On Current Trends in Forward Model-driven SHM

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ABSTRACT

Forward model-driven approaches to Structural Health Monitoring (SHM) are a category of methods in which validated physics-based models are used to generate data for machine learning classifiers. These approaches were developed to address the lack of available damage state data; which is often a problem in many SHM contexts due to it being impractical or infeasible to collect. Many data-driven approaches to SHM are successful when the appropriate damage state data are available, however the problem of obtaining data for various damage states of interest restricts their use in industry. With this aim, several forward model-driven techniques have been developed in recent years focusing on issues in generating validated physical-models that produce damage state predictions without the need for a damage state data. This paper presents the current state of forward model-driven techniques within the literature. In addition, several key technology areas are highlighted with a demonstration of the benefits and challenges a forward model-driven framework provides.

INTRODUCTION

The integration of physics-based modelling and machine learning technologies for use with within Structural Health Monitoring (SHM) has seen considerable research interest in recent years. Distinct from the two established categories of SHM method: model-driven and data-driven [1–4], the approach offers potential solutions to key challenges within SHM.

To clarify this distinction, current model-driven methods use law-based models in combination with inverse techniques in order to infer or 'update' a set of parameters [5, 6]. Changes in these updated parameters are subsequently used in making structural health decisions; accordingly this category of methods is herein defined as *inverse model-driven*. On the other hand, data-driven methods 'learn' relationships be-

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tween measured response data and structural damage states based on pattern recognition or machine learning-based models; all without the construction of a physics-based model [1, 3, 7, 8]. The combination of physics-based model and machine learning algorithms is therefore a fusion of technologies from the two established frameworks. This third category, where physics-based models are used in training machine learning techniques, can be seen as utilising physics-based models in a *forward* manner, and hence have been defined as *forward model-driven* within the literature [9–11].

SHM technologies that utilise physics-based models have often approached structural health diagnosis by using changes to inferred parameters as a method for making health statements, part of the inverse model-driven category of approaches. These techniques often suffer from non-identifiability issues, difficulties in parametrisation of the model and interpretation of the updated parameters [12, 13]. In contrast, forward model-driven SHM provides a framework whereby validated models, employed in a forward manner, generate predictions of damage sensitive features that are statistically representative of health state data obtained from the operational structure. The emphasis in this class of methods is that models can be used as a proxy in order to generate damage state data that would otherwise not be economically viable or practically infeasible to obtain from the in-service structure, a major challenge for supervised data-driven approaches. These health state predictions from models are subsequently incorporated in training pattern recognition or machine learning classification technologies, which can be implemented online in order to make health diagnostic decisions. In addition, forward model-driven methods provide a practical and cost-effective technique for designing sensor networks, performing feature selection, obtaining health state data and achieving prognosis [14], all of which are challenges to data-driven methods.

This paper presents existing forward model-driven approaches, before outlining the key benefits and challenges with this category of approach. Three main technology areas are subsequently highlighted, namely calibration and validation, feature selection and monitoring system design, and health state decision strategies, where appropriate methods are discussed.

A HISTORY OF FORWARD MODEL-DRIVEN SHM

Several approaches to utilising physics-based models and machine learning algorithms in SHM have been investigated in recent years. The majority of these approaches seek to use physics-based models in providing training data for supervised machine learning classifiers, or in inferring baseline conditions for novelty detectors. Here the literature is presented with a discussion of the challenges these approaches face.

Early examples of forward model-driven SHM can be seen within damage identification in bridges where Finite Element Analysis (FEA) models were used to generate features for an Artificial Neural Network (ANN) classifier [15, 16]. In contrast, Pawar and Jung combined a series of damage models, composite blade models under various analysis scenarios to provide training data for an Support Vector Machine (SVM) such that damage classification could be performed [17]. Integrating FEA models and SVMs was also explored in the context of a cable-stayed bridge [18]; a scenario where obtaining damaged state data is often infeasible. A framework for combing FEA models and machine learning algorithms was formalised by Barthorpe in [9]. In terms of performing the levels of Rytter's hierarchy, Sbarufatti et al. proposed a method whereby an FEA model provided training data for an ANN such that detection, localisation and quantification of fatigue cracks in an aerospace structure could be performed [19]. Outside of utilising FEA models, lamb wave propagation models have also been used to train ANNs for crack characterisation [20]. Physics-based models have also been used to inform novelty detection algorithms. Sbarufatti et al. inferred a mahalanobis-based baseline condition from FEA models of strain field under fatigue damage scenarios [21]. Studies comparing inverse model-driven, data-driven and forward model-driven approaches have also been conducted. One study, in which FEA predictions, corrected for model form errors, were used in training an SVM, was shown to be comparable to an SVM trained using observational data. The method also outperformed a sensitivity-based model updating approach, showing forward model-driven methods are a viable option for performing SHM [22]. Satpal et al. implemented a combined model updating and SVM approach where model predictions trained the classier [23] with Hariri-Ardebili and Pourkamali-Anaraki applying a similar methodology to concrete dams [24]. Another analogous approach whereby sensitivity-based model updating was combined with a Gaussian mixture model has also been applied to a bridge case study [25, 26].

Most of these approaches utilise deterministic FEA model outputs, with a few adding arbitrary noise terms to replicate variability, whilst others propagate 'known' parameter uncertainties through Monte Carlo realisations. Most of these methods do not consider model form errors, and either do not attempt to validate their models or implement full-system damage state data in the validation process. As a consequence these approaches fail to tackle the key challenges facing SHM technologies. This provides motivation in presenting the main difficulties and providing technological solutions to these issues discussed within this paper.

FORWARD MODEL-DRIVEN SHM

Forward model-driven SHM is the combination of two main components: generating physics-based models that produce representative damage state features, and using those predictions to train machine learning or pattern recognition algorithms. Specifically, the first component is a combination of techniques for developing models that are validated, such that trust has been established in confidently predicting damage state features of interest. The output predictions from these models can be used in conventional supervised learning methods that are well studied within SHM, such as SVMs, Gaussian mixture models, ANNs or any other classification technique [3,27]. Generally these methods will remain algorithmically the same, with the only difference arising in the training source. However, utilising these models in a semi-supervised approach [28], providing a level of labelling to data and reducing the number of inspections required, may require larger adaptations of current techniques.

This section will discuss the key benefits and challenges within these two main components. Specific methods for performing each task will be outlined with emphasis on outstanding areas of further research.

Benefits and Challenges

Benefits of a forward model-driven approach, besides offering a method for generating damage state data, are that the creation of physics-based models provides opportunities for new technologies within an SHM workflow. Two key benefits are that monitoring system design and feature selection can all be performed via computer experiments, whereby the physics-based model is interrogated such that the type, number and location of sensors can be determined. This is a significant advantage, as one of the most challenges aspects of data-driven SHM is determining what features will be sensitive to damage before any data has been collected from the structure in operation. Furthermore, the development of physics-based models also means that risk-based classifiers and decision algorithms, that account for the physics of the problem, can be incorporated. These techniques would lead to more robust decisions, providing more information for asset management, and models that could be used for prognosis.

However, key challenges exist in utilising physics-based models within a forward model-driven SHM framework. Many of these challenges arise from difficulties in producing accurate predictions of damage states from models. Here questions such as,

- How do I trust my model?
- How do I validate my model when observational damage data is unavailable for the full-system?
- How do I account for model form errors as well as environmental and observational variations?

all must be answered if confidence is to be established in the trained classifier. All these questions fundamentally ask how best to calibrate and validate physics-based models for use in SHM, discussed in [11]. This is therefore seen as the biggest difficulty in the uptake of forward model-driven methods in industrial applications and discussed further in the following subsection.

In light of these benefits and challenges three main technology areas are discussed in the following subsections, namely methods for performing calibration and validation, such that trust in the model is established, feature selection and monitoring system design, a major benefit of developing validated models in SHM, and lastly, health decisions strategies.

Calibration and Validation

Generating predictions of damage state features from physics-based models that are statistically representative is a significant challenge for forward model-driven SHM. This process will require some level of data from the structure in order to develop confidence in the model. A second challenge is accounting for model-form errors and environmental variation, as no model will ever capture all physics, even if known. These problems lead to the need for two technologies: a multi-level uncertainty integration strategy, where a physics-based model is validated without data from the full-system, but with data from sub-systems and components, and calibration methods that inferring model form errors and environmental variability. Technologies for performing these two methods are discussed below.

One of the key goals of forward model-driven SHM is the replacement of in-service damage state data with predictions from physics-based models. Unfortunately, to have any confidence in any physics-based model data from the real-world structure is required, leaving the conundrum of how to calibrate and validate the physics-based model given that health state data is neither feasible to obtain nor cost-effective in the majority of applications. If this question is not tackled, forward model-driven approaches simply become an expensive and demanding way to perform sub-standard data-driven SHM, introducing further approximations and modelling challenges. One solution to this problem is the division of the structure in question, and hence the physics-based model, into a set of components, sub-assembly etc., for which obtaining health state data is feasible and economically viable. In this scenario a full-system, such as an aeroplane, is divided into various sub-systems, e.g. wing panels, riveted joints, landing gear assemblies, coupons etc., where each sub-system can be tested under damage types which are expected to be likely causes of failure in the full-system. Small scale test strategies can then be developed, or existing certification tests used to collect data sets that can be implemented in calibrating and validating the set of physics-based model. The usefulness of forward model-driven technologies rest on the ability to utilise and integrate these sub-system data sets into calibrating and validating sub-system level physics-based models, which when propagated through to the full-system, via an algebra of models and uncertainty management, produce valid, i.e. statistically representative predictions, which have required no full-system health state data. Obviously this is an incredibly ambitious goal, nonetheless methods such as multi-level uncertainty integration strategies offer techniques for undertaking such a challenge. Bayesian networks have been proposed as a method for performing this type of analysis [29, 30], where multiple models and sub-system data sets are used to perform inferences that are robust at a full-system level. Another approach is a subfunction discrepancy approach, inspired from a technique utilised in combining medical drugs trails and health models in financial planning [31]. This method seeks to calibrate and account for model form errors at each sub-system, where the corrected model outputs and associated uncertainties are propagated through to the full-system, providing greater confidence in the output. Essentially the technique relies on the ability to capture damage physics at a sub-system level where data is obtainable and propagate this through the model levels whilst inferring and accounting for other model form errors [11, 32].

In order to produce robust statistical predictions, procedures for calibrating physics-based models should involve mechanisms for handling multiple sources of uncertainty, especially those from model form errors, known as model discrepancy. Statistically representative predictions will not often be achievable without capturing observational variability, along with parameter uncertainties and accounting for any functional model discrepancy — the differences between model outputs and observational data. Several modelling approaches exist that try to capture model form errors with the majority using a bias correction framework proposed by Kennedy and O'Hagan [33], where the parameters are calibrated in a Bayesian manner whilst model discrepancy is inferred as a Gaussian Process (GP). Since then the method has been applied to several engineering applications [34–37], including SHM [11, 38]. However issues of non-identifiability are present between the discrepancy and parameter estimates. Another method for inferring model form errors is Bayesian history matching, an approximate Bayesian computation approach that is 'likelihood free' [39–42]. The method performs calibration under the assumption of model discrepancy and can be combined with an importance sampling technique to infer the functional model discrepancy [11]. These techniques provide a potential solution to the problem of accounting for multiple sources of uncertainty, including model form errors.

Feature Selection and Monitoring System Design

A clear benefit of forward model-driven SHM is that by developing physics-based models of the structure various outputs and their mathematical transforms can be investigated as potential damage sensitive features. Furthermore, once a particular damage feature has been selected, the physics-based model(s) can be utilised in selecting and optimising a sensor network all before physical testing or in-service data has been collected. This offers significant cost benefits and risk reduction as monitoring setups can be considered virtually.

One approach to damage feature selection using physics-based models is via sensitivity analysis techniques, specifically Global Sensitivity Analysis (GSA) which aims to determine the variation of an output quantity in terms of the variation in the inputs [43–45]. This would lead to an assessment of the sensitivity of a given set of outputs and their transforms to changes in the inputs, specifically the extent and location of each particular damage type being considered. In addition, the proposed features can be assessed for sensitivity to other inputs, aiming to identify a feature that is sensitive to damage alone, rather than other confounding influences.

Monitoring system design, i.e. selecting the number, location and type of sensors to implement on a structure, has been attempted using a variety of methods, e.g. energetic techniques [46], information [46, 47] and risk-based approaches [48]. A positive by-product of forward model-driven SHM is that the availability of physics-based models means that these techniques become applicable within the framework.

Health State Decision Strategies

Classification algorithms have been well studied within data-driven SHM. The main difference in forward model-driven SHM is that the physics-based models provide the training data. However, other methods also become applicable in a forward model-driven approach.

Bayes risk classifiers are one method for making decisions about the health state of a structure [48]. The technique aims to weight known outcome probabilities of events (i.e. undamaged, de-lamination, cracks etc.) by the costs of that outcome occurring. This allows a process whereby decision bounds are formulated as a function of the likelihood of particular damage scenarios, their associated maintenance costs and the cost of structural failure. A difficulty with implementing Bayes risk for SHM is the ability to obtain the conditional probabilities of the chosen feature vector given local damage states in particular regions, i.e. the probability of a feature vector given some form of damage event. Forward model-driven SHM provides a potential solution to this challenge by using full-system physics-based model predictions of feature vectors for different damage events, i.e. output predictions from the full-system physics-based model for the range of damage scenarios being considered. Furthermore, decisions can be made as part of a larger Bayesian network [49, 50], where both calibration, inspection and maintenance decisions can be formed within a forward model-driven approach.

CONCLUSION

Forward model-driven SHM has been performed multiple times within the literature. This category of approach, distinct from the two existing categories, utilises physics-based models such that they generate training data for machine learning classifiers.

It has been shown that the majority of approaches within the literature fail to target several key challenge. These difficulties are centred around the issue of gaining trust in the physics-based models, especially when damage state data is not available at a full-system level. In addition, all forms of uncertainty must be incorporated such that model predictions are statistically representative of real-world responses. This paper has discussed several existing technologies that provide solutions to these issues, namely multi-level uncertainty integration, Bayesian calibration and bias correction, and Bayesian history matching.

The generation of validated models offers several additional benefits. Two key benefits discussed in this paper were the ability to design a monitoring system and select damage sensitive features *a priori*, as well as the ability to use more informative and robust decision frameworks, such as a Bayes risk approach. These offer significant advantages over conventional data-driven approaches potential making SHM more commercially viable.

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