Nondimensionalization of Haseloff et al. 2018

Pierce Hunter

1/24/2019

1 Definitions

A few of the scales jump out right away:

$$[z] = h_s \qquad [T] = T_m - T_s$$

These will be used in the following:

$$y = [z]\,Y \qquad z = [z]\,Z$$
$$T = [T]\,\mathcal{T} + T_s$$

Note: Haseloff et al. reads: $T = [T]\,\mathcal{T} + T_m$ which is incorrect.

The three velocity scales can also be defined:

$$u = [u]\,U \qquad v = [v]\,V \qquad w = [v]\,W$$

I will, however, make no assumptions in regards to the values of these scales.

$$[s'] = \frac{n+2}{n+1}\frac{q_r}{A\tau_s^n h_s^2}\frac{\tau_s}{\rho g} \qquad [u] = A h_s \tau_s^n$$
$$[v] = \frac{n+2}{n+1}\frac{q_r}{h_s}$$
$$s' = [s']\,S' \qquad p = \rho g\,[s']\,P$$
$$\alpha = \frac{A\tau_s^{n+1}h_s^2}{k\,(T_m - T_b)} \qquad Pe = \frac{n+2}{n+1}\frac{\rho c_p q_r}{k} \qquad \nu = \frac{T_b - T_s}{T_m - T_s}$$
$$\tau = \frac{\tau_c}{\tau_s} \qquad \varepsilon = \frac{n+2}{n+1}\frac{q_r}{A\tau_s^n h_s^2}$$

2 Nondimensionalization of Equations

2.1 $4 \to 24$

$$\eta = \frac{A^{-1/n}}{2^{1/n}}\left[\left|\frac{\partial u}{\partial y}\right|^2 + \left|\frac{\partial u}{\partial z}\right|^2 + \left|\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right|^2 + 2\left|\frac{\partial v}{\partial y}\right|^2 + 2\left|\frac{\partial w}{\partial z}\right|^2\right]^{\frac{1-n}{2n}}$$

$$\frac{A^{-1/n}}{2^{1/n}}\left[\frac{[u]^2}{[z]^2}\left(\left|\frac{\partial U}{\partial Y}\right|^2 + \left|\frac{\partial U}{\partial Z}\right|^2\right) + \frac{[v]^2}{[z]^2}\left(\left|\frac{\partial V}{\partial Z} + \frac{\partial W}{\partial Y}\right|^2 + 2\left|\frac{\partial V}{\partial Y}\right|^2 + 2\left|\frac{\partial W}{\partial Z}\right|^2\right)\right]^{\frac{1-n}{2n}}$$

$$\frac{A^{-1/n}}{2^{1/n}}\left(\frac{[u]}{[z]}\right)^{\frac{1-n}{n}}\left[\left|\frac{\partial U}{\partial Y}\right|^2 + \left|\frac{\partial U}{\partial Z}\right|^2 + \frac{[v]^2}{[u]^2}\left(\left|\frac{\partial V}{\partial Z} + \frac{\partial W}{\partial Y}\right|^2 + 2\left|\frac{\partial V}{\partial Y}\right|^2 + 2\left|\frac{\partial W}{\partial Z}\right|^2\right)\right]^{\frac{1-n}{2n}}$$

Which for now can be written as: $[\eta]\,\mu$

2.2 2 → 22

$$
\begin{gathered}
\frac{\partial}{\partial y}\left(\eta\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial z}\left(\eta\frac{\partial u}{\partial z}\right)=0 \\
\frac{1}{[z]}\frac{\partial}{\partial Y}\left([\eta]\,\mu\frac{[u]}{[z]}\frac{\partial U}{\partial Y}\right)+\frac{1}{[z]}\frac{\partial}{\partial Z}\left([\eta]\,\mu\frac{[u]}{[z]}\frac{\partial U}{\partial Z}\right)=0 \\
\frac{[\eta]\,[u]}{[z]^2}\left[\frac{\partial}{\partial Y}\left(\mu\frac{\partial U}{\partial Y}\right)+\frac{\partial}{\partial Z}\left(\mu\frac{\partial U}{\partial Z}\right)\right]=0 \\
\frac{\partial}{\partial Y}\left(\mu\frac{\partial U}{\partial Y}\right)+\frac{\partial}{\partial Z}\left(\mu\frac{\partial U}{\partial Z}\right)=0
\end{gathered}
$$

2.3 3a → 23a

$$
\frac{\partial}{\partial y}\left(\right)
$$