Kernelised Bayesian transfer learning for population-based structural health monitoring

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Abstract

Population-based structural health monitoring is the process of utilising information from a group of structures in order to perform and improve inferences that generalise to the complete population. A significant challenge in inferring a general representation for structures is that feature spaces will be inconsistent for a wide variety of populations and datasets. This scenario, where the dimensions of the feature spaces for each structure are different, occurs for a variety of reasons. Firstly, the group of structures themselves may be a *heterogeneous population*, where differences occur due to topology, leading to inconsistency in modal-based features. Secondly, feature spaces may be inconsistent across the population due to differences in the raw data (i.e. different sample frequencies etc.) and feature extraction processing. In this context, where feature spaces are inconsistent between different structure in a population, a general model that describes their behaviours becomes challenging to infer. This issue is because dimensionality reduction must be performed such that each domain's feature set projects to a consistent shared latent space where a model can be inferred. This paper introduces a technique, *kernelised Bayesian transfer learning*, that seeks to learn a projection matrix and kernel embedding that map to a latent space where a discriminative classifier can be inferred in a Bayesian manner, using variational inference. This algorithm allows a general discriminative classifier to be inferred across a population where the feature spaces for each structure are inconsistent. A numerical case study is presented, demonstrating the effectiveness of this approach and for providing a discussion of its implications for population-based structural health monitoring.

Keywords: Population-based structural health monitoring, transfer learning, multi-task learning

1 Introduction

Performing health monitoring collectively for a group of structures is known as *Population-based* Structural Health Monitoring (PBSHM) [1]. A requirement of performing PBSHM is inferring a general representation of a population of structures that applies to the complete population [2, 3]. By inferring a general machine learning model, label knowledge can be exchanged between structures in the population, improving the overall accuracy of any health monitoring approach for the population. Several benefits are provided by a population-view of Structural Health Monitoring (SHM), such as improved knowledge of feature and label relationships, a removal of confounding influences across the population in a latent space representation, and an increase in classification performance for each member of the population. Another aim of PBSHM is to overcome a lack of available damage state data for each member of the population by pooling and transferring label knowledge between the population, as demonstrated in [3].

This paper is concerned with the scenario in PBSHM in which the aim is to generate an improved general representation of the population of structures, related to a *form* [2]. Specifically, the context considered is when the feature spaces are inconsistent, which arises for several reasons. Firstly, the population may be heterogeneous [4, 3], based on differences in topology (i.e. differences in physical connections between parts of the structure) [4], leading to feature spaces that are different dimensions for each structure, especially when features are modal-based. The second scenario is when the raw data and feature extraction process lead to inconsistent feature spaces. This may occur due to different signal processing and data acquisition processes for each member of the population. As a result, any general model must include a projection into a consistent shared feature space, such that label information can be shared. Here kernelised Bayesian transfer learning is introduced as a multi-task learning algorithm [5]. This technique seeks to use information from multiple sources with inconsistent feature spaces, such that an improved discriminative classifier can be inferred for the complete population. This technique is beneficial when several structures have large overlap between clusters in their feature space, and when limited label knowledge is known for each structure.

Health monitoring for wind farms provides an illustration of this approach. Typically, an asset manager of a wind farm is required to monitor the complete fleet, where each turbine has a distinct individual behaviour due to unique environmental conditions and manufacturing tolerances. This results in the finite datasets from each turbine having a different data distribution and will cause a machine learner trained on one member of the population to fail to generalise to another member of the population. In fact, the company will often have access to a variety of wind farms, each of which might have a different model of wind turbine in them. Within all these data sources (from members of the population) a variety of labelled health state data will often be available. Obtaining a complete label set for each turbine in the population is clearly not feasible; however, the asset manger wishes to use their label knowledge from their complete population of turbines from multiple wind farms in health monitoring, as labelled data is a valuable asset. However, the feature spaces may not be the same dimension for each member of the population, due to different data acquisition processes and feature extraction. In this context, kernelised Bayesian transfer learning is applicable in inferring a model in a consistent latent feature space in which all members of the population can be mapped, and their label information utilised.

The outline of this paper is as follows. Firstly, kernelised Bayesian transfer learning is introduced in Section 2 within the context of transfer learning and multi-task learning. Section 3 demonstrates kernelised Bayesian transfer learning on a numerical example in which different sets of shear buildings with varying degrees-of-freedom are utilised in creating a general classification model of the population. Finally, conclusions are presented highlighting areas of further research.

2 Kernel-based transfer learning

Kernelised Bayesian transfer learning is a supervised multi-task learning algorithm for inconsistent feature domains [5], i.e. the dimension d of the feature space $\mathcal{X}$ for each domain is not equal e.g. $d_1 \neq d_2$ for a two-domain problem. Before formally introducing multi-task learning, the definition of two objects are required:

> A **domain** $\mathcal{D} = \{\mathcal{X}, p(X)\}$ is an object that consists of a feature space $\mathcal{X}$ and a marginal probability distribution $p(X)$ over a finite sample of feature data $X = \{\boldsymbol{x}_i\}_{i=1}^{N} \in \mathcal{X}$ from $\mathcal{X}$.
>
> A **task** $\mathcal{T} = \{\mathcal{Y}, f(\cdot)\}$ is the combination of a label space $\mathcal{Y}$ and a predictive function $f(\cdot)$.

Multi-task learning is a branch of transfer learning in which the aim is to use knowledge from *multiple domains* to improve a particular task [6, 7]. The main difference to other transfer learning methods is that multi-task learning weights knowledge transfer equally for all domains. The goal therefore, is to generate an improved prediction function $f(\cdot)$ for one consistent label space $\mathcal{Y}$ using the feature data from several source domains. Kernelised Bayesian transfer learning is a particular form of multi-task learning that can be defined as heterogeneous transfer learning i.e. at least one feature space $\mathcal{X}_j$ for a domain $\mathcal{D}_j$ is not the same dimension as another feature space $\mathcal{X}_k$ in the domain set such that $d_j \neq d_k$. The assumptions are that there is a relationship between the feature space for each domain and the label space and that each domain provides informative knowledge that will improve the predictive function $f(\cdot)$. Several other heterogeneous transfer learning methods exist [8, 9]; however, kernelised Bayesian transfer learning is a multi-task learning that performs two tasks: 1) finding a shared latent subspace for each domain and 2) inferring a discriminative classifier in the shared latent subspace, in a Bayesian manner, are visualised in Figure 1.

Kernelised Bayesian transfer learning assumes T domains $\{\mathcal{D}_j\}_{j=1}^{T}$ with consistent label spaces $\mathcal{Y}_j = \mathcal{Y}_k \forall\, j \in T$ and $\forall\, k \in T$ but with features spaces that are not consistent. For each domain, an associated finite label set is available $\boldsymbol{y}_j = \{y_{j,i} \in \{-1, +1\}\}_{i \in \mathcal{I}_j}$ where $\mathcal{I}_j$ is the index set for each data point in domain j. For each domain, the data are embedded in a kernel matrix $\{K_j \in \mathbb{R}^{N_j \times N_j}\}_{j=1}^{T}$ using a particular kernel function, where N_j is the number of data point in domain j. An optimal projection matrix $\{A_j \in \mathbb{R}^{N_k \times R}\}_{j=1}^{T}$ is constructed for each domain and projects the kernel embedding into a a shared latent subspace $\{H_j = A_j^{\mathsf{T}} K_j\}_{j=1}^{T}$. In this subspace a discriminative functional

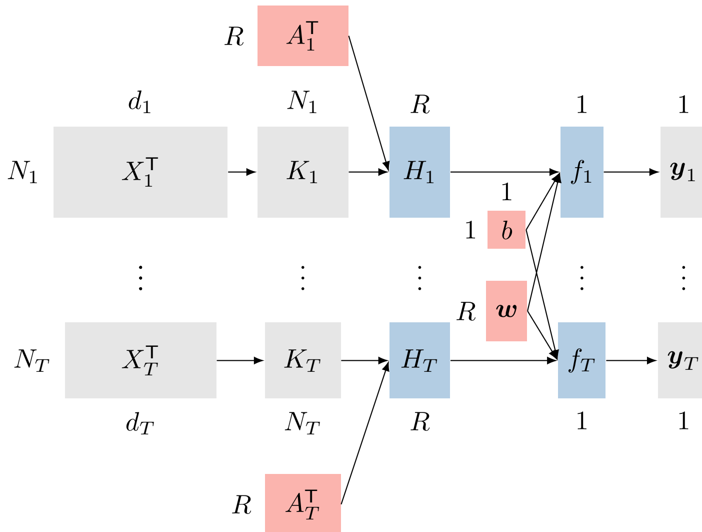
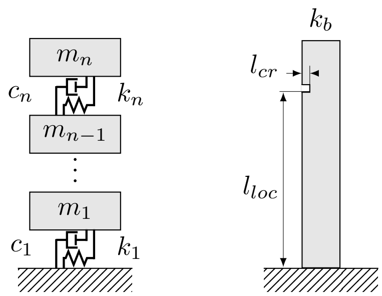


Figure 1: Visual representation of kernelised Bayesian transfer learning.



(a) (b)

Figure 2: Schematic of the shear structures: panel (a) is a nominal representation of the five systems and panel (b) depicts the cantilever beam component where $\{k_i\}_{i=1}^n = 4k_b$ i.e. the stiffness coefficients in (a) are generated from four times the tip bending stiffness in (b).

classifier $\{\boldsymbol{f}_j = H_j^\mathsf{T}\boldsymbol{w} + \mathbf{1}b\}_{j=1}^T$ is inferred that predicts the outputs in the shared latent subspace with shared classification parameters $\{b \in \mathbb{R}, \boldsymbol{w} \in \mathbb{R}^R\}$. The quantities are inferred in a Bayesian manner, utilising variational techniques and conjugate analysis. In keeping with the brevity of the current paper the reader is referred to [5] for mathematical details.

3 Shear Building Case Study

In order to demonstrate the applicability of kernelised Bayesian transfer learning for PBSHM, a numerical case study is demonstrated. The SHM problem is a two-class location problem (which can be seen as binary damage detection as there are only two classes). The population of structures is composed of five different shear building structures, schematically depicted in Figure 2 as lumped-mass models in bending, forming a heterogeneous population [4, 3].

Each structure is represented by n mass $\{m_i\}_{i=1}^n$, stiffness $\{k_i\}_{i=1}^n$ and damping $\{c_i\}_{i=1}^n$ coefficients. The masses are specified by a length l_m, width w_m, thickness t_m and density ρ. The stiffness elements are calculated from four cantilever beams in bending $4k_b = 4(3EI/l_b^3)$, where E is the elastic modulus, I the second moment of area and l_b the length of the beam. The damping coefficients are directly specified and are not derived from a physical model. Damage is introduced to the structure via an open crack, using a reduction in EI (using the model in [10]) across one of the beams in k_1, i.e. $k_1 = 3(3EI/l_b^3) + k_d$, where k_d is the tip stiffness of a cantilever beam subject to an open crack of length l_{cr} at location l_{loc} along the length of the beam. Each observation, for a particular structure, is

Table 1: Properties of the five structures in the heterogeneous population case study. Degrees-of-freedom (DOF) are denoted by n.

Domain	DOF n	Beam geometry $\{l_b,\, w_b,\, t_b\}$ m	Mass geometry $\{l_m,\, w_m,\, t_m\}$ m	Elastic modulus E GPa	Density ρ kg/m^3	Damping coefficient c Ns/m
1	4	$\{5.0, 0.35, 0.35\}$	$\{12, 12, 0.35\}$	$\mathcal{N}\left(210, 1.0 \times 10^{-9}\right)$	$\mathcal{N}\left(8000, 10\right)$	$\mathcal{G}\left(50, 0.1\right)$
2	10	$\{4.3, 0.50, 0.50\}$	$\{10, 10, 0.50\}$	$\mathcal{N}\left(200, 2.0 \times 10^{-9}\right)$	$\mathcal{N}\left(7800, 50\right)$	$\mathcal{G}\left(8, 0.8\right)$
3	20	$\{4.8, 0.40, 0.40\}$	$\{11, 11, 0.40\}$	$\mathcal{N}\left(205, 1.5 \times 10^{-9}\right)$	$\mathcal{N}\left(7950, 25\right)$	$\mathcal{G}\left(25, 0.2\right)$
4	3	$\{4.5, 0.45, 0.45\}$	$\{11, 11, 0.45\}$	$\mathcal{N}\left(208, 1.0 \times 10^{-9}\right)$	$\mathcal{N}\left(7900, 15\right)$	$\mathcal{G}\left(20, 0.1\right)$
5	5	$\{4.2, 0.46, 0.46\}$	$\{12, 12, 0.46\}$	$\mathcal{N}\left(201, 2.0 \times 10^{-9}\right)$	$\mathcal{N}\left(7850, 20\right)$	$\mathcal{G}\left(50, 0.1\right)$

Table 2: Number of data points in each class for each domain.

Domain	**Training**		**Testing**	
	$y = -1$	$y = +1$	$y = -1$	$y = +1$
1	250	100	500	500
2	100	25	500	500
3	120	50	500	500
4	200	150	500	500
5	500	10	500	500

composed from random draws from a base distribution for E, ρ and c. The properties of the five structures in the population are shown in Table 1.

The features in this case study are damped natural frequencies and damping ratios i.e. $X = \{\omega_i, \zeta_i\}_{i=1}^{n}$. The first two principal components for each domain are depicted in Figure 3, indicating the level of overlap in the classes, where the number of training and testing data point for each domain are presented in Table 2. Each domain has a different degree of class imbalance in the training dataset (Table 2), reflecting scenarios where certain structures may provide more informative data about a particular damage class.

Figure 4 presents the classification function and margin (set as zero) for the test data (top panel) and the probability of the data point belong to the +1 class (bottom panel) compared to the true label. These plots show the value of a probabilistic approach, as a soft margin reduces the number of misclassified points. It can also be seen, that due to imbalance in the class (weighted towards -1 the undamaged class for each structure), there is greater uncertainty in the damaged class (+1). The figures also depict more confident predictions for domains 1 and 4, which can be expected due to their relatively large number of data points and separability.

The classification result are presented in Table 3. Accuracies and F1 macro scores are based on data points belonging to class +1 if $p(y_{*,j} = +1)f_{*,j} \geq 0.5$. Considering classification performance, kernelised Bayesian transfer learning has accurately inferred a general model for the five domains. Although there is a large performance decrease for domain 2, this is expected due to the few data points used to train the method, compared with the other domains, and a relatively large degree of overlap in the classes. Performance remains high for the two separable domains (1 and 4), meaning that the method has not negatively impacted classification in these domains by learning a multi-task model.

Table 3: Classification results on test data for each domain.

Domain		1	2	3	4	5
Training	Accuracy	99.7%	96.8%	97.6%	100.0%	99.6%
	F1 Macro	0.997	0.947	0.971	1.000	0.943
Testing	Accuracy	99.6%	87.9%	92.8%	99.8%	92.1%
	F1 Macro	0.996	0.877	0.928	0.998	0.921

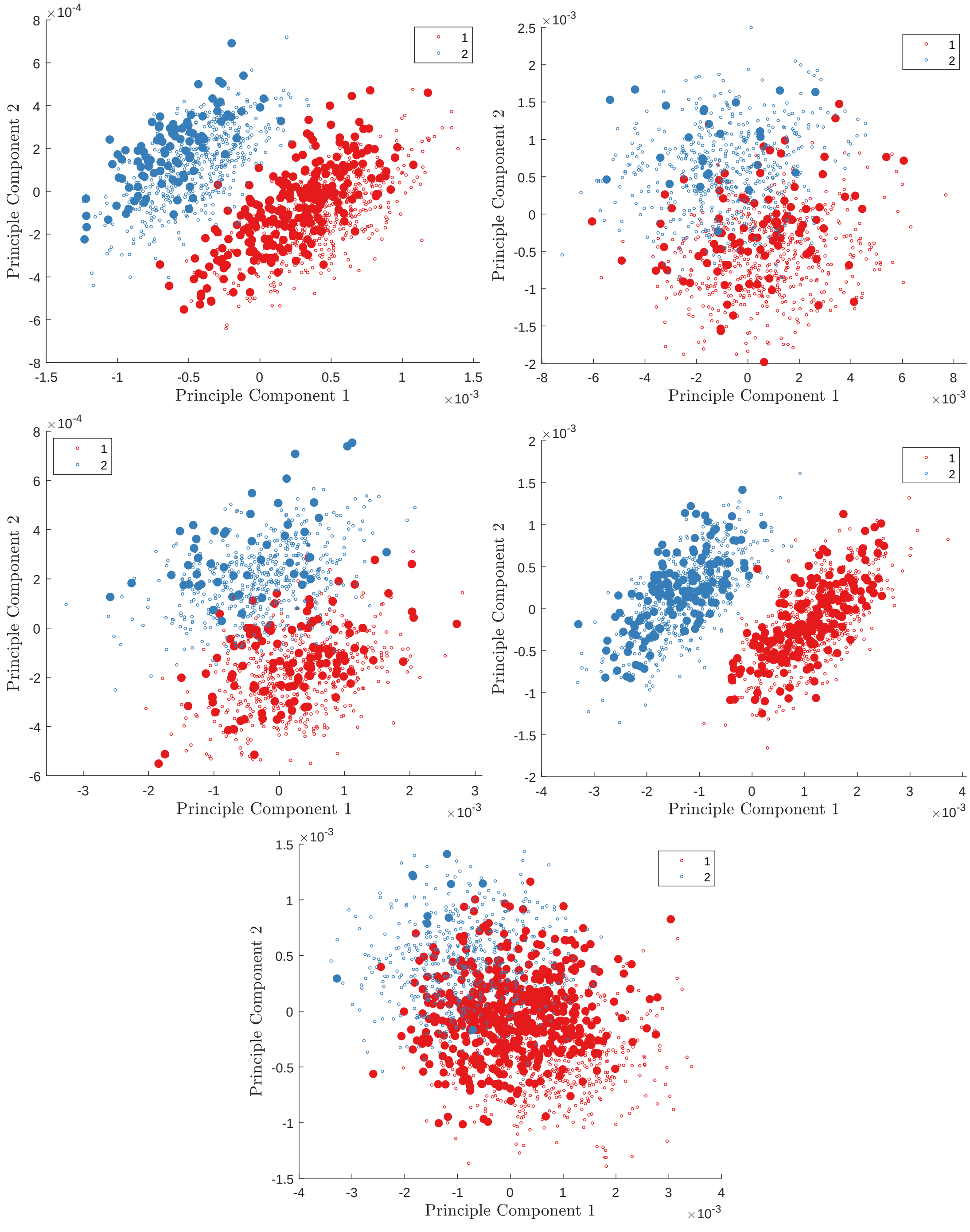


Figure 3: First and second principal components for each of the five domains, where training data are denoted by $(\cdot)$ and testing data by $(\circ)$ (where domains are presented top left to bottom right).

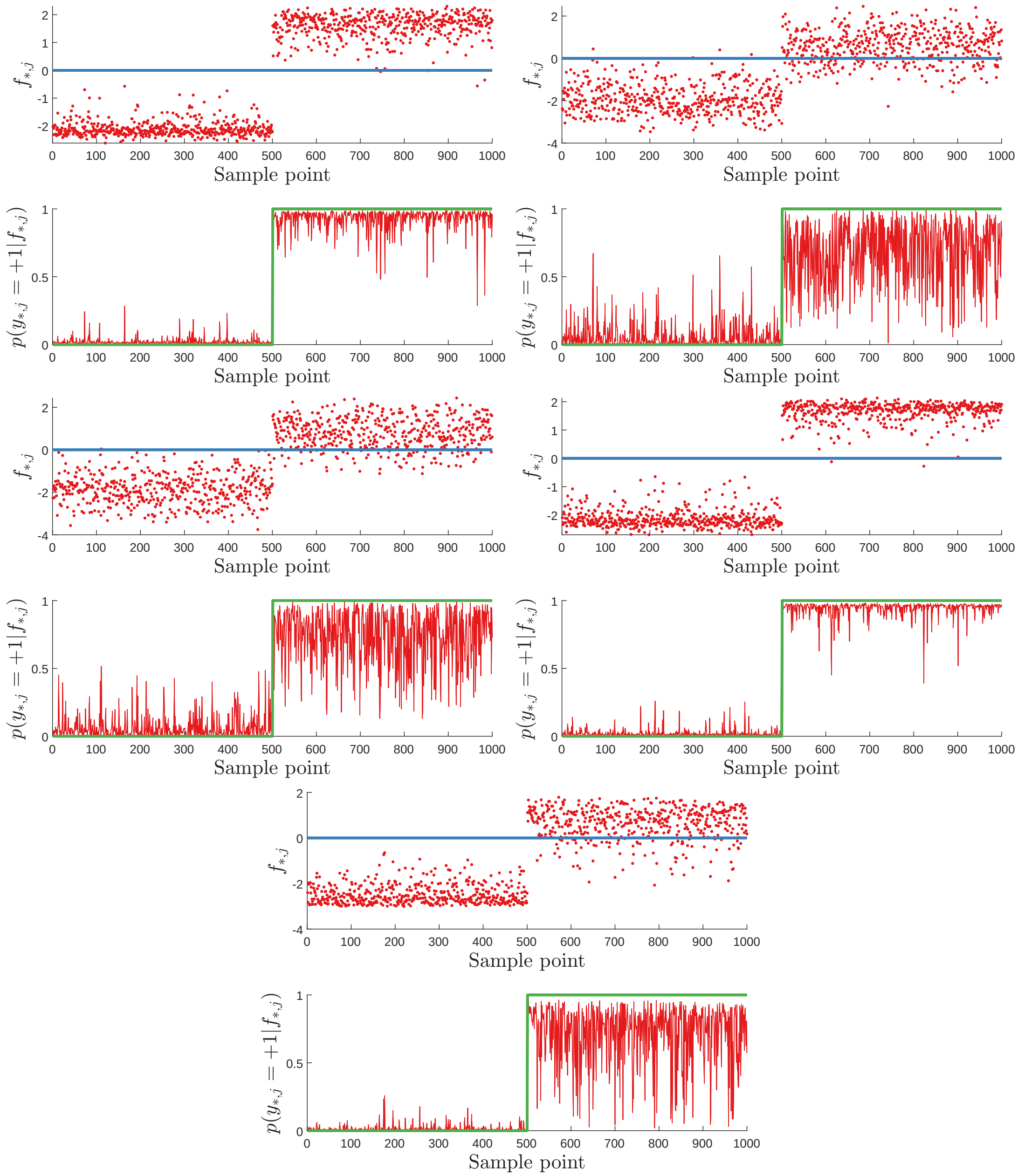


Figure 4: Functional classifier (red $(\cdot)$) and margin (blue) (top panels) and probability of $y_{*,j} = +1$ (red) and true label (green) (bottom panels) for the test data for each of the five domains (where domains are presented top left to bottom right).

4 Conclusions

Population-based SHM requires machine learning techniques that utilise knowledge from a range of sources in order to construct improved general models of the population. One challenge is creating general models when feature spaces are inconsistent. This paper demonstrates one approach to multi-task learning, kernelised Bayesian transfer learning, where each domain has an inconsistent feature space. This approach is applicable to heterogeneous populations (with topological differences) and when data processing has led to feature inconsistencies.

Kernelised Bayesian transfer learning, has been demonstrated to provide a good level of classification performance for a numerical case study involving a heterogeneous population of shear structures with a different number of storeys. The approach aids classification in scenarios of high class overlap, and in scenarios with a large class imbalance.

Further research is required in investigating the multi-class problem, and in extending the method to scenarios in which one domain has no labels for a particular class. This will extend the approach from a multi-task learner to a full transfer learning method.

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