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Working Memory Strategies During Rational Number Magnitude Processing

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Rational number understanding is a critical building block for success in more advanced mathematics; however, how rational number magnitudes are conceptualized is not fully understood. In the current study, we used a dual-task working memory (WM) interference paradigm to investigate the dominant type of strategy (i.e., requiring verbal WM resources vs. requiring primarily visuospatial WM resources) used by adults when processing rational number magnitudes presented in both decimal and fraction notation. Analyses revealed no significant differences in involvement of verbal and visuospatial WM, regardless of notation (fractions vs. decimals), indicating that adults rely upon a mix of strategies and WM resources when processing rational number magnitudes. However, this pattern interacted with algebra ability such that those performing better on the algebra assessment relied upon both verbal and visuospatial WM when engaging in rational number comparisons, whereas rational number performance by adults with low algebra fluency was affected only by a simultaneous verbal WM task. Together, results support previous work implicating the involvement of WM resources in rational number processing and is the first study to indicate that the involvement of both verbal and visuospatial WM, as opposed to relying primarily on verbal WM, when processing rational number magnitudes may be indicative of higher mathematical proficiency in the domain of algebra.

Keywords: working memory, rational numbers, fractions, decimals, algebra

An understanding of rational number concepts has been shown to be critical for further math learning. For example, early fraction and decimal knowledge is a unique predictor of arithmetic ability and general math achievement in elementary and middle school (e.g., Bailey, Hoard, Nugent, & Geary, 2012; Schneider, Grabner, & Paetsch, 2009), as well as algebra ability in older children and adults (e.g., Booth, Newton, & Twiss-Garrity, 2014; Hurst & Cordes, 2016b; Siegler et al., 2012). Although substantial evidence suggests that there is some relationship between algebra and rational number ability, what aspect of rational number knowledge is most critical and the mechanisms through which this relationship develops are only just beginning to be explored. Recent evidence has suggested that one of the critical aspects of fraction knowledge is an understanding of rational number magnitudes (Booth & Newton, 2012; Booth et al., 2014). For example, Booth et al. (2014) found that eighth graders' ability to map fractions onto number lines was predictive of improvement in their equation solving after an algebra course. However, processing rational number magnitudes is not a straightforward task, as it potentially involves distinct strategies, ranging from holistic processing (i.e., getting a feel for the numerical size of a value) to computational processing (i.e., transforming fractions into decimal values to get a sense for the size of the value). Yet no work has investigated whether specific rational number magnitude processing strategies may be stronger predictors of algebraic processing. Thus, in order to better understand the relationship between rational number magnitude understanding and algebra ability, we must also investigate how people go about processing rational number magnitude information and whether there are differences in how these magnitudes are understood across individuals with differing algebra abilities.

Rational Number Magnitudes

To investigate how people think about the magnitudes associated with symbolic numbers, researchers often use number comparison tasks. In these tasks, participants are asked to rapidly judge which of two numbers is greater. Work with whole numbers has revealed that performance on these tasks is predictive of more general math fluency (e.g., Holloway & Ansari, 2009) and correlated with math anxiety (e.g., Maloney, Ansari, & Fugelsang, 2011), suggesting that performance on these tasks can provide insight into how these values are processed. More recently, researchers have begun to use these tasks with other types of numbers like fractions and decimals, with results similarly revealing performance on these rational number magnitude comparisons predicting math ability in other domains (e.g., Hurst & Cordes, 2016b; Schneider et al., 2009; Siegler, Thompson, & Schneider, 2011).

However, little is known about the specific strategies children and adults may invoke to access the magnitudes associated with

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rational number notation. Some evidence suggests that in a rational number comparison task (e.g., "Which is larger [1/2] vs. [3/4]?"), adults are able to access magnitude information from both fractions and decimals (e.g., DeWolf, Grounds, Bassok, & Holyoak, 2014; Hurst & Cordes, 2016a; Schneider & Siegler, 2010), but only when they are prevented from using other component-based strategies (Bonato, Fabbri, Umiltà, & Zorzi, 2007). When explicitly asked to report their strategy use, Faulkenberry and Pierce (2011) found that adults' strategies could be grouped into one of five different categories (although a small percentage reported strategies that did not fit into one of these categories): just knowing it, cross-multiplication, benchmarking (e.g., comparing the values with [1/2]), visualization, and converting fractions into decimals. Although many of these strategies involve understanding magnitude, they may also involve other procedures—including arithmetic and calculation. Given that fractions and decimals are complicated symbols that involve a combination of Arabic numerals and non-numeric symbols (i.e., the vinculum, or dividing line, in fractions and the decimal point in decimals), it may not be surprising that some adults engage in calculation-based strategies (i.e., cross-multiplication, converting fractions into decimals, and possibly benchmarking). In addition, even the strategy of "just knowing it" (the single strategy with the highest reported use—30.7% of trials; Faulkenberry & Pierce, 2011) may have encompassed more than one type of implicit strategy, including ones the participants could not readily describe using self-report. Thus, given that fraction and decimal magnitudes may be interpreted and processed in different ways, and that fraction and decimal magnitude understanding is related to algebra ability, it is important to explore whether differences in how rational number magnitudes are approached may be related to algebra ability. However, because self-report may not be the most accurate way to assess strategies, and because the reporting of strategies on each trial could potentially impact the future use of those strategies within the task, it is ideal to investigate rational number magnitude strategies using implicit measures.

Working Memory

To implicitly assess rational number processing strategies, the current study explored how distinct components of WM (i.e., the phonological loop [i.e., verbal WM] and the visuospatial sketchpad; Baddeley, 1992; Baddeley, 2012; Baddeley & Hitch, 1974) are implicated during a rational number magnitude task. Although studies have identified a relationship between WM capacity and rational number processing abilities (e.g., Jordan et al., 2013; Vukovic et al., 2014), no studies have explored how these distinct components of WM may individually contribute to rational number magnitude processing. Importantly, understanding the involvement of these distinct components of WM during mathematical tasks can provide insight into the nature of the strategies invoked when performing these tasks (e.g., Caviola, Mammarella, Cornoldi, & Lucangeli, 2012; DeStefano & LeFevre, 2004; Raghubar, Barnes, & Hecht, 2010).

Significant research has explored how these WM components are implicated in other numerical and math tasks, such as mental arithmetic, revealing distinct patterns of involvement for the phonological loop and the visuospatial sketchpad. Given that the phonological loop is thought to be involved in temporarily storing verbal information in memory (Baddeley, 1992, 2012; Baddeley & Hitch, 1974), it is not surprising that the phonological loop is implicated in mental arithmetic tasks in which verbal strategies are invoked, such as when children use counting strategies and/or perform calculations that involve maintaining operands or an interim solution (DeStefano & LeFevre, 2004). Even in adults, verbal WM has been shown to be involved in solving complex arithmetic problems (Hitch, 1978). The visuospatial sketchpad, on the other hand, is deemed responsible for maintaining visual information in memory, including creating mental pictures and diagrams (Baddeley, 1992, 2012; Baddeley & Hitch, 1974). Thus, the visuospatial sketchpad has been found to play a role when mental transformation of the problem may be necessary for solving the problem (e.g., carrying in multidigit addition presented vertically; Caviola et al., 2012). In addition, theories positing that individuals use a "mental blackboard" to solve mathematical problems (e.g., Hayes, 1972) suggest that the visuospatial sketchpad may be involved to some extent in most situations of arithmetic, although the specific role of the visuospatial sketchpad in mental arithmetic, and particularly complex arithmetic, is unclear (e.g., Hubber, Gilmore, & Cragg, 2014).

Other work reveals that numerical magnitudes themselves, specifically for whole numbers, are visuospatially encoded in both adults and children (Simmons, Willis, & Adams, 2012; van Dijck, Gevers, & Fias, 2009). This is consistent with findings from other tasks suggesting that both children and adults represent whole numbers along a spatially encoded mental number line (e.g., Dehaene, Bossini, & Giraux, 1993; Moyer & Landauer, 1967, 1973), which may suggest that magnitude processing—distinct from mental arithmetic—may rely primarily on visuospatial WM and only minimally involve verbal WM. Whether this is also the case for rational numbers—whose magnitudes can be accessed through visualization (i.e., envisioning a pie chart), through verbally based strategies (e.g., direct computation, such as converting a fraction to a decimal or step-by-step digit comparisons in decimals), or a mix of strategies (e.g., estimating on a number line using place value or spatially demanding computations, such as cross-multiplication)—is an open question.

To investigate how distinct WM components may be implicated in rational number processing, in the current study, we employed a dual-task WM paradigm, which involves performing a primary task of interest (e.g., number comparison) while performing a secondary task that intentionally taxes WM resources (e.g., remembering four letters). The dual-task WM paradigm has been used to investigate how these various components of WM may be implicated during the primary task in order to identify implicit strategies involved (e.g., DeStefano & LeFevre, 2004; Raghubar et al., 2010). Importantly, the assumption of dual-task paradigms is that if processing in both the primary and the secondary tasks rely upon the same cognitive resources (e.g., verbal WM), then performance on the primary task will be impaired in the dual-task paradigm relative to that of a single-task control. On the other hand, if both tasks can be performed simultaneously without any interference, then they must not rely upon the same cognitive resources.

Given that symbolic notation for both fraction and decimal magnitudes involve both numeric (i.e., Arabic numerals) and non-numeric (i.e., decimal point, vinculum/division bar) symbols, there is reason to expect the involvement of both visuospatial and verbal resources when adults conceive of rational number magnitudes. On the one hand, given the rampant use of visual representations in the classroom when teaching rational numbers, coupled with evidence suggesting that fractions and decimals are spatially encoded (Faulkenberry & Pierce, 2011; Hurst & Cordes, 2016a; Schneider & Siegler, 2010), values in fraction and decimal notation may be processed holistically as magnitudes, without engaging explicit computations (e.g., envisioning a pie chart and/or as values falling along a line). If so, then rational number magnitudes should be primarily visuospatially encoded, only minimally requiring the use of the phonological loop. Alternatively, fraction notation implies the division of two whole numbers and decimals involve multiple components (i.e., values before and after the decimal point), making interpretation of the magnitudes associated with these symbols less transparent. In turn, this might suggest that both fraction and decimal magnitudes may be only accessible via direct computation and/or component-to-component comparisons (e.g., cross-multiplication, comparing values in the tenths digit, then in the hundredths, and so on). If this is the case, then processing of these values should require a greater reliance upon the phonological loop to maintain interim solutions in the calculation for each comparison.

Notation Differences

Furthermore, although fraction and decimal notation are used to represent the same numerical magnitudes, processing of values in these distinct notations may not rely on the same WM resources. It has been argued that decimal notation is more similar to whole numbers (e.g., Johnson, 1956), and recent evidence suggests that magnitudes are more easily accessed in decimal notation relative to fraction notation (e.g., DeWolf et al., 2014; Hurst & Cordes, 2016a). If so, then judgments of decimal magnitudes (as opposed to fraction magnitudes) may be more likely to be spatially encoded (similar to whole-number magnitudes), and thus rely primarily upon visuospatial WM, whereas fraction magnitudes may reveal a greater reliance upon verbal resources (reflecting increased computations, i.e., translating into decimal notation). Alternatively, evidence suggests that adults conceive of fraction and decimal magnitudes as falling along a single integrated mental continuum (Hurst & Cordes, 2016a), suggesting that underlying similarities in the numerical concepts these distinct notations represent may be salient to adults. If so, then these symbolic systems may receive similar treatment, resulting in consistent strategies employed across notations.

In addition, if distinct strategies are employed when comparing magnitudes exclusively in decimal notation (by comparing two decimals; e.g., 0.5 vs. 0.75) and exclusively in fraction notation (by comparing two fractions; e.g., [1/2] vs. [3/4]), then investigating the strategies used when comparing two values presented in different notation (by comparing a fraction with a decimal; e.g., [1/2] vs. 0.75) can provide important insight. For example, if the level of verbal WM recruitment increases as a function of the number of fractions involved in the comparison (with the lowest level of recruitment involved in comparisons between two decimals [zero fractions], with slightly more for comparisons between a decimal and a fraction [one fraction], and the highest level for those between two fractions), then this would suggest that each fraction requires additional computational processing. Alternatively, if comparisons involving two fractions yield the same pattern as comparisons involving a decimal and a fraction, then it may be that merely the presence of a fraction invokes a distinct set of strategies not employed when there are only decimals.

Relationship to Algebra Ability

Most critically, however, is investigating whether the involvement of visuospatial and verbal WM resources may differ as a function of algebra ability. The relationship between algebra ability and rational number understanding has been explained through a number of mechanisms, including having a strong understanding of the rational number system, being proficient with both algebraic and arithmetic procedures, understanding the conceptual aspects of fraction units (e.g., the denominator), and so on (e.g., Booth & Newton, 2012; Hurst & Cordes, 2016b; Kilpatrick & Izsak, 2008; Wu, 2001). Although studies have investigated algebra (e.g., Booth & Davenport, 2013; Koedinger, Alibali, & Nathan, 2008; Landy, Brookes, & Smout, 2014) and fraction problem solving (e.g., Faulkenberry & Pierce, 2011) separately, how specific strategies for approaching fraction problems may be related to algebra proficiency is an unexplored area. Given that understanding fraction magnitudes may be critical for algebra understanding (e.g., Booth & Newton, 2012; Kilpatrick & Izsak, 2008), we might expect those proficient in algebra to engage in fewer computational strategies when processing fraction and decimal magnitudes (having a more intuitive understanding of the magnitudes associated with those symbols) and those less proficient in algebra to rely more upon calculation-based strategies. If this is the case, then we would expect to see individuals with lower algebra fluency to rely more upon verbal WM resources and less so upon visuospatial WM resources. On the other hand, other evidence suggests that performance on rational number arithmetic assessments is also predictive of algebra ability and may be an essential part of the relationship (Hurst & Cordes, 2016b; Kilpatrick & Izsak, 2008). Thus, it may be that those who are more fluent with algebraic processing are more likely to process fraction and decimal magnitudes arithmetically, executing calculations in order to make the comparison, for example, cross-multiplying two fractions or converting values into a common notation for purposes of comparison. In this case, we might expect that those individuals with higher algebra ability to have greater reliance upon verbal WM resources (evidence of engaging a calculation based strategy), whereas those with lower algebra ability may not.

The Current Study

In summary, there is a growing literature investigating how people think about rational number magnitudes and how rational number understanding may be related to algebra ability. However, there are several open questions about the strategies invoked when processing rational number magnitudes presented in both fraction and decimal notation. In the current study, we used a dual-task WM paradigm to assess how visuospatial and verbal WM are implicated during a rational number magnitude comparison task. We then assessed whether individual differences in WM involvement (indicative of distinct processing strategies) were associated with performance on an algebraic assessment. We explored these relationships in a group of educated young adults who have had several years of schooling beyond the introduction of basic rational number and algebra concepts. Given that rational number and algebra concepts are introduced in different school grades and taught throughout a large range of grades (Common Core State Standards: National Governors Association Center for Best Practices & Council of Chief of State School Officers, 2010), adult participants allow us to investigate these relationships once they have already received basic educational instruction on these topics. By doing so, we are able to take a first look at the pattern of these relationships, providing insight into individual differences in rational number and algebra understanding and opening up new avenues for further investigation into children who are in the process of learning these concepts.

Specifically, this study addresses three research questions (RQs):

1. Is WM differentially implicated in rational number magnitude understanding based on WM type (visuospatial vs. verbal WM)?

RQ #1 will be investigated by looking at whether performance on the rational number task differs depending on WM load type. If rational number magnitudes are processed in terms of visuospatial representations (i.e., visualizing the quantities), then we would expect visuospatial WM to show more interference than verbal WM. On the other hand, if rational number magnitudes are primarily processed in terms of their computational features (i.e., arithmetic manipulation of the symbols), then we would expect primarily verbal WM interference.

2. Is WM differentially implicated in rational number magnitude understanding based on rational number notation (fractions vs. decimals)?

RQ #2 will be investigated by looking at whether the level of verbal and/or visuospatial WM interference depends upon the notation being used. If fractions and decimals are processed similarly, we would expect no differences across notation. Alternatively, given substantial literature suggesting adults consider these notations to be qualitatively different (e.g., DeWolf et al., 2014; Hurst & Cordes, 2016a), we might expect WM interference to differentially impact decimals and fractions.

3. Do individuals with different levels of algebra ability show distinct patterns of WM resource use in a rational number magnitude task? That is, does the pattern of findings in RQ #1 depend on the algebra ability of the individual?

RQ #3 will be investigated by looking at how the pattern of results discussed in RQ #1 may differ across those with high and low algebra ability. If the often-reported relationship between algebra ability and rational number understanding (e.g., Booth et al., 2014; Hurst & Cordes, 2016b; Siegler et al., 2012) is dependent upon the type of resource-based strategies used by the individual, then we would expect to see different levels of verbal and visuospatial WM involvement between those with high and low algebra ability. Furthermore, in order to isolate algebra ability in particular, we will include performance on a rational number arithmetic assessment as a covariate in our analyses in order to control for individual differences in procedural ability with rational number notation more generally.

By investigating individual differences in WM recruitment, we may be able to look at differences in how those with varying algebra abilities approach rational number magnitudes. By relying on previous research with WM recruitment during mental arithmetic, we may be able to shed some light on the kinds of strategies adults may employ based on their patterns of WM recruitment.

Method

Participants

Seventy-nine adults participated for course credit or $10.00. Adults were recruited from a university campus through introductory psychology courses and flyers, resulting in a sample primarily made up of undergraduate and graduate college students. Nineteen adults were not included in the analyses because of computer error resulting in the loss of all data ($n = 13$) or because their data exceeded our exclusion criteria ($n = 6$; see the Exclusion Criteria section for details). Thus, data from a final sample of 60 adults ($M_{age} = 20.9$ years; age range = 18 to 33 years old; 35% males) were included in the WM analyses. Additionally, data from three adults were excluded from analyses involving the math assessments (see Exclusion Criteria), resulting in data from 57 adults ($M_{age} = 21.0$ years; age range = 18 to 33 years old; 37% males) used for analyses relating WM involvement to algebra performance.

Procedure

All adults completed the number comparison task, participating in four within-subject blocks (visuospatial dual-task, visuospatial control, verbal dual-task, verbal control), with the order of the blocks counterbalanced across subjects. Each individual block of the WM dual-task took approximately 5 min. Following the number comparison task, adults completed two math assessments: a rational number arithmetic assessment and an algebraic assessment. The entire session took no longer than 60 min.

Each block of the WM dual-task began with three practice problems (one of each number comparison type), during which the experimenter sat next to the participant to ensure that the participant understood and followed the instructions. During the task, the experimenter left the room and only reentered to provide instructions for the next condition. All tasks (number comparison task and math assessments) were presented on a 22-in. monitor connected to an Apple computer.

Measures

WM dual-task. The WM dual-task procedure (modeled after Caviola et al., 2012) contained four within-subject blocks: two dual-task blocks (visuospatial and verbal) and two control blocks (visuospatial and verbal). On the dual-task blocks, participants were asked to remember visuospatial or verbal information (secondary task) while performing a numerical comparison (primary task). The control blocks were designed to be perceptually and temporally identical to the dual-task blocks (i.e., to have the same temporal spacing between trials and the same perceptual distractors)—the only difference was that participants were instructed not to remember the information from the secondary task, and they were never asked to recall that information. Trials in each block followed the same basic procedure: (a) center fixation cross (1,000 ms; 1.5 cm × 1.5 cm); (b) secondary task memory stimulus (2,500 ms); (c) blank screen (1,000 ms); (d) number comparison stimuli (until response); (e) blank screen (1,000 ms); and (f) memory recall (in dual blocks) or memory stimulus reappearance (in control blocks; until the participant responded to move on to next trial; see Figure 1).

Primary task. The primary task of interest was the number comparison task (similar to tasks used in DeWolf et al., 2014; Hurst & Cordes, 2016a). Across all four blocks, participants were presented two rational numbers and were instructed to indicate which of the two numbers was larger in numerical value. There were three different types of numerical comparison trials in which participants were asked to judge the relative magnitude of: (a) two fractions (FvF; e.g., [1/2] vs. [3/4]); (b) two decimals (DvD; e.g., 0.5 vs. 0.75); or (c) one decimal and one fraction (DvF; e.g., 0.5 vs. [3/4]). Participants indicated their response by selecting the corresponding key (right arrow for right stimulus and left arrow for left stimulus) on the keyboard as quickly as possible.

In each of the four blocks, eight trials of each number comparison type (FvF, DvD, and DvF; all intermixed) were randomly presented, for a total of 96 trials (8 trials × 3 comparison types × 4 blocks).

On the FvF and DvF trials, the two numerical values presented differed, on average, by a ratio of 2.4 (range = 2.1 to 2.8), and the numerical values presented in DvD comparisons differed by an average ratio of 1.12 (range = 1.07 to 1.15). The ratio of the DvD comparisons was set lower than the DvF and FvF comparisons, because previous work (Hurst & Cordes, 2016a) suggested that a ratio around 1.12 in DvD comparisons would result in a comparable level of performance as DvF and FvF trials at a 2.4 ratio.

Numerators and denominators of the fraction stimuli only involved single-digit values ranging from 1 to 9 and had magnitudes between 0 and 2 (exact range = 1/7 to 9/5), resulting in a mix of proper unit fractions, proper nonunit fractions, and improper fractions. In FvF trials, the two numerators and two denominators used to make up the two fractions were always four distinct integers to prevent adults from using an exclusively numerator or denominator comparison strategy (as in Schneider & Siegler, 2010).

Decimal values ranged from 0.15 to 1.69 (approximately the same range as the fractions), making the unit value in the decimal notation (i.e., value to the left of the decimal point) either a 0 or a 1. Every DvD trial included one numerical stimulus with digits to the thousandths place (i.e., had three digits after the decimal; e.g., 0.714) and the other included digits only to the hundredths place (i.e., two digits after the decimal; e.g., 1.75). On half the trials, the correct (larger value) decimal was the longer decimal (decimal with three digits), and on the other half, the correct decimal value was the shorter decimal (decimal with two digits), in order to make decimal length (i.e., number of digits after the decimal point) not a reliable indicator of which magnitude was larger. The Appendix provides the full set of numerical stimuli.

All stimuli were made in 100-point Myriad Pro font (in Adobe Illustrator) and were approximately 5 cm apart (from right edge of the left stimulus to the left edge of the right stimulus), centered on the screen. Fraction stimuli were approximately 3.5 cm wide × 5 cm high; decimal stimuli to the thousandths place were approximately 5.5 cm wide × 2 cm high; and decimal stimuli to the hundredths place were approximately 4.25 cm wide × 2 cm high.

Reaction time (RT) was used as the primary dependent variable. Only RTs from correct trials and those within three standard deviations of the individual participant's average RT on that notation and WM condition were included.

Secondary tasks. During the dual-task blocks, participants engaged in a secondary task at the same time as performing the primary task.

Secondary visuospatial task. In the visuospatial dual-task block, participants performed a secondary visuospatial task (while performing the primary task) in order to tax visuospatial memory

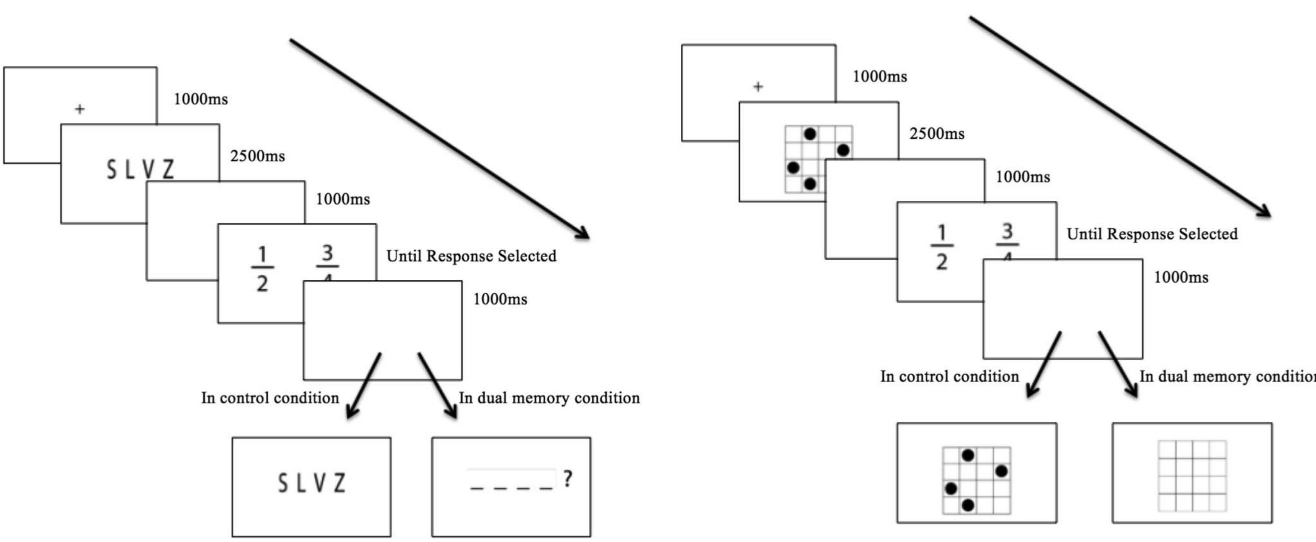

Figure 1. The procedure for the verbal working memory (WM) conditions (left) and the visuospatial WM conditions (right).

resources. The secondary visuospatial task required participants to remember visuospatial information on every trial. Participants were presented with a 240.25-cm^2 4 $\times$ 4 grid (made up of 16 3.75 cm $\times$ 3.75 cm squares). The grid was centered on the screen and four of the locations on the grid contained a black circle (2.75 cm in diameter, centered within the square on the grid).

In the dual-task block, on every trial, participants were instructed to remember the location of the four circles shown on the grid while performing a numerical comparison. Although participants were told to remember the information on every trial, they were only asked to recall this information on a random half of the trials (in order to shorten the experiment length; as in Caviola et al., 2012). On trials in which participants were asked to recall the visuospatial information, after selecting their response to the numerical comparison, participants were shown an empty 4 $\times$ 4 grid and were instructed to click on the four locations in the grid (using the computer mouse) in which they recalled there being a black circle. After selecting the four locations, they pressed the up-arrow key on the keyboard to submit their response and move on to the next trial. On those trials in which they were not asked to recall the location of the circles, participants were shown the same 4 $\times$ 4 grid (including the four circles) they had seen prior to the number comparison. On these trials, they simply had to press the up-arrow to move on to the next trial. Whether the participant had to recall the grid stimulus or not was randomly determined on a trial-to-trial basis, such that participants could not reliably pick and choose when to remember the information and when not to remember. Thus, in order to succeed in the task, they were required to remember the grid on every trial. Accuracy on those trials in which subjects were asked to recall the visuospatial information (in dual blocks) was scored to ensure that participants actually remembered the information during the dual-task conditions. Visuospatial WM accuracy was measured as the number of trials in which the participant indicated the correct location of at least three of the four dots in the grid.

In the visuospatial control block, trials were identical to the visuospatial dual-task block, except that participants were never asked to recall the locations of the circles. That is, on every trial, participants were shown a 4 $\times$ 4 grid with four circles before the number comparison task, and were reshown the same 4 $\times$ 4 grid after the number comparison task, and had to push the up-arrow to move on to the next trial. Importantly, participants were told that they did not need to remember the location of the circles on the grid, thus making it irrelevant to the task (though identical to the dual-task block in every other way).

None of the visuospatial memory stimuli were presented more than once to each person, resulting in 48 (plus six practice) different visuospatial stimuli. However, the same 24 numerical comparison stimuli were used for both of the visuospatial blocks (but were different than the verbal block numerical stimuli), and all the visuospatial stimuli were randomly paired with their accompanying number comparison stimuli for each participant.

Secondary verbal task. In the verbal dual-task block, participants performed a secondary verbal task (while performing the primary numerical comparison task) in order to tax verbal memory resources. The secondary verbal task involved remembering verbal information (as in Caviola et al., 2012). Participants were presented with four consonant letters from the English alphabet in a random order centered on the screen (e.g., XQRT). The total length of the four letters was approximately 12 cm $\times$ 3 cm.

The verbal dual-task block was identical to the visuospatial dual-task block, except that on every trial, instead of a grid, participants were shown four consonant letters (presented horizontally) and were instructed to read them out loud and remember them. Following the number comparison task, on half the trials, participants were provided with an empty text box and instructed to type in the four letters they saw previously, then press the up-arrow to submit their response and move on to the next trial. On the other half of the trials, participants were not asked to recall the verbal information, but instead were reshown the same four letters and just had to press the up-arrow to move on to the next trial. Accuracy on those trials in which subjects were asked to recall the verbal information was scored to ensure that participants remembered the required information during the dual-task blocks. Verbal WM accuracy was scored as the number of trials in which the participant correctly recalled all four letters.[1]

The verbal control block was perceptually and temporally identical to the half of the trials in the verbal dual-task block that did not require the participant to recall the letters they had seen previously. Thus, participants were instructed to read the four letters aloud but not to remember them. After the number comparison task, participants were reshown the same four letters and simply had to press the up-arrow to move on to the next trial.

None of the verbal memory stimuli were presented more than once to each person, resulting in 48 (plus six practice) different verbal stimuli. However, the same 24 numerical comparison stimuli were used for both of the verbal blocks (but were different than the visuospatial block numerical stimuli), and all of the verbal stimuli were randomly paired with the accompanying number comparison stimuli for each participant.

Math assessments. Following the dual-task procedure, participants completed math assessments given in two parts: rational number arithmetic (involving both fractions and decimals) and algebra, in that order. For all assessments, questions were presented one at a time on a computer screen, and participants were given a paper workbook to do as much work as they needed and to record their answers. The use of aid devices (e.g., calculators) was not allowed. Participants were told they had as much time as they needed, but to work as quickly as they could because they were being timed (by the computer).

The fraction and decimal assessment consisted of eight decimal arithmetic problems and eight fraction arithmetic problems, presented in two blocks with order counterbalanced (see the Appendix for a full list of problems). There were two each of addition, subtraction, division, and multiplication problems for each notation type. The fraction problems always contained four distinct integers, meaning none of the problems contained a common denominator or a common numerator. For the decimal problems, one problem of each arithmetic type involved two decimal values with the same number of digits (i.e., to the hundredths digit, e.g., 0.48 + 0.56). The other problem within each arithmetic type

[1] Slightly different criteria were used for the visuospatial and verbal blocks in order to approximately match accuracy between visuospatial (average 91% correct as opposed to 79% when the same criteria were used) and verbal (average 94% correct) working memory.

involved two decimal values with a different number of digits (e.g., 0.67 + 0.843).

The algebra assessment consisted of 12 problems adapted from the Trends in International Mathematics and Science Study (TIMSS) Grade Eight assessment (International Association for the Evaluation of Educational Achievement [IEA], 2005, 2013). The assessment involved a variety of problems involving solving expressions, using values in an expression, and finding the relation between values in a table or expressed in a word problem (see the Appendix for a full list of problems). Importantly, correctly solving the algebra problems only required manipulation of whole numbers (noninteger values were not included in this assessment). Thus, although whole-number division was occasionally required (e.g., 24/8 = 3), no knowledge of arithmetic or procedures associated with fractions and decimals was required.

Two independent coders scored each of the math assessments to determine accuracy (99% agreement on both assessments, with a third coder resolving the disagreements), and the computer recorded completion time. Accuracy was fairly high on both the algebra ($M = 9.8$ of 12) and rational number arithmetic assessment ($M = 12.6$ of 16), with relatively low variability (e.g., 50% of adults scored 10, 11, or 12 out of 12 on the algebra measure). Thus, as our dependent measure, we used completion time as a measure of fluency for both the rational number arithmetic and the algebra measures. The internal reliability of the algebra assessment was fairly good, with Cronbach's alpha of 0.744 for this sample (based on completion times).

Exclusion Criteria

WM dual task. As per Caviola et al. (2012; also see Conway et al., 2005), participants were required to score above 60% on both WM types to be included in the analyses in order to ensure they had actually invoked WM during the task. Two participants did not meet this criterion, and so their data were excluded from analyses.

Magnitude comparison task. At the group level, participants who scored below chance on the comparison task ($n = 2$) or who had RTs greater than three standard deviations away from the group RT performance ($n = 2$) were excluded. Only RTs from trials in which participants provided the correct response to the numerical comparison were included in analyses. Thus, data from 60 participants were included in analyses of the WM dual-task.

Math assessments. In order to make differences in completion time comparable across individuals (and to avoid issues of speed/accuracy trade-offs), participants who performed worse than 50% correct on the math assessments were excluded from analyses involving the math assessments ($n = 3$). Thus, data from 57 participants were included in analyses involving the assessments.

Results

Descriptive statistics for each task are presented in Table 1. On all tasks, speed (response time [comparison task] or completion time [assessments]) was used as the primary dependent variable as a measure of fluency. Preliminary analyses suggest no significant main or interaction effects involving gender, and therefore gender was not included in any of the analyses.

Table 1
Means (SDs) for Time and Accuracy for Each Measure

	Verbal WM						Visuospatial WM						Math assessments	
	Control task			Dual task			Control task			Dual task				
	FvF	DvD	DvF	FvF	DvD	DvF	FvF	DvD	DvF	FvF	DvD	DvF	Algebra fluency	Rational number arithmetic fluency
Time	1,277 (295)	1,029 (142)	1,263 (275)	1,491 (525)	1,156 (197)	1,433 (505)	1,252 (473)	1,080 (194)	1,221 (340)	1,333 (387)	1,164 (207)	1,364 (358)	571 (243)	693 (289)
Accuracy	91.7 (8.9)	92.2 (9.5)	95.6 (8.2)	91.7 (8.5)	96.0 (6.7)	94.1 (8.4)	98.3 (5.3)	92.7 (9.8)	95.8 (8.2)	98.1 (6.0)	97.5 (6.4)	97.5 (5.0)	82.5 (12.5)	80.3 (12.4)

Note. Times reported are response times (ms) for the working memory task and completion times (s) for the math assessments. Accuracy is reported as percent correct. WM = working memory; DvD = Decimal vs. Decimal; FvF = Fraction vs. Fraction; DvF = Decimal vs. Fraction.

RQ #1 and RQ #2: Interference Across Notation and WM Type

In order to investigate RQ #1 and RQ #2, we were interested in performance on the dual task to determine whether there were differences in how the secondary WM tasks may have interfered with performance on the primary task depending on the type of WM (RQ #1: visuospatial or verbal) or the notation of the comparison (RQ #2: fractions, decimals, or both). Thus, we used a 2 (task: control vs. dual) × 2 (WM type: visuospatial vs. verbal) × 3 (notation: fraction, decimal, mixed) repeated measures ANOVA on RT.

Consistent with previous research, there was a main effect of task, $F(1, 59) = 38.959$, $p < .001$, $\eta_p^2 = 0.398$, with the dual-tasks ($M_{\text{dual}} = 1{,}324$ ms) taking longer than the control tasks ($M_{\text{control}} = 1{,}187$ ms). However, task did not interact with WM type ($p = .3$, $\eta_p^2 = 0.02$), suggesting that, in general, there was no evidence of a statistically significant difference in how much the secondary task interfered with RT performance on the primary task across the verbal and visuospatial conditions. Thus, in reference to RQ #1, neither verbal nor visuospatial strategies appeared to dominate over the other. In addition, there was a main effect of notation, $F(1.5, 30.6) = 46.14$, $p < .001$, $\eta_p^2 = 0.44$, with average RT on the DvD trials (1,107 ms) being significantly faster than performance on the FvF (1,338 ms) or DvF (1,320 ms; follow-up *t* tests, *p*s $<$ 0.001) trials, which did not significantly differ from each other. However, analyses did not reveal a statistically significant three-way interaction (Type × Notation × WM, $p = .23$, $\eta_p^2 = 0.02$). Thus, in response to RQ #2, there is not statistically significant evidence to suggest that the use of verbal or visuospatial memory differed across the notations.[2]

There was a very small, marginal main effect of WM type, $F(1, 59) = 3.44$, $p = .07$, $\eta_p^2 = 0.06$, with responses in the verbal conditions taking slightly more time ($M_{\text{verbal}} = 1{,}275$ ms) than in the visuospatial conditions ($M_{\text{visuo}} = 1{,}236$ ms). WM also interacted with notation, $F(1.8, 107.3) = 11.5$, $p < .001$, $\eta_p^2 = 0.16$, such that the difference in response speed between DvD trials and those trials involving fractions was greater in the verbal condition ($M_{\text{DvD}} = 1{,}092$ ms, $M_{\text{FvF}} = 1{,}384$ ms, $M_{\text{DvF}} = 1{,}348$ ms) than in the visuospatial condition ($M_{\text{DvD}} = 1{,}122$ ms, $M_{\text{FvF}} = 1{,}292$ ms, $M_{\text{DvF}} = 1{,}293$ ms). However, none of these variables interacted with task type (Task × Notation, $p = .14$, $\eta_p^2 = 0.03$; Task × Notation × WM, $p = .23$, $\eta_p^2 = 0.02$). Although this pattern may suggest some differences in performance when adults were presented with verbal or visuospatial information during the magnitude comparison task, the critical comparison in our design involved differences between the control and dual-task blocks (i.e., when the verbal/visuospatial information is relevant to the task or not). Therefore, any differences between verbal and visuospatial tasks overall (collapsing across control and dual-task tasks) may be because of perceptual distractions that are not likely related to direct memory effects, making it difficult to interpret these performance differences as meaningful within the current study.

RQ #3: Differences Across Individual Differences in Algebra Fluency

In addition to looking at overall performance on the dual task, we were interested in whether strategy use with rational number magnitudes differed between individuals who were highly proficient in algebra and those who were less proficient, controlling for general math ability. Thus, we divided participants into two groups based on a median split of completion times on the algebra assessment (median completion time = 539 s, range = 252 s to 1,525 s) to create groups that differed in their algebra fluency. We then conducted a 2 (task: control vs. dual) × 2 (WM: visuospatial vs. verbal) × 2 (algebra fluency: high fluency [$N = 28$] vs. low fluency [$N = 29$]) mixed measures ANCOVA on RT on the rational number comparison task, including rational number arithmetic completion time as a covariate (see Figure 2). Because notation did not interact with WM interference in the previous analysis, it was not included as a factor in these analyses. Because completion time on the algebra assessment could assess both general mathematical skills as well as skills specific to algebraic reasoning, completion times on the rational number arithmetic assessment were included as a covariate to allow us to investigate the relationship to algebra ability specifically and not more general math or rational number ability.

In this secondary ANCOVA, which included algebra ability as a factor (and controlled for rational number arithmetic ability), we did not find an overall effect of task ($p = .7$, $\eta_p^2 = 0.003$), WM ($p = .19$, $\eta_p^2 = 0.03$) or an overall Task × WM interaction ($p = .6$, $\eta_p^2 = 0.004$). The rational number arithmetic fluency covariate was related to task type ($p = .04$, $\eta_p^2 = 0.08$), with follow-up analyses suggesting that slower arithmetic performance was positively correlated with overall levels of WM interference (i.e., difference between control and dual-tasks, $r[57] = 0.3$, $p = .03$), indicating that people with higher rational number arithmetic ability were less impacted by the secondary WM task when engaging in rational number comparisons. The rational number arithmetic covariate did not interact with algebra fluency (*p*s > 0.1).

Critically, however, addressing RQ #3, the pattern of WM interference did interact with algebra fluency, suggesting distinct strategy use across those with relatively high versus low algebra fluency. Specifically, there was a WM × Task × Algebra Fluency interaction, $F(1, 54) = 4.76$, $p = .03$, $\eta_p^2 = 0.08$).

A follow up WM × Task ANOVA on higher algebra fluency ($N = 28$) individuals revealed that those with high algebra fluency showed a statistically significant task effect ($p < .001$, $\eta_p^2 = 0.3$, with longer RTs on the dual conditions ($M = 1{,}233$ ms) than on the control conditions ($M = 1{,}109$ ms). However, the task type did not interact with WM type ($p = .16$, $\eta_p^2 = 0.07$), suggesting that data from people with higher algebra fluency showed no evidence of differential involvement of verbal and visuospatial resources, and thus these individuals may have used both verbal and visuospatial strategies approximately equally. On the other hand, when looking at the data from those with lower algebra fluency (2 × 2 ANOVA; $N = 29$), there was a main effect of task ($p < .001$, $\eta_p^2 = 0.43$) as well as a statistically significant Task × WM interaction ($p < .05$, $\eta_p^2 = 0.14$). Specifically, data from participants in the low algebra fluency group only revealed a statistically significant interference effect in the verbal WM condition ($M_{\text{Control}} = 1{,}257$ ms, $M_{\text{Dual}} =$

[2] Analyses involving the notation factor showed evidence of heterogeneity among the variances of the differences between possible pairs of levels of notation, and thus required a correction for sphericity. Thus, the Huynh-Feldt correction was used (although the correction did not change the statistical significance of any of the results).

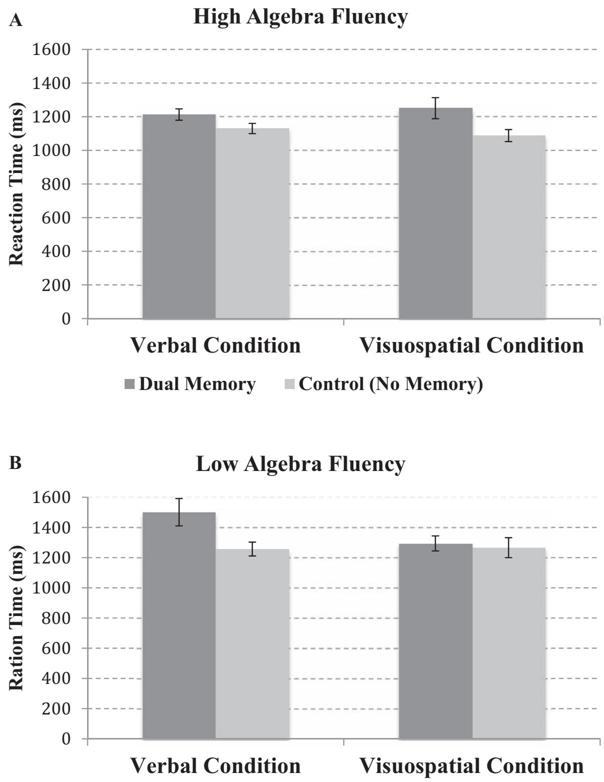

Figure 2. Reaction times on the magnitude comparison task across all notations (Fraction vs. Fraction [FvF], Decimal vs. Decimal [DvD], Decimal vs. Fraction [DvF], separated by working memory type (verbal vs. visuospatial) and interference (dual memory vs. no memory control). Individuals with high algebra fluency showed significant interference by both verbal and visuospatial working memory (WM; slower Reaction time [RT] in the dual relative to the control), whereas individuals with low algebra fluency showed interference only by verbal WM and not visuospatial WM.

1,501 ms; paired t test, $p < .001$) and not in the visuospatial condition ($M_{\text{Control}} = 1{,}266$ms, $M_{\text{Dual}} = 1{,}293$ ms; paired t test, $p = .6$).

Discussion

The current study used a dual-task paradigm to address adults' dominant strategy use during a rational number magnitude comparison task. We then explored the relationship between algebra ability and the differential engagement of distinct WM resources. Data revealed that, on average, adults tended to engage both verbal and visuospatial WM for both fraction and decimal comparison tasks. However, this pattern differed between individuals with higher and lower levels of algebra fluency.

Rational Number Notation

Although differences in the speed of processing distinct notation were found—such that decimal magnitudes were accessed significantly faster than those presented in fraction notation (as found in DeWolf et al., 2014; Hurst & Cordes, 2016a)—the use of visuospatial and verbal WM strategies did not vary as a function of notation type. Thus, we did not find evidence that the type of WM resources required to perform the task differed, on average, between fraction and decimal notation. Rather, adults seemed to use strategies that similarly relied upon verbal and visuospatial WM resources for both decimal and fraction notation and/or there was substantial individual variability in preferred strategy choice such that a dominant preference did not emerge. This is particularly striking given notable differences in structure between the two notations, claims that decimal notation is much more similar to whole number notation compared with fraction notation (e.g., Johnson, 1956), and findings that decimal magnitudes are more easily accessed than fraction magnitudes (the current study; DeWolf et al., 2014; Hurst & Cordes, 2016a). Despite these noted differences, results of our task did not reveal a distinction in the kinds of resources (visuospatial and verbal WM) recruited for fraction and decimal notation, suggesting that individuals may use similar types of strategies for both notations.

Although WM involvement differences across fractions and decimals were not obtained, the patterns of WM involvement when processing rational numbers found in the current study point to a potential distinction between whole numbers and non-whole-number rational numbers. Whereas both visuospatial and verbal WM were equally implicated when decimal and fraction magnitudes were judged, previous research suggests that magnitude judgments of whole numbers rely primarily on visuospatial resources suggesting that whole number magnitudes are spatially encoded (e.g., Simmons et al., 2011; van Dijck, Gevers, & Fias, 2009). Thus, despite parallels between decimal notation and whole number notation (Johnson, 1956), and despite evidence suggesting that all rational numbers (fractions, decimals, and whole numbers) are represented as falling along an integrated mental continuum in adults (Hurst & Cordes, 2016a), our data indicate that rational number magnitude processing may not perfectly parallel that of whole numbers. Instead, the involvement of verbal WM when processing decimal and fraction magnitudes indicates a role for symbolic calculation when adults process (nonwhole number) rational number magnitudes.

These findings shed light on cognitive models of rational number magnitude processing, suggesting that adults process rational number magnitudes more similarly to mental arithmetic (requiring both verbal and visuospatial WM resources) than to whole-number magnitudes (which likely require visuospatial resources and limited verbal resources). However, an interesting open question is how this pattern may change across development as a function of education. Research on WM recruitment during mental arithmetic tasks indicate a developmental trend such that children first rely primarily on visuospatial strategies, and then with greater experience, they use a mix of both visuospatial and verbal resources (e.g., McKenzie, Bull, & Gray, 2003; Raghubar et al., 2010). A similar developmental pattern may be found for fraction and decimal magnitude judgments as well. On the other hand, given how notoriously difficult rational number concepts are for children to acquire, coupled with the common whole number bias errors children show (e.g., treating fractions as two whole numbers as opposed to a coherent unit; Ni & Zhou, 2005), we may expect to see a reverse pattern of development in which children first learning rational numbers may initially show a greater reliance upon computational (verbal based) strategies when accessing rational number magnitudes, with a later emerging reliance upon a mix of strategies. In fact, our finding that low algebra fluency adults relied primarily upon verbal strategies indicates that verbal strategies may be associated with low expertise in rational numbers and/or math ability more generally, consistent with the idea that children might initially have a primary reliance on verbal strategies.

Individual Differences in the Use of WM Resources

Results revealed that it was those adults with higher algebra fluency that were impacted by both verbal and visuospatial WM interference, but those with lower algebra fluency were only impacted by verbal WM interference. Although we did not directly measure the strategies employed by individual participants, assessing adults' reliance upon distinct WM resources allowed us to address the general kinds of strategies that may have been engaged during the rational number comparison task. In particular, the current findings suggest that those individuals who were relatively more fluent in algebra engaged both visuospatial-based strategies (i.e., reliance on the visuospatial sketchpad) and verbal strategies. However, lower algebra fluency was associated with the engagement of primarily verbal-based strategies (i.e., reliance on the phonological loop, but not the visuospatial sketchpad).

Given previous work looking at WM resource use during mental arithmetic, verbal WM interference in the current study is thought to be indicative of computational strategies requiring the memory of operands and/or interim solutions (see DeStefano & LeFevre, 2004). The use of verbal strategies, without also engaging visualization strategies (as was the case with low algebra individuals), may be particularly indicative of calculation or verbal rule-based strategies that do not also involve visualizing the magnitude or using complex calculations that involve mentally manipulating digits/components (which would involve both visuospatial and verbal WM). For example, these adults may have made comparisons based on component parts (i.e., numerators and denominators; tenths and thousandths place), converted fractions into decimals, or executed other verbal, calculation-based strategies. Importantly, however, our findings suggest that it is not simply the use of verbal WM during a rational number task that is associated with poor algebra fluency (because those with higher algebra fluency also engaged verbal resources), but rather a higher reliance on verbal WM with little reliance on visuospatial WM. This suggests that the engagement of particular kinds of strategies that rely primarily on verbal WM and not on visuospatial WM may be a critical predictor of poorer understanding of algebra. Those individuals, in the current study, who opted to use such computational strategies without also engaging visualization strategies likely have a poor understanding of how rational number symbols (in either decimal or fraction notation) translate to analog numerical magnitudes—a skill that may be important for success in algebra.

Individuals with high fluency, on the other hand, may have relied upon both visuospatial resources and verbal resources to process rational number magnitudes. The specific role of visuospatial WM is less clearly understood (relative to verbal WM) in the domain of mathematics. However, visuospatial WM has been implicated in visualization, such as in complex mental arithmetic that requires spatial movement (e.g., "carrying" in arithmetic; Raghubar et al., 2010). Thus, in the context of rational number processing, visuospatial and verbal WM engagement may be found when visualizing a proportional model (such as a pie chart or number line) or complex visual arithmetic (e.g., cross-multiplying, a strategy that presumably requires retaining verbal and spatial information).

Regardless, our data suggest that relying on both verbal and visuospatial strategies may be indicative of higher level conceptual processing. As such, this finding provides support for current policy recommendations that rational number instruction should highlight visuospatial representations of rational numbers as magnitudes (National Governors Association for Best Practices & Council of Chief of State School Officers, 2010; National Mathematics Advisory Panel, 2008). For example, one common recommendation is to emphasize the spatial organization of rational numbers along a number line, a recommendation drawn from other studies revealing training with number lines can be a successful intervention for understanding whole number magnitudes (Siegler & Ramani, 2009). Extending these recommendations, our results emphasize the importance of encouraging people to use strategies that incorporate both symbolic representations (requiring verbal resources) and visuospatial representations for thinking about rational number magnitudes. Although our findings cannot pinpoint exactly which strategies or representations adults in the high algebra fluency group engaged (e.g., pie charts, part–whole representations, number lines, discrete objects, complex visual arithmetic), they highlight the importance of engaging both visuospatial and verbal representations when thinking about rational numbers, rather than relying on exclusively verbal, calculation-based strategies and representations. As such, it may be that including many representations in the classroom may lead to a greater chance of the individual incorporating both visuospatial and verbal strategies. Future research should investigate which representations are most likely to be employed by mathematically fluent adults during rational number processing and, in addition, whether promoting particular visual representations in the classroom can lead to more efficient processing of rational numbers. Results of these studies will have implications both for understanding the cognitive processes underlying rational number processing, while also having important implications for educational practices.

Specificity to Algebra Fluency

Importantly, because rational number arithmetic fluency was entered as a covariate in our analysis, this pattern of WM resource use is not simply indicative of the speed of mathematical processing more generally. Rational number arithmetic fluency did predict the overall level of interference, suggesting that math fluency may be related to overall WM use. This is consistent with work suggesting that WM is involved in many areas of mathematics, including fraction conceptual knowledge and procedural ability (e.g., De Smedt, Verschaffel, & Conway et al., 2009; Geary, 2011; Jordan et al., 2013; Vukovic et al., 2014). Moreover, evidence suggests that as people gain practice with an activity, they require fewer WM resources to complete the activity (Gevins, Smith, McEvoy, & Yu, 1997; Jonides, 2004). Thus, those individuals who were most impacted in our dual-task (revealing the greatest amount of interference) were likely less practiced or fluent in rational number magnitude processing, suggesting that some aspect of our results may stem from overall differences in expertise. However, differences in the pattern of resource use (across verbal and visuospatial WM) for those with differing levels of algebra fluency emerged even when controlling for performance on our rational number arithmetic assessment, indicating a specific relationship between rational number magnitude processing and algebra proficiency. That is, findings involving differences between individuals with relatively high and low algebra fluency do not reflect overall differences in cognitive or mathematical ability (which should be implicated about equally for Grade 8 algebra and rational number arithmetic), but rather are indicative of specific links between the processing of rational number magnitudes and algebra performance.

Limitations

There are some aspects of the current design that are worth noting. First, contrary to predictions, we did not find differences across rational number notation. One possibility is that the differences across notation are very small and that the current study did not have sufficient power to evaluate the three-way interaction required to see differences across notation within the current design. Alternatively, it may be that notation differences were not obtained due to our experimental design. The different notation trial types (FvF, DvD, and DvF) were intermixed within the same block, which may have impacted the types of strategies adults engaged in between notations. It is possible that if these distinct trial types were presented in separate blocks (i.e., all FvF trials were presented in a single block), then participants may have been more likely to settle upon a single strategy for working with the notation presented within that block of trials. If so, then notational differences may arise in contexts in which specific notations are consistent. Regardless, investigating differences across notation remains an open question for future research.

Additionally, although the current study used a variety of stimuli to investigate general processing of fraction and decimal magnitudes, it may be that the same strategies are not consistently used even within the same notation. For example, values on opposite sides of common bench marks, like 0.5 (or [1/2]) or 1, may lead to different strategies than comparing values on the same side of a common bench mark. In addition, there are several other component-based strategies (e.g., serially comparing decimals based on place value or comparing fractions based on numerators alone), heuristic-based strategies (e.g., choosing the longest decimal as the largest[3]), and format differences (e.g., vertical vs. horizontal alignment; presenting numbers one at a time instead of simultaneously) that may impact the kinds of strategies and resources that adults tend to engage. Because our study was not designed to specifically explore these issues, it was not possible to isolate the effects of these manipulations in our data, though this may be a topic for future research.

Lastly, our control task was designed in order to be perceptually and temporally identical to the dual task, and thus included verbal or visuospatial information between trials. Although adults were specifically instructed not to do so, it is possible that adults engaged some memory resources during the control task (i.e., there may be carry over effects between blocks). Importantly, however, our analyses do reveal significantly more interference during the dual tasks than the control tasks, making it unlikely that this greatly impacted our results.[4]

The Format of the Relationship

The current study leaves open the question of whether a *causal* relationship exists between the use of visuospatial strategies and algebra abilities. Given that children are taught rational number concepts prior to algebra, it may be that engaging complex visuospatial and verbal strategies when learning rational numbers may promote learning in more advanced math domains such as algebra. For example, representing rational numbers as holistic magnitudes (which would require symbolic and visuospatial strategies and not just verbal calculations) may be indicative of a more direct representation of the rational number system which algebra notation and manipulation is built (e.g., variables, unknown values, values along a line or curve). On the other hand, however, it may be that the learning of algebraic concepts can provide individuals with additional tools or strategies needed to engage both verbal and visuospatial strategies when processing rational numbers. For example, proficiency with algebraic manipulation (across both sides of an equation) may promote the use of visualization strategies when processing rational numbers as well. Lastly, it may be that visuospatial rational number strategies and high algebra ability are both associated with a third, general variable, such as visuospatial WM capacity or a general tendency toward abstract thinking. The correlational design of our study does not allow for a disentanglement of these accounts of mathematical learning. Therefore, future research should investigate this issue of causal direction in the relation between rational number processing strategies and algebra across various educational levels in order to better understand how these patterns may change across various stages in education and in young children whose WM capacities may not be at an adult level.

Conclusions

In sum, the current study used a dual-task WM paradigm to investigate individual differences in WM recruitment (verbal and visuospatial) during a rational number magnitude comparison task between individuals with relatively high and low algebra fluency. On average, adults were equally likely to engage visuospatial and verbal strategies when assessing the relative magnitudes of both decimals and fractions. Interestingly, however, individual variability in these strategies was associated with algebraic performance. Although individuals with relatively high algebra fluency relied on both verbal and visuospatial WM, individuals with relatively low algebra fluency relied more heavily on verbal WM to engage with rational numbers in both fraction and decimal notation. Thus, the use of strategies that involve both verbal and visuospatial resources (i.e., complex computations; visualization involving symbols) was associated with higher algebra performance, whereas using simple calculations or verbal rule-based strategies was as-

[3] In our study, 50% of trials were consistent with decimal length (meaning, longer decimal was the largest decimal), whereas 50% were inconsistent with length. In line with other work, there was an overall difference in performance, with inconsistent trials taking significantly longer than consistent trials ($p < 0.001$).

[4] Moreover, when we analyze each participant's first block only as a between-subject design, we get the same pattern of results involving WM interference and differences across algebra ability (although with much smaller sample sizes per cell).

sociated with lower algebra performance. These results add to the growing literature investigating the relationship between algebra ability and rational number understanding (e.g., Bailey et al., 2012; Hurst & Cordes, 2016b; Siegler et al., 2012), and further clarify the relationship by suggesting that individual differences in algebra ability may be associated with the use of different kinds of resource-based strategies in a rational number magnitude task. Additionally, results provide strong support for current recommendations to incorporate more visuospatial representations of rational number magnitudes, alongside symbolic representations, in the classroom (National Governors Association for Best Practices & Council of Chief of State School Officers, 2010; National Mathematics Advisory Panel, 2008), as the engagement of both verbal and visuospatial strategies was associated with advanced mathematical proficiency.

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Appendix

Stimuli and Measures

Complete List of Magnitude Comparison Task Stimuli

	FvF	DvD	DvF
Verbal blocks	3/2 vs 5/7	.15 vs .168	3/2 vs .714
	4/3 vs 5/9	.196 vs .22	4/3 vs .56
	7/5 vs 2/3	.67 vs .594	7/5 vs .67
	7/4 vs 5/8	.835 vs .74	1.75 vs 5/8
	6/5 vs 4/9	.987 vs 1.12	1.201 vs 4/9
	3/5 vs 2/7	1.115 vs .99	3/5 vs .286
	2/5 vs 1/6	1.49 vs 1.687	.391 vs 1/6
	2/3 vs 1/4	1.54 vs 1.368	.67 vs 1/4
Visuospatial blocks	8/5 vs 2/3	.24 vs .256	8/5 vs .667
	9/5 vs 6/7	.33 vs .293	9/5 vs .86
	7/6 vs 5/9	.534 vs .47	1.19 vs 5/9
	6/5 vs 3/7	.639 vs .72	1.193 3/7
	8/5 vs 4/7	.98 vs 1.124	8/5 vs .57
	5/6 vs 3/8	1.075 vs 1.21	.83 vs 3/8
	4/9 vs 1/5	1.08 vs .948	.456 vs 1/5
	2/5 vs 1/7	1.493 vs 1.32	2/5 vs .143

Complete List of Rational Number Arithmetic Questions (8 Fraction and 8 Decimal)

Decimal Arithmetic Questions

$$0.5 + 0.13 \quad 1.27 + 0.89 \quad 0.36 - 0.12 \quad 1.74 - 1.321$$

$$0.63 \div 0.12 \quad 1.452 \div 0.480.456 \times 0.32 \quad 1.75 \times 0.21$$

Fraction Arithmetic Questions

$$\frac{3}{4} + \frac{7}{9} \quad \frac{2}{3} + \frac{5}{7} \quad \frac{4}{5} - \frac{4}{8} \quad \frac{6}{7} - \frac{1}{4}$$

$$\frac{8}{9} \times \frac{1}{3} \quad \frac{4}{5} \times \frac{3}{8} \quad \frac{4}{7} \div \frac{3}{5} \quad \frac{3}{9} \div \frac{3}{8}$$

Complete List of the 12 Algebra Questions

Question:
There are two pipes. The first pipe is x meters long. The second pipe is y times as long as the first one. How long is the second pipe?

Question:
In Zedland, total shipping charges to ship an item are given by the equation $y = 4x + 30$ where x is the weight in grams and y is the cost in zeds. If you have 150 zeds, how many grams can you ship?

Question:
Simplify the expression $2(x + y) - (2x - y)$

Question:
Give two points on the line $y = x + 2$

Question:
Simplify the expression $2a^2 \times 3a$

(*Appendix continues*)

Question:
The table below shows a relation between x and y
What is the relation between x and y?
Appendix (follows the sentence "The table below shows a relation between x and y" and before "What is the relation between x and y?").

x	1	2	3	4	5
y	1	3	5	7	9

Question: $3(2x - 1) + 2x = 21$ What is the value of x?

Question:
The number of jackets that Haley has is 3 more than the number Anna has. If n is the number of jackets Haley has, how many jackets does Anna have in terms of n?

Question:
$a = 3$ and $b = -1$ What is the value of $2a + 3(2 - b)$?

Question:
Joe knows that a pen costs 1 zed more than a pencil. His friend bought 2 pens and 3 pencils for 17 zeds. How many zeds will Joe need to buy 1 pen and 2 pencils?

Question:
Simplify the expression $4x - x + 7y - 2y$

Question:
If $\frac{x}{3} > 8$ then what does x equal?

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