Haseloff Conversion from Temperature Independent to Temperature Dependent

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1 Velocity

1.1 Governing Equations

The governing equations for velocity remain untouched when changing from independent to dependent:

$$\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)=0$$
$$\frac{\partial}{\partial x}\left(2\mu\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial v}{\partial y}+\frac{\partial w}{\partial x}\right)\right]-\frac{\partial p}{\partial x}=0$$
$$\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial v}{\partial y}+\frac{\partial w}{\partial x}\right)\right]+\frac{\partial}{\partial y}\left(2\mu\frac{\partial w}{\partial y}\right)-\frac{\partial p}{\partial y}=0$$
$$\frac{\partial v}{\partial x}+\frac{\partial w}{\partial y}=0$$

1.2 Viscosity

There are two options with how to convert the viscosity equation from temperature independent to temperature dependent:

1. Use the viscosity equation presented in Suckale et al. 2014
2. Change only the parameter A from constant to variable

1.2.1 Suckale et al. 2014 Viscosity Model

Suckale presents μ as:

$$\mu=\frac{1}{2}\left(D_c+\frac{1}{3}G_c\tau_E^2\right)^{-1}$$

Which relies on the three variables D_c, G_c, and τ_E:

$$D_c=\frac{42\Omega B}{k_B T d^2}\exp\left(-\frac{Q}{RT}\right)$$
$$G_c=AE\exp\left[-\frac{Q}{R}\left(\frac{1}{T}-\frac{1}{T^*}\right)\right]\tau_E^2$$
$$\tau_E=\sqrt{\frac{1}{3}\left[\frac{F}{\sqrt[3]{2}G_c^2}+\frac{\sqrt[3]{2}D_c^2}{F}-2\frac{D_c}{G_c}\right]}$$

These, in turn, introduce two additional variables: F and Q

$$F = \sqrt[3]{27\dot{\epsilon}_E^2 G_c^4 + 3\sqrt{3\left(27\dot{\epsilon}_E^4 G_c^8 + 4\dot{\epsilon}_E^2 D_c^3 G_c^7\right)} + 2D_c^3 G_c^3}$$
$$Q = Q_l + \frac{Q_h - Q_l}{2}\text{heavi}\left(T - T^*\right)$$

1.2.2 Change to a Variable form of A

$$A\left(T\right) = A\exp\left[-\frac{Q}{R}\left(\frac{1}{T} - \frac{1}{T^*}\right)\right]$$

Which requires Q as above:

$$Q = Q_l + \frac{Q_h - Q_l}{2}\text{heavi}\left(T - T^*\right)$$

1.3 Boundary Conditons and Additional Equations

All other equations and all boundary conditions remain unchanged.

2 Temperature

2.1 Governing Equation

The governing equation for temperature will not change when conversion takes place unless the temperature dependent forms of k and c_p are used:

$$k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) = \rho c_p\left(v_m\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial x} + w\frac{\partial T}{\partial y}\right) - a$$

With temperature dependent k (and optional c_p):

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) = \rho c_p\left(v_m\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial x} + w\frac{\partial T}{\partial y}\right) - a$$

2.2 Shear Heating a

Again, there are two options for converting shear heating:

1. Use the Suckale model $a = 2\tau_E\dot{\epsilon}_E$
2. Change only parameter A from constant to variable

2.2.1 Suckale et al. 2014 Shear Heating Model $a = 2\tau_E\dot{\epsilon}_E$

The shear heating presented in Suckale's paper requires τ_E and $\dot{\epsilon}_E$:

$$\dot{\epsilon}_E = \frac{1}{2}\sqrt{\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2} \tag{1}$$

with τ_E as in the viscosity model. It could also be important to update the equation for $\dot{\epsilon}_E$ to include the advection terms:

$$\dot{\epsilon}_E = \sqrt{\left|\frac{\partial u}{\partial y}\right|^2 + \left|\frac{\partial u}{\partial z}\right|^2 + \left|\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right|^2 + 2\left|\frac{\partial v}{\partial y}\right|^2 + 2\left|\frac{\partial w}{\partial z}\right|^2}$$

2.2.2 Change to a Variable form of A

As above.

2.3 Boundary Conditions and Additional Equations

All other equations and all boundary conditions remain unchanged (except possibly with a temperature dependent k).