Review: Build a Sporadic Group in Your Basement

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February 28, 2022

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  2. associativity: $a * (b * c) = (a * b) * c \quad \forall a, b, c \in G$
  3. existence of identity: $\exists e \in G$ s.t. $a * e = e * a = a \; \forall a \in G$
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Examples: $(\mathbb{Z}, +)$

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A *sporadic group* is a special kind of finite simple group.

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   1. Mathieu Groups $M_{11}, M_{12}, M_{22}, M_{23}, M_{24}$

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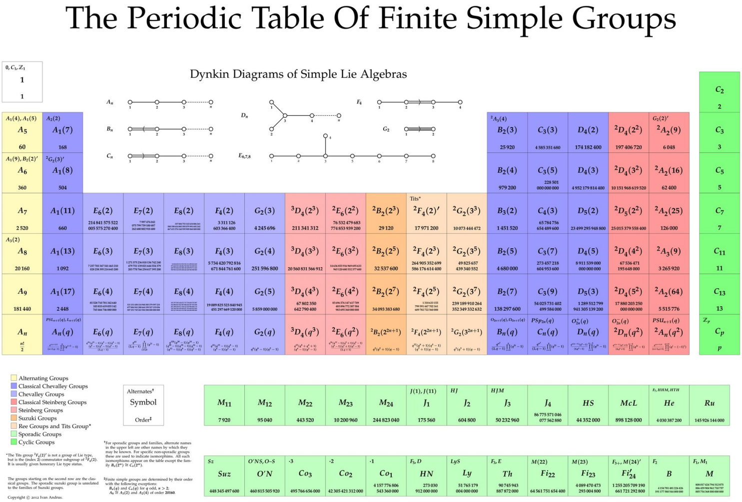

What is the Mathieu Group M_{24}

different

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