Rational-Number Comparison Across Notation: Fractions, Decimals, and Whole Numbers

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Although fractions, decimals, and whole numbers can be used to represent the same rational-number values, it is unclear whether adults conceive of these rational-number magnitudes as lying along the same ordered mental continuum. In the current study, we investigated whether adults' processing of rational-number magnitudes in fraction, decimal, and whole-number notation show systematic ratio-dependent responding characteristic of an integrated mental continuum. Both reaction time (RT) and eye-tracking data from a number-magnitude comparison task revealed ratio-dependent performance when adults compared the relative magnitudes of rational numbers, both within the same notation (e.g., fractions vs. fractions) and across different notations (e.g., fractions vs. decimals), pointing to an integrated mental continuum for rational numbers across notation types. In addition, eye-tracking analyses provided evidence of an implicit whole-number bias when we compared values in fraction notation, and individual differences in this whole-number bias were related to the individual's performance on a fraction arithmetic task. Implications of our results for both cognitive development research and math education are discussed.

Keywords: rational numbers, decimals, fractions, number representation, number comparison

Whether comparing prices, mortgage rates, or health-care options, and/or when pursuing STEM-related fields (science, technology, engineering, and mathematics) (Sadler & Tai, 2007), an understanding of the magnitudes associated with numbers presented in fraction and decimal notation is essential. However, the task of mastering rational-number concepts is notoriously difficult. A number of studies have revealed robust and systematic errors when individuals process rational numbers, reflecting poor procedural and conceptual competence even after years of schooling (e.g., National Mathematics Advisory Panel, 2008; Ni & Zhou, 2005; Vamvakoussi & Vosniadou, 2010). Moreover, rational-number concepts are particularly difficult for people diagnosed with mathematics learning disability (dyscalculia; affecting approximately 6–11% of the population; Shalev, 2007), with both the nature and developmental trajectory of difficulties in learning about these concepts differing substantially from typically developing peers (Mazzocco, Myers, Lewis, Hanich, & Murphy, 2013). Thus, explorations into how rational-number magnitudes are processed and how particular strategies or biases may be related to arithmetic proficiency are of particular importance for identifying ways to target better rational-number learning in the classroom.

Although researchers over decades have investigated the characteristics of whole-number magnitude processing, only recently have similar investigations been undertaken with rational numbers in other notations, such as fractions and decimals. Although fractions and decimals are part of the same rational-number system as whole numbers, the difficulties and systematic errors shown by children and adults suggest that the processing of rational numbers in these notations may show clear distinctions from whole numbers. One potentially important distinction between fractions, decimals, and whole numbers may be the way individuals represent the magnitudes associated with the rational-number notation. In particular, a substantial literature with whole numbers suggests that adults and children represent whole-number magnitudes as approximate magnitudes in an ordered system. Similarly, a recent surge of researchers looking at fractions (e.g., Faulkenberry & Pierce, 2011; Kallai & Tzelgov, 2012; Schneider & Siegler, 2010; Sprute & Temple, 2011) and decimals (e.g., DeWolf, Grounds, Bassok, & Holyoak, 2014; Kallai & Tzelgov, 2014; Varma & Karl, 2013) has provided similar evidence of an ordered and approximate analogue system. However, whether the magnitudes represented using fraction, decimal, and whole-number notation are represented using two or three distinct mental representations or alternatively using an integrated representation is an open question.

In whole-number magnitude comparisons, one robust characteristic is known as the ratio effect (or similarly, distance effect)[1]. In particular, when asked to quickly judge which of two numbers (Arabic numerals) is largest, response times and accuracy vary as

[1] Although distance effects and ratio effects have been examined somewhat interchangeably in the literature, it should be noted that ratio effects provide a more accurate account of the data as they take into account both distance and numerical size. For example, comparing 10 and 15 (ratio of 2:3) is easier and faster than comparing 20 and 25 (ratio of 4:5), despite a comparable numerical distance between the two pairs of integers.

a function of the numerical distance between the two numbers (e.g., discriminating 5 from 6 is harder than 5 from 10) and the numerical size of the numbers (e.g., discriminating 7 from 8 is easier than 19 from 20, despite similar distances; Moyer & Landauer, 1967, 1973). Together, these distance and numerical-size effects describe ratio effects, such that comparisons are ratio-dependent, becoming faster and easier when the ratio between the two numbers is larger (Moyer & Landauer, 1967, 1973), consistent with Weber's Law. These ratio effects are robust: Not only are they found in reaction-time (RT) data, but they have also been observed in eye-gaze fixation patterns during whole-number symbolic comparison tasks. In particular, participants tend to engage in longer fixations when the ratio between the two values approaches one (Merkley & Ansari, 2010), suggesting that these ratio effects are present and observable using multiple implicit measures of numerical processing. It is important to note, the presence of ratio effects has been taken as evidence to suggest that whole numbers are represented as ordered approximate magnitudes within an analog (i.e., continuous) system or continuum (e.g., Dehaene, Bossini, & Giraux, 1993; Gallistel & Gelman, 1992; Meck & Church, 1983; Moyer & Landauer, 1967, 1973), although the specific mathematical model for this representation is still debated (see Balci & Gallistel, 2006; Brannon, Wusthoff, Gallistel, & Gibbon, 2001; Cordes, Gallistel, Gelman, & Latham, 2007; Dehaene, 1992, 2001; Verguts, Fias, & Stevens, 2005 for alternative accounts). Many cognitive models posit that this analog system can be thought of as being spatially organized, akin to a mental number line (e.g., Siegler & Opfer, 2003). However, it should be noted that the presence of ratio effects alone does not require that numerical magnitudes be spatially organized, as in a mental number-line model (Cantlon, Cordes, Libertus, & Brannon, 2009). Moreover, whether ratio effects result from noise in memory for these numerical values (i.e., slight inaccuracies in the reference value stored in memory for numerical values; e.g., Meck & Church, 1983) or in the decision-making process (e.g., Gallistel, King, & McDonald, 2004) is also unclear. It is important to note, however, it is generally agreed that ratio effects are indicative of numerical information being represented by an ordered analog magnitude system, analogous to a mental continuum.

Recent research has begun to investigate whether similar ratio-dependent characteristics are seen in rational-number magnitude tasks when presented in fraction and decimal notation. These investigations have provided mixed evidence, suggesting that the processing of both fraction and decimal notation is context-dependent (Bonato, Fabbri, Umilta, & Zorzi, 2007; Ganor-Stern, Karasik-Rivkin, & Tzelgov, 2011; Ganor-Stern, 2013; Iuculano & Butterworth, 2011; Meert, Gregoire, & Noel, 2009; Zhang, Fang, Gabriel, & Szucs, 2014). Researchers investigating fraction notation specifically have attempted to disentangle whether the numerical magnitudes depicted by fraction notation are automatically accessed (similar to whole numbers) and whether the magnitudes are accessed holistically or componentially (i.e., based on the whole-number components in the numerator or denominator, and not the entire fraction). As a whole, findings from this research suggest that the numerical magnitudes represented by fractions are not automatically activated (Kallai & Tzelgov, 2012; Obersteiner, Van Dooren, Van Hoof, & Verschaffel, 2013; although see Jacob, Vallentin, & Nieder, 2012), but can be holistically accessed in some contexts when the processing of magnitude is deliberate

(Gabriel, Szucs, & Content, 2013). For example, fraction magnitudes may be accessed holistically in situations in which the range of fractions between 0 and 1 is emphasized, priming participants to focus on the relative magnitude of the unit fractions within this range (Ganor-Stern et al., 2011). In addition, holistic processing is more likely to occur when the components of the fractions are difficult to compare (e.g., when the fractions are presented sequentially, as opposed to simultaneously; Ganor-Stern, 2013) and when the fractions are designed to make the components less meaningful (e.g., nonmatching numerators and/or denominators preventing a "larger numerator equals larger fraction" strategy; Meert et al., 2009; Schneider & Siegler, 2010; Zhang et al., 2014). It is important to note, in these contexts that emphasize holistic processing, both adult responses (Faulkenberry & Pierce, 2011; Schneider & Siegler, 2010; Sprute & Temple, 2011) and children's responses (Meert, Gregoire & Noel, 2010; Wang & Siegler, 2013) show systematic ratio effects when the magnitudes represented by two fractions are compared.

Though fewer studies have focused on decimal-magnitude understanding, some evidence suggests that similar to fractions, decimal magnitudes may not be automatically accessed (Kallai & Tzelgov, 2014). However, when decimal magnitude information is deliberately accessed (e.g., in a magnitude-comparison task) decimals also show evidence of ratio-dependent responding, suggesting that decimal magnitudes are also represented in an ordered and approximate system (DeWolf et al., 2014; Varma & Karl, 2013; Wang & Siegler, 2013).

Thus, when children and adults compare fractions to fractions or decimals to decimals, responses are generally ratio-dependent, with smaller numerical differences between values leading to lower accuracy and longer RTs for these within-notation judgments. This suggests that, despite difficulties with rational-number understanding, adults and children are able to think about the magnitudes of fractions and the magnitudes of decimals each as ordered, approximate magnitudes along a mental continuum.

It is important to note, however, there is yet to be a systematic investigation of rational-number magnitude comparisons across different notations, leaving open the question of whether whole-number, fraction, and decimal magnitudes are represented as falling along the *same* mental continuum, or instead are conceived of as three distinct ordered systems. That is, are magnitude comparisons made *across* notation types (e.g., 3/4 vs. 0.5) ratio-dependent? On the one hand, the mental continuum has been posited to be a continuous system (e.g., Gallistel, Gelman, & Cordes, 2006) and a recent theory of number-knowledge development highlights the continuities in broadening whole-number knowledge to include other rational numbers (Siegler & Lortie-Forgues, 2014; Siegler, Thompson, & Schneider, 2011). If this is the case, then all real-valued numerical magnitudes, including rational numbers, should be represented in an integrated fashion, independent of symbolic notation (i.e., decimal or fraction), resulting in across-notation ratio effects. In line with this hypothesis, evidence suggests that unit fractions (i.e., $1/n$, where n ranges from 2 to 9) and basic decimals (i.e., $0.n$, where n ranges from 1 to 9) are represented along the same mental continuum as positive whole numbers (Ganor-Stern, 2013; Ganor-Stern, 2012). However, magnitude processing of these simple fractions and decimals does not necessarily require holistic processing of magnitudes; observed ratio effects could instead be the result of componential

processing of the denominator or other potential heuristics associated with these commonly used fractions. In addition, some evidence suggests that people treat all fractions as being a general value less than one (Kallai & Tzelgov, 2009), suggesting that "improper" fractions (those greater than 1) may not show the same magnitude-based responding as those that are less than one and/or unit fractions in particular. Thus, this leaves open the question of whether fractions in general are treated similarly or if those below one are given an advantage by aligning with intuitions about fraction magnitudes.

Similarly, despite the fact that these notations come from the same number system and, in some cases, represent identical magnitudes (e.g., 0.5 and 1/2), the robust and systematic errors children and adults make when processing rational numbers may be indicative of a failure to conceive of those values as falling within the same rational-number system, but instead as representing two or three distinct numerical systems (Christou & Vosniadou, 2012; Vamvakoussi & Vosniadou, 2010). For example, when asked what kinds of numbers are between two fractions, many high school students will deny that either decimals or whole numbers can be between them, but instead insist that only fractions can be between two fractions (Vamvakoussi & Vosniadou, 2010), indicating students do not conceive of an integrated rational-number system. Moreover, findings that the ratio effects measured when comparing two fractions are higher (i.e., more greatly impacted by the ratio) than those of comparisons between two decimals (DeWolf et al., 2014), provide further evidence to support that fractions and decimals may refer to distinct mental representations.

Along these same lines, work investigating magnitude representations of different kinds of numerical information have revealed marked differences in the presence or magnitude of ratio effects, providing support for the claim that children and adults may conceive of numbers as falling along multiple distinct continuums. For example, when comparing fractions to integers (positive and negative whole numbers), ratio effects were found when positive whole numbers were compared with fractions, but not between positive and negative numbers (Ganor-Stern, 2012), suggesting that negative numbers may not be represented on the same mental continuum as whole numbers. Similarly, evidence suggests that relative frequencies (e.g., 540 in 1000) do have analog representations akin to a mental continuum, but these representations depend on the scale being used (as evidenced by higher ratio effects when comparing across scales than when comparing within a scale; Warren & Cohen, 2013). In addition, Kallai and Tzelgov (2009) provided evidence to suggest that adults may consider fractions in general as only representing values less than one, regardless of their actual magnitude, that is, both proper (i.e., <1) and improper (i.e., >1) fractions are treated as though they are smaller than one. If this is the case, then adults should not show magnitude-based responding when comparing a fraction to a whole number, but instead should show responding consistent with these more general heuristics. For example, when using the "fractions are small" heuristic, comparing a fraction to a whole number should be easier when the fraction represents a smaller value than the whole number (consistent with the bias) and more difficult when the fraction represents a larger value.

Together, these previous findings suggest that children and adults may not perceive whole numbers, fractions, and decimals as an integrated system, but rather may see them as distinct numerical

systems. If so, then judgments across notation types (e.g., 3/4 vs. 0.5) may not result in systematic responding characteristic of an ordered, approximate number representation, even though magnitude judgments within notation (e.g., 3/4 vs. 1/2) may reveal systematic ratio effects. Moreover, by using a variety of fraction values, i.e., both smaller than and larger than whole numbers, in these across-notation comparisons, we can further investigate whether people process fractions, decimals, and whole numbers as general magnitudes along an integrated continuum or alternatively, if heuristics like "fractions are smaller than whole numbers" are primarily used instead.

In addition to ratio-effect performance characteristics, notational preferences and relative affordances of fractions and decimals are open questions that have only recently begun to be addressed. Few studies have attempted to characterize how rational-number processing may differ as a function of notation or whether the use of two distinct notation systems may serve as an impediment to learning rational-number concepts. Recent evidence suggests that decimal notation may provide better access to magnitude information compared with fraction notation (DeWolf et al., 2014; DeWolf, Bassok, & Holyoak, 2015; Wang & Siegler, 2013; Zhang, Wang, Lin, Ding, & Zhou, 2013). These findings are in line with arguments that decimal notation is a more transparent notation system for children to learn (Johnson, 1956) and that curriculums emphasizing decimals and percentages before fractions, may lead to a better understanding of rational-number concepts (Moss & Case, 1999). However, in U.S. curricula, fractions are generally introduced before decimals, resulting in greater exposure to this notation over the course of schooling, suggesting that there may be some contexts in which fraction notation is preferred, as opposed to decimal notation. For example, recent evidence suggests that relational (i.e., part–whole) information is more readily accessed using fraction notation, relative to decimal notation (DeWolf et al., 2015). During the current study, we further clarified the relative difficulty of fraction and decimal notation in tasks requiring magnitude processing and basic procedural arithmetic involving rational numbers.

In addition to investigating performance differences between the two notations, it is unclear how individuals incorporate the use of both notations simultaneously (i.e., compare 0.5 and 3/4). One possibility is that symbolic processing of rational-number magnitude should be easier when values are presented within a common notation system (i.e., decimals to decimals or fractions to fractions), regardless of the notation type, much like it is easier to compare apples to apples than apples to oranges. If so, then comparisons of rational numbers presented in distinct notations (decimal vs. fraction) should be relatively more difficult than those presented in a common notation (fraction vs. fraction or decimal vs. decimal). Alternatively, it may be that numerical magnitudes are less accessible using one of these notations compared with the other (e.g., understanding fractions may be more difficult than decimals). In this case, magnitude comparisons across notation systems (decimal vs. fraction) should still be relatively more difficult (slower, less precise) than those involving two values presented in the preferred notation system (e.g., decimal vs. decimal), but it is important to note, they should be relatively *easier* than comparisons involving two values in the less preferred notation (e.g., fraction vs. fraction); that is, relative difficulty should scale with the number of values presented in the less preferred

notation. The current investigation sheds light on these alternative predictions by examining the relative difficulty of across-notation and within-notation magnitude comparisons.

In addition to behavioral data, we explored eye-gaze patterns during rational-number processing to characterize implicit levels of processing. Eye-tracking methods used during whole-number, magnitude-comparison tasks have provided evidence of systematic ratio effects in eye-fixation patterns (Merkley & Ansari, 2010). However, whether a similar ratio dependency would be found for fractions and decimals is unclear. Given the increased perceptual complexity of fraction and decimal notation, i.e., multiple components including nonnumerical components (i.e., decimal point or vinculum), it is unclear whether eye fixation patterns would be similarly sensitive to the ratio between the values.

Furthermore, although ratio effects in both within- and across-notation comparisons would suggest that adults represent these values (regardless of notation) on a single integrated magnitude continuum, componential biases (i.e., whole-number biases) may still be evident at an implicit level. Previous studies have used eye-tracking methods to study componential processing of both fraction (Huber, Moeller, & Nuerk, 2014) and decimal (Huber, Klein, Willmes, Nuerk, & Moeller, 2014) notation. These studies have revealed novel insights into the processing of rational numbers, for example suggesting similarities in the processing of decimals and natural numbers (Huber, Klein et al., 2014) and showing that whether and how componential processing is used for fraction-notation changes depending on the context (Huber, Moeller et al., 2014). However, whether these implicit biases interfere with rational-number processing, resulting in poorer performance on other math assessments, is an open question. Thus in the current study, we explored whether relative eye-fixation patterns to the two fraction components (i.e., numerator and denominator) may provide insight into individual numerical processing strategies (e.g., implicit whole-number biases unavailable via standard behavioral response time and accuracy measures) by investigating whether these implicit strategies are predictive of performance on a procedural fraction arithmetic task.

In sum, this study specifically addressed three main research goals: (a) to determine whether fractions, decimals, and whole numbers are represented as using a single integrated mental continuum; in other words, whether across-notation comparisons show systematic responding characteristic of an ordered approximate magnitude representation (i.e., ratio effects); (b) to assess the relative difficulty of rational-number processing in fraction notation versus decimal notation in both magnitude (e.g., 1/2 vs. 3/4 compared with 0.5 vs. 0.75) and arithmetic context (1/2 + 3/4 compared with 0.5 + 0.75), as well as to explore how magnitude processing across notations (e.g., 1/2 vs. 0.75) might clarify the picture; and (c) to investigate how individual implicit strategies during fraction processing, as measured via eye-tracking techniques, might predict competence in fraction arithmetic.

Method

Participants

Sixty-two Boston College students ($M = 19.4$ years, range = 17 to 24 years, 15 men) participated for course credit or $10. Fifty participants had accurately calibrated eye-tracking data in addition to the behavioral data.[2]

Stimuli and Apparatus

Number-comparison task. On every trial of the number-comparison task, subjects were presented two rational numbers (decimal, fraction, and/or whole number) side by side and were asked to judge which of the numbers was larger. There were six distinct notation-comparison types, each presented within its own block: fraction versus fraction (FvF; e.g., 1/2 vs. 3/4), decimal versus decimal (DvD; e.g., 0.50 vs. 0.75), whole number versus whole number (NvN; e.g., 5 vs. 2), whole number versus decimal (NvD; e.g., 5 vs. 2.10), whole number versus fraction (NvF; e.g., 4 vs. 9/2), and decimal versus fraction (DvF; e.g., 0.89 vs. 4/5). The order of the blocks was counterbalanced between participants. The ratio between the two numerical values (larger value/smaller value) presented on each trial was chosen from one of four approximate numerical ratio bins: 1.125 (exact ratios tested ranged from 1.11 to 1.17), 1.25 (exact ratios tested ranged from 1.24 to 1.27), 1.5 (exact ratios tested ranged from 1.35 to 1.67), and 2.5 (exact ratios tested ranged from 2.2 to 2.92). Each ratio bin for each comparison type included four unique comparisons shown twice (once with the largest on the left and once with the largest on the right). Thus, each block contained a total of 32 trials (4 comparisons × 2-shown-twice comparisons × 4 ratio bins), making the entire task 192 trials (32 trials × 6 blocks).

All stimuli were made using Adobe Illustrator in 72-pt regular Arial font (with a height of approximately 2 cm) and presented on a 1400.08-cm^2 screen (1024 × 768 pix). For fraction stimuli, the numerator and denominator values were between 1 and 15 and no number appeared twice in the same comparison (e.g., 2/4 vs. 4/5 would not occur), meaning all four integers used to make up the two fractions were distinct so as to prevent the use of overt componential, whole-number strategies (as in Schneider & Siegler, 2010). In addition, on FvF trials, the relative magnitude of the fraction was congruent with the relative magnitude of the numerator (i.e., the larger valued fraction had the larger valued numerator) on 14 of the 32 trials and incongruent on the other 18 trials, making a numerator-based strategy unreliable. For decimal stimuli, all numbers were presented to the hundredths digit (i.e., two digits after the decimal point), with a whole number (sometimes 0) always before each decimal point (e.g., 0.20; 1.25). Decimal values ranged from 0.20 to 22.50; fraction values ranged from 1/5 to 15/2; whole numbers ranged from 1 to 45. Between each trial, a 32-pt font (1 cm^2) fixation point appeared in the center of the screen.

Eye-tracking apparatus. Participants' eye movements were recorded with a SensoMotoric Instruments (SMI, Boston, MA) mobile eye tracker. Participants were seated at eye level with the center of a 22-in. computer monitor at a distance of approximately 24 in. (although exact distance, seating level, and lighting level was adjusted for each participant for optimal calibration). The SMI system recorded participants' left eye gaze at 120 Hz via an SMI RED mobile tracking system. Recording accuracy was within a 1° visual angle.

Fraction and decimal procedural assessments. The fraction procedural assessment and decimal procedural assessment each

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[2] The other 12 participants were not calibrated within 1° (the set criterion) for reasons such as excessive movement, problems with lighting, and/or problems with height and adjustments.

consisted of eight items involving fraction or decimal arithmetic (two items each of addition, subtraction, multiplication, and division) presented in two separate blocks (order counterbalanced across participants; see the Appendix for the full set of questions). None of the fraction arithmetic problems contained two fractions with a common denominator. On the decimal assessment, half the problems contained two decimals to the hundredths digit (e.g., 1.27 + 0.89) and half the problems contained one decimal to the hundredths digit, and the other to either the tenths or thousandths digit (e.g., 0.5 + 0.13; 1.74 − 1.321). Answers to all problems resulted in positive values. The questions within each block were presented in random order. All questions were presented on a Macintosh laptop, which was different from the eye-tracking computer used for the comparison task.

Procedure

Participants were seated in a quiet room. The experimenter entered the room to explain the instructions at the beginning of each task. During the tasks, the experimenter sat just outside the room out of the participants' view and could monitor the eye-tracker during the task.

All participants first completed the number-comparison task followed by the two arithmetic assessments (fraction and decimal, order counterbalanced). Participants also completed an algebra assessment prior to the comparison task, however, data from the algebra assessment will be reported elsewhere (Hurst & Cordes, 2015) and will not be discussed here further. For all tasks, participants were encouraged to perform as quickly and accurately as possible.

The number-comparison task was performed on the eye-tracker. The eye tracking session began with a 4-point calibration procedure. Participants were instructed to follow a fixation circle that moved to four quadrants on the computer screen. Participants were instructed to hold their fixation onto the circle until it moved to the next quadrant. After the calibration, participants were provided with additional instructions for the number-comparison task and then told to begin whenever they were ready. The experimenter monitored eye-tracking status throughout the experiment, and if needed, the setup was adjusted between comparison blocks.

Each of the six numerical comparison types (i.e., DvD, NvN, FvF, NvF, NvD, DvF) was presented in a block of 32 trials. The blocks were presented in six different fixed orders between participants, with participants randomly assigned to a particular order (keeping the distribution across orders even). Participants selected which of the two numbers was larger by selecting one of the two neighboring brackets on the keyboard with their right hand: left stimulus = "[", right stimulus = "]". After selecting his or her answer with the right hand, the participant advanced to the next trial by pushing the F4 key on the keyboard with the left hand. Participants were instructed to keep both hands on the keyboard throughout the session. Numbers remained on the screen until the participant advanced to the next trial. A fixation cross appeared between each trial in the middle of the screen to direct attention back to the middle before new numbers became visible. Before the task began, participants performed two nonnumerical practice trials in which they indicated the side corresponding to the location of a circle on the screen to ensure they understood which buttons to select. In addition, each block of trials began with two block-specific practice trials, on which the participant received feedback from the computer (e.g., they received two FvF practice trials prior to the FvF block).

For both the fraction and decimal procedural assessments, participants were seated in front of Macintosh laptop computers and given pencil and paper. Questions were presented one at a time on the computer screen (allowing for the timing of individual questions) but participants were asked to solve all problems using the pencil and paper. After solving each problem, the participant pressed a button on the computer to advance to the next question. Participants were told that their responses would be timed but that there was no time limit for responding.

Data Analyses

Number-comparison task measures. Accuracy analyses include data from all trials on the number-comparison task. For RT analyses, only RTs shorter than 4000 ms, within 3 *SD*s of that individual's mean RT for that Comparison Type × Ratio Combination, and from trials in which the participant gave the correct response were included. Following these exclusion criteria, in order for data from a Comparison Type × Ratio Combination to be included, the participant had to have scored above chance (5 out of 8 correct) on that subset of trials and have a minimum of two included data points (correct, within 3 *SD*s of individual mean, and under 4000 ms).

Reaction-time ratio effects were calculated for each comparison type as the slope of line of best fit across numerical ratio (i.e., *x* values = 1.125, 1.25, 1.5, 2.5; *y* values = average RTs) for each individual. Similarly, to ensure equal treatment of RTs across the four ratio, average RTs for each comparison type were calculated as the average of the average RTs for each ratio. To be included in these slope and average data analyses for a particular comparison type, participants must have had usable data (as per the above criteria) from at least three of the four ratios (i.e., at most one missing data point per type). Ten participants had missing data from only one comparison type (FvF $n = 6$; DvF $n = 3$; NvF $n = 1$) and these data points were replaced with the group mean of that comparison type. Data from two participants were excluded from the analyses for having missing data from at least two comparison types, resulting in a total of 60 participants being included in all number-comparison analyses.

Eye tracking. BeGaze software (SMI, Boston, MA) was used to analyze eye-tracking data during the number-comparison task. Eye-tracking data were analyzed using preset rectangular areas of interest (AOIs) fit around the numerical symbol. All types were analyzed using two AOIs: larger number (i.e., correct response) and smaller number (i.e., incorrect response). AOI size depended on the type of number: fractions (31,248 pix^2; 55.6 cm^2), decimals (28,854 to 29,484 pix^2; 51.3 to 52.4 cm^2), and whole numbers (14,364 to 16,330 pix^2; 25.6 to 29.1 cm^2). Within each type of trial, the AOI size for each number type remained the same, but the AOI size differed slightly between trial types. Additional AOI analyses examining eye-gaze patterns to the numerator and denominator on FvF trials were also performed (each AOI 13,566 pix^2; 24.2 cm^2).

Fixations were defined as a participant's gaze resting within 1° of a predetermined AOI for at least 80 ms. Fixation duration was calculated as the total length of fixations within the specified area

and only trials that included at least one fixation within the specified area were included in the fixation duration analyses. Reported here are results of analyses involving the total fixation durations, an indicator of cognitive processing, with more difficult processing resulting in longer fixations (see Rayner, 1998 for review), however analyses of fixation counts (number of fixations made) and saccades yielded very similar patterns. Data inclusion was identical to the RT data. Of the individuals who had accurately calibrated eye-tracking data, eight individuals had data replaced by the mean for only one comparison type and one participant was excluded for having missing data in more than one comparison type. This resulted in the data from 49 participants being included in all eye-tracking analyses.

Procedural assessments. For the fraction and decimal procedural assessments, accuracy was near ceiling levels ($M_{fraction} =$ 7.08 of 8, $M_{decimal} = 6.08$ of 8) with little variability. Thus, completion times were used as our dependent variable as a measure of performance, with longer completion times being indicative of poorer performance.

Corrections. In all analyses, when a correction for sphericity was required, the Huynh–Feldt correction was used. To address the issue of multiple comparisons and control the family-wise error rate, we used the Holm's sequential method within each family of tests (Holm, 1979; Howell, 2013). This method involves performing each individual test to calculate the uncorrected p value, and then comparing this p value to a specified adjusted comparison level. To calculate the adjusted comparison level, we arranged the uncorrected p values sequentially from smallest to largest (with the index k labeled from 1 to n, the number of tests) and comparing each uncorrected p value to $\alpha/(k - [n + 1])$. Because this was a sequential procedure, as soon as a null result was not rejected, all the following tests in the family were also deemed nonsignificant. We used the α level of .05. In each of the tests, we reported the Holm's adjusted comparison level for each test, until nonsignificance was reached for that particular family.

Results

Number-Comparison Task: Potential Response Biases

To explore the possibility of response biases in across-notation comparisons (such as expecting the fraction to always be smaller than the whole number), preliminary analyses investigated whether responding in the across-notation comparisons differed as a function of which notation was the larger (i.e., correct) value. We found no significant differences in RT to select the larger value (i.e., a correct response) in DvF comparisons when the fraction was larger than when the decimal was larger, NvD comparisons when the decimal or whole number was larger, and in NvF comparisons when the whole number was larger versus when the fraction was (in contrast to Kallai & Tzelgov, 2009; all $ps > .2$).

Number-Comparison Task: Ratio Effects

To assess the plausibility of an integrated mental continuum model of rational-number representation, we investigated whether ratio effects were present in our number-comparison task data. At first inspection of the data, it was observed that average RTs were generally monotonically decreasing as numerical ratios increased

(1.125, 1.25, 1.5, and 2.5), consistent with the existence of ratio effects. The only exception to this rule was data from the NvF and DvF comparisons, which had inexplicably shorter RTs for the hardest ratio bin (1.125) relative to the next hardest ratio bin (1.25; NvF $M_{1.125} = 2075$ ms; $M_{1.25} = 2176$ ms, $M_{1.5} = 2122$ ms, $M_{2.5} = 2039$ ms; DvF $M_{1.125} = 2150$ ms; $M_{1.25} = 2221$ ms, $M_{1.5} = 2234$ ms, $M_{2.5} = 2036$ ms).

Critically, however, we were interested in looking at the pattern of performance for each individual for each comparison type. Thus, ratio-effect measures were calculated for each comparison type on average RTs as the slope of the line of best fit across numerical ratio (i.e., x values = 1.125, 1.25, 1.5, 2.5; y values = average RTs) for each individual such that a slope of 0 indicated no ratio effect and a negative slope indicated the predicted ratio effect (i.e., responses to smaller ratios took longer than responses to larger ratios). Thus, for each comparison type we compared the ratio effect slope measures to a slope of 0 (using single-sample t tests). These comparisons revealed significant ratio effects in the predicted direction for all six comparison types (controlling for family-wise error rate with six tests), FvF $M = -110.7$ ms, $t(59) = -2.657$, $p = .01$ (Holm's level = .0167), Cohen's $d =$ 0.327; DvD $M = -42.55$ ms, $t(59) = -3.599$, $p < .001$ (Holm's level = .008), Cohen's $d = 0.424$; NvN $M = -81.63$ ms, $t(59) = -14.027$, $p < .001$ (Holm's level = .01), Cohen's $d =$ 0.877; DvF $M = -78.06$ ms, $t(59) = -2.334$, $p = .023$ (Holm's level = .025), Cohen's $d = 0.29$; NvD $M = -48.68$ ms, $t(59) = -4.506$, $p < .001$ (Holm's level = .0125), Cohen's $d =$ 0.51; NvF $M = -68.67$ ms, $t(59) = -2.081$, $p = .04$ (Holm's level = .05), Cohen's $d = 0.26$; see Figure 1, Panel A. A repeated measures ANOVA on these slopes revealed no main effect of type, $F(3.06,180.27) = 0.896$, $p = .45$, $\eta_p^2 = 0.015$, suggesting that the magnitude of these ratio effects were comparable across comparison types. It is important to note, ratio effects were observed in comparisons involving common notations (i.e., FvF, DvD, NvN, replicating previous findings), and in those involving distinct notations (i.e., DvF, NvF, NvD), providing strong evidence of a systematic responding characteristic of an ordered, approximate mental representation, regardless of notation type.

Eye-tracking measures of ratio effects. Similar to the behavioral RT data, only fixation durations on correct trials were analyzed and ratio effects of fixation durations were calculated as the slope across ratios (x value = 1.125, 1.25, 1.5, 2.5; y values = fixation duration). To assess whether evidence of an integrated mental continuum was also found at the implicit level using eye-tracking measures, we investigated whether there were significant ratio effects found in fixations on the larger (correct) and on the smaller (incorrect) numbers in each of the comparison types. Single-sample t tests revealed a significant ratio effect (with a slope of 0 suggesting no ratio effect and a negative slope suggesting a ratio effect in the predicted direction) in fixation durations on the smaller number, but not on the larger number, across all comparison types except NvD (NvD looking at larger and smaller stimulus, $p > .5$), and only marginally for NvF, which did not hold up to multiple comparisons for looking at the smaller stimulus ($p = .026$, Holm's adjusted level = .025; see Table 1 for a full set of test statistics and effect sizes and see Figure 2, Panel A). Of note, although ratio effects were only apparent in looking at the smaller number, participants looked significantly longer at the
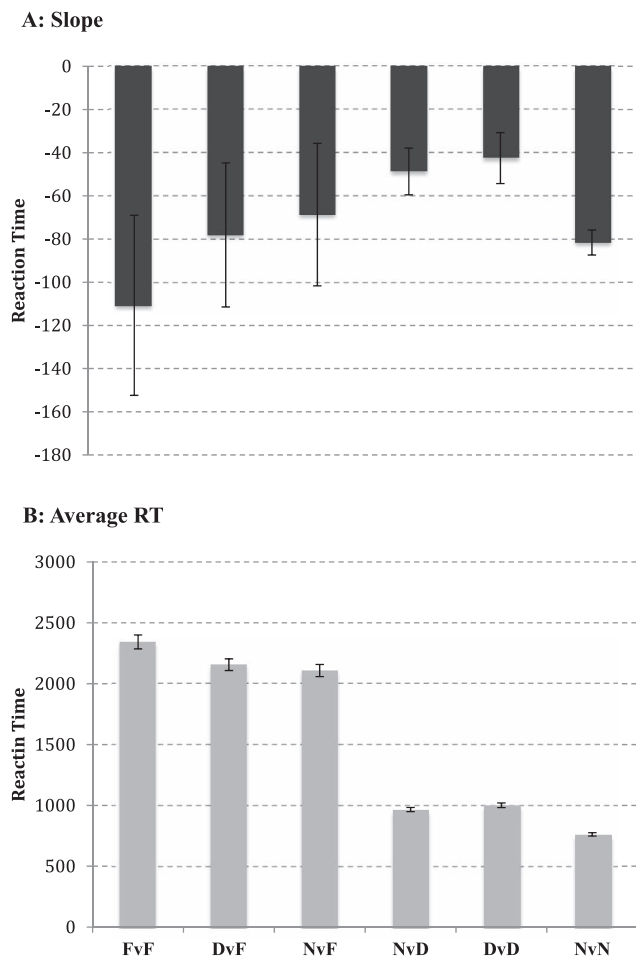
A: Slope

B: Average RT

Figure 1. Panel A: Ratio effects, as measured by the slope in RT across ratios, in all comparison types (error bars represent standard error). There were significant ratio effects observed in all comparison types but no difference in the magnitude of ratio effects across comparison types. Panel B: Average RT in all comparison types in the number-comparison task (error bars represent standard error). There is a clear distinction between comparisons involving fractions and those not involving fractions, with all comparisons involving fractions taking significantly longer than those involving decimals comparisons.

larger (compared with the smaller) number across all trial types (ps < .001; see Figure 2, Panel B).

Comparing Notations: Average RT and Accuracy

Next, we investigated the relative difficulty of comparing the various comparison types using two repeated-measures ANOVAs with a single within-subject factor of comparison type (6: FvF, DvD, NvN, DvF, NvD, NvF) on average RT and on accuracy.

There was a significant main effect of type on RT, $F(2.8, 163.2) = 625.4$, $p < .001$, $\eta_p^2 = 0.914$, and accuracy, $F(3.069, 187.2) = 40.554$, $p < .001$, $\eta_p^2 = 0.399$. Comparisons involving fractions took the longest and had the lowest accuracy, followed by those involving decimals, followed by whole-number comparisons, RT: $M_{\text{FvF}} = 2343$ ms, $M_{\text{DvF}} = 2156$ ms, $M_{\text{NvF}} = 2108$ ms, $M_{\text{DvD}} = 1001$ ms, $M_{\text{NvD}} = 966$ ms, $M_{\text{NvN}} = 762$ ms; accuracy (%

correct): $M_{\text{FvF}} = 84.1\%$, $M_{\text{NvF}} = 88.1\%$, $M_{\text{DvF}} = 90\%$, $M_{\text{DvD}} = 96.3\%$, $M_{\text{NvD}} = 96.3\%$, $M_{\text{NvN}} = 98.1\%$; see Figure 1, Panel B.

One of our primary questions was the relative difficulty of comparisons involving two fractions, two decimals, or a fraction and a decimal. Thus, additional planned comparisons on accuracy and average RT of FvF, DvD, and DvF comparisons indicated that comparisons involving two fractions (FvF) were significantly more difficult (slower and less accurate) than comparisons between a decimal and a fraction (DvF), which in turn were significantly more difficult than comparisons involving two decimals (DvD; all ps < .001, for both accuracy and RT; medium effect sizes between FvF and DvF, Cohen's $d = .54$ (RT) and .59 (accuracy); large effect sizes for DvF versus DvD, Cohen's ds > .8 for both; DvD versus FvF, Cohen's ds > 3.0 for both).

Whereas magnitude processing of values presented in decimal notation was significantly faster and more accurate than that of fractions, the converse was found when it came to arithmetic using each of these distinct notations. Analyses revealed that participants were both faster, $t(60) = 8.072$, $p < .001$; $M_{\text{fraction}} = 248.1$ s, $M_{\text{decimal}} = 420.4$ s, and more accurate, $t(60) = 5.15$, $p < .001$; $M_{\text{fraction}} = 88.5\%$, $M_{\text{decimal}} = 76\%$, when performing the fraction procedural arithmetic assessment than when performing the decimal procedural arithmetic assessment.

Fraction AOI Analyses

In addition to using traditional RT and accuracy measures to compare rational-number processing in different notations, we were particularly interested in whether evidence of whole-number biases could be found in fraction processing specifically at the implicit level using eye-fixation patterns, despite the fact that our stimuli were created with the intention of limiting whole-number strategies. Thus, specific AOI analyses were performed on FvF trials to examine the pattern of fixations on specific parts of the fraction (fixations on the numerators and denominators of the larger and smaller fractions).

First, a Stimulus (2: larger vs. smaller fraction) × AOI (2: numerator vs. denominator) repeated-measures ANOVA was performed on overall fixation durations. Results revealed that fixation duration varied across stimulus and AOI, such that (consistent with previous analyses) participants fixated longer on the larger fraction than the smaller fraction, $M_{\text{larger}} = 755.67$ms; $M_{\text{smaller}} = 704.76$ms; $F(1, 39) = 10.425$, $p < .003$, $\eta_p^2 = 0.211$. In addition, consistent with predictions of whole-number biases, participants fixated longer on the numerator than the denominator, $M_{\text{denominator}} = 669.7$ms; $M_{\text{numerator}} = 790.7$ ms; $F(1, 39) = 4.418$, $p < .05$, $\eta_p^2 = 0.096$. There was also a marginal interaction between variables; the difference in looking at the numerator and denominator of the larger fraction, $M_{\text{denominator}} = 712.6$ms, $M_{\text{numerator}} = 798.8$ ms was less dramatic (i.e., smaller) than the difference in looking at these two components for the smaller fraction, $M_{\text{denominator}} = 626.9$ ms, $M_{\text{numerator}} = 782.6$ ms; $F(1, 39) = 4.009$, $p = .052$, $\eta_p^2 = 0.093$.

To explore whether specific patterns of looking at fractions were indicative of better fraction processing, we computed an individual measure of implicit numerator bias (potentially indicative of whole-number bias) by subtracting fixation durations on the denominator (DEN; averaged across the smaller and larger fractions)

Table 1

Investigating the Existence of Ratio Effects in Fixation Durations to Both the Correct and Incorrect Stimulus in Each of the Comparison Types.

Trial type	Ratio effect of fixation duration to larger stimulus			Ratio effect of fixation duration to smaller stimulus			Holm's adjusted criteria
	Mean	Statistical test	Cohen's *d*	Mean	Statistical test	Cohen's *d*	
FvF	.61	$t(48) = .012, p > .99$	.002	−235.4	$t(48) = −6.7, p < .001$	.96	.0083
NvN	−13.9	$t(48) = −1.67, p > .10$	.24	−24.8	$t(48) = −2.75, p < .008$	.39	.0167
DvD	−3.89	$t(48) = −.49, p > .6$	.07	−33.5	$t(48) = −4.13, p < .001$	.59	.0125
DvF	−63.5	$t(48) = −1.449, p > .15$	.21	−177.3	$t(48) = −4.77, p < .001$	.68	.01
NvF	−62.42	$t(48) = −1.44, p > .15$	.21	−82.9	$t(48) = −2.29, p < .026$	.33	.025
NvD	1.18	$t(48) = .118, p > .9$	.02	−6.0	$t(48) = −.63, p > .53$	.09	.05

Note. Fraction versus fraction (FvF; e.g., 1/2 vs. 3/4), decimal versus decimal (DvD; e.g., .50 vs. .75), whole number versus whole number (NvN; e.g., 5 vs. 2), whole number versus decimal (NvD; e.g., 5 vs. 2.10), whole number versus fraction (NvF; e.g., 4 vs. 9/2), and decimal versus fraction (DvF). The statistical tests are single-sample *t* tests on the slope of fixation durations across the four ratios. A slope of 0 indicates no evidence of ratio-dependent responding and a negative slope indicates ratio-dependent responding with increased fixations for smaller ratios (i.e., higher difficulty). Holm's adjusted criteria is the adjusted α level used for each comparison to maintain the family-wise error rate of 5%. All looking time ratio effects to the smaller stimulus are significant except for NvF, which shows marginal significance that does not hold up to multiple comparisons, and NvD.

from fixation durations on the numerator (NUM; averaged across the smaller and larger fractions). We then measured the correlation between performance in the fraction procedural assessment and the numerator bias score. Consistent with predictions that greater relative looking at the numerator was indicative of a whole-number bias, and thus less efficient fraction processing, this difference score was positively correlated with fraction procedural completion time, $r(40) = 0.386, p = .014$ (see Figure 3). Thus, looking longer at the numerator than at the denominator (indicative of stronger implicit whole-number biases) was associated with poorer performance on the fraction procedural assessment.

To confirm that this relationship was specific to fraction processing and not arithmetic understanding or rational-number understanding more generally, we used a regression analysis to control for performance on the decimal arithmetic assessment. Overall, the model was significant, $R^2 = 0.173, F(2, 37) = 3.88, p = .029$. It is important to note, however, even when controlling for decimal arithmetic completion time, $\beta = 0.163, p = .3$, looking at the numerator relative to the denominator on the magnitude task was a significant and unique predictor of fraction arithmetic, $\beta = 0.342, p = .03$.

Discussion

In this study, we thoroughly investigated rational-number magnitude understanding across notations using both a magnitude-based task and a procedural arithmetic task. In addition, we used more general fraction and decimal values beyond common unit fractions and included comparisons within a single notation as well as across two notations, allowing for a more complete picture of adults' processing of rational numbers. Further, using implicit eye-gaze patterns, we were able to identify implicit whole-number biases present in fraction-magnitude tasks (even when we used carefully controlled stimuli to discourage componential processing) and used the individual bias measure to predict performance on a fraction arithmetic task.

Our primary research question addressed whether the systematic response pattern indicative of an ordered, approximate magnitude representation would be seen across fraction, decimal, and whole-number notations. In other words, are these values represented in

a notation-independent fashion along an integrated mental continuum? In line with previous studies, we found significant RT ratio effects for comparisons involving exclusively whole numbers (NvN; e.g., Merkley & Ansari, 2010; Moyer & Landauer, 1967, 1973), exclusively fractions (FvF; Faulkenberry & Pierce, 2011; Meert et al., 2010; Schneider & Siegler, 2010), and exclusively decimal values (DvD; DeWolf et al., 2014; Varma & Karl, 2013). Most significantly, however, our data are the first to simultaneously reveal ratio effects in data from comparisons involving distinct notations across all three notation types for values both above and below 1 (i.e., DvF, DvN, NvF). So, despite the prevalence of whole-number biases (Ni & Zhou, 2005) and difficulties in understanding the relationship between values presented in distinct notations (Vamvakoussi & Vosniadou, 2010), when judging numerical magnitudes, results indicate that adults represent rational numbers in a notation-independent fashion, considering fraction, decimal, and whole-number magnitudes as falling within the same mental continuum. Providing further support for this claim, we found no evidence of particular biases or heuristics in the across-notation comparisons (e.g., adults were no faster at responding when the whole number was larger than the fraction than when the fraction was larger than the whole number), suggesting that participants processed the magnitude of each value without notation-dependent heuristics (such as "fractions are always less than one;" Kallai & Tzelgov, 2009). That is, not only do we find evidence of ratio-dependent responding, but also by investigating performance across different notations, we showed no evidence of biases or heuristics about the general size of particular types of numbers. Thus, in the context of fraction, decimal, and whole-number notations that span a range of numerical values, adults appear to process the magnitude associated with the numerical symbol for all rational-number notations.

Further support for an integrated continuum was found in the eye-gaze data, in which ratio effects in average fixation durations were found in looking at the incorrect stimulus across all comparison types (except, inexplicably, for NvD and only marginally for NvF). Thus, even when making magnitude judgments across no-
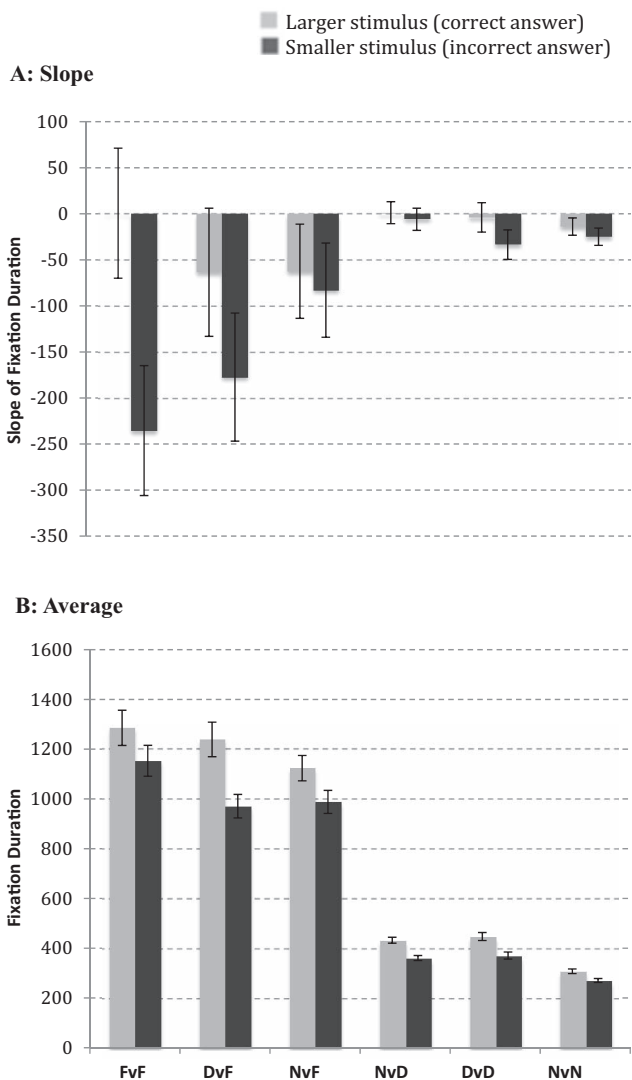
Figure 2. Panel A: Ratio effects of looking time to the larger (light grey; on the left) and smaller (dark grey; on the right) stimuli, as measured by the slope across ratios, in all comparison types. Significant ratio effects in fixations on the smaller number stimuli, but not to the larger number, were found in all comparison types (except NvD and only marginally for NvF). Panel B: Average looking time to the smaller number and larger number in all comparison types (error bars represent standard error). Looking at the larger number (dark grey; on the right) was significantly higher than to the smaller number (light grey; on the left) in all comparison types. FvF = fraction versus fraction; DvF = decimal versus fraction; NvF = number versus fraction; NvD = number versus decimal; DvD = decimal versus decimal; NvN = number versus number.

tation types, as the ratio between the two stimuli decreased, the average length of fixation on the smaller number increased. In contrast, average fixation durations on the larger number were longer than on the smaller number and not dependent upon ratio for all the comparison types. As such, the longer fixations on the larger number, regardless of ratio, were most likely a function of response selection—that is, participants simply looked longer at the value they were selecting (because only correct trials were

included). On the other hand, whereas looking at the larger number was generally longer and driven by response selection, fixations on the smaller number appeared to be dependent upon trial difficulty (and thus ratio dependent). These results suggest that fixations on the smaller (i.e., incorrect) number may be a more sensitive measure of the decision-making processes involved in numerical comparison judgments.

Thus, using both behavioral response and eye-tracking methods, the current data extend previous literature looking at rational-number comparisons by providing strong evidence for an integrated mental continuum of rational numbers, independent of notation in educated adults. While the specific interpretation of ratio effects may differ slightly between theoretical models, one key finding, independent of the interpretation of ratio effects, was the similarity in performance patterns across both within-notation (e.g., between two fractions) and across-notation (e.g., between one decimal and one fraction) comparisons, suggesting that magnitude processing may function in a similar manner, regardless of notation. Although previous studies have provided some evidence to support this, these previous studies have used common fractions, most of which were less than 1. Thus, given that some evidence suggests that all fractions are treated as being less than 1 (Kallai & Tzelgov, 2009), it is feasible that the magnitude-based responding shown for common fractions that are less than 1 would not be seen for other more general fractions. However, our study used more uncommon fractions with a range of numerical values, providing more general evidence of magnitude-based responding for fraction, decimal, and whole-number notations. These findings align with recent evidence comparing symbolic and nonsymbolic fractions, suggesting that responding in this context is also based on the magnitude of the values presented, and not on other heuristics (Matthews & Chesney, 2015). Thus, although these data do not speak to the developmental trajectory of these number systems, they provide support for an integrated theory of number-knowledge development (e.g., Siegler & Lortie-Forgues, 2014; Siegler et al., 2011) by sug-
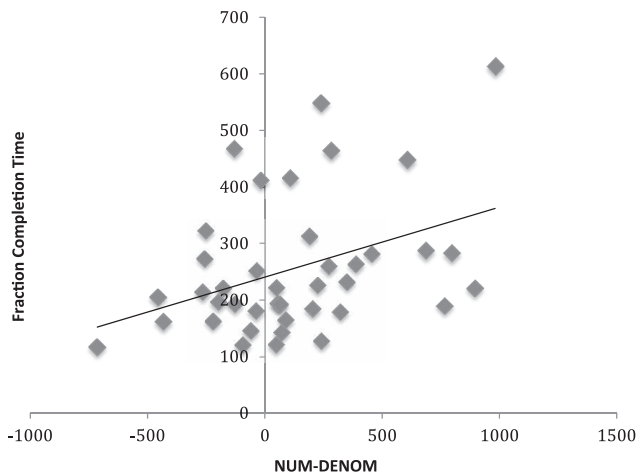

Figure 3. Fraction arithmetic ability (measured by completion time) was significantly correlated with NUM–DEN, a difference score assessing fixation durations to the numerator minus fixation durations to the denominator in FvF trials (positive values are indicative of greater relative looking at numerator and negative values are indicative of more looking at the denominator).

gesting that, at least by adulthood, adults have a unified understanding of rational numbers presented in fraction, decimal, and whole-number notation.

In light of these findings, educational program designers may be able to take advantage of the integrated continuum model by using an integrated number line as an external visual reference in the classroom. This has been done successfully with whole-number learning; a group of at-risk preschoolers showed more whole-number learning when they played linear board games than when they played circular board games (Siegler & Ramani, 2008; Siegler & Ramani, 2009). A similar approach can be taken with introducing rational numbers in fraction and decimal notation as values along a single number line. Moreover, given that fractions are frequently described as a part–whole relationship (e.g., pie charts), future researchers should investigate the relative usefulness of linear (e.g., number line) and nonlinear (e.g., pie charts) visual representations in teaching rational numbers, and determine whether either of these representations promote or hinder an integrated understanding of rational numbers.

In addition, we were interested in the relative difficulty of processing rational numbers in fraction versus decimal notation, as well as whether comparing across notation types would pose additional challenges. In line with other data suggesting that decimal-magnitude comparisons are easier than fraction-magnitude comparisons (e.g., DeWolf et al., 2014), results revealed notably lower performance on magnitude problems involving fractions compared with decimals. Specifically, results of our number-comparison task revealed that magnitude judgments involving fractions (FvF, FvD, FvN) were more challenging (longer RTs and lower accuracy) than comparisons *not* involving fractions (DvD, NvN, NvD). Moreover, responding became slower as a function of the number of fractions involved in the comparison, with performance on FvF trials (involving two fractions) being significantly worse than performance on DvF trials (involving a single fraction), which, in turn, was significantly worse than performance on DvD trials (involving no fractions). Thus, the difficulty in dealing with fraction magnitudes seems to lie in the fraction notation itself, not in comparing across notation.

One possibility is that adults may have converted fractions into approximate decimal magnitudes to judge the relative magnitudes of the numbers during the comparison task. Because this conversion process takes time and is error-prone, trials requiring the conversion of two fractions (FvF trials) necessarily took longer and were less accurate than those requiring the conversion of only a single fraction (DvF trials). Given that adults in this study found decimal-magnitude processing easier than fraction-magnitude processing, they may have chosen this strategy to maximize performance on the task. However, it should be noted that in a previous study using a similar FvF comparison task, adult participants self-reported using a decimal conversion strategy on only 5.5% of trials, making this possibility seem unlikely (Faulkenberry & Pierce, 2011).

More work directly investigating the strategies used for rational-number processing in different contexts, e.g., comparing across notations (0.5 vs. 3/4) or within the same notation (1/2 vs. 3/4), will help to determine why it is that comparisons involving fractions take significantly longer than those involving only decimals. In addition, we did not address possible strategies used during the comparison of two decimals (DvD). Previous researchers have suggested that, like multidigit whole numbers, decimals may be processed componentially, digit by digit (e.g., Huber, Mann, Nuerk, & Moeller, 2014; Varma & Karl, 2013), making it possible for decimals not to be treated as distinct from whole numbers at all (e.g., Huber, Klein et al., 2014). Future studies should further investigate these strategies and performance differences across various notations, including how notation patterns and preferences may change throughout education.

In contrast to the magnitude task, performance on the rational-number arithmetic assessment revealed a very different pattern, with faster and more accurate performance on the fraction arithmetic compared with the decimal arithmetic. This suggests that these educated adults were better able to remember and correctly carry out the rules and procedures associated with fraction arithmetic compared with decimal arithmetic, despite encountering more difficulty processing fraction magnitudes. This pattern of findings may be attributed to differences in the educational focus found in the classroom. Generally speaking, U.S. math curricula often place a greater emphasis on fractions than decimals (Common Core State Standards Initiative, 2010), thus resulting in greater practice with fraction arithmetic procedures. As such, although fraction magnitudes may be less transparent, fraction arithmetic procedures may be more likely to be remembered correctly, especially in adults who have long since completed instruction in rational-number arithmetic procedures. Together, these findings provide support for the idea that fraction notation is less transparent than decimal notation in providing magnitude information, whereas fraction notation may provide an advantage in arithmetic calculations, at least for educated adults. However, which aspect of fraction notation and procedures leads to this arithmetic and procedural advantage remains unclear.

One way to approach this question is to investigate whether these preferences for decimal notation in magnitude comparisons and fraction notation in arithmetic are present across development (i.e., from the time they are first introduced in the classroom). On the one hand, fraction notation is generally introduced first in the classroom and given greater emphasis in educational settings early in development. In adulthood, however, experience with decimal notation is much more frequent (e.g., through the monetary system). Thus, it may be the case that children just learning these notations will show a preference for fraction notation in both magnitude and arithmetic tasks, and the processing preference for accessing magnitudes via decimal notation may emerge later as a function of greater experience with this notation type. On the other hand, evidence indicating that decimal notation may actually provide an important stepping stone between whole numbers and fractions (e.g., Johnson, 1956; Moss & Case, 1999) would suggest that the observed decimal preference for magnitude information is due to the increased cognitive simplicity offered by decimal notation, regardless of experience. If so, then children should also demonstrate a preference for decimal notation as soon as these notations are introduced. Similarly for arithmetic procedures, given the emphasis on fraction learning in the classroom and the experience adults have with decimal information, it is unclear how and why adults show a preference for fraction notation in arithmetic contexts. Furthermore, whether the context-dependent preferences for fraction notation and decimal notation are apparent as children learn fraction and decimal arithmetic rules, or alternatively, whether educated adults show a distinct pattern only well after they have com-

pleted instruction in these procedures is an open question. Investigating these open questions could provide important insights into how the introduction of decimal notation and fraction notation may impact the acquisition of rational-number concepts.

Last, using eye-tracking data, we were able to look more closely at how adults process fraction magnitudes specifically. When comparing fractions, fixations on the numerators were significantly longer, on average, than on the denominators. Although stimuli were designed in such a way as to prevent componential, whole-number strategies, this looking pattern is consistent with a numerator-based whole-number bias at the implicit level. Given the inverse relationship between the value of the denominator and the value of the fraction, optimal responding should have involved either longer or at least comparable looking at the denominator relative to the numerator (Huber, Moeller et al. 2014). Thus, our finding of an implicit numerator bias is in line with other evidence suggesting that both adults and children often treat fraction components like separate whole numbers (e.g., Ni & Zhou, 2005). Furthermore, those participants who demonstrated a stronger implicit whole-number bias by fixating longer on the numerator than the denominator also performed worse on the fraction procedural assessment, even when controlling for rational-number procedural ability in decimal notation. This is the first evidence to suggest that, even when the use of whole-number strategies is prevented (by not presenting fractions with common denominators or numerators), evidence of a whole-number bias in implicit looking patterns was still found. It is important to note, the strength of this implicit bias was negatively associated with procedural performance with fractions even in educated adults who perform fairly well on both tasks.

In conclusion, data from the current study provide strong evidence of an integrated mental continuum of rational numbers, regardless of notation, while also suggesting that fraction magnitudes are much more difficult than either decimal and whole-number magnitudes. In contrast to the magnitude task, fraction notation provided an advantage for arithmetic calculation. In addition, while the adults in this study fixated significantly longer at the numerator of the fractions relative to the denominator, individual differences in this measure of whole-number bias suggested that looking more at the numerator than at the denominator suggested poorer understanding of fraction procedures. Together, these findings call for more educational and developmental investigations into rational numbers in fraction, decimal, and whole-number notation in both children and adults, to clarify exactly how these rational number values are represented in the mind and to determine how educational practices can benefit from this greater understanding. These investigations could provide important directions for the development of educational tools to promote rational-number understanding through an integrated representation, and to advance math learning in other subsequent areas, like arithmetic and algebra.

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Appendix

Arithmetic Questions

Decimal Arithmetic Questions

$0.5 + 0.13$ $1.27 + 0.89$ $0.36 - 0.12$ $1.74 - 1.321$ $0.63 \div 0.12$ $1.452 \div 0.48$ 0.456×0.32 1.75×0.21

Fraction Arithmetic Questions

$\frac{3}{4} + \frac{7}{9}$ $\frac{2}{3} + \frac{5}{7}$ $\frac{4}{5} - \frac{4}{8}$ $\frac{6}{7} - \frac{1}{4}$ $\frac{8}{9} \times \frac{1}{3}$ $\frac{4}{5} \times \frac{3}{8}$ $\frac{4}{7} \div \frac{3}{5}$ $\frac{3}{9} \div \frac{3}{8}$