

Question 7

a)

X_n is a MC because the number of female of a family is dependently exclusively on the last generation.

Also because the MC has a absorbing state, that is $X_n=0$. So the MC is reducible and not recurrent.

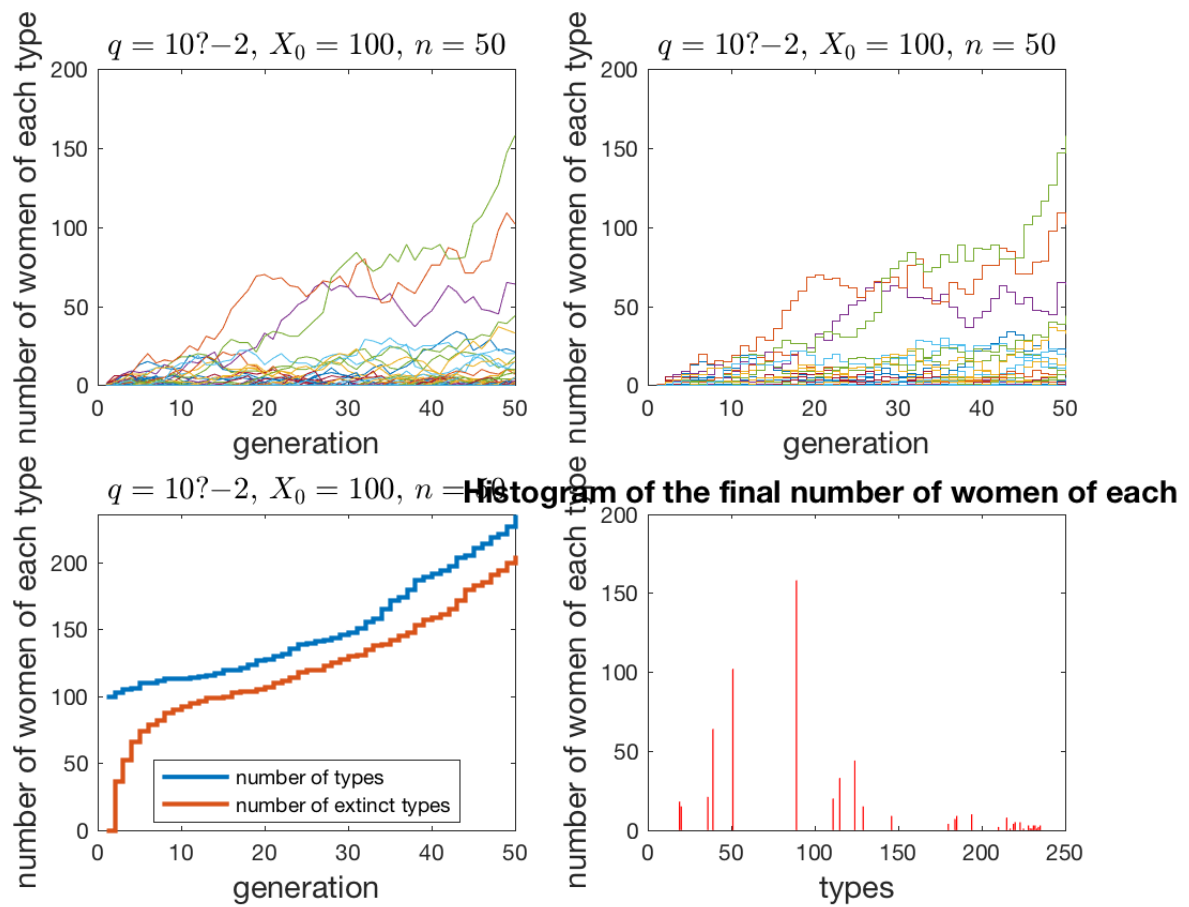
b)

This is not a MC because if $X_n=0$ and the probability that X_{n+1} depends on information that whether r has occurred in the past. In this special example, $X_n=0$ does not proved enough information for us to decide the outcome of X_{n+1} .

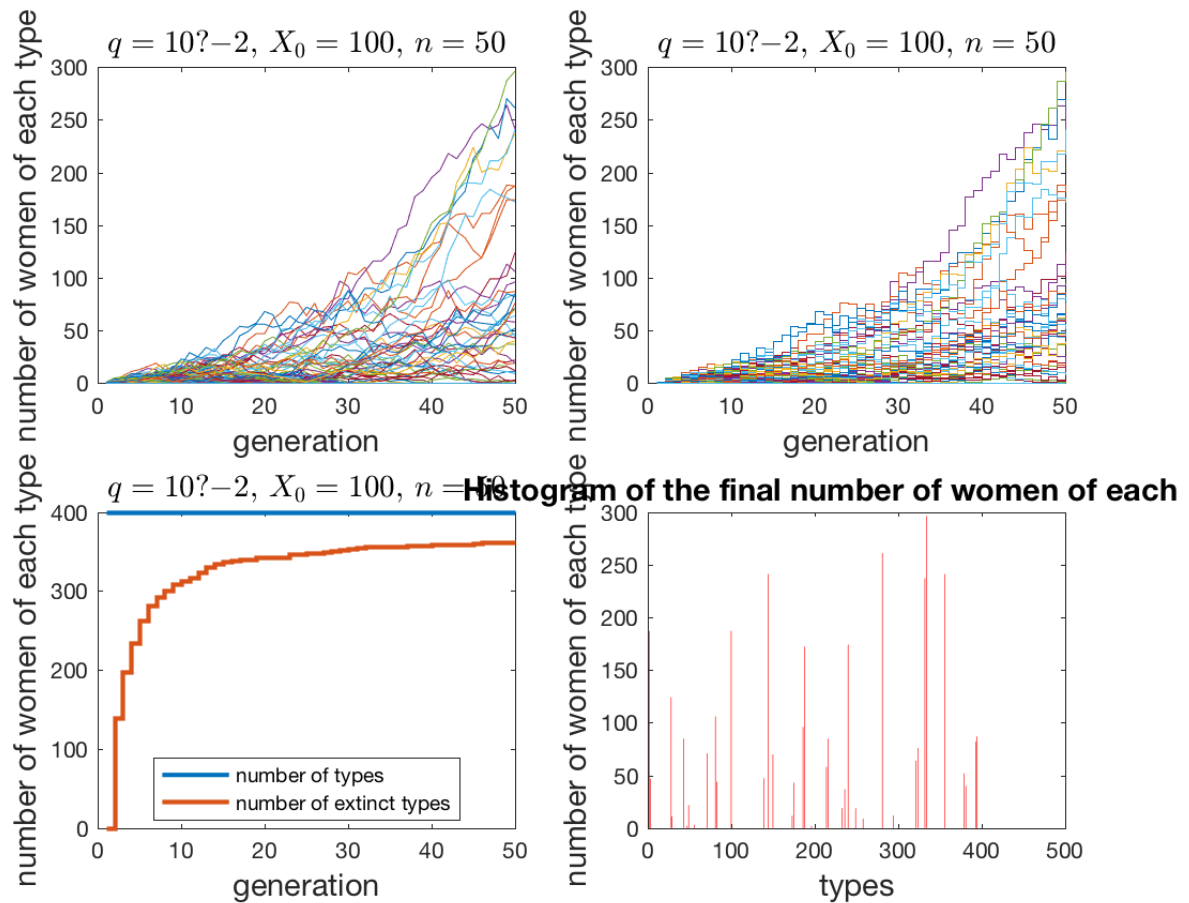
If given $X_n>0$ then The problem is a MC for sure. The reason is that we can then transform the question into a). and study only the gene type r population.

c)See code

d)



e)



We see that most of the gene are extinct. Some survival lasts to the end with high fraction of number.

Code:

```
function hw4
close all; clear; clc;
x0 = 100;
max_t = 50;
max_types = 1000;

mu = 1.05;
q = 10^-2;
X=zeros(max_types, max_t);
number_of_types=zeros(1, max_t);
X(1:x0,1) = 1;
number_of_types(1)=x0;
number_of_extinct_types=zeros(1,max_t);

for n=2:max_t
    number_of_types(n)=number_of_types(n-1);
    for type = 1:number_of_types(n-1);
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    for i = 1:X(type,n-1)
        daughters = poissrnd(mu,1,1);
        mutation = binornd(1,q,1,1);
        if mutation
            number_of_types(n) = number_of_types(n)+1;
            X(number_of_types(n),n) = daughters;
        else
            X(type,n) = X(type,n) + daughters;
        end
    end
    if X(type,n)== 0
        number_of_extinct_types(n)=number_of_extinct_types(n)+1;
    end
end

figure(1)
subplot(2,2,1)
plot(1:max_t, X)
xlabel('generation','FontSize',14)
ylabel('number of women of each type','FontSize',14)
title('$q=10\{-2\},$ $X_{\{0\}}=100,$ $n=50$', 'FontSize',14,'Interpreter','latex')

subplot(2,2,2)
stairs(1:max_t, X')
xlabel('generation','FontSize',14)
ylabel('number of women of each type','FontSize',14)
title('$q=10\{-2\},$ $X_{\{0\}}=100,$ $n=50$', 'FontSize',14,'Interpreter','latex')

subplot(2,2,3)
stairs(1:max_t, [number_of_types;number_of_extinct_types'],'LineWidth',2)
xlabel('generation','FontSize',14)
ylabel('number of women of each type','FontSize',14)
title('$q=10\{-2\},$ $X_{\{0\}}=100,$ $n=50$', 'FontSize',14,'Interpreter','latex')
axis([0 50 0 number_of_types(end)])
legend('number of types','number of extinct types','Location','Best')

subplot(2,2,4)
bar(1:number_of_types(end), X(1:number_of_types(end),max_t),'r')
xlabel('types','FontSize',14)
ylabel('number of women of each type','FontSize',14)
title('Histogram of the final number of women of each type','FontSize',14)
saveas(figure(1), './figure1.png')

close all; clear;clc;
x0 = 400;
max_t = 50;
max_types = 1000;

mu = 1.05;
q = 0;
X=zeros(max_types, max_t);
number_of_types=zeros(1, max_t);
X(1:x0,1) = 1;

```

```

number_of_types(1)=x0;
number_of_extinct_types=zeros(1,max_t);

for n=2:max_t
    number_of_types(n)=number_of_types(n-1);
    for type = 1:number_of_types(n-1);
        for i = 1:X(type,n-1)
            daughters = poissrnd(mu,1,1);
            mutation = binornd(1,q,1,1);
            if mutation
                number_of_types(n) = number_of_types(n)+1;
                X(number_of_types(n),n) = daughters;
            else
                X(type,n) = X(type,n) + daughters;
            end
        end
        if X(type,n)== 0
            number_of_extinct_types(n)=number_of_extinct_types(n)+1;
        end
    end
end

figure(2)
subplot(2,2,1)
plot(1:max_t, X)
xlabel('generation','FontSize',14)
ylabel('number of women of each type','FontSize',14)
title('$q=10\{-2\},$ $X_{\{0\}}=100,$ $n=50$', 'FontSize',14, 'Interpreter', 'latex')

subplot(2,2,2)
stairs(1:max_t, X')
xlabel('generation','FontSize',14)
ylabel('number of women of each type','FontSize',14)
title('$q=10\{-2\},$ $X_{\{0\}}=100,$ $n=50$', 'FontSize',14, 'Interpreter', 'latex')

subplot(2,2,3)
stairs(1:max_t, [number_of_types;number_of_extinct_types]', 'LineWidth',2)
xlabel('generation','FontSize',14)
ylabel('number of women of each type','FontSize',14)
title('$q=10\{-2\},$ $X_{\{0\}}=100,$ $n=50$', 'FontSize',14, 'Interpreter', 'latex')
axis([0 50 0 number_of_types(end)])
legend('number of types','number of extinct types','Location','Best')

subplot(2,2,4)
bar(1:number_of_types(end), X(1:number_of_types(end),max_t),'r')
xlabel('types','FontSize',14)
ylabel('number of women of each type','FontSize',14)
title('Histogram of the final number of women of each type','FontSize',14)
saveas(figure(2), './figure2.png')

end

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