Question4

A)

- For state i to be transient, it only needs state i connect to a recurrent state and never no other way to come back.

- For all states to be transient, it is impossible.

- For all states to be recurrent, it only need P^n (where P is the transition matrix) as n-> infinity has all non-zero entry.

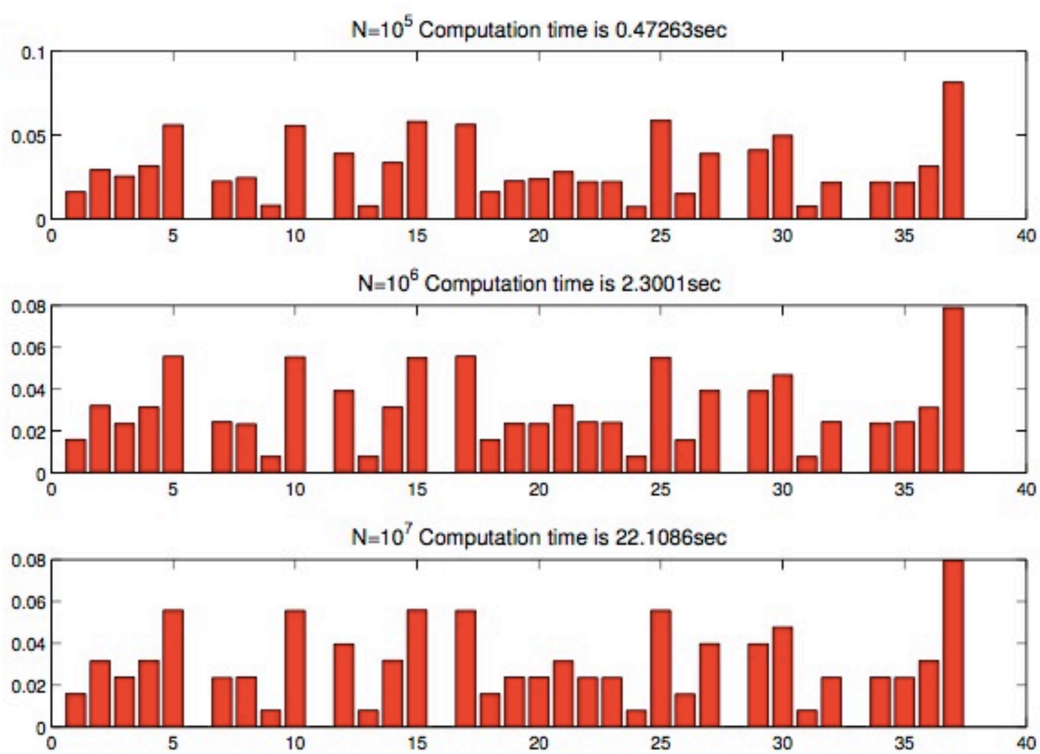
- For all states to be aperiodic, it is reasonable to assume that for all Pij is strictly less than 1. That is all pages have at least 2 links. This condition will strictly remove the possibility of periodic.

- For all states positive recurrent, it only has to satisfy that the MC is strongly connected, and recurrent, because that states are finite.

- For all states to be Ergotic, it only needs to satisfy recurrent, connected, aperiodic conditions above.

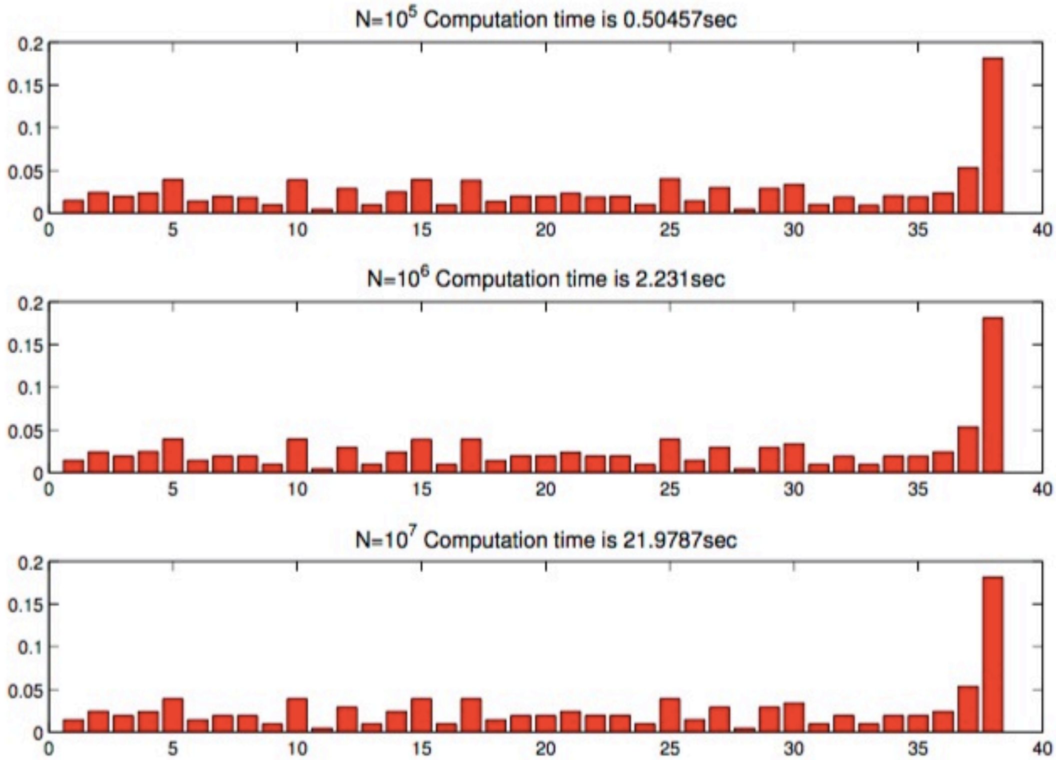
- For a MC to be irreducible, it only needs to satisfy that the MC is strongly connected.

B)



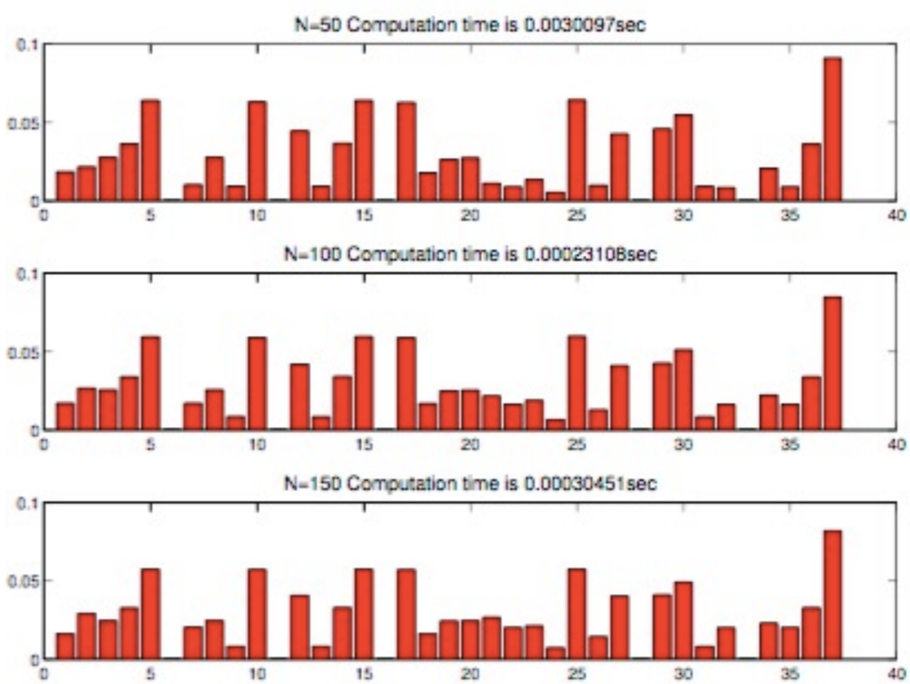
From the plot above we see that some pages have 0 ranking. The reason is because the are of different class of state 1. And if an agent starts there they will never reach states of 0 ranking.

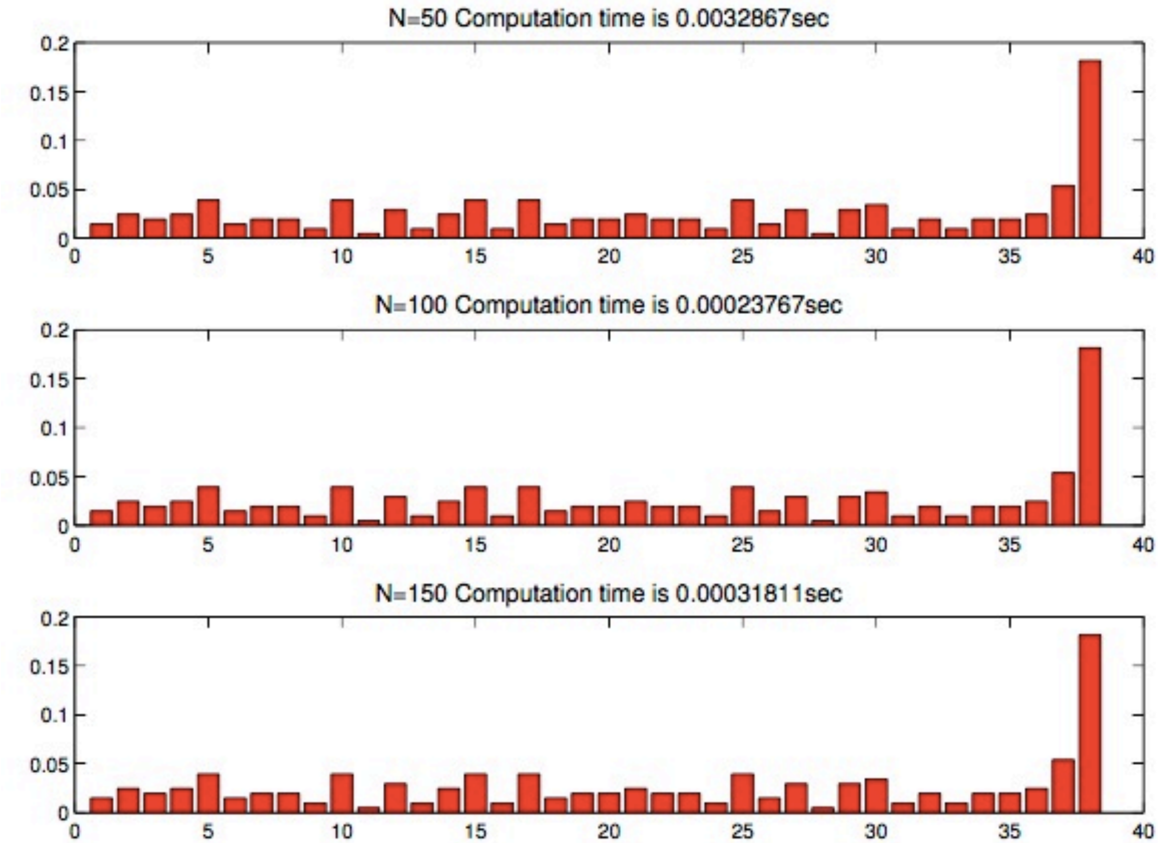
After adding the professor:



We see that all pages have rank now. And professor has the highest ranking, which is a reasonable explanation.

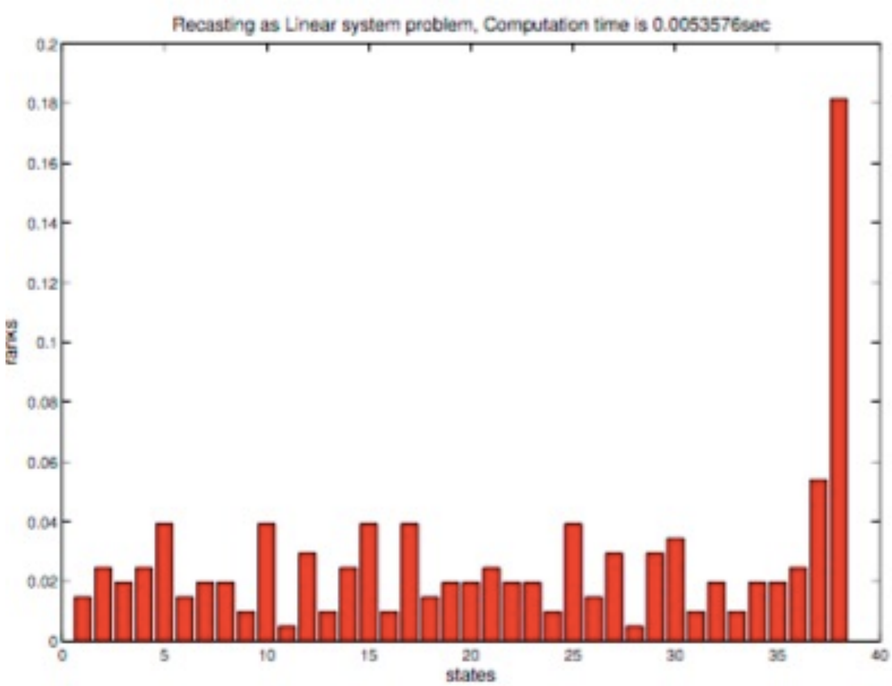
D)

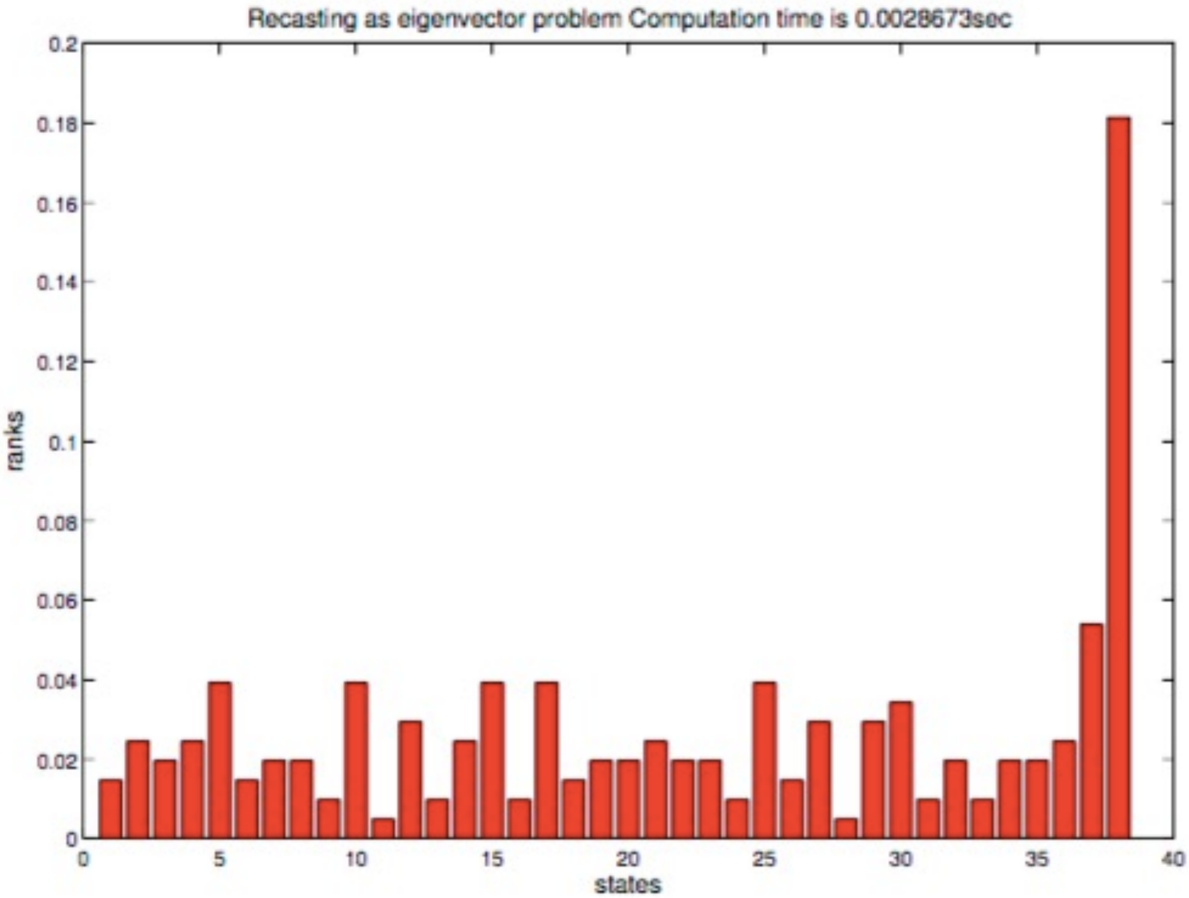




The two graph are very similar to the result we obtain in the questionB however, it is clear that the convergence time is much shorter than B after we use random walk model.

E),F)





The two graphs are identical, this means that the page rank problem can be solved with two different approaches, calculate the eigen-value, vector or matrix multiplication.

G)

- Random walk, this approach is simple and easy to carryout, however, this approach takes much longer time to converge. And thus less preferable than other method.

- Probability update, this approach is much faster than random walk in terms of convergence. Because it allows the multiplication of matrix and equal to sending m random agents to the network. This clearly is a better approach.

- Linear system:
  This approach requires much more memory than any approach mentioned above. However, it avoids the problem of convergence. The worst case is the computation time is O(J^3), so for large matrix this approach is not executable.

- Eigen-value,vector
  This approach is very similar to linear system, more computation, memory, and convergence-free. Not feasible to large matrix.