

Review: Build a Sporadic Group in Your Basement

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 3. existence of identity: $\exists e \in G$ s.t. $a * e = e * a = a \quad \forall a \in G$
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Examples: $(\mathbb{Z}, +)$

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A *sporadic group* is a special kind of finite simple group.

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 1. Mathieu Groups $M_{11}, M_{12}, M_{22}, M_{23}, M_{24}$

The Periodic Table Of Finite Simple Groups

- Alternating Groups
- Classical Chevalley Groups
- Chevalley Groups
- Classical Steinberg Groups
- Steinberg Groups
- Suzuki Groups
- Bee Groups and Tits Group
- Sporadic Groups
- Cyclic Groups

Alternatives ¹	Symbol	Order ²
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M_{11}	M_{12}	M_{22}	M_{23}	M_{24}	I_1	I_2	I_3	I_4	HS	McL	He	Ru
7920	55 040	443 520	10 200 960	244 825 040	175 560	604 800	50 232 960	86 773 871 066 677 961 960	44 352 000	898 128 000	4 006 307 200	145 926 544 000

^aFor sporadic groups and families, alternate names in the upper left; no other names by which they may be known. For quasisimple sporadic groups these are used to indicate isomorphism. All such isomorphisms appear in the table except the family $A_n(2^f) \cong G_2(2^f)$.

[†]The Tio-group 2U_4H is not a group of Lie type, but is the (order 2-)commutator subgroup of ${}^2U_4(2)$.

The groups starting on the second row are the classical groups. The sporadic Suzuki group is unrelated to the families of Suzuki groups.

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S_z	O'N, O-S	-3	-2	-1	$F_0 D$	$L_3 S$	$F_0 E$	M(22)	M(23)	$F_{12}, M(24)^a$	F_3	$F_0 M_1$
S_{WZ}	O'N	Co_3	Co_2	Co_1	HN	L_y	Th	F_{122}	F_{123}	F'_{12}	B	M
448 547 497 480	4185 585 519	40576 636 808	42 305 421 312 804	4130776 606	233 638	311 763 179	90 745 943	280 489 479	283 205 789 190	663 731 120 800	4 108 766 606 600	4080 766 606 600
				543 500 000	912 800 000	90 768 000	107 872 000	44 761 751 654 400	2 001 000 000			4080 766 606 600

What is the Mathieu Group M_{24}

different

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