

Haseloff Conversion from Temperature Independent to Temperature Dependent

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1 Velocity

1.1 Governing Equations

The governing equations for velocity remain untouched when changing from independent to dependent:

$$\begin{aligned}\frac{\partial}{\partial x} \left(\mu \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right) &= 0 \\ \frac{\partial}{\partial x} \left(2\mu \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial y} \left[\mu \left(\frac{\partial v}{\partial y} + \frac{\partial w}{\partial x} \right) \right] - \frac{\partial p}{\partial x} &= 0 \\ \frac{\partial}{\partial x} \left[\mu \left(\frac{\partial v}{\partial y} + \frac{\partial w}{\partial x} \right) \right] + \frac{\partial}{\partial y} \left(2\mu \frac{\partial w}{\partial y} \right) - \frac{\partial p}{\partial y} &= 0 \\ \frac{\partial v}{\partial x} + \frac{\partial w}{\partial y} &= 0\end{aligned}$$

1.2 Viscosity

There are two options with how to convert the viscosity equation from temperature independent to temperature dependent:

1. Use the viscosity equation presented in Suckale et al. 2014
2. Change only the parameter A from constant to variable

1.2.1 Suckale et al. 2014 Viscosity Model

Suckale presents μ as:

$$\mu = \frac{1}{2} \left(D_c + \frac{1}{3} G_c \tau_E^2 \right)^{-1}$$

Which relies on the three variables D_c , G_c , and τ_E :

$$\begin{aligned}D_c &= \frac{42\Omega B}{k_B T d^2} \exp \left(-\frac{Q}{RT} \right) \\ G_c &= A E \exp \left[-\frac{Q}{R} \left(\frac{1}{T} - \frac{1}{T^*} \right) \right] \tau_E^2 \\ \tau_E &= \sqrt{\frac{1}{3} \left[\frac{F}{\sqrt[3]{2} G_c^2} + \frac{\sqrt[3]{2} D_c^2}{F} - 2 \frac{D_c}{G_c} \right]}\end{aligned}$$

These, in turn, introduce two additional variables: F and Q

$$F = \sqrt[3]{27\epsilon_E^2 G_c^4 + 3\sqrt{3(27\epsilon_E^4 G_c^8 + 4\epsilon_E^2 D_c^3 G_c^7)} + 2D_c^3 G_c^3}$$

$$Q = Q_l + \frac{Q_h - Q_l}{2} \text{heavi}(T - T^*)$$

1.2.2 Change to a Variable form of A

$$A(T) = A \exp \left[-\frac{Q}{R} \left(\frac{1}{T} - \frac{1}{T^*} \right) \right]$$

Which requires Q as above:

$$Q = Q_l + \frac{Q_h - Q_l}{2} \text{heavi}(T - T^*)$$

1.3 Boundary Conditions and Additional Equations

All other equations and all boundary conditions remain unchanged.

2 Temperature

2.1 Governing Equation

The governing equation for temperature will not change when conversion takes place unless the temperature dependent forms of k and c_p are used:

$$k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) = \rho c_p \left(v_m \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial x} + w \frac{\partial T}{\partial y} \right) - a$$

With temperature dependent k (and optional c_p):

$$\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) = \rho c_p \left(v_m \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial x} + w \frac{\partial T}{\partial y} \right) - a$$

2.2 Shear Heating a

Again, there are two options for converting shear heating:

1. Use the Suckale model $a = 2\tau_E \dot{\epsilon}_E$
2. Change only parameter A from constant to variable

2.2.1 Suckale et al. 2014 Shear Heating Model $a = 2\tau_E \dot{\epsilon}_E$

The shear heating presented in Suckale's paper requires τ_E and $\dot{\epsilon}_E$:

$$\dot{\epsilon}_E = \frac{1}{2} \sqrt{\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial u}{\partial y} \right)^2} \quad (1)$$

with τ_E as in the viscosity model. It could also be important to update the equation for $\dot{\epsilon}_E$ to include the advection terms:

$$\dot{\epsilon}_E = \left(\left| \frac{\partial u}{\partial y} \right|^2 + \left| \frac{\partial u}{\partial z} \right|^2 + \left| \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right|^2 + 2 \left| \frac{\partial v}{\partial y} \right|^2 + 2 \left| \frac{\partial w}{\partial z} \right|^2 \right)$$

2.2.2 Change to a Variable form of A

As above.

2.3 Boundary Conditions and Additional Equations

All other equations and all boundary conditions remain unchanged (except possibly with a temperature dependent k).