Assignment 03: EM for Gaussian Mixture Models (GMMs): Image Segmentation Application

1. For this problem, we will use Gaussian mixture models (GMMs) for image segmentation and study the behavior of the algorithm on a simple image. The file "TestImage.jpg" available on the website provides an image that is used as a test for color blindness[1]. Color normal individuals viewing the image should see a clear message in the image revealed by a partitioning of the image into distinct regions that are characterized by homogeneity of some color attributes. However, these regions also show variation in other attributes.

   We will segment the image using Gaussian mixture modeling and assess the results we obtain. For the GMM fitting in this lab, you may choose to use your own program using the equations provided during our discussion of GMMs in the context of the EM algorithm in class, or you may choose to use an existing implementation from the web, or from the appropriate MATLAB™toolbox.

   (a) To simplify things and allow us to visualize our results, we will work in a 2-dimensional space instead of directly using the 3-dimensional RGB image data Convert the image RGB data from its native 3-dimensional RGB color space into a 2-dimensional rg chromaticity space by applying the transformation

   $$\begin{aligned} r &= \frac{R}{R+G+B} \\ g &= \frac{G}{R+G+B} \end{aligned} \tag{1}$$

   Transform the test image into rg chromaticity space and visualize your transformed image by mapping it back into an 8-bit RGB image via the transformation

   $$\begin{aligned} \alpha &= \frac{255}{\max\left(r, g, 1-(r+g)\right)} \\ R_{out} &= \text{round}\left(\alpha r\right) \\ G_{out} &= \text{round}\left(\alpha g\right) \\ B_{out} &= \text{round}\left(\alpha(1-(r+g))\right) \end{aligned} \tag{2}$$

   Comment on what attributes are preserved and what attributes are lost in the process of conversion from the input RGB to the output RGB (equivalently from RGB space to rg chromaticity space.

   Hint: The MATLAB functions imread() and imshow(), will be helpful for you in this exercise.

   (b) Cluster the pixels in the image by fitting a $K = 3$ component GMM to the chromaticities in the image. Specifically, model the observed rg chromaticity values across the pixels in the image as a set of iid $d = 2$-dimensional rg vector observations from a Gaussian mixture with unknown mean and covariance and use the Expectation Maximization (EM) algorithm for GMM parameter estimation to estimate the unknown mean vectors and covariance matrices for the GMM. Because the EM algorithm is only locally convergent, you need to perform the GMM parameter estimation with multiple random initializations and select between these to obtain a "good" result. For determining a "good" result, you can either evaluate the likelihood function, or (more readily) you can visualize your results as described in the following parts and assess the fit based on the visualizations.

   Hint: The MATLAB function reshape() will be helpful for re-organizing the data from the 2D two rg channel chromaticity image representation into a sequence of 2D rg vectors in this exercise.

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[1] The image was obtained from: http://www.nature.com/nature/journal/v447/n7146/full/447782a.html.

i. Visualize the GMM fit you have obtained by making a scatter-plot of the rg chromaticities for the pixels in the image and superimposing contours for the components in your estimated GMM on this plot. In particular, for each mixture component in the GMM, indicate mean value of the mixture component on the scatter-plot by a "$+$" and plot the "3σ" contour that corresponds to locations at which the components' probability falls to a level that is a fraction $\exp(-3^2/2)) = \exp(-9/2) \approx 0.0111$ of its peak value. You may find it useful to use the singular value decomposition (SVD) to obtain your contours - although for this 2D setting, you can also compute these analytically using the formula for the roots of a quadratic equation.

ii. Use the GMM to compute, for each pixel, the posterior probability that it came from the mixture component j, for $j = 1, 2, \ldots K$. Visualize the results of the "soft" or probabilistic segmentation you have obtained in this process as a series of images. Specifically, for each mixture component j, create and display a normalized image that represents the posterior probability that a pixel belongs to the class j as an 8-bit gray-scale value, where gray-scale values of 0 corresponds to a probability of 0 and a gray-scale value of 255 corresponds to a probability of 1. Present one set of K such images for one of the "good" mixture model fits that you obtain. Comment on your results.

iii. Comment on the effect of different initializations on your GMM fit and how these appear in the visualizations presented in the preceding two parts.

(c) Repeat the exercise in Step 1b for $K = 4$ and $K = 5$. Comment on the results.