1.8

Ar vektoriai tiesiškai nepriklausomi?

$$\vec{v_1} = \begin{bmatrix} 4+4i \\ 4-4i \\ -5-2i \\ 2-i \end{bmatrix} \qquad \vec{v_2} = \begin{bmatrix} -2-i \\ -2+i \\ 4-3i \\ 3-i \end{bmatrix} \qquad \vec{v_3} = \begin{bmatrix} -4-4i \\ -2-2i \\ 3+2i \\ 2-4i \end{bmatrix}$$

Tiesiškai nepriklausomi, jei:

$$\alpha_1 \begin{bmatrix} 4+4i \\ 4-4i \\ -5-2i \\ 2-i \end{bmatrix} + \alpha_2 \begin{bmatrix} -2-i \\ -2+i \\ 4-3i \\ 3-i \end{bmatrix} + \alpha_3 \begin{bmatrix} -4-4i \\ -2-2i \\ 3+2i \\ 2-4i \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 4+4i & -2-i & -4-4i & 0 \\ 4-4i & -2+i & -2-2i & 0 \\ -5-2i & 4-3i & 3+2i & 0 \\ 2-i & 3-i & 2-4i & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 4+4i & -2-i & -4-4i & 0 \\ 0 & -1-i & 2-6i & 0 \\ 0 & \frac{15}{8}-\frac{25}{8}i & 2 & 0 \\ 0 & \frac{29}{8}-\frac{13}{8}i & 4-5i & 0 \end{array}\right] \sim$$

$$\left[\begin{array}{ccc|c} 4+4i & -2-i & -4-4i & 0 \\ 0 & -1-i & 2-6i & 0 \\ 0 & 15-25i & 16 & 0 \\ 0 & 29-13i & 24-40i & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 4+4i & -2-i & -4-4i & 0 \\ 0 & -1-i & 2-6i & 0 \\ 0 & 0 & -111-10i & 0 \\ 0 & 0 & -86-130i & 0 \end{array}\right]$$

$$(-86-130i)\alpha_3 = 0 \to \alpha_3 = 0$$
$$(-1-i)\alpha_2 + (2-6i)\alpha_3 = 0 \to \alpha_2 = 0$$
$$(4+4i)\alpha_1 + (-2-i)\alpha_2 + (-4-4i)\alpha_3 = 0 \to \alpha_1 = 0$$

$\alpha_1 = \alpha_2 = \alpha_3 = 0$, todėl yra tiesiški nepriklausomi.

2.8

$A^2 + 3B^\dagger A^{-2} + C^{-1}$

$$A = \begin{bmatrix} 4-7i & 1+5i \\ -7-6i & -5-6i \end{bmatrix} \qquad B = \begin{bmatrix} -5-7i & -5-7i \\ 3+5i & 5+4i \end{bmatrix} \qquad C = \begin{bmatrix} -4-8i & 8+8i \\ 1-8i & 2-9i \end{bmatrix}$$

$$A^2 = AA = \begin{bmatrix} 4-7i & 1+5i \\ -7-6i & -5-6i \end{bmatrix} \begin{bmatrix} 4-7i & 1+5i \\ -7-6i & -5-6i \end{bmatrix} = \begin{bmatrix} -10-97i & 64-18i \\ -71+97i & 12+19i \end{bmatrix}$$

$$3B^\dagger = \begin{bmatrix} -15+21i & 9-15i \\ -15+21i & 15-12i \end{bmatrix}$$

$$A^{-1} = \frac{1}{-85+52i} \begin{bmatrix} -5-6i & 7+6i \\ -1-5i & 4-7i \end{bmatrix}$$

$$A^{-2} = \frac{1}{-85+52i} \begin{bmatrix} -5-6i & 7+6i \\ -1-5i & 4-7i \end{bmatrix} \frac{1}{-85+52i} \begin{bmatrix} -5-6i & 7+6i \\ -1-5i & 4-7i \end{bmatrix} =$$

$$= \frac{1}{4521-8840i} \begin{bmatrix} 12+19i & 71-97i \\ -64+18i & -10-97i \end{bmatrix}$$

$$C^{-1} = \frac{1}{-152+76i} \begin{bmatrix} 2-9i & -1+8i \\ -8-8i & -4-8i \end{bmatrix}$$

$$A^2 + 3B^\dagger A^{-2} + C^{-1} = \frac{1}{98585041} \begin{bmatrix} 975595211,1-9555697997i & 6289795114-1773437390i \\ -7012327919+9561823686i & 1167233070+1882564516i \end{bmatrix}$$

3.8

Užrašyti $\vec{v_4}$ bazėje $\{\vec{v_1}, \vec{v_2}, \vec{v_3}\}$.

$$\vec{v_1} = \begin{bmatrix} 2-3i \\ -3+2i \\ 2-i \end{bmatrix} \quad \vec{v_2} = \begin{bmatrix} 4-4i \\ -3-2i \\ 1+i \end{bmatrix} \quad \vec{v_3} = \begin{bmatrix} 2+4i \\ 2+3i \\ 3+i \end{bmatrix} \quad \vec{v_4} = \begin{bmatrix} 4+4i \\ 4+3i \\ -5-2i \end{bmatrix}$$

$$\vec{v_4} = x\vec{v_1} + y\vec{v_2} + z\vec{v_3}$$

$$\left[\begin{array}{ccc|c} 2-3i & 4-4i & 2+4i & 4+4i \\ -3+2i & -3-2i & 2+3i & 4+3i \\ 2-i & 1+i & 3+i & -5-2i \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & \frac{20}{13}+\frac{4}{13}i & -\frac{8}{13}+\frac{14}{13}i & =\frac{4}{13}+\frac{20}{13}i \\ 0 & \frac{29}{13}-\frac{54}{13}i & \frac{30}{13}+\frac{97}{13}i & \frac{80}{13}+\frac{107}{13}i \\ 0 & -\frac{31}{13}+\frac{25}{13}i & \frac{41}{13}-\frac{23}{13}i & -\frac{77}{13}-\frac{70}{13}i \end{array}\right] \sim$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & \frac{444}{289}-\frac{110}{289}i & \frac{496}{289}-\frac{352}{289}i \\ 0 & 1 & -\frac{336}{289}+\frac{341}{289}i & -\frac{266}{289}+\frac{571}{289}i \\ 0 & 0 & \frac{766}{289}+\frac{948}{289}i & -\frac{1248}{289}+\frac{317}{289}i \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{2607}{1285}-\frac{3676}{1285}i \\ 0 & 1 & 0 & -\frac{773}{2570}+\frac{4642}{1285}i \\ 0 & 0 & 1 & -\frac{567}{1285}+\frac{2467}{2570}i \end{array}\right]$$

$$\vec{v_4} = (\tfrac{2607}{1285} - \tfrac{3676}{1285}i)\vec{v_1} + (-\tfrac{773}{2570} + \tfrac{4642}{1285}i)\vec{v_2} + (-\tfrac{567}{1285} + \tfrac{2467}{2570}i)\vec{v_3}$$

4.8

$proj_u(v) = \frac{\langle v,u \rangle}{\langle u,u \rangle} u, \qquad u_1 = v_1$

$u_k = v_k - \sum j = 1k - 1 proj_{u_j}(v_k)$

$$u_1 = \begin{bmatrix} 2-3i \\ -3+2i \\ 2-i \end{bmatrix}$$

$$u_2 = \begin{bmatrix} 4-4i \\ -3-2i \\ 1+i \end{bmatrix} - \frac{\left\langle \begin{bmatrix} 4-4i \\ -3-2i \\ 1+i \end{bmatrix}, \begin{bmatrix} 2-3i \\ -3+2i \\ 2-i \end{bmatrix} \right\rangle}{\left\langle \begin{bmatrix} 2-3i \\ -3+2i \\ 2-i \end{bmatrix}, \begin{bmatrix} 2-3i \\ -3+2i \\ 2-i \end{bmatrix} \right\rangle} \begin{bmatrix} 2-3i \\ -3+2i \\ 2-i \end{bmatrix} =$$

$$= \begin{bmatrix} 4-4i \\ -3-2i \\ 1+i \end{bmatrix} - \frac{26-19i}{31} \begin{bmatrix} 2-3i \\ -3+2i \\ 2-i \end{bmatrix} = \frac{1}{31} \begin{bmatrix} 129-8i \\ -53-171i \\ -2+95i \end{bmatrix}$$

$$u_3 = \begin{bmatrix} 2+4i \\ 2+3i \\ 3+i \end{bmatrix} - \frac{\left\langle \begin{bmatrix} 2+4i \\ 2+3i \\ 3+i \end{bmatrix}, \frac{1}{31} \begin{bmatrix} 129-8i \\ -53-171i \\ -2+95i \end{bmatrix} \right\rangle}{\left\langle \frac{1}{31} \begin{bmatrix} 129-8i \\ -53-171i \\ -2+95i \end{bmatrix}, \frac{1}{31} \begin{bmatrix} 129-8i \\ -53-171i \\ -2+95i \end{bmatrix} \right\rangle} \frac{1}{31} \begin{bmatrix} 129-8i \\ -53-171i \\ -2+95i \end{bmatrix} -$$

$$- \frac{\left\langle \begin{bmatrix} 2+4i \\ 2+3i \\ 3+i \end{bmatrix}, \begin{bmatrix} 2-3i \\ -3+2i \\ 2-i \end{bmatrix} \right\rangle}{\left\langle \begin{bmatrix} 2-3i \\ -3+2i \\ 2-i \end{bmatrix}, \begin{bmatrix} 12-3i \\ -3+2i \\ 2-i \end{bmatrix} \right\rangle} \begin{bmatrix} 2-3i \\ -3+2i \\ 2-i \end{bmatrix} = \begin{bmatrix} 2+4i \\ 2+3i \\ 3+i \end{bmatrix} - \frac{-\frac{304}{31} - \frac{428}{31} i}{\frac{1864}{31}} \frac{1}{31} \begin{bmatrix} 129-8i \\ -53-171i \\ -2+95i \end{bmatrix} - \frac{-3-6i}{31} \begin{bmatrix} 2-3i \\ -3+2i \\ 2-i \end{bmatrix} =$$

$$= \frac{1}{14446} \begin{bmatrix} 49024+44771i \\ -3219+30421i \\ 58943+26074i \end{bmatrix}$$

$$u_1 = \begin{bmatrix} 2-3i \\ -3+2i \\ 2-i \end{bmatrix}, \qquad u_2 = \frac{1}{31} \begin{bmatrix} 129-8i \\ -53-171i \\ -2+95i \end{bmatrix}, \qquad u_3 = \frac{1}{14446} \begin{bmatrix} 49024+44771i \\ -3219+30421i \\ 58943+26074i \end{bmatrix}$$

5.8

$$A = \begin{bmatrix} -i & -4 & 2 \\ 0 & -3i & 5i \\ 0 & 0 & -5i \end{bmatrix}$$

$$(A - \lambda I)x = 0$$

$$\begin{vmatrix} -i-\lambda & -4 & 2 \\ 0 & -3i-\lambda & 5i \\ 0 & 0 & -5i-\lambda \end{vmatrix} = 0$$

$(-i-\lambda)(-3i-\lambda)(-5i-\lambda)+0+0-0-0-0 = 0 \quad \rightarrow \quad \lambda_1 = -i, \lambda_2 = -3i, \lambda_3 = -5i$

,kai $\lambda = -i$

$$\begin{pmatrix} -i & -4 & 2 \\ 0 & -3i & 5i \\ 0 & 0 & -5i \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -xi \\ -yi \\ -zi \end{pmatrix}$$

$$\begin{cases} -xi - 4y + 2z = -xi \\ -3iy + 5iz = -yi \\ -5iz = -zi \end{cases} \Rightarrow \begin{cases} z = 2y \\ 5z = 2y \\ z = 0 \end{cases}$$

,kai $\lambda = -3i$

$$\begin{pmatrix} -i & -4 & 2 \\ 0 & -3i & 5i \\ 0 & 0 & -5i \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3xi \\ -3yi \\ -3zi \end{pmatrix}$$

$$\begin{cases} -xi - 4y + 2z = -3xi \\ -3iy + 5iz = -3yi \\ -5iz = -3zi \end{cases} \Rightarrow \begin{cases} 2y = x \\ z = 0 \\ z = 0 \end{cases}$$

,kai $\lambda = -5i$

$$\begin{pmatrix} -i & -4 & 2 \\ 0 & -3i & 5i \\ 0 & 0 & -5i \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -5xi \\ -5yi \\ -5zi \end{pmatrix}$$

$$\begin{cases} -xi - 4y + 2z = -5xi \\ -3iy + 5iz = -5yi \\ -5iz = -5zi \end{cases} \Rightarrow \begin{cases} 3z = -x \\ 5z = -2y \\ 0 = 0 \end{cases}$$

Ats.: kai $\lambda = -i$, $\vec{v} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$,kai $\lambda = -3i$, $\vec{v} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$

,kai $\lambda = -5i$, $\vec{v} = \begin{pmatrix} -6 \\ -5 \\ 2 \end{pmatrix}$ (tai tik vieni iš galimu vektoriu)

6.8

$$\begin{bmatrix} -i & -4 & 2 \\ 0 & -3i & 5i \\ 0 & 0 & -5i \end{bmatrix}$$

a) AX=H , kur H=H†
X=A^{-1}H

$$H = \begin{bmatrix} 1 & -i & 1 \\ i & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \qquad A^{-1} = \frac{1}{15i}\begin{bmatrix} -15 & -20i & -14i \\ 0 & -5 & 5 \\ 0 & 0 & -3i \end{bmatrix}$$

$$X = \frac{1}{15i}\begin{bmatrix} -15 & -20i & -14i \\ 0 & -5 & 5 \\ 0 & 0 & -3i \end{bmatrix}\begin{bmatrix} 1 & -i & 1 \\ i & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{14}{15} - \frac{1}{3}i & -\frac{19}{15} & -\frac{34}{15} + i \\ -\frac{1}{3} - \frac{1}{3}i & 0 & 0 \\ \frac{1}{5}i & \frac{1}{5}i & \frac{1}{5}i \end{bmatrix}$$

b) AY=U, kur UU†=I
Y=A^{-1}U

$$U = \begin{bmatrix} i & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{bmatrix}$$

$$Y = \frac{1}{15i}\begin{bmatrix} -15 & -20i & -14i \\ 0 & -5 & 5 \\ 0 & 0 & -3 \end{bmatrix}\begin{bmatrix} i & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{bmatrix} = \begin{bmatrix} -1 & -\frac{14}{15}i & -\frac{4}{3}i \\ 0 & \frac{1}{3} & -\frac{1}{3} \\ 0 & -\frac{1}{5} & 0 \end{bmatrix}$$

c) $Z = \begin{bmatrix} -1 & 3i \\ -i & 1 \end{bmatrix}$

$$Z \otimes A = \begin{bmatrix} -1 & 3i \\ -i & 1 \end{bmatrix} \otimes \begin{bmatrix} -i & -4 & 2 \\ 0 & -3i & 5i \\ 0 & 0 & -5i \end{bmatrix} = \begin{bmatrix} -1\begin{bmatrix} -i & -4 & 2 \\ 0 & -3i & 5i \\ 0 & 0 & -5i \end{bmatrix} & 3i\begin{bmatrix} -i & -4 & 2 \\ 0 & -3i & 5i \\ 0 & 0 & -5i \end{bmatrix} \\ -i\begin{bmatrix} -i & -4 & 2 \\ 0 & -3i & 5i \\ 0 & 0 & -5i \end{bmatrix} & 1\begin{bmatrix} -i & -4 & 2 \\ 0 & -3i & 5i \\ 0 & 0 & -5i \end{bmatrix} \end{bmatrix} =$$

$$= \begin{bmatrix} i & 4 & -2 & 3 & -12i & 6i \\ 0 & 3i & -5i & 0 & 9 & -15 \\ 0 & 0 & 5i & 0 & 0 & 15 \\ -1 & 4i & -2i & -i & -4 & 2 \\ 0 & -3 & 5 & 0 & -3i & 5i \\ 0 & 0 & -5 & 0 & 0 & -5i \end{bmatrix}$$

$$A \otimes Z = \begin{bmatrix} -i & -4 & 2 \\ 0 & -3i & 5i \\ 0 & 0 & -5i \end{bmatrix} \otimes \begin{bmatrix} -1 & 3i \\ -i & 1 \end{bmatrix} = \begin{bmatrix} -i\begin{bmatrix} -1 & 3i \\ -i & 1 \end{bmatrix} & -4\begin{bmatrix} -1 & 3i \\ -i & 1 \end{bmatrix} & 2\begin{bmatrix} -1 & 3i \\ -i & 1 \end{bmatrix} \\ 0\begin{bmatrix} -1 & 3i \\ -i & 1 \end{bmatrix} & -3i\begin{bmatrix} -1 & 3i \\ -i & 1 \end{bmatrix} & 5i\begin{bmatrix} -1 & 3i \\ -i & 1 \end{bmatrix} \\ 0\begin{bmatrix} -1 & 3i \\ -i & 1 \end{bmatrix} & 0\begin{bmatrix} -1 & 3i \\ -i & 1 \end{bmatrix} & -5i\begin{bmatrix} -1 & 3i \\ -i & 1 \end{bmatrix} \end{bmatrix} =$$

$$= \begin{bmatrix} i & 3 & 4 & -12i & -2 & 6i \\ -1 & -i & 4i & -4 & -2i & 2 \\ 0 & 0 & 3i & 9 & -5i & -15 \\ 0 & 0 & -3 & -3i & 5 & 5i \\ 0 & 0 & 0 & 0 & 5i & 15 \\ 0 & 0 & 0 & 0 & -5 & -5i \end{bmatrix}$$