The development of a damage model for the use in machine learning driven SHM and comparison with conventional SHM Methods

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Abstract

Approaches to damage detection can be categorised into two main approaches: model-driven methods and data-driven methods. Model driven methods pose the risk of the model departing from real physical meaning and are generally computationally expensive. Data driven methods *per contra* are limited by the experimental data available for all likely damage scenarios, and therefore can be impractical and costly. This paper presents the development of a damage model using finite elements (FEs) for the use in machine learning driven structural health monitoring (SHM). This method maintains a model that has physical meaning thereby removing the need for numerous experimental damage scenarios as a validated FE model can be used to simulate a plethora of likely damage scenarios. Two case studies are presented; a cantilever beam and a representative three-story building structure, for which the novel method is compared to both data-driven and model-driven methods.

1 Introduction

The process of monitoring a structure with the aim of damage identification is referred to as SHM. Currently two main categories of approaches exist in solving the problem of SHM, model-driven and data-driven methods. Model-driven methods are those which use an inverse approach applied to a law-based model, the most common method is FE model updating [1]. Data-driven methods on the other hand, use pattern recognition and machine learning techniques that are applied to statistical representations of the system from data [2].

Model updating is the process of adjusting certain model parameters in order to reduce the residual between experimental measurements and model predictions. The change in the parameters of the updated model can then be used to infer damage in the structure. Several problems arise when using these methods for damage detection. Among these is selection of the number of parameters to use, which can become very large when the damage location is unknown. Correct parametrisation of the model is often difficult due to the many potential input parameters to a FE model. An accurate understanding of the underlying physics and knowledge that the updated model has not departed from physical meaning due to unrealistic updated parameters also poses challenges especially when using parameters that directly affect stiffness to infer damage. Further challenges are presented by the inevitable presence of model errors; the effect of uncertainties associated with non-calibrated model parameters; and the possible presence of variability in the experimental 'target' data arising, for example, from environmental effects. The methods are also computationally expensive as they require multiple executions of an FE model in order to make predictions for a single new data set.

Data-driven methods, in contrast, remove the need for complete physical understanding of the structure. These methods use response data that captures the complete loading environment to establish a normal undamaged condition. Using statistically based machine learning approaches, the structures departure from the normal condition is used to infer damage. These methods require data from each damage state in the training of the machine learning process in order to work effectively. This need for data in all possible damage states is not always feasible or economically viable in real structures and therefore many of these methods are not practical.

A third method has been developed to address these issues, one in which combines the use of a law-based model of all damage states created from FEs (herein referred to as a *damage model*) to generate predictive features for statistical machine learning approaches. This approach allows FE models to be generated in a forward method, which allows the models to maintain physical meaning and removes the need for complex parameter selection. This method also allows the generation of data for potentially all possible damage states without the need for numerous complex and expensive testing. The steps of the process are as shown in Figure 1. A damage model is created and validated against a sample of subsystem-level experimentally obtained data, and hence used to create predictions for potential damage states. The predictions from the damage model are modified and corrected for model bias and experimental variability is incorporated; the model is then applied to make system-level predictions. The modified damage model is used to create feature vectors for machine learning processes. The trained machine learning model can then be shown in-service data and subsequently used to make predictions about the state of the structure.

Figure 1: A process flowchart for using a damage model in machine learning driven SHM.

This paper develops a damage model for the use of generating features for machine-learning driven SHM (herein referred to as *forward model-driven* SHM) and compares the method against both data-driven and model-driven methods (herein referred to as *inverse model-driven* SHM) for two case studies; a cantilever beam and a representative three-story building structure. The outline of the paper is as follows. Section 2 contains the cantilever beam case study and Section 3 displays the representative three-story building structure case study, both of which are divided into five sections. The first section is on the experimental methodology used to obtain the data; the second on the development of the damage model for forward model-driven SHM; the third outlining data-driven SHM predictions; the fourth outlining inverse model-driven SHM predictions; and the fifth a discussion of the three methods. Conclusions and areas for further research are outlined in Section 4.

2 Cantilever beam case study

A cantilever beam (part of the representative three-story building structure) is used as a test structure to develop and compare the use of a damage model in forward model-driven SHM against the conventional methods of data-driven and inverse model-driven approaches for damage detection.

2.1 Experimental methodology

Modal testing was conducted on five uniformly prismatic cross-section cantilever beams for various damage states. For each damage state, defined here as the introduction of damage via a saw cut at different depths, the first six bending modes were obtained. The beams were constrained in a fixed-free boundary condition, as shown in Figure 2 using a 100mm clamp plate, l_c, fastening the beam to a rigid steel block. Damage was incrementally introduced in the beam width (z-direction), w, at a location of 175mm, l_d, from the right hand edge of the clamp. The beams were first tested in the undamaged condition before saw cuts were added in increments of 2.5mm to a total depth of 20mm. The beams were made from Aluminium 6082 with the nominal physical and geometric properties as shown in Table 1.

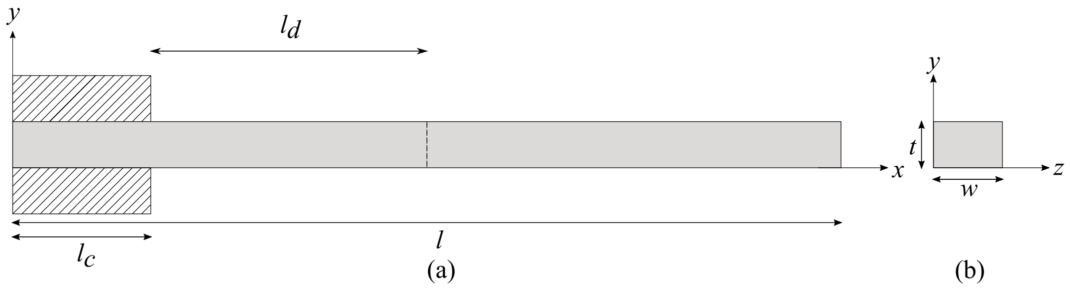

Figure 2: A schematic of the experimental set-up for the fixed-free beam; (a) front view, (b) side view.

Beam Properties	Length, l	Width, w	Thickness, t	Young's Modulus, E	Density, ρ	Poisson's Ratio, ν
Values	655mm	25.5mm	6.5mm	71GPa	2700kg/m^3	0.33

Table 1: Geometric and material properties of the beams used in the experiments.

For each damaged and undamaged state a roving impact hammer was used to excite the structure in the y-direction at twelve points along the beam. Two accelerometers were used to measure the response, one at 225mm and the other at 475mm from the right hand edge of the clamp. The acquisition and analysis of the data was performed using a LMS Test.Lab system with a bandwidth 6400Hz consisting of 4096 spectral lines (0.78Hz resolution) and an exponential response window. Five repeats were performed at each damage state in order to capture the inherent variability in the beam. For each damage state the mode shapes were calculated and the modal frequencies were used to calculate the normalised percentage difference.

2.2 Damage model for forward model-driven classification

A damage model was developed using a law-based approach, namely FEs. This model produced a deterministic prediction of the natural frequencies of the beam in various damage states, this therefore does not allow the inherent variability present in the experimental data to be represented. This removal of variability and the bias caused by a simplification of the physics, results in inaccurate feature vector predictions for machine learning and can result in misclassification. For this reason the damage model needs to be modified to incorporate the variability and account for residual bias. This is performed using a discrepancy model. The modified damage model is then used to produce features that can be used in machine learning, specifically in training a Support Vector Machine (SVM) model with two classes; damaged and undamaged. The trained SVM model is subsequently used to classify data obtained from the experimental testing of the beam.

2.2.1 Damage modelling

A robust damage model is required that closely displays the same behaviour as the experimental data in order to produce robust features and classifiers for damage detection. The relationship between the observed experimental data, z, and the model output, $\eta(\cdot)$ is defined as shown in equation (1):

$$z(\mathbf{x}) = \eta(\mathbf{x}) + \delta(\mathbf{x}) + e \quad (1)$$

where $\mathbf{x}$ are the model inputs, $\delta(\cdot)$ is a model inadequacy or bias function, and e is the experimental variability. By constructing the components of equation (1) a damage model can be created that better displays the behaviour observed by experimental measurements. It is noted that equation (1) is a restricted form of the equation Kennedy and O'Hagan proposed in [3], without the pursuit of model calibration.

The model output, $\eta(\cdot)$, has been modelled using FE models of the beam under different damaged-state conditions using the commercial solver ANSYS. There are three main ways of modelling a crack: local stiffness reduction, discrete spring models and complex models in two or three dimensions, as outline by Friswell and Penny [4]. Two dimensional FE models were used where the damage was modelled as a removal of geometry, the width of the cut (beam x-direction) is 1mm (the same width as that of the saw cut in the experiments). Second order shell elements (SHELL281) were used for each damage state. The FE models were used to obtain the first six bending modes for each damage state and the modal assurance criterion (MAC) was used to check that the FE modal frequencies were assigned to the same bending mode as the experiment. The modal frequencies were subsequently used to calculate the normalised percentage difference. Formal model calibration is not pursued for this study as a good degree of correlation between the FE and experimental normalised percentage differences has been achieved. This is shown as for the relative errors $\Delta\omega_{1-6}$, the mean square error (MSE) between the mean experimental data and the FE model predictions across the full damage range was 0.043.

The bias function, $\delta(\mathbf{x})$, can be approached by using regression techniques. The relationship between the FE model outputs and the mean experimental measurements needs to be defined in order to correct the models bias. Regression can be a complex problem to solve as deciding on the functional form of the relationship between inputs and outputs leaves an infinite choice, i.e. it could be linear, quadratic, periodic etc. In this study Gaussian Process Regression (GPR) is used, as it assigns a probability to every function that is possible from a chosen kernel that could describe the input-output relationship, where higher probabilities are given to functions that are considered more likely. The modeller therefore only needs to describe the mean, $m(\mathbf{x})$, and the kernel (or covariance function), $k(\mathbf{x}, \mathbf{x}')$ in order to describe the family of possible functional forms for the relationship between the inputs and outputs. This is formally defined as in equation (2):

$$f(x) \sim GP\left(m(\mathbf{x}), k(\mathbf{x}, \mathbf{x}')\right) \quad (2)$$

where $f(x)$ is the function of the inputs that will produce the target outputs, GP is a GP with a mean and kernel, and $\mathbf{x}$ and $\mathbf{x}'$ are two points in the input space. Due to the limitations in the scope of this paper, the reader is referred to [5] for more information and mathematical definitions. The kernel used in this study is the squared exponential kernel as it is a universal approximation and is a good kernel for modelling smooth functions [5]; it has the hyperparameters; length scale and standard deviation, set in this study as 0 and 0 respectively. A zero mean function was also used.

The normalised percentage difference for each of the six bending modes were used as features for GPR. GPR models were generated using inputs from the FE models predictions for all six bending modes, and the target output for each model was the average of each bending mode from the experimental testing of the five beams. Only the 0mm, 10mm and 20mm damage state data were used in the training stage, showing the methods ability to interpolate between selected training points and that a reduced experimental dataset can be used to generate robust feature vectors over a wider range of damage states using this methodology. The trained GPR models were then used to make predictions when shown the FE models predictions for all damage states. The maximum MSE between the average experimental results and the GPR models predicted outputs was 6.673 which shows a good correlation. It is noted that the bias function requires damaged state experimental data, and while it is unlikely this data will be available at system-level, it may be available at component level. The incorporation of validated component-level models into system-level simulations is left as a topic for future work.

The experimental variability, e, is assumed to be independent multivariate Gaussian distributions. The variability was quantified from the experimental observations from the undamaged state only, 0mm, (as for this study only 0mm, 10mm and 20mm data is assumed available in generating the damage model) and a covariance was calculated. Samples were generated from multivariate Gaussian distributions where the mean was the damage state prediction from the GPR and the covariance was that calculated from the 0mm damage state.

The damage model was finally constructed using equation (1). A two dimensional visualisation of the process is shown in Figure 3 for the first two bending modes, $\Delta\omega_1$ and $\Delta\omega_2$. The outputs from the damage model are shown to produce a better correlation with the observed experimental data and therefore are expected to produce similar features for classification as a data-driven approach.

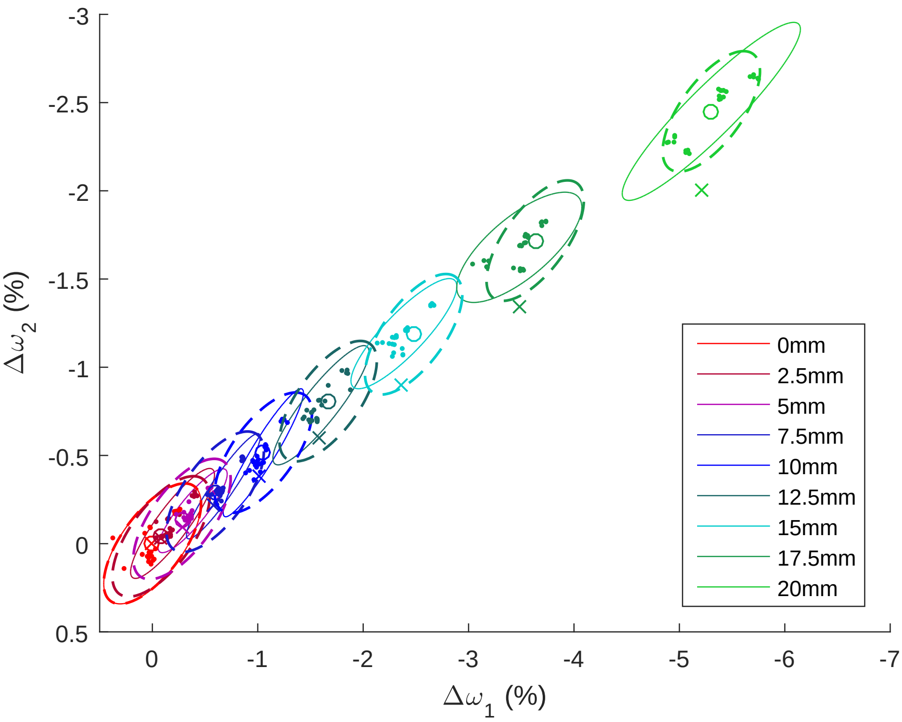

Figure 3: Comparison of damage state distributions for mode 1, $\Delta\omega_1$, and mode 2, $\Delta\omega_2$, for the observed experimental data, z, (· -); the model output, $\eta(\mathbf{x})$, (×); the model output and bias function, $\eta(\mathbf{x}) + \delta(\mathbf{x})$, (○); and the complete damaged state model, z_{model}, (- -).

The damage model from Section 2.2.1 was subsequently used to generate features, normalised percentage difference of natural frequencies for the use as training data for SVM classification. SVM is the problem of creating an optimised linear function (one resulting in the maximum margin between the two classes) that separates a two class data set. The input-output pairs for SVM classification are known as a feature vector, which describes the object in each dimension of feature space, and class label, which represents to what class the object belongs. Once trained, the SVM model is shown new data and a subsequent class label is assigned. SVMs are a powerful tool as they not only provide nonlinear separable hyperplanes by mapping the vectors to higher dimensional feature space using kernel functions, but they can also incorporate a soft margin allowing inseparable data to be classed by incorporating slack variables. Due to the limitations in the scope of this paper, the reader is referred to [6] for more information and mathematical definitions.

A two dimensional example for $\Delta\omega_1$ and $\Delta\omega_2$ is shown in Figure 4. Five samples of data were drawn from the damage model distributions for simplification of the illustration. The undamaged class has been defined here as 0-5mm damage, and the damaged class is defined as 7.5-20mm damage. A Gaussian radial basis function (RBF) has been used as the kernel due to the nonlinear nature of the data near the two classes. The RBF kernel has a scaling factor parameter, γ_{RBF}, which describes the influence of a single training sample where very small values overly constrain the model and reduce the ability to capture the boundary's complexity. A soft margin approach is used (allowing for misclassification of data in order to avoid overfitting), parameter C defines the trade off between misclassification of the training data and the simplicity of the decision bound. The larger the value of C the less misclassification and the more complex the decision bound. For this example γ_{RBF} and C are, 1 and 100 respectively. The SVM model, once trained on the damage model feature vectors, was then tested on the experimental data where 97.8% was correctly classified.

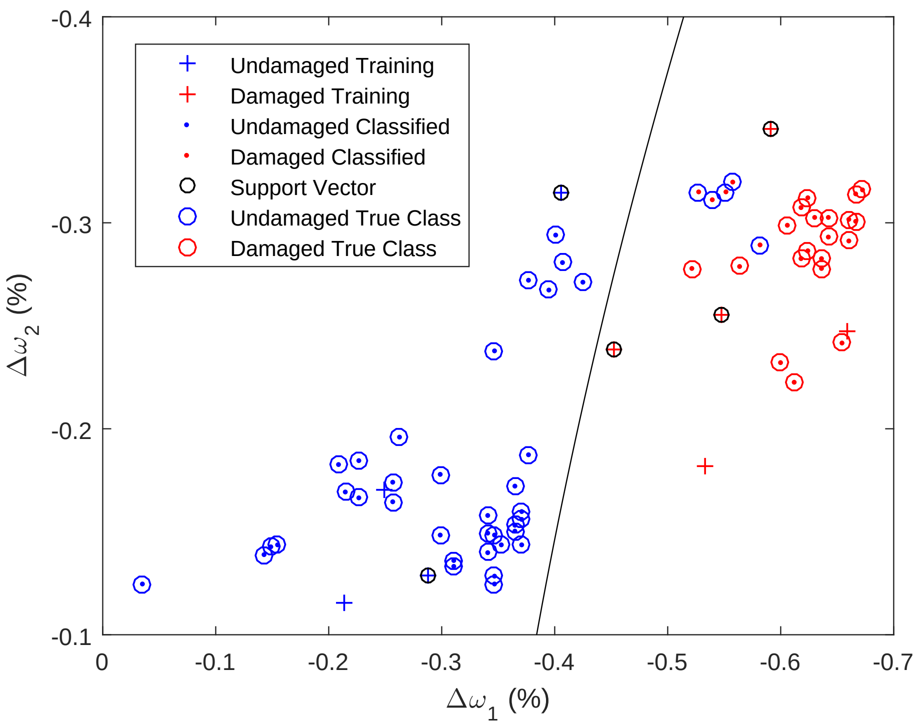

Figure 4: A zoomed in view of the SVM classification of the damage and undamaged classes.

Damage detection was performed using 50 samples drawn from the damage model distributions (for the six bending modes) as feature vectors. The undamaged class has been defined here as 0-5mm damage, and the damaged class as 7.5-20mm damage. A RBF kernel was used. The trained SVM model was tested using the experimental data. The percentage of correct classifications are shown in Table 2 along with the parameters values for γ_{RBF} and C. The approach was compared against the FE model output alone as the feature vector. As shown in Table 2, the modified damage model increases the percentage correct classification of data in SVM learning.

Training Data	**Damage Model**	**FE Model**
Correctly Classified	100.0%	91.1%
γ_{RBF}	10	10
C	1	20

Table 2: Summary of classification results using the complete damage model and the FE model outputs as training data.

2.3 Data-driven classification

Data-driven approaches use signal response data, in this case study the observed experimental data, as the feature vectors in machine learning classification. Providing that all damaged state data is available, this approach represents the 'best case' scenario, one where the actual systems response to damage is used rather than the response being modelled. The method used here is the same as that described in Section 2.2.1, SVM classification using a RBF kernel. The feature vector used is the normalised percentage difference from the experimental data. The undamaged class is again defined as 0-5mm damage and the damaged class as 7.5-20mm damage. The data was split into five sets, one for each beam tested. SVM Models were trained using each of the five beams and subsequently tested on the remaining data. Table 3 shows the results for each model.

Training Data	**Beam 1**	**Beam 2**	**Beam 3**	**Beam 4**	**Beam 5**
Correctly Classified	100.0%	97.8%	93.3%	98.3%	97.8%
γ_{RBF}	1	1	1	10	1
C	10	1	10	10	1

Table 3: Summary of classification results using each beam as training data.

2.4 Inverse model-driven damage detection

Inverse model-driven damage detection has been performed using model updating, which seeks to update cer-tain parameters, $\boldsymbol{\theta}$, of an FE model in order to reduce the residual between the model predictions, $\mathbf{z}(\boldsymbol{\theta})$, and the corresponding measurements from the physical structure, $\mathbf{z_m}$. More specifically, an iterative sensitivity based method has been used which uses a linearisation of the nonlinear relationship between measurements of the physical structure and the model parameters. The method uses a Taylor series expansion truncated after the first order term so that the residual, ϵ, for the ith iteration, is as shown in equation (3).

$$\epsilon = \mathbf{z_m} - \mathbf{z}(\boldsymbol{\theta}) \approx \mathbf{S}_i \delta\theta_i - \delta\mathbf{z}_i \tag{3}$$

where $\delta\mathbf{z}_i$ is,

$$\delta\mathbf{z}_i = \mathbf{z_m} - \mathbf{z}(\boldsymbol{\theta}_i) \tag{4}$$

and $\mathbf{S}_i$, the sensitivity matrix, is,

$$\mathbf{S}_i = \frac{\partial \mathbf{z}}{\partial \theta}|_{\theta=\theta_i} = \frac{\mathbf{z}(\theta_i + \Delta\theta_i) - \mathbf{z}(\theta_i)}{(\theta_i + \Delta\theta_i) - (\theta_i)} \tag{5}$$

The problem can formally be presented as finding a parameter set that minimises the cost function, for the ith iteration, $J(\delta\boldsymbol{\theta})$, as shown in equation (6).

$$J(\delta\boldsymbol{\theta}) = \|\mathbf{S}_i \delta\theta_i - \delta\mathbf{z}_i\|_2^2 \tag{6}$$

Weighting matrices and a side constraint regularisation are applied to help account for the importance of parameters and the treatment of ill-conditioned equations. At each iteration, the derivative of the cost function, $J(\delta\boldsymbol{\theta})$, with respect to each parameter is calculated and the result is set to zero which leads to equation (7).

$$\delta\theta_i = (\mathbf{S}_i^T \mathbf{W}_\epsilon \mathbf{S}_i + \boldsymbol{\lambda}^2 \mathbf{W}_\theta)^{-1} \mathbf{S}_i^T \mathbf{W}_\epsilon \left(\mathbf{z_m} - \mathbf{z}(\boldsymbol{\theta})\right) \tag{7}$$

where $\delta\theta_i$ is the change in parameter at each iteration, superscript T is the Hermitian transpose, $\mathbf{W}_\epsilon$ is a weighting matrix, $\boldsymbol{\lambda}^2$ is the regularisation parameter and $\mathbf{W}_\theta$ is the parameter weighting matrix. The reader is referred to [7, 8] for more information and mathematical definitions.

There are two steps in using model updating for damage detection, the first is to update the global model for the undamaged case, and the second is to update localised parameters of the correlated model from step one with data from the possible damage states. Model updating was performed using the experimental modal frequencies obtained from testing the first beam. The beam was divided into ten areas each with an independent Young's modulus that could be updated using multiplier parameters, $\boldsymbol{\theta}$. The model was updated globally in accordance to step one with an initial Young's modulus of 71GPa for every parameter. Each of the ten areas was updated using the average response of the undamaged condition (0mm damage state). The updated parameters $\boldsymbol{\theta}$ are shown in Table 4.

Parameters	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9	θ_{10}
Value	0.935	0.936	0.936	0.935	0.935	0.935	0.935	0.936	0.935	0.937

Table 4: Updated model parameters for the undamaged case.

The localised parameters that were updated in step two were parameters θ_4 and θ_8, all other parameters were excluded from the updating process and held at the values shown in Table 4. The damage in the experimental data was located within the area governed by θ_4. Parameters θ_4 and θ_8 were updated using the response data from beam one for all damage states. Figure 5 shows the updated parameters for each damage state in step two.

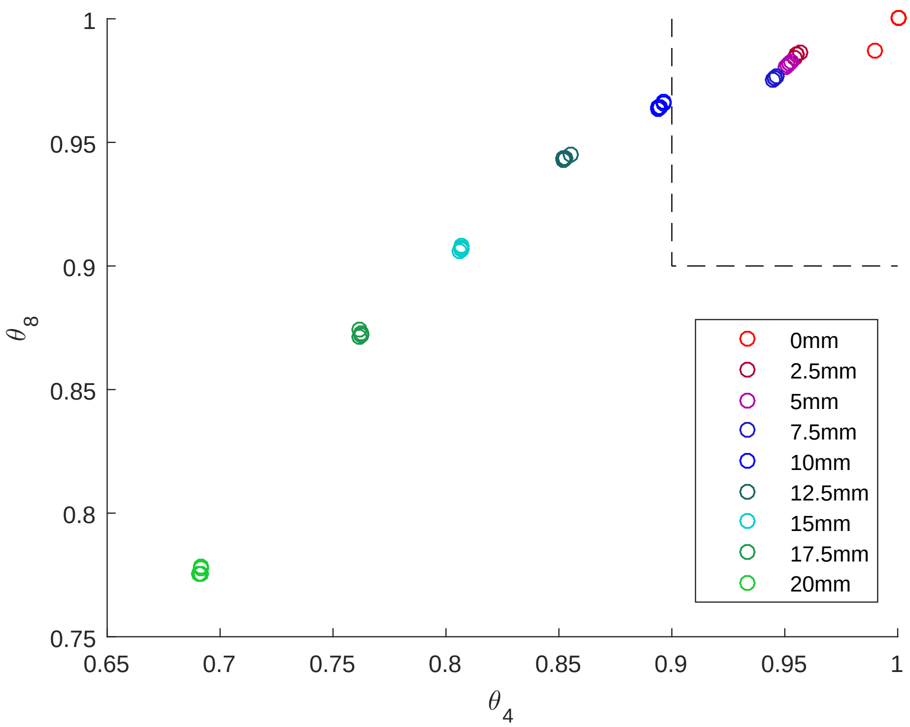

Figure 5: A comparison of updated model parameters for different damage states. (- -) indicates the undamaged state.

The results in Figure 5 highlight one (of several) issues with model updating, namely non-uniqueness. The true health of the structure is single-site damage, yet the updated model indicates damage of comparable severity at two locations as θ_8 reduces when it would be expected to remain constant i.e. it has misidentified the health state. The increased reduction in θ_4 over θ_8 can be used to infer that only single-site damage has occurred, as it is known prior to model updating that the structure contained only single-site damage. However in problems where unknown damage locations exist this would be problematic. An additional problem with using model updating for damage detection is that of selecting a damage threshold based on parameter changes, as parameters can change due to statistical variation (temperature effect for example). It is assumed for this study that a 10% reduction in stiffness ($\theta_4 \leq 0.9$) signifies that the structure is damaged, under this definition damage over 10mm has been identified as damaged. It is noted that although model updating can be problematic, even for very simple structures with well controlled experiments and a small number of updating parameters, such as the cantilever beam, extensions of the method have been applied reduce these problems. Examples of these extensions are Bayesian model updating using Markov-Chain Monte Carlo (MCMC) sampling which allows for exploring the parameter space in more detail, revealing other likely health states. These methods allow for the incorporation of prior beliefs about the likely damage distribution e.g. that only single-site damage will occur, and therefore can improve confidence in the updating parameters. Additionally, it is possible to incorporate such belief into a sensitivity based update by running multiple updates for a single parameter (or subset of parameters) at a time and selecting the most likely.

2.5 Discussion

Damage detection was performed with a high degree of accuracy for all three methods, which data-driven and forward model-driven methods both producing correct classifications of 100.0%. It has been shown that using just the FE model in forward model-driven SHM produces less robust predictions (91.1%) when compared to those using the damage model outlined in Section 2.2.1 (100.0%). This therefore shows that the inclusion of the bias function and experimental variability produces better and more robust feature vectors for classification using SVM models. It is noted that for this trivial case, forward model-driven SHM using the damage model produced 100.0% accurate predictions, this is due to the simplicity in the boundary between the undamaged and damaged states for this case study.

Data-driven classification produced accurate predictions (93.3%-100.0%) however variability was seen depending on which experimental data was used to train the SVM model, this shows that even with data-driven methods the quality and ability of the feature vector to generalise for all possible data will affect the ability of the SVM model to correctly classify new data. The inverse model-driven method is difficult to directly compare as classification of damage states is not possible as the criteria for damage is different. Damage detection is difficult using inverse model-driven methods as defining the change in stiffness that identifies damage can be relatively arbitrary, especially when referring to Young's modulus. It has also been shown that the location of damage is an issue due to the problem of non-uniqueness.

Forward model-driven SHM was less computationally expensive when compared to the inverse model-driven method as only nine FE models were required, compared to the multiple updates for each set of data that the inverse model-driven method was shown. The data-driven method was the least computationally expensive however, multiple sets of testing were required in order to apply this method. Forward model-driven SHM method has therefore been shown for this case study to be as robust as data-driven methods without the need for numerous expensive testing and more robust than inverse model-driven approaches with less computational expense.

3 Representative three-story building structure case study

A representative three-story building structure is used as a more complex test structure in order to develop and compare the use of forward model-driven SHM against the conventional methods of data-driven and inverse model-driven approaches for damage detection.

3.1 Experimental Methodoloy

Modal testing was conducted on a representative three-story building structure for various damage states. Damage was introduced via a saw cut at different depths and the first three bending modes were obtained. The structure was fastened to a rigid table. The setup is shown in Figure 6. The front right beam of the structure (in Figure 6) was damaged incrementally in a like manner to the beam, with the damage location being 85mm from the top of the structure. The representative three-story building structure was first tested in the undamaged condition before incremental cuts were added of 2.5mm to a total depth of 20mm. The front right beam was subsequently replaced and the procedure repeated so that data was obtained from two beams.

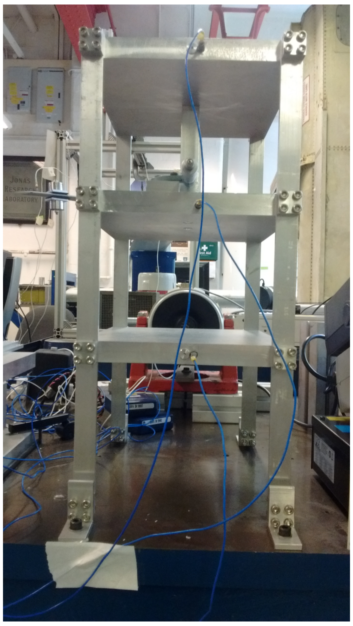

Figure 6: The representative three-story building structure experimental setup.

An electrodynamic shaker was used to excite the structure and three accelerometers, one attached to the base of each floor, were used to obtain the response. The acquisition and analysis of the data was performed using a LMS Test.Lab system with a bandwidth 6553.6Hz containing 16384 spectral lines (0.2Hz resolution) with a Hanning window on the excitation and response. The procedure was repeated five times for each state.

3.2 Damage model for forward model-driven classification

A damage model was created according to equation (1). The model output, $\eta(\cdot)$, was generated from multiple three dimensional FE models under different damage-state conditions using the commercial solver ANSYS. Damage was modelled as a removal of geometry using the same dimensions as Section 2.2.1. The first three bending modes were obtained for each FE model and the normalised percentage difference calculated. Once again formal model calibration was not pursued for this study as a good correlation with the experimental results has deemed to be achieved. This is shown as for the relative errors $\Delta\omega_{1-3}$, the MSE between the mean experimental data and the FE model predictions across the full damage range was 0.006. The bias function, $\delta\ (\cdot)$, was modelled using GPR with a squared exponential kernel using the hyperparameters; length scale and standard deviation, set as 0 and 0 respectively. A zero mean function was also used. The GPR models were trained using only the 0mm, 10mm and 20mm damage state data. The trained GPR models were subsequently used to make predictions when shown the FE model results as inputs. The maximum

MSE between the experimental results and the GPR predicted output was 8.488 which was seen to be a reasonable correlation. The experimental variability, e, was assumed to be an independent multivariate Gaussian distribution. A covariance was calculated from the experimental data for the 0mm undamaged state. Samples were then drawn from these distributions to generate the damage model data.

A two dimensional visualisation of the process is shown in Figure 7 for the first two bending modes $\Delta\omega_1$ and $\Delta\omega_2$. Figure 7 displays that a high degree of overlap exists between the different damage states, it also shows that changes in modal frequencies are fairly insensitive to the introduction of damage. This poses problems in generating a robust classifier as the boundaries are complex. For this case study it is shown that changes in modal frequency is a relatively poor feature for damage classification. Due to the complexity in boundaries, Figure 7 shows that the damage model appears to deviate from the experimental distributions.

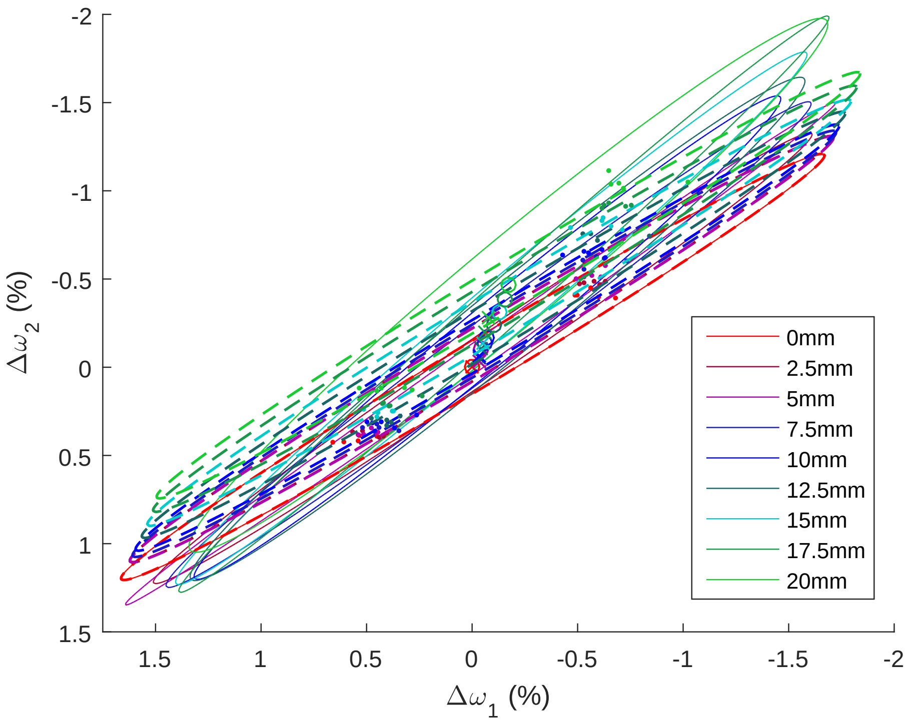

Figure 7: Comparison of damage state distributions for mode 1, $\Delta\omega_1$, and mode 2, $\Delta\omega_2$, for the observed experimental data, z, (· -); the model output, $\eta(\mathbf{x})$, ($\times$); the model output and bias function, $\eta(\mathbf{x}) + \delta(\mathbf{x})$, ($\circ$); and the complete damaged state model, z_{model}, (- -).

The damage model was used to perform damage detection using SVM with an RBF kernel. 50 samples were drawn from the damage model distributions and used as feature vectors. The undamaged class has been defined here as 0-5mm damage and the damaged class is defined as 7.5-20mm damage. The SVM model was tested using the experimental data and compared to using just the FE model as feature vectors. The percentage of correct classifications are shown in Table 5. Table 5 shows that the modified damage model produces a better percentage classification to the FE model alone.

Training Data	Damage Model	FE Model
Correctly Classified	90.0%	66.7%
γ_{RBF}	1	10
C	100	1

Table 5: Summary of classification results using the complete damage model and the FE model outputs as training data.

3.3 Data-driven classification

SVM models were trained using the normalised percentage difference from the experimental data and an RBF kernel. The undamaged class was defined as 0-5mm damage and the damaged class 7.5-20mm damage. The data was split into two sets one for each beam tested, where the data was trained using one beam and tested on the other. Table 6 shows the results for each model.

Training Data	Beam 1	Beam 2
Correctly Classified	95.6%	84.4%
γ_{RBF}	2	1
C	0.3	1

Table 6: Summary of classification results using each of the two sets of data from the representative three-story building structure as training data.

3.4 Inverse model-driven damage detection

Iterative sensitivity based model updating was used as outlined in Section 2.4. Damage detection was performed using a two step updating approach using the average experimental model frequencies for each damage state. Each part of the structure was given a multiplier parameter, $\boldsymbol{\theta}$, that governed it's Young's modulus. Nuts, bolts and the end plates were grouped with their like parts, with each group assigned an updating parameter. This reduced the number of parameters in the model, reducing the computational time. For step one the global model was updated using the average response of the undamaged conditions (0mm damage state). The updated parameters $\boldsymbol{\theta}$ are shown in Table 7.

Parameters	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9
Value	0.910	0.901	0.902	0.902	0.902	0.902	0.902	0.902	0.902
Parameters	θ_{10}	θ_{11}	θ_{12}	θ_{13}	θ_{14}	θ_{15}	θ_{16}	θ_{17}	θ_{18}
Value	0.902	0.902	0.902	0.902	0.902	0.902	0.902	0.902	0.902

Table 7: Updated model parameters for the undamaged case.

Each of the four legs were used as localised parameters in step two, these parameters were θ_{12-15}, all other parameters were excluded from the updating process and held at the values shown in Table 7. The experimental damage was located in the leg governed by parameter θ_{15}. Model updating was performed using the average response for each damage state. Figure 8 shows the updated parameters for each damage state in step two.

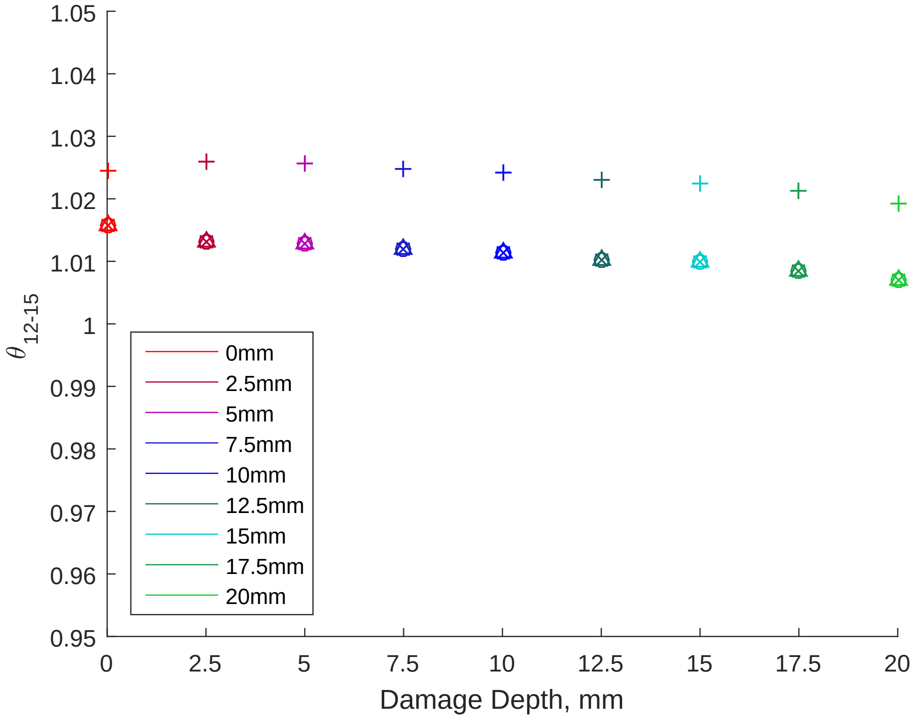

Figure 8: A comparison of updated model parameters for different damage states; θ_{12} ($\circ$), θ_{13} ($\times$), θ_{14} ($+$), θ_{15} ($\triangle$).

Figure 8 highlights the difficultly of implementing model updating for a complex structure such as the representative three-story building structure. The relatively small changes in natural frequency mean that very small changes in stiffness are required in order to 'correct' the model. This means that detection can be difficult due to the small parameter changes. Figure 8 also shows the non-uniqueness of the solution and difficulty in parametrisation of the model in order to locate damage. The method has failed to identify the location of damage as θ_{12-15} all show the same reduction trend with θ_{14} showing an initial stiffening compared to the other parameters. The reduction in parameters is also not great enough for damage to be inferred as parameters will change small amounts due to other effects, such as temperature.

3.5 Discussion

For the representative three-story building structure damage detection is a more complex problem. This therefore has led to a reduction in correct classification and detection of damage in the structure when compared to the cantilever beam. Each method has also highlighted the issue of using modal frequencies as a measure of determining damage, as it can be relatively insensitive to damage. It has been shown that for forward model-driven methods, using the modified damage model produces more robust predictions (90.0%) when compared to using the FE model alone (66.7%). The inclusion of the bias function and experimental variability is therefore crucial to the implementation of this technique.

Data-driven classification accurately and correctly predicted between 84.4-95.6% of the data. This shows the importance of the data used in the training process, data that does not generalise will produce less robust classifiers. The inverse model-driven method failed to locate the damage as all parameters reduced with each damage state, this is due to the non-uniqueness of the problem. The parameters did not reduce by any significant amount implying that damage was either insignificant or was not present in the structure for all damage states. It is therefore shown that more robust model updating techniques such as the Bayesian model updating technique are required for more complex structures.

Forward model-driven SHM has been shown to be comparable to data-driven methods as it made similar correct classifications. It is also easier to implement and less computationally expensive when compared with inverse model-driven methods when used for more complex structures.

4 Conclusion

Damage detection was performed for two case studies: a cantilever beam and a representative three-story building structure. Three methods were used: forward model-driven, data-driven and inverse-model driven. The forward model-driven approach proposed in this paper was found to be comparable to data-driven techniques with similar percentage correct classifications. The inverse model-driven approach has shown to be complex to implement due to the non-uniqueness in solutions and the complexity of parameter selection. Additionally, the problem of damage detection is difficult as a threshold can be relatively arbitrary. Damage detection was performed more accurately on the cantilever beam than the representative three-story building structure for all methods.

All three methods have shown the difficulty in using modal frequencies as features and targets for damage detection. The measure is relatively insensitive to damage and therefore produces problems for damage detection. It is therefore proposed that other features such as windows of the frequency response function (FRF) of a structure are used.

The forward model-driven approach is promising, however further research is required to reduce the amount of damage states required to build the model. At present, data is needed at a few damage states in order to use GPR. Further study is also required in the incorporation of validated component-level models into a system-level simulation. Furthermore calibration of the model in the discrepancy function should be perused as outlined by Kennedy and O'Hagan [3].

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