the Z axis of the Spacecraft Spinning (SS) reference frame.

5 Polarization variables definition

A transverse electromagnetic wave is fully described by the electric fields E_X and E_Y in two directions orthogonal to the propagation direction. A general radiation is an incoherent superposition of waves with the same wave vector $\vec{k}$. Choosing two basis vectors X and Y orthogonal to $\vec{k}$, for a quasi-monochromatic wave of frequency ν, we have

$$E_X(t) = A_X(t)\cos(\phi_X - 2\pi\nu t) \tag{22}$$

$$E_Y(t) = A_Y(t)\cos(\phi_Y - 2\pi\nu t) \tag{23}$$

5.1 Stokes parameters

The polarisation state of incoherent radiation is completely and conveniently defined by four intensity-like quantities, the Stokes parameters I, Q, U, and V. These can be defined as:

$$I = \langle A_X^2 \rangle + \langle A_Y^2 \rangle \tag{24}$$

$$Q = \langle A_X^2 \rangle - \langle A_Y^2 \rangle \tag{25}$$

$$U = \langle 2A_X A_Y \cos(\phi_Y - \phi_X) \rangle \tag{26}$$

$$V = \langle 2A_X A_Y \sin(\phi_Y - \phi_X) \rangle \tag{27}$$

where $\langle . \rangle$ denotes the ensemble average. $A_X(t)$ and $A_Y(t)$ change in time, slowly as compared to the period $2\pi/\nu$, but rapidly as compared to the typical detection time. Hence, Planck detectors measure time–averaged values assumed to be equal to ensemble averages. Planck detectors are directly sensitive to combinations of the Stokes parameters.

The Stokes parameters I, Q, U, V satisfy the inequality

$$I^2 \geq Q^2 + U^2 + V^2 \tag{28}$$

which means that the polarized energy cannot exceed the total energy. It becomes an equality for a fully polarized radiation.

$P = \sqrt{Q^2 + U^2 + V^2}$ is the total polarized intensity, and $p = P/I$ the degree of polarization.

Note that the Stokes parameters only make sense when associated with an orientation convention. Indeed, it is easy to see that the U Stokes parameters change sign when changing between the IAU and COSMO conventions.

In the following, we will call $S = (I, Q, U, V)$ the Stokes vector.

5.2 Coherence matrix

Stokes parameters are conveniently introduced alternatively using the coherence matrix C which conveys all statistical information about the polarisation state of the wave. Using now complex field representation

$$E_x(t) = A_x(t) \exp i(\phi_x - 2\pi\nu t) \tag{29}$$

$$E_y(t) = A_y(t) \exp i(\phi_y - 2\pi\nu t) \tag{30}$$

The 2×2 coherence matrix is given by

$$C = \begin{pmatrix} \langle |E_x|^2 \rangle & \langle E_x E_y^* \rangle \\ \langle E_y E_x^* \rangle & \langle |E_y|^2 \rangle \end{pmatrix} = \frac{1}{2} \begin{pmatrix} I + Q & U - iV \\ U + iV & I - Q \end{pmatrix}, \tag{31}$$

5.3 Transformation of Stokes parameters

The measured Stokes parameters depend on the reference system in the plane orthogonal to the propagation direction XY. In a frame XY' obtained from XY by a rotation by an angle ψ around the propagation direction, the Stokes parameters are obtained by the following transformation:

$$S_{XY'} = \mathcal{R}(\psi)\, S_{XY} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\psi & \sin 2\psi & 0 \\ 0 & -\sin 2\psi & \cos 2\psi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} S_{XY} \tag{32}$$

or

$$Q \pm i\,U \rightarrow Q' \pm iU' = e^{\mp 2\,i\,\theta}\,(Q \pm i\,U) \tag{33}$$

This is the transformation of a spin 2 object.

5.4 Measurement by a single polarimeter

The transformation of a polarized radiation going thru an optical instrument is easily parametrized by the mean of the Muller Matrix. If we call S_i the Stokes vector describing the radiation before the transformation and S_f after it, the Muller Matrix encode all the rotation between the different Stokes parameters

$$S_f = M\ S_i. \tag{34}$$

In the case of a perfect detector, and assuming the detector is aligned with the XY plane in the direction orthogonal to the propagation, this matrix reduces to the Identity matrix.

A rotation of an angle ψ of the detector relative to the XY plane can be described by a rotation on the Muller matrix measured in the XY plane

$$M' = \mathcal{R}(\psi)^t\ M\ \ \mathcal{R}(\psi). \tag{35}$$

If we now describe a polarized detector as a polarizer, forming an angle ψ in the XY plane, in front of a total power detector, the signal s_ψ measured is given by the first element of S_f, which in the case of a perfect instrument reduces to

$$s_\psi = \frac{1}{2}\left[I + Q\cos 2\psi + U \sin 2\psi\right]. \tag{36}$$

Note that we have

$$I = s_\psi + s_{\psi+\pi/2} \tag{37}$$

$$Q = s_0 - s_{\pi/2} \tag{38}$$

$$U = s_{\pi/4} - s_{3\pi/4} \tag{39}$$

In practice, an imperfect polarimeter has some amount of cross polar leakage and is not perfectly efficient. This can be modeled by saying that the optical system has imperfect transmission components $\eta \neq 1$ and $\delta \neq 0$ along the axis of the polarizer. It is easy to show that this translates into the Muller matrix

$$M(\eta,\delta) = \mathcal{R}(\psi)^t\ \frac{\eta^2}{2}\begin{pmatrix} 1+\frac{\delta^2}{\eta^2} & 1-\frac{\delta^2}{\eta^2} & 0 & 0 \\ 1-\frac{\delta^2}{\eta^2} & 1+\frac{\delta^2}{\eta^2} & 0 & 0 \\ 0 & 0 & 2\left|\frac{\delta}{\eta}\right| & 0 \\ 0 & 0 & 0 & 2\left|\frac{\delta}{\eta}\right| \end{pmatrix}\mathcal{R}(\psi). \tag{40}$$

Modelling now our polarized detector as an imperfect optical system followed by a total power detector of efficiency g/η^2, and definig the polarization leackage $\epsilon = \delta^2/\eta^2$ the signal measured is

$$s_\psi = \frac{g}{2}\left[(1+\epsilon)\, I + (1-\epsilon)\left(Q\cos 2\psi + U\sin 2\psi\right)\right]. \tag{41}$$

The value $1-\epsilon$ is also called the polarization efficiency.

5.5 Definitions for Beam patterns

For a given polarisation sensitivity direction at the receiver, the direction of co-polarization at the beam center (on–axis) is conveniently defined as the image of the sensitivity direction through the optics. The cross polarisation sensitivity direction is orthogonal to the co–polarization.

However, the computation of the integrals of sky Stokes parameters in instrumental beam patterns requires a definition of co-polarization and cross-polarization over the whole surrounding sphere.

The most widely used convention (third convention of Ludwig, or Ludwig III) is obtained by parallel–transport of the on–axis conventions.

By convention, the direction of co–polarization sensitivity (the co–polar orientation) is along the Y axis (this corresponds to a specific orientation of the polarimeter in the focal plane). The Ludwig III reference system is thus

$$\mathbf{X}_{\mathrm{L3}} = (\text{cross-polar} \quad \text{direction}) \tag{42}$$

$$\mathbf{Y}_{\mathrm{L3}} = (\text{co-polar} \quad \text{direction}) \tag{43}$$

$$\mathbf{Z}_{\mathrm{L3}} = (\text{radial} \quad \text{direction}) \tag{44}$$

At any point on the sphere, the L3 reference frame is connected to the spherical reference frame as follows

$$\begin{pmatrix} x^{\mathrm{L3}} \\ y^{\mathrm{L3}} \\ z^{\mathrm{L3}} \end{pmatrix} = \begin{pmatrix} \cos(\varphi) & -\sin(\varphi) & 0 \\ \sin(\varphi) & \cos(\varphi) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e_\theta \\ e_\varphi \\ e_r \end{pmatrix} \tag{45}$$

where $(e_\theta, e_\varphi, e_r)$ are the usual vectors connected to the spherical coordinate system on the sphere (see figure 8).

A big advantage of the Ludwig III reference frame is that there is a single singularity on the sphere, at the opposite of the main beam center.

5.6 Polarized beam patterns

We now enrich our modelization of the measurements of a PSB by taking into account the optical response of the horn and telescope system. We define the co and cross beam ($\tilde{B}_\parallel$ and $\tilde{B}_\perp$) of the telescope and horn optics system in the Ludwig III reference frame. We will use normalized beams $B_\parallel$ and $B_\perp$, such that

$$B_\parallel \equiv \frac{\tilde{B}_\parallel}{\int \mathrm{d}^2\theta \; \tilde{B}_\parallel + \tilde{B}_\perp}, \tag{46}$$

and the same for $B_\perp$. Furthermore, we define B_0 the normalization

$$B_0 \equiv \int \mathrm{d}^2\theta \; \tilde{B}_\parallel + \tilde{B}_\perp. \tag{47}$$

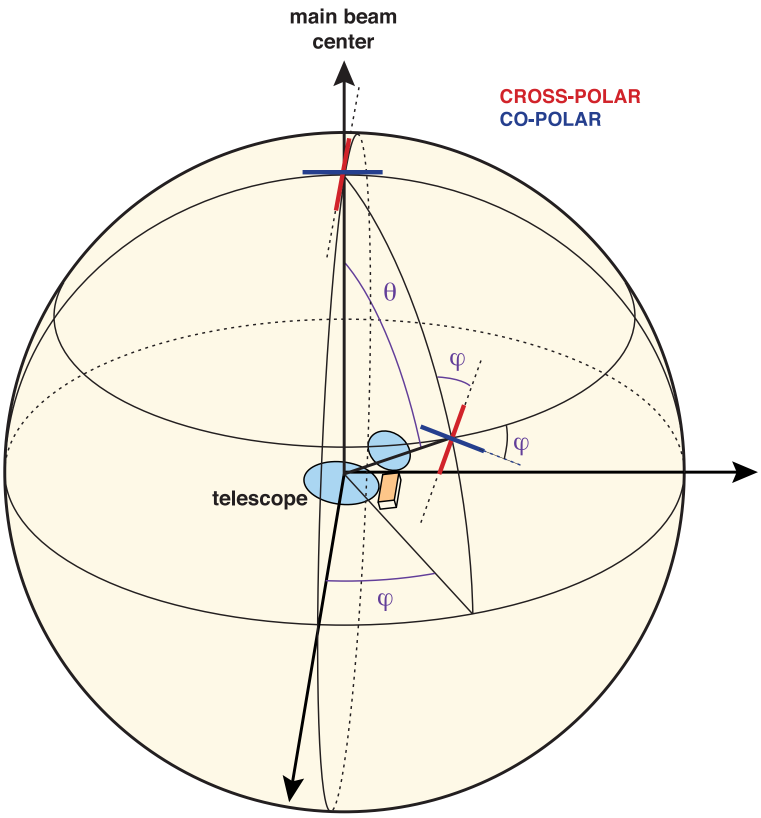

Figure 8: The Ludwig III (L3) reference frame defining the co–polarization and cross–polarization at each point of the sphere.

The signal measured on the bolometer, pointing in the direction $\boldsymbol{\omega}$, is now the result of the convolution of the sky I, Q and U by the co and cross beam, transfered through an imperfect polarizer, and measured by a total power detector

$$\begin{aligned} s(\boldsymbol{\omega}) &= \frac{g}{2} B_0 \int \mathrm{d}^2\theta \, \left[B_{\parallel} \left(\boldsymbol{\omega} - \boldsymbol{\theta}\right) + B_{\perp} \left(\boldsymbol{\omega} - \boldsymbol{\theta}\right) \right] (1+\epsilon) \times \\ & \times \left[I\left(\boldsymbol{\theta}\right) + \gamma\left(\epsilon\right) \mathcal{B}\left(\boldsymbol{\omega} - \boldsymbol{\theta}\right) \left(Q\left(\boldsymbol{\theta}\right) \cos 2\psi + U\left(\boldsymbol{\theta}\right) \sin 2\psi \right) \right], \end{aligned} \tag{48}$$

where we defined

$$\mathcal{B} \equiv \frac{B_{\parallel} - B_{\perp}}{B_{\parallel} + B_{\perp}} \tag{49}$$

and

$$\gamma(\epsilon) = \frac{1-\epsilon}{1+\epsilon}. \tag{50}$$

Note that the equations above only hold for a mono frequency bolometer. In the case where the frequency bandpass is not a dirac, the equation becomes

$$s(\boldsymbol{\omega}) = \frac{g}{2} \int \mathrm{d}\nu \; F_\nu B_{\nu,0} \int \mathrm{d}^2\theta \, \left[B_{\nu\parallel} + B_{\nu\perp} \right] (1+\epsilon_\nu) \left[I_\nu + \gamma\left(\epsilon_\nu\right) \mathcal{B}_\nu \left(Q_\nu \cos 2\psi + U_\nu \sin 2\psi \right) \right]. \tag{51}$$

We have removed the explicit $\boldsymbol{\omega}$ and $\boldsymbol{\theta}$ dependency. The ν indice reflects the dependency of the parameters on the frequency, F_ν being the bandpass.

From this point, we can define the effective $\tilde{I}$, $\tilde{Q}$ and $\tilde{U}$ beams such that as

$$\tilde{I}_\nu \equiv B_{\nu,0}\left[B_{\nu\|} + B_{\nu\perp}\right](1+\epsilon_\nu), \tag{52}$$

$$\begin{aligned}\tilde{Q}_\nu &\equiv B_{\nu,0}\left[B_{\nu\|} + B_{\nu\perp}\right](1+\epsilon_\nu)\,\gamma(\epsilon_\nu)\,\mathcal{B}_\nu \cos 2\psi \\ &\equiv B_{\nu,0}\left[B_{\nu\|} - B_{\nu\perp}\right](1-\epsilon_\nu)\cos 2\psi,\end{aligned} \tag{53}$$

$$\begin{aligned}\tilde{U}_\nu &\equiv B_{\nu,0}\left[B_{\nu\|} + B_{\nu\perp}\right](1+\epsilon_\nu)\,\gamma(\epsilon_\nu)\,\mathcal{B}_\nu \sin 2\psi \\ &\equiv B_{\nu,0}\left[B_{\nu\|} - B_{\nu\perp}\right](1-\epsilon_\nu)\sin 2\psi.\end{aligned} \tag{54}$$

In such case, we ca rewrite the signal received by the bolometer as

$$\begin{aligned}s(\boldsymbol{\omega}) = \frac{g}{2}\int \mathrm{d}\nu\, F_\nu \Bigg[\int \mathrm{d}^2\theta\, \widetilde{I}_\nu(\boldsymbol{\omega}-\boldsymbol{\theta})\, I_\nu(\boldsymbol{\omega}) \\ + \int \mathrm{d}^2\theta\, \widetilde{Q}(\boldsymbol{\omega}-\boldsymbol{\theta})\, Q_\nu(\boldsymbol{\omega}) + \int \mathrm{d}^2\theta\, \widetilde{U_\nu}(\boldsymbol{\omega}-\boldsymbol{\theta})\, U_\nu(\boldsymbol{\omega})\Bigg]\end{aligned} \tag{55}$$

We recall here that we used the L3 reference frame. Computing the responses for another reference frame for polarisation is straightforward, but requires a rigorous definition of the conventions for directions and frames. If the electromagnetic wave is considered to propagate *inwards* ($\vec{k}$ in the direction of $-\widehat{e}_r$), electric fields are changed to their complex conjugates. This impacts the sign of the V term. Rotations of the frame used for defining Q and U mix the Q and U responses accordingly (see *Challinor et al. 2000, PRD, 62, 13002*, where responses are expressed in terms of the spherical coordinates frame).

6 Units

6.1 Angle Units

All units for angles are assumed to be radians. All the angles in the inputs and outputs of DPC programs shall be radians.

For easy order-of-magnitude conversion, 10^{-1} radians is about 5.7 degrees, and 10^{-2} radians is about 34 arcminutes. Planck detector resolutions are typically a few 10^{-3} radians, pointing accuracies a few 10^{-4} radians.

6.2 Time Definition

The Spacecraft clock will distribute the On-Board Time (OBT) to all subsystems. This time will be synchronized on TAI (Temps Atomique International). The resolution of the OBT will be $1/2^{16}$s $= 15\mu$s. The different instruments (LFI, HFI and AOCS – Attitude and Orbit Control System) will synchronize their LOBT (Local On-Board Time) on the OBT. The maximum shift between two different LOBT will never exceed 0.5ms. The OBT will be already available before launch, during ground phases, thanks to a Spacecraft Simulator.

All HFI data are time-sampled at a rate defined by the HFI readout clock. This clock period drives the period of the 4K cooler compressor, and can be adjusted through telecommands to minimize parasitic effects due to this cooler. Hence, it is subject to variations of as much as a few per cent during the mission, but will remain very stable for very long periods of time.

Due to data compression considerations, HFI raw data will be sliced in packets of successive time–samples, each second or so. Every packet will contain only one time value, corresponding to the data–taking time of the first sample of the packet. Level 1 data processing will regenerate the right time value for each time sample.

6.3 Brightness and flux conventions

Planck brightness and fluxes in physical units (i.e. after calibration) should be quoted using a single colour correction convention, in order to avoid confusion. Note that this convention does not apply to brightness expressed in units with intrinsic spectral assumptions, such as mK, or mKRJ. This convention is chosen to be the same as used for the IRAS and COBE satellites data. Justification for using such a convention can be found in the IRAS supplement series. The brightness (for extended sources) and fluxes (for point sources) B_0 and I_0 respectively should be given at the reference frequency ν_0 for each channel (see table 1) so that a spectrum with $\nu I_\nu = \nu_0 I_{\nu_0} = const.$ ($\nu I_\nu = const.$ convention) produces the same integrated power P as measured by the instrument, i.e.

$$P = \int I_0(\nu/\nu_0)^{-1} T_\nu d\nu = \int I_\nu T_\nu d_\nu \tag{56}$$

where T_ν is the spectral transmission of the channel, I_0 is the quoted brightness (or flux) at ν_0 in the $\nu I_\nu = const.$ convention and I_ν is the true astrophysical spectrum. From the in-band power P measured by the instrument (e.g. in $Wm^{-2}sr^{-1}$), I_0 can be computed using the left hand side of the above equation, once the spectral transmission of the considered channel and the reference frequency ν_0 are specified. The left-hand side is used to derive a colour correction to be applied to I_0 to retrieve the actual spectrum intensity at ν_0, for a given spectral shape of the actual astrophysical spectrum (which can be constrained by multiple channel measurements; or be known a priori, as for the CMB). By convention, we call colour-correction factor (cc) the value by which I_0 needs to be multiplied to give the actual physical intensity, i.e.

$$I_0 \times cc = I(\nu_0) \;\text{ and }\; cc = \int (\nu/\nu_0)^{-1} T(\nu) d\nu / \int I(\nu)' \, T(\nu) d\nu \tag{57}$$

where $I(\nu)'$ is the assumed astrophysical spectrum normalized at $\nu = \nu_0$.

Channel	857 GHz	545 GHz	353 GHz	217 GHz	143 GHz	100 GHz
ν_0 (GHz)	857	545	353	217	143	100

Table 1: reference frequencies for each Planck HFI channel