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# *Effects of general relativity on habitable zone particle dynamics under the influence of an internal perturber*

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# INTRODUCTION

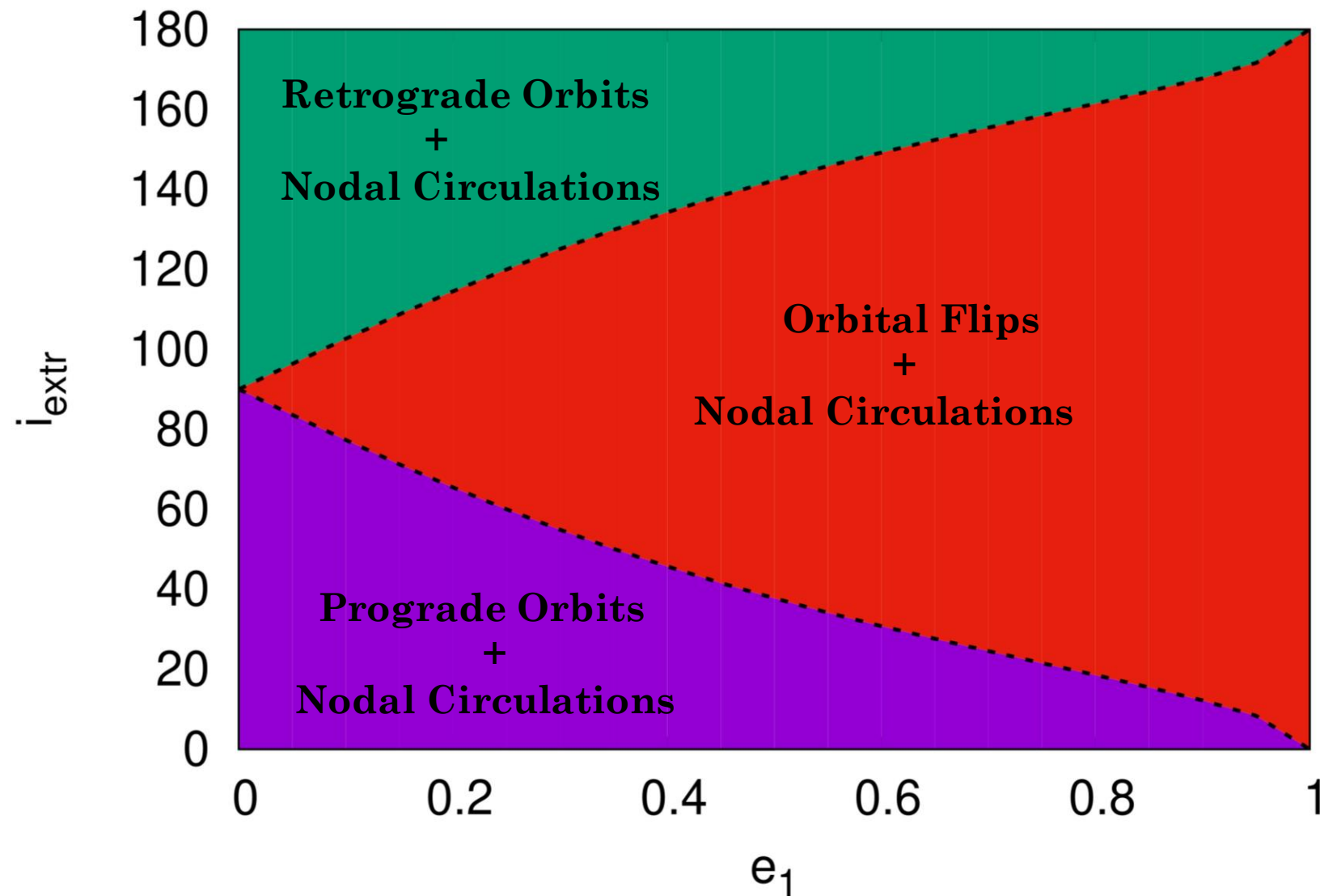
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In literature, there exist several studies which focus on the dynamical evolution of planetary systems with the goal of having a better understanding of their stability regions.

In the present work we analyze the elliptical restricted three-body problem for an external massless particle. Initially we considered results of previous investigations such as those developed by Ziglin (1975) and Naoz et al. (2017), who studied the dynamical behavior of a distant particle under the presence of an internal binary.

The authors considered the secular evolution of the particle up to the quadrupole level of approximation. Their results show that when relativistic forces are not included, the dynamical behavior of the particle can be classified in two different motion regimes: nodal circulations with retrograde or prograde orbits, and nodal librations associated with orbital flips.

It is important to remark that the nodal libration region is determined by the inner perturber's eccentricity ( $e_1$ ) (**Fig. 1**).



$$i_2^e = \arccos \left( \pm \sqrt{\frac{5e_1^2}{(1 + 4e_1^2)}} \right)$$

**Figure 1:** Regimes of motion of an external massless particle in the elliptical restricted three-body problem.

Recently, Zanardi et al. (2018) studied the secular evolution of a distant test particle orbiting an inner massive binary up to the quadrupole level of approximation taking into account general relativity (GR) effects.

In fact, the authors showed that the dynamical behavior of these particles evidences significant differences in comparison with that obtained in absence of general relativity.

In this new scenario of work, the nodal libration region is more complex since it is determined by the masses of the binary as well as by the orbital parameters of the inner perturber and of the external massless particle.

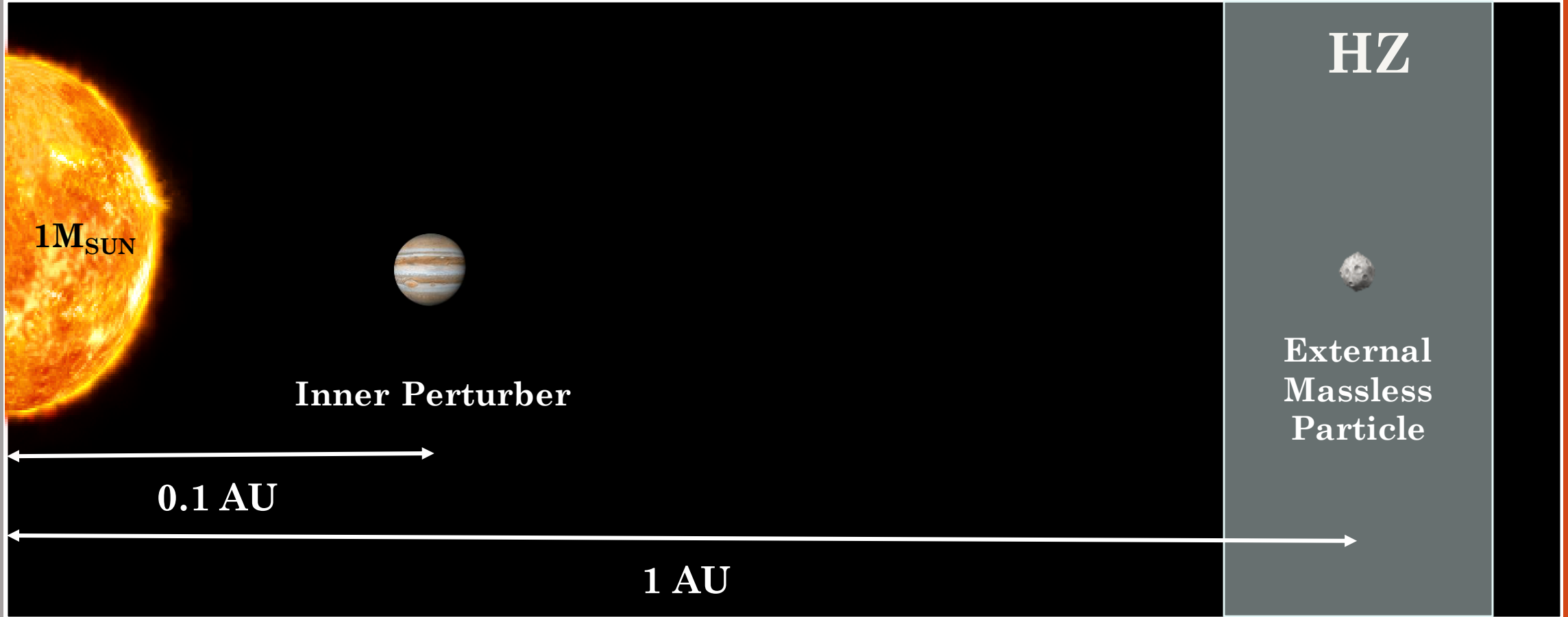
In this work, we are interested in studying the dynamical evolution of external massless particles within the framework of the elliptical three-body problem including GR effects. The particles are located at the habitable zone (HZ) of the system which is composed of a Sun-like star and an inner eccentric perturber.

To carry out our study we make use of the analytical approach derived by Zanardi et al. (2018).

# SCENARIOS OF WORK

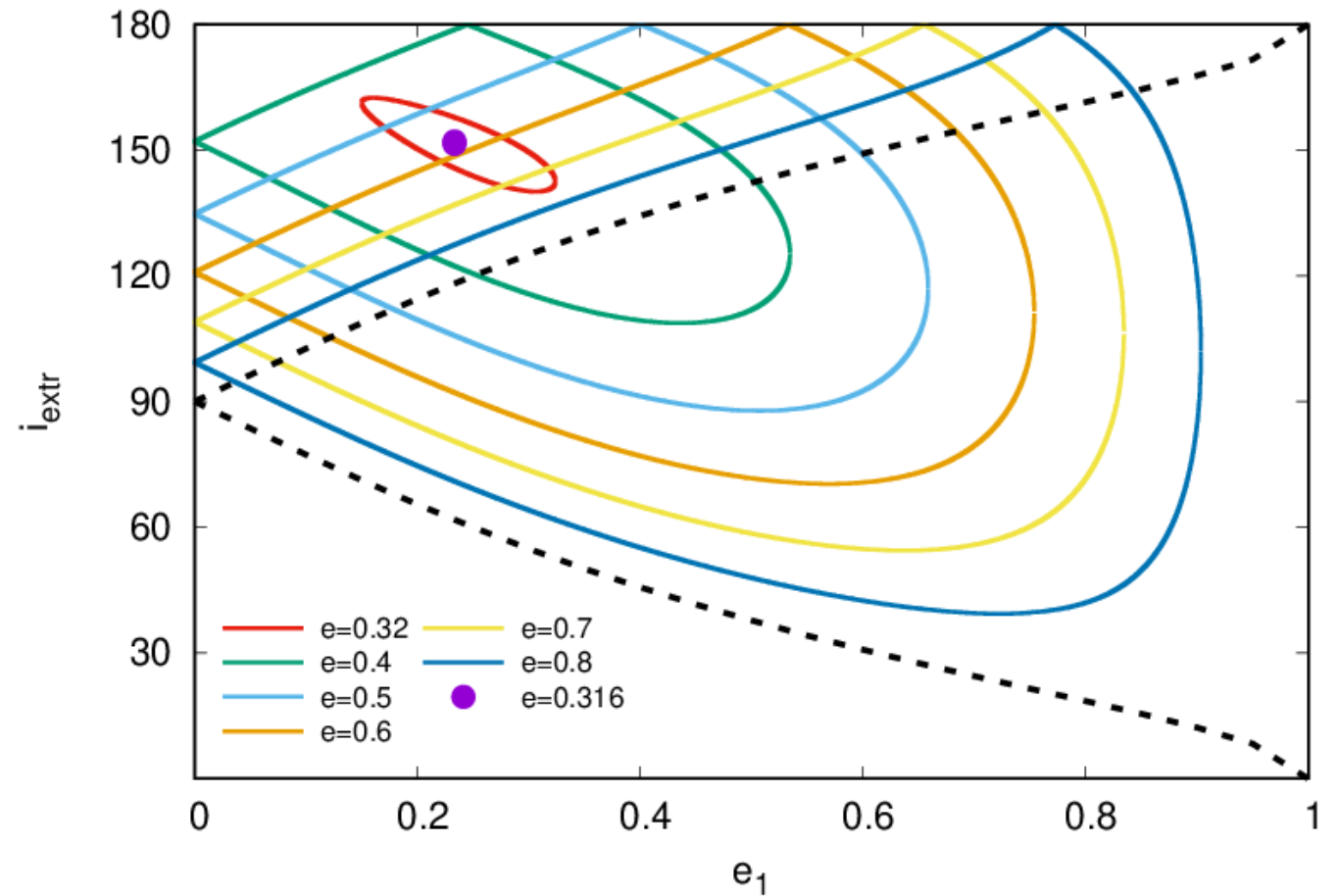
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In our scenarios of work, the inner perturber and the HZ external massless particle have a semi-major axis of 0.1 AU and 1 AU respectively.



In the first scenario of study, we consider an inner Jupiter-mass planet and calculate the nodal libration region in the  $(e_1, i_2)$  plane for different values of the external massless particle's eccentricity ( $e_2$ ). These regions are illustrated in the **Figure 2**.

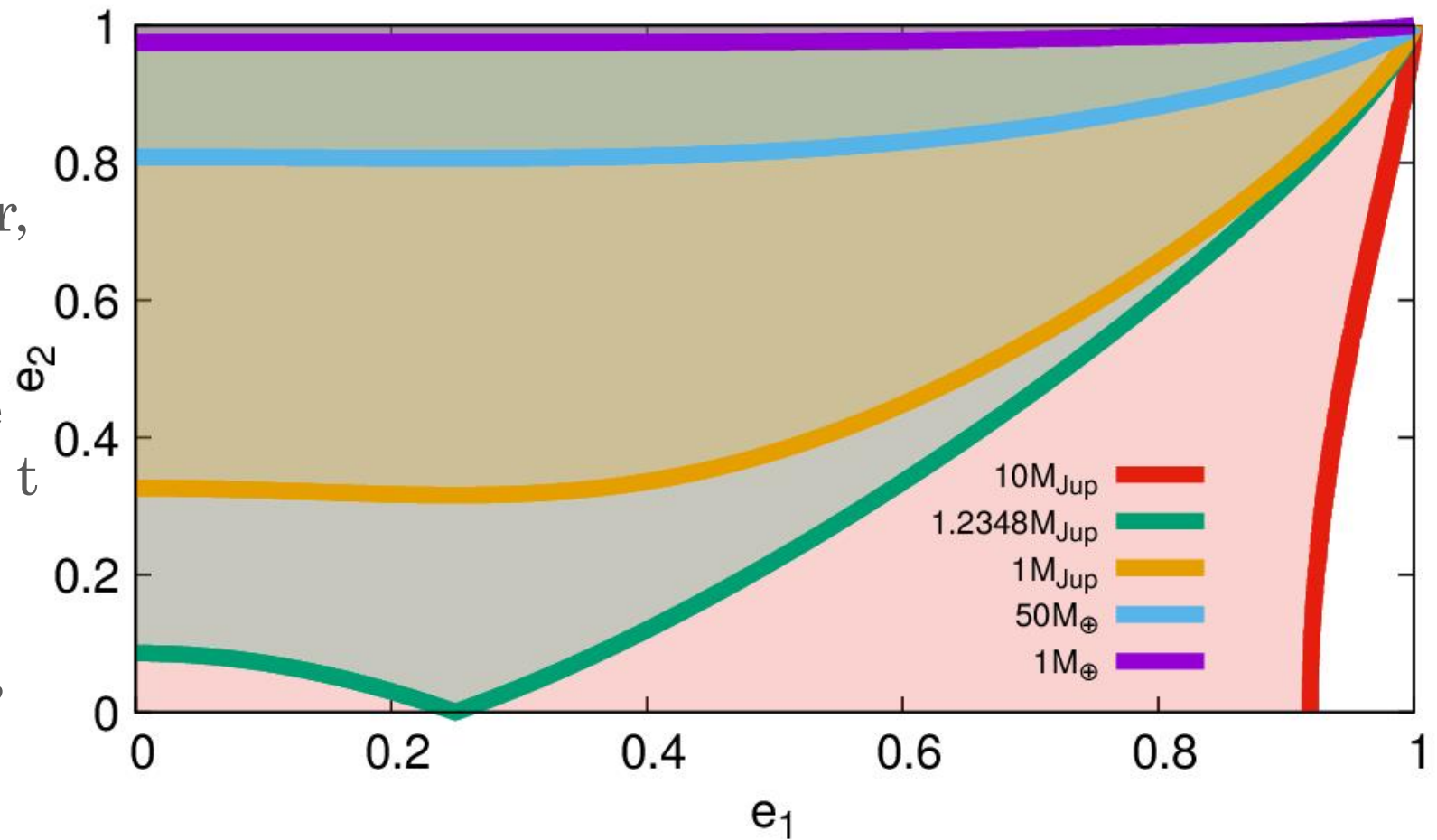
On the one hand, the range of prograde inclinations of the nodal libration region is reduced in comparison with that obtained without GR effects for any value of  $e_1$  and  $e_2$ , which is consistent with that previously derived by Zanardi et al. (2018). In fact, our results indicate that a HZ test particle with prograde inclinations can not experience nodal librations for  $e_2 < 0.5$  in this scenario of work. On the other hand, the greater the orbital eccentricity  $e_2$ , the wider the range of values associated with the inner planet's eccentricity  $e_1$  that lead to nodal librations of the HZ test particle, which is in agreement with the results from Zanardi et al. (2018).



**Figure 2:** Each color curve is given by a different value of particle's eccentricity ( $e_2$ ). Dotted black curve represent the scenario without GR.

At the same way, we were able to obtain an expression which defines the minimum value of the particle's eccentricity ( $e_2$ ) that leads to nodal librations as a function of the inner planet's eccentricity ( $e_1$ ) (**Fig. 3**).

This relationship allows us to derive two important results. First, the more massive the inner perturber, the greater the nodal libration region in the ( $e_1, e_2$ ) plane. Second, there exists a minimum mass of the inner perturber above which the particle can experience nodal librations, for any value of  $e_2$  considered. Such a value is equal to  $1.23M_{\text{Jupiter}}$ .



*Figure 3: Set of eccentricities ( $e_1, e_2$ ) which leads to nodal librations of the particle, for different masses of the inner perturber.*

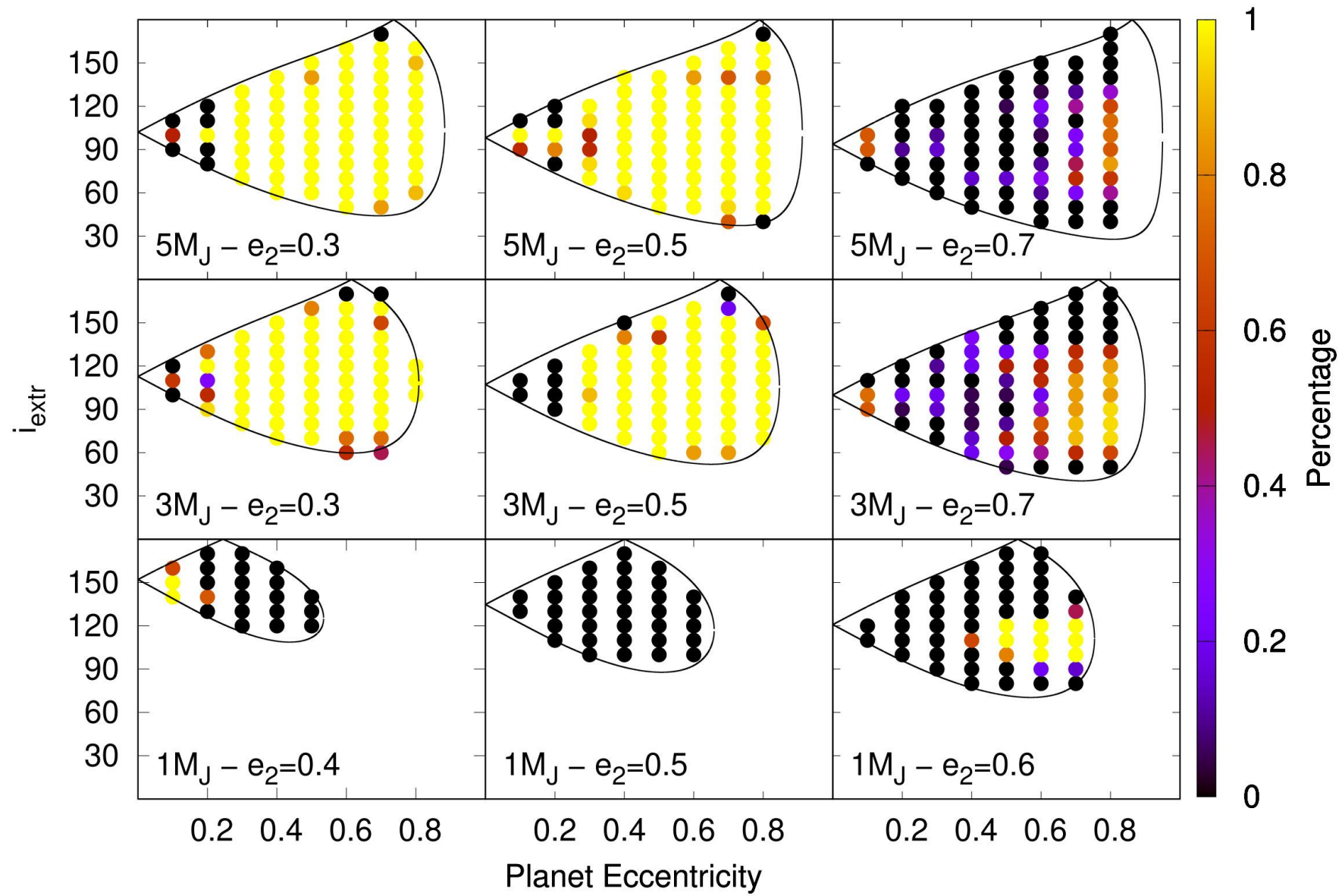


Finally, we carried out N-body simulations with the goal of analyzing the validity of our analytical treatment. To do this, we made use of an modified version of the Mercury Code (Chambers 1999), in which we included an external routine aimed to model general relativity effects based on the corrections proposed by Anderson et al. (1975). Each simulation was integrated for 10 Myr.

**Figure 4** shows the results of the simulations developed in a (e1,i2) plane for different values of the massless particle eccentricity (e2) and masses of the inner perturber. In particular we adopt three different value masses for the inner perturber:  $1 M_{\text{Jupiter}}$ ,  $3 M_{\text{Jupiter}}$  and  $5 M_{\text{Jupiter}}$ . The colorbar represents the percentage correlation between simulations and analytic treatment considered. Each dot represents a set of 20 external massless particles with the corresponding initial parameters.

As can be seen, exists a good agreement between the analytical method and the numerical simulations developed for the scenarios considered. In particular, this is more evident for high mass planets and medium-to-low particles eccentricity (e2).





**Figure 4:** Results of the simulations developed in a  $(e_1, i_2)$  plane for different values of the massless particle eccentricity ( $e_2$ ) and different masses of the inner perturber. The colorbar represents the percentage correlation between simulations and the analytical treatment considered.

# CONCLUSIONS

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Our results show that general relativity (GR) effects play a key role in the dynamical behaviour of massless particles located at the HZ around a solar-type star, which evolve under the influence of an inner perturber. We remark that this result is valid for a value of the mass of the inner perturber ranging from sub-Earths to super-Jupiters.

The analytical treatment based on the secular Hamiltonian expanded up to the quadrupole level of the approximation is consistent with results derived from N-body simulations for an appropriate range of orbital eccentricities associated with the inner perturber and the HZ massless particle. The limit values of such orbital eccentricities that show the validity of the analytical secular treatment depend on the mass of the inner perturber.