Impact of Spatial Filtering with a Pinhole on Wavefront Errors and the Resulting Null in Nulling Interferometers

Semester Project

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1 Wavefront Errors

Wavefront errors can be modeled using the Zernike polynomials. Zernike polynomials are defined as

$$Z_n^m(u,v) = R_n^{|m|}(u,v) \begin{cases} \cos\left(m \operatorname{arctan2}(v,u)\right) & m \geqslant 0 \\ \sin\left(|m| \operatorname{arctan2}(v,u)\right) & m < 0 \end{cases}, \tag{2.1}$$

where the radial part is given by

$$R_n^m(u,v) = \begin{cases} \sum_{k=0}^{\frac{n-m}{2}} \frac{(-1)^k (n-k)!}{k!\left(\frac{n+m}{2}-k\right)!\left(\frac{n-m}{2}-k\right)!} \left(\frac{u^2+v^2}{u_{\max}^2+v_{\max}^2}\right)^{\frac{n}{2}-k} & n-m \text{ even} \\ 0 & n-m \text{ odd} \\ 1 & u^2+v^2 = u_{\max}^2+v_{\max}^2 \end{cases}. \tag{2.2}$$

Wavefront errors can then be modeled as a sum of multiple Zernike polynomials, i. e.

$$\Delta W(u,v) = \sum_{n,m} z_n^m Z_n^m(u,v), \tag{2.3}$$

where z_n^m are coefficients of units length.

2 Electric Field After Aperture

Essentially, the following will be a derivation of Fraunhofer diffraction for the case of an aberrated wavefront. The amplitude of the electric field of a monochromatic, spherical wavefront at time t and distance r from its source and subject to a wavefront error $\Delta W(u,v)$ is given by

$$E(t,r,u,v) = \frac{E_0}{r} e^{\frac{2\pi i}{\lambda}(ct-r-\Delta W(u,v))},$$

where λ is the wavelength, c the speed of the wave and E_0 the initial amplitude. Note that an ideal wavefront can be modeled similarly simply by setting $\Delta W(u,v)$ to zero. Now, if planar waves hit an aperture the aperture itself can be interpreted as the source of a wavefront. Consider the setup illustrated in Figure 2.1. To calculate the electric

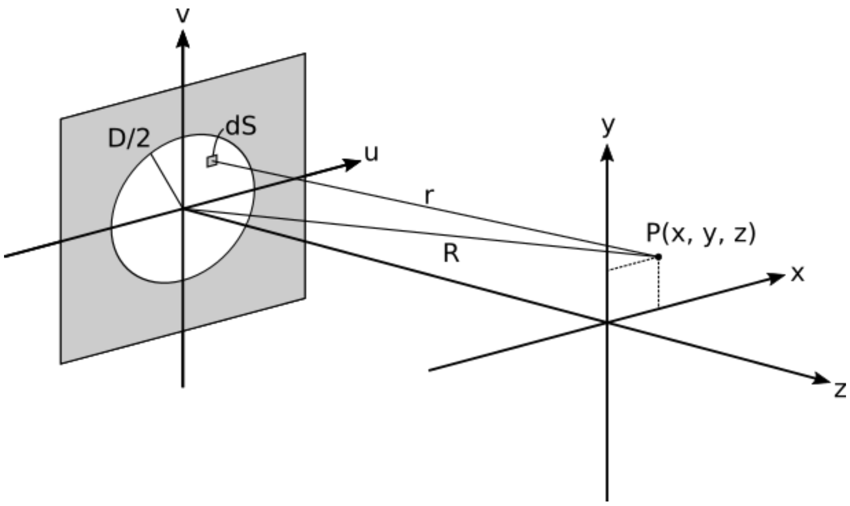

Figure 2.1: Illustration of the setup with one aperture.

field at a point $P(x, y, z)$ after such an aperture, the following manipulations will become useful. The distance R from the center of the aperture to the point P can simply be written as

$$R = \sqrt{x^2 + y^2 + z^2}. \tag{2.4}$$

For the distance of the infinitesimal surface element $\mathrm{d}S$ to point P we find

$$\begin{aligned} r &= \sqrt{z^2 + (x-u)^2 + (y-v)^2} && (2.5)\\ &= R\left(1 - \frac{2(ux+vy)}{R^2}\right)^{1/2} && (2.6)\\ &\approx R\left(1 - \frac{ux+vy}{R^2}\right) && (2.7)\\ &\approx R, && (2.8) \end{aligned}$$

where in (2.6) we have used $R \gg D/2$, in (2.7) we have performed a Taylor series expansion and only kept the two first terms and in (2.8) we have assumed that $R \gg x, y$. The amplitude of the electric field at point $P(x, y, z)$ after the aperture, originating from the surface element $\mathrm{d}S$ within the aperture can now be calculated as

$$\begin{aligned} \mathrm{d}E'(t, r, u, v) &= A(u, v)E(t, r, u, v)\,\mathrm{d}u\,\mathrm{d}v && (2.9)\\ &\approx A(u, v)\frac{E_0}{R}e^{\frac{2\pi i}{\lambda}\left(ct - R + \frac{ux+vy}{R} - \Delta W(\rho,\theta)\right)}\mathrm{d}u\,\mathrm{d}v && (2.10)\\ &\approx \frac{E_0}{R}e^{\frac{2\pi i}{\lambda}(ct-R)}A(u, v)e^{\frac{2\pi i}{\lambda}\left(\frac{ux+vy}{R} - \Delta W(u,v)\right)}\mathrm{d}u\,\mathrm{d}v && (2.11) \end{aligned}$$

3 Electric Field After Aperture and Pinhole