Quantitative metrics of performance for different localization algorithms

190115-239
*cpagniello@ucsd.edu

Camille M.L.S. Pagniello*, Gerald L. D'Spain, Jules S. Jaffe, and John A. Hildebrand

Scripps Institution of Oceanography, University of California San Diego, La Jolla, CA 92093, USA

Objective

- **compare the performance of four different localization algorithms:** (1) plane-wavefront frequency-domain beamforming, (2) curved-wavefront frequency domain beamforming, (3) time-difference-of-arrivals (TDOAs) using waveform cross-correlation, and (4) TDOAs using spectrogram cross-correlation
- **use bias** (equal to the mean of estimated positions minus the true position) **and variance** (variability in the measured positions about their mean) **of the estimated localizations**

Array Geometry

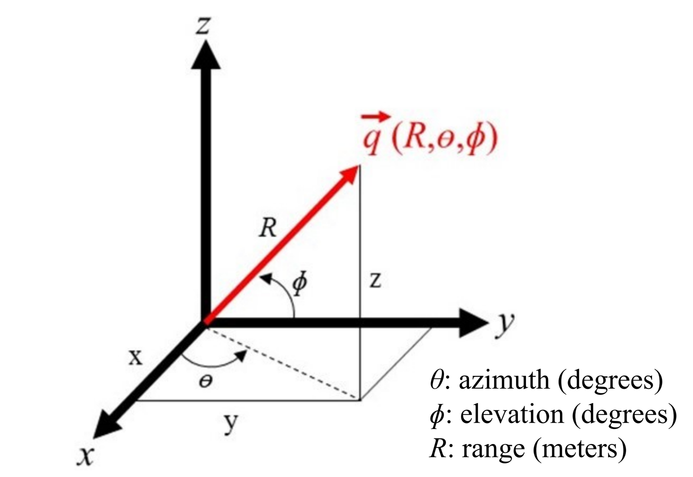


Figure 1. Right-handed coordinate system used for the localization scenario.

Passive Acoustic System:
- SoundTrap ST4300 four-channel recorder sampling at 48 kHz
- four HTI-96-MIN hydrophones arranged in the geometry shown in Figure 2 in approx. 14 m water depth

Controlled Source:
- Lubell Labs LL916C underwater loudspeaker
- played 210, 305, 395, 495, 610, 690, 805, 910 and 995 Hz tones at 4 playback locations

- position vectors $\vec{p_m},\ m = 0,\ldots,M-1$ are defined as:

$$\begin{cases} \vec{p_0} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}, \\ \vec{p_1} = \begin{bmatrix} \frac{d}{2} & \frac{-\sqrt{3}}{2}d & 0 \end{bmatrix}, \\ \vec{p_2} = \begin{bmatrix} \frac{d}{2} & \frac{\sqrt{3}}{2}d & 0 \end{bmatrix}, \\ \vec{p_3} = \begin{bmatrix} -d & 0 & 0 \end{bmatrix}, \end{cases}$$

where d = 20 m is the length of each arm of the array and M = 4 is the number of hydrophones

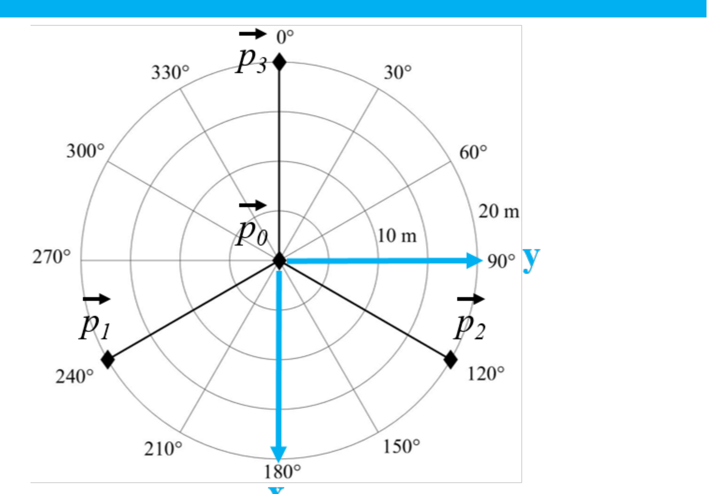


Figure 2. 2D configuration of passive acoustic system. Black diamonds are array element locations.

Localization Methods

Frequency-Domain Beamforming

$$K(\omega_i) = \frac{1}{L}\sum_{l=1}^{L} \vec{d_l}(\omega_i)\vec{d_l}^{\,*}(\omega_i)$$ is the cross-spectral density matrix where $\vec{d}(\omega_i)$ is the complex data vector for the ith frequency bin

1. replica vector for **plane-wavefront**

$$\vec{r}_m(\omega,\theta,\phi) = \exp\left(i\frac{2\pi f}{c}\vec{p}_m \cdot \vec{q}\right)$$

where the source position vector is $\vec{q}(\theta_s,\phi_s) = \begin{bmatrix} \cos(\theta_s)\cos(\phi_s) \\ \sin(\theta_s)\cos(\phi_s) \\ \sin(\phi_s) \end{bmatrix}$

2. replica vector for **curved-wavefront**

$$\vec{r}_m(\omega,\theta,\phi,R) = \frac{1}{\|\vec{q}-\vec{p}_m\|}\exp\left(i\frac{2\pi f}{c}\|\vec{q}-\vec{p}_m\|\right)$$

where the source position vector is $\vec{q}(\theta_s,\phi_s,R_s) = \begin{bmatrix} R_s\cos(\theta_s)\cos(\phi_s) \\ R_s\sin(\theta_s)\cos(\phi_s) \\ R_s\sin(\phi_s) \end{bmatrix}$

Output Magnitude Squared of Bartlett Beamformer

$$B_{Bartlett}(\omega) = \vec{r}^{\,*}K(\omega)\vec{r}$$

Time-Domain Beamforming

3. TDOAs from **waveform** cross-correlation

$$C(l) = \sum_{j=0}^{J-1} x(j)y(j-l),\ \text{over range of } l$$

4. TDOAs from **spectrogram** cross-correlation

$$C(k,l) = \sum_{i=0}^{I-1}\sum_{j=0}^{J-1} S_x(i,j)S_y(i-k,j-l),\ \text{over range of } k \text{ and } l$$

measured TDOA

$$\Delta t_{nm} = \frac{\|\vec{p}_m - \vec{p}_n\|}{c}$$

calculated TDOA

$$\Delta\hat{t}_{nm}(\vec{q}) = \hat{t}_n(\vec{q}) - \hat{t}_m(\vec{q})$$

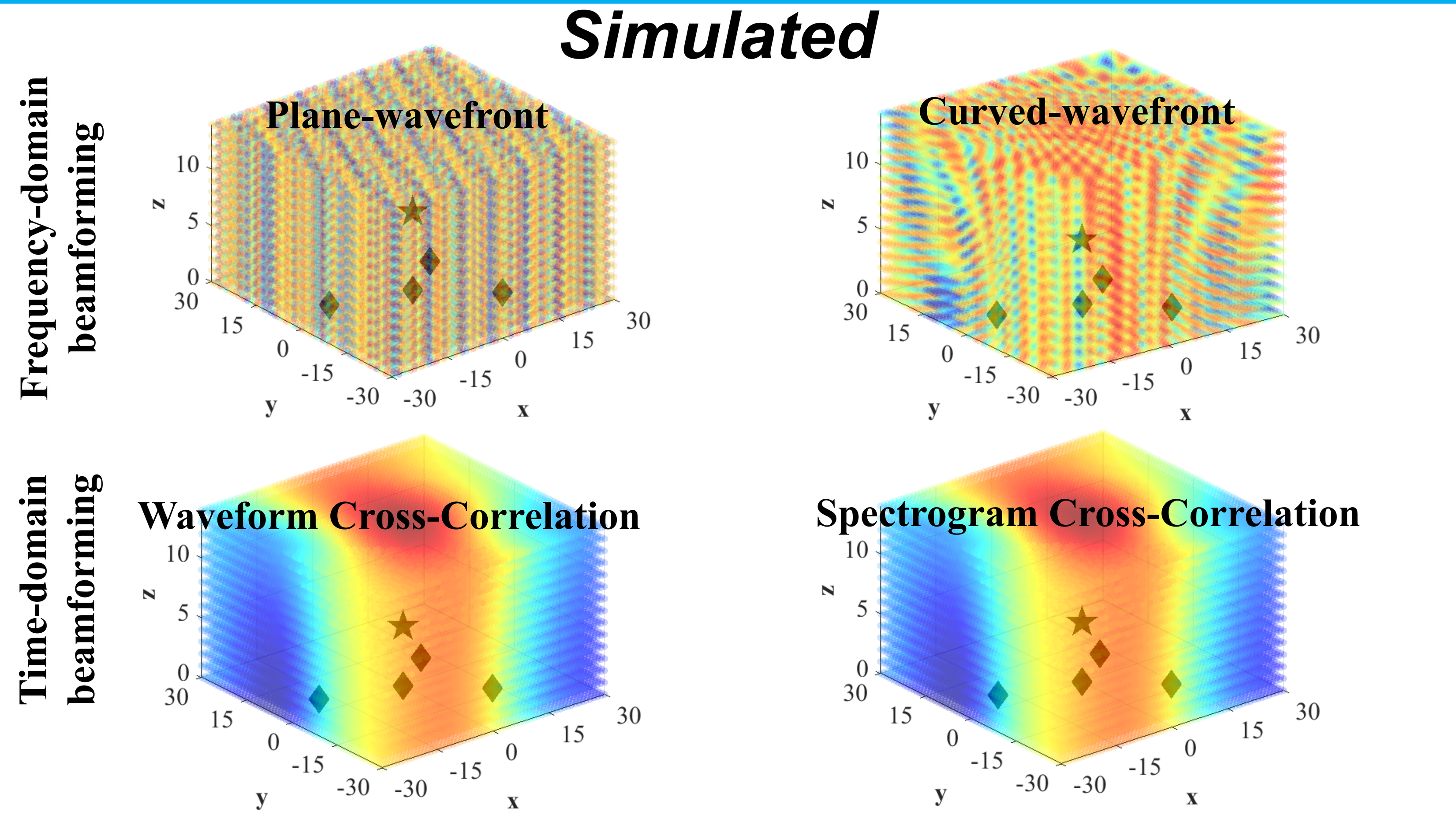
where $\hat{t}_m(\vec{q}) = \frac{\|\vec{q}-\vec{p}_m\|}{c}$

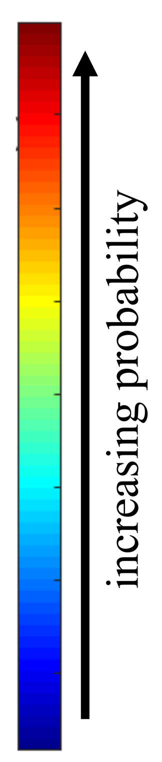
Maximum Likelihood Estimate of the True Position

$$LS(\vec{q}) \propto \prod_{n,m}\left\{\max_k\left(\exp\left[\frac{-1}{2\sigma_{nm}^2}\left(\Delta t_{nm}(k) - \Delta\hat{t}_{nm}(\vec{q})\right)^2\right]\right)\right\}$$

Spatial Response of Array

Simulated





Experimental

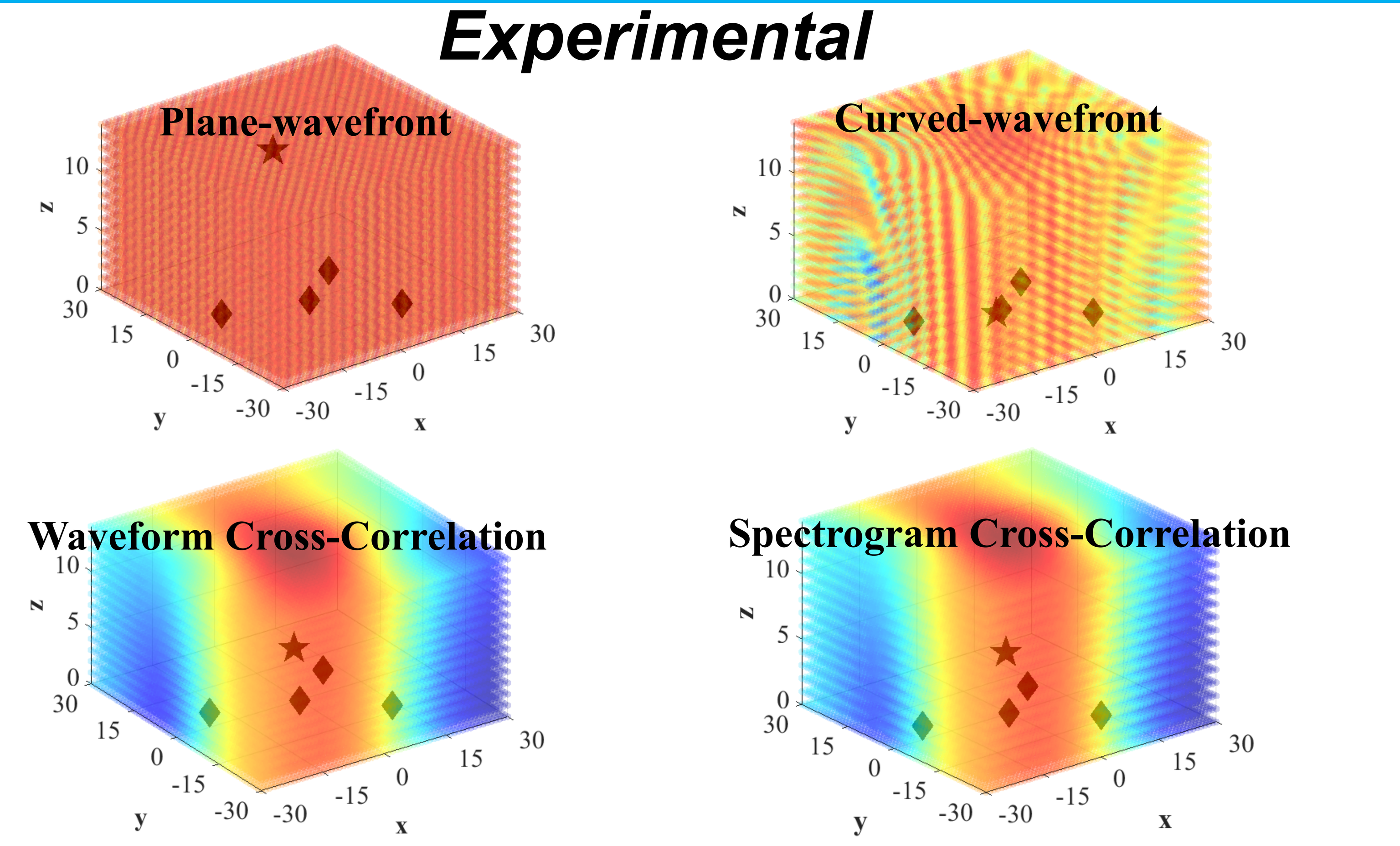


Figure 3. Ambiguity surface for simulated 610 Hz source at (x,y,z) = (0,0,5) (star). Diamonds are array element locations.

Figure 4. Ambiguity surfaces for 610 Hz source at (x,y,z) = (0,0,5) and estimated position (star). Diamonds are array element locations.

Histograms of Localization Error from Experimental Data

Frequency-Domain Beamforming

Time-Domain Beamforming

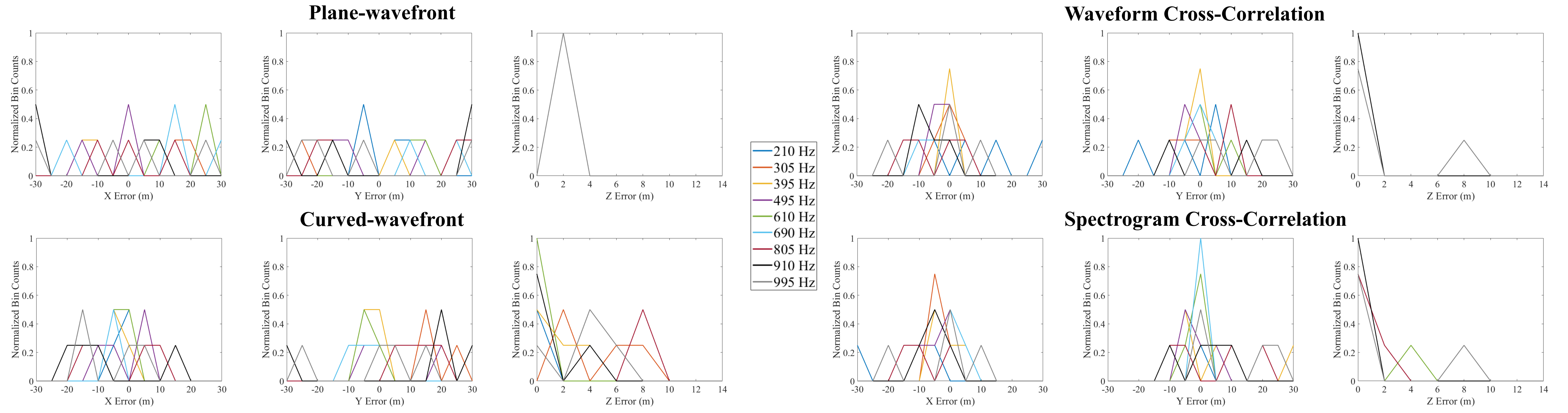


Figure 5. x-, y- and z-position estimation performance results. Error distributions were calculated by subtracting the GPS-derived position from the beamformer-derived position.

Conclusions

- bias and variance in estimated positions derived from localization not only depend on source position and frequency, but also localization method
- for this array design and singular, low-frequency tones, TDOAs using spectrogram cross-correlation minimizes error in the estimated position derived from localization

Next Steps

- more realizations (in terms of x, y and z positions) of underwater loudspeaker playback of tones and sweeps to characterize localization algorithm error within a 60 m^2 area
- implement filtering and interpolation methods to increase accuracy of TDOA estimates
- localize biological sounds