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Spontaneous and Directed Attention to Number and Proportion

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Although difficulties processing both symbolic and nonsymbolic proportion compared with absolute number are well established, the mechanisms involved remain unclear. We investigate four potential explanations to account for better number processing in adulthood: (a) number is more salient than proportion, (b) number is encoded more automatically than proportion, (c) proportion is more effortfully processed than number, and (d) number competes with proportion during decision making. Across three experiments, we used a delayed match-to-sample paradigm in which adults were asked which of two alternatives matched a sample set of red and blue dots. We systematically manipulated which dimension of the sample participants matched (number of red dots, total number of dots, proportion of red dots), the presence/absence of the competing quantity in the choice alternatives, and when they were told which quantitative dimension to encode (before vs. after the sample presentation, or not at all). Overall, data reveal that proportion was less salient than the numerical subset. Additionally, the number of items within the subset, but not the total number of items in the superset, interfered with proportion-based responding. Last, even in the absence of response competition and costly task demands, proportion matching took longer than number matching, highlighting basic processing differences. Together, results reveal pervasive difficulties in representing proportion compared with number, even when task demands are unambiguous. However, this varied depending on the numerical set involved and across encoding, processing, and decision processes. We discuss the implications of these findings for theories of ratio processing and of quantity more generally.

Keywords: proportion, numerical interference, discrete quantity, fractions

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Research in both psychology and education has highlighted children's difficulty with formal fractions (Lortie-Forgues et al., 2015; National Governors Association Center for Best Practices, 2010). At least some of these difficulties have been attributed to the interference of whole number information in the way children interpret proportion (Braithwaite & Siegler, 2018; Ni & Zhou, 2005; Siegler & Lortie-Forgues, 2014). Although most of this work has focused on difficulties with interpreting symbolic proportion (i.e., fractions and decimals represented with numerals), recent studies have noted interesting parallels with nonsymbolic representations of proportion (e.g., pie charts, groups of items). That is, when interpreting both symbolic and nonsymbolic proportional information, whole number information has been found to interfere with proportional judgments (Boyer et al., 2008; Fabbri et al., 2012; Hurst & Cordes, 2018a; Jeong et al., 2007). Given that this whole number interference arises even before children encounter fraction or decimal instruction (by around 6 years old; Hurst & Cordes, 2018a) and continues into adulthood (even for visual nonsymbolic representations; Fabbri et al., 2012), one step toward better understanding the challenges posed by formal fractions is to understand the difficulties children and adults encounter when interpreting proportional information in nonsymbolic contexts.

Some evidence suggests that the saliency of counting for young children increases their attention to whole numbers when that countable numerical information is available. These studies reveal

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that when presented with continuous proportional information (in which proportional information is available but numerical information is not—that is, there are no countable units), children are able to make judgments based upon proportion (e.g., Boyer et al., 2008; Hurst & Cordes, 2018a; Jeong et al., 2007). However, when the proportional information is presented discretely with demarcated units, making it possible for children to count them, children tend to use the number of salient pieces to make judgments (e.g., judging that a display with five of eight blue pieces has a greater proportion of blue than another containing three of four because five is greater than three), despite being asked to make judgments based on proportion (Boyer et al., 2008; Hurst & Cordes, 2018a; Jeong et al., 2007). This overreliance on whole numbers at the expense of proportion in discrete contexts appears to be due to children's use of counting strategies; only when these strategies are not available (either because the information is not countable or because children are young enough to not yet have mastered counting) will they then rely on proportion instead (Boyer et al., 2008; Hurst & Cordes, 2018a).

This numerical interference pattern is found not only in children who are still developing their understanding of number and proportion, but also in adults who have substantial experience with whole numbers and fractions (Fabbri et al., 2012). For example, Fabbri and colleagues (2012) asked adults to judge the ratios between two sets of dots and found that the number of dots in the subsets interfered with the processing of the ratio. Furthermore, although adults are able to process ratio information fairly flexibly across representations (Matthews & Chesney, 2015), some evidence suggests that it may be more difficult to represent ratio information than absolute numerical information (Fabbri et al., 2012). Thus, both early in development and into adulthood, absolute numerical information appears to be privileged over proportional information.

This numerical advantage and interference, however, may not be an inherent or necessary aspect of proportional reasoning.[1] In support of this, some evidence suggests that in the context of discrete quantities, where numerical and proportional information are both readily available, this overattention to numerical information may be quite malleable early in development. For example, with only a short period of experience comparing continuous proportional amounts (i.e., judging proportion in a context in which contrasting whole-number information is not available), children are less likely to rely on a numerical strategy in subsequent trials where discrete quantities are available (Boyer & Levine, 2015; Hurst & Cordes, 2018a). In addition, introducing children to equivalent proportions using categorical, as opposed to numerical, language may help them attend to the relations over absolute number (Hurst & Cordes, 2019). Thus, although numerical interference is evident early in development and remains in adulthood, the malleability of this numerical interference suggests that it may not be an inherent or necessary aspect of thinking about proportion.

An important next step to studying proportional reasoning is to investigate why numerical information appears to have an advantage over proportional information. Is numerical information more readily encoded and/or more difficult to inhibit? To address this question, it is important to determine at what point during quantitative processing numerical information supersedes proportional information. In the current study, we consider four possible mechanisms that could contribute to these numerical biases: (a) differences in the relative salience of numerical and proportional information, (b) differences in the automaticity of encoding number and proportion, (c) differences in the ease of processing number and proportion, and (d) differences in the ability to inhibit alternative responses during the decision-making process. Importantly, these potential contributors are not mutually exclusive and are potentially overlapping, yet we can consider specific behavioral predictions in which each factor may contribute. To do so, we developed a delayed match-to-sample task (MTS), where participants viewed an array of bicolored dots and then chose another array from two alternatives that matched either the sample's absolute number or proportion. Importantly, across three experiments, we manipulated (a) the ambiguity of task instructions, allowing us to investigate the relative salience of these two dimensions without explicit instructions; (b) the timing of the prompt to encode a specific quantity dimension (i.e., instructions to attend to number vs. proportion occurred either before or after the presentation of the sample, providing relevant information to the participant prior to, or just after, encoding); and (c) whether or not the two choice alternatives matched the sample along the relevant and irrelevant quantity dimension, thus requiring inhibition at the time of decision making. These manipulations allowed us to shed light on the point(s) in the cognitive process during which absolute number and relative proportion compete.

One possibility is that numerical information is inherently more salient. There is some evidence that attending to numerical information does depend on the relative salience of other relevant features (Chan & Mazzocco, 2017) and that there are individual differences in how readily we mentally access rational number magnitude (Alibali & Sidney, 2015). For example, children are less likely to use number to match two stimulus arrays when number competes with color (a highly salient alternative) compared with when number competes with orientation (a less salient alternative; Chan & Mazzocco, 2017). In addition, research directly comparing proportion and number suggests that number consistently interferes with proportion, but proportion less consistently interferes with number (Fabbri et al., 2012). This suggests that although certain aspects of both number and proportion may be automatically encoded, number may still be more salient than proportion, leading to more consistent attention to numerical information. If number is more salient than proportion, then in the absence of direct instruction when task demands are ambiguous, adults will spontaneously rely on numerical information more than proportional information. This is a possibility we explore in Experiment 1.

Another deeply related possibility is that numerical information is more automatic, whereas attending to proportion is more effortful. Fabbri and colleagues (2012) found that both number and proportion interfered with each other, suggesting they are both automatically encoded to some extent. However, participants were also much slower at matching on proportion than on number, suggesting the proportional information required more effortful processing. On the other hand, other researchers have suggested that proportional information is processed through a perceptual ratio processing system (RPS; Lewis et al., 2016) and that even symbolic fractions may not require computation to access magnitude

[1] Throughout, we use the term *numerical* to refer to whole number information.

(Binzak & Hubbard, 2020). Here, we investigate two separate ways in which proportional information may be more effortful and/or less automatic: automaticity at the level of encoding and differences in the effortful cost of processing outside encoding.

At the level of encoding, we investigate whether numerical and proportional information are both automatically encoded, or alternatively if numerical information is more automatic, by manipulating the timing of encoding. If number and proportion are differentially encoded, then whether individuals are instructed to encode proportional or numerical information prior to seeing an array compared with after seeing it may impact the accuracy with which this information is encoded. More specifically, if there is a cognitive cost of encoding both number and proportion, then we should observe a difference in performance as a function of whether the instruction to attend to number or proportion is provided prior to versus after the presentation of the sample. Specifically, we should see an effect of the probe timing; the prompt occurring before the sample resolves ambiguity about which dimension to attend to, but when the prompt occurs after the sample, and thus participants do not know which quantity is the relevant one, they must simultaneously attend to both number and proportion. If proportion is less automatically encoded than number, then probe timing will exert a greater effect when matching on proportion than on number. We explore this in Experiment 2.

At a more general level, we can also ask whether number and proportion show differences in processing demands. That is, regardless of the level of encoding, if there is a more general cognitive cost to reasoning about proportion than number, then we should see differences in performance on trials in which they must attend to number compared with those when they must attend to proportion. That is, if adults engage in effortful computational processes when encoding proportional information, but not for number, then it may take longer and/or be more error prone to make proportional matches, even on trials in which there is an unambiguous correct choice (i.e., simple match trials, as will be described later). The contribution of this possible mechanism is tested in Experiments 2 and 3.

The final possibility we explore here is the role of interference. That is, differences in how the information is prioritized during the decision-making process. If numerical information is prioritized and more difficult to inhibit than proportional information, then we would expect performance to drop when matching proportion in the presence of competing numerical response options, relative to when a numerical response option is not available. Specifically, performance on the match-to-sample task is expected to be worse when the incorrect response option matches the sample on absolute number (referred to as *competition trials*) compared with when the incorrect response option does not match on numerical features (referred to as *simple match trials*). Notably, this explanation is somewhat dependent on differences in saliency or encoding discussed earlier. For there to be the opportunity for response competition, numerical information must be automatically encoded from the response options. However, if there is an additional cost of the difficulty in inhibiting a numerical response, we would expect that in the presence of competing response options, proportional reasoning would be *even more* difficult. We examine this possibility in Experiments 2 and 3.

As summarized in Table 1, we consider four mechanisms that may contribute to the observed numerical interference in proportional reasoning tasks. We do so across three experiments measuring how adults process proportional and numerical information in the context of discrete, nonsymbolic displays, when given no explicit directions (to explore differences in saliency via spontaneous preferences; Experiment 1) and when explicitly directed to attend to numerical or proportional information (Experiments 2 and 3). Much of the research that has investigated numerical interference in nonsymbolic contexts has focused primarily on interference from the "numerator" information, such as the number of target items out of the total set (Fabbri et al., 2012; Hurst & Cordes, 2018a). Furthermore, research that has compared interference from both the total amount and the relevant subset in nonsymbolic proportional reasoning contexts suggest that, at least for young children, the numerical subset (i.e., numerator) may interfere more strongly with proportional information than the total amount (i.e., denominator; Boyer et al., 2008). Thus, in the current

Table 1
Operationalization of the Questions for Each Hypothesized (and Overlapping) Mechanism

Testable mechanism	Specific question	Experiment(s)	Operationalization
Saliency	Are adults more likely to match on the number of red than proportion of red in the absence of instruction?	Experiment 1: Subset, no probe	Preference for number on competition trials without a probe
Encoding automaticity	Is there a difference in the additional cost to performance when adults must encode both number and proportion versus can focus on the relevant dimension only?	Experiment 2: Subset, with probe	Interaction between probe timing and quantitative dimension
Processing costs	Is there a general cost to accuracy or reaction time when matching on proportion, in the absence of encoding or competition demands?	Experiments 2 and 3: Subset and total, with probe	Better performance on simple-match trials when matching on number compared with proportion
Response competition	Is there an additional cost to accuracy when matching on proportion in the presence of a numerical match (vs. not)?	Experiments 2 and 3: Subset and total, with probe	When matching on proportion, better performance on simple-match trials than on competition trials

study, we focused primarily on the potential mechanisms explaining differences in reasoning about the numerical subset versus proportion (Experiments 1 and 2). However, there is some evidence in symbolic fractions that both the numerator and the denominator can interfere with fraction magnitude processing (Bonato et al., 2007). So, in Experiment 3, we compared proportion to numerical information of both the relevant subset and the total number (the "denominator"), to provide some initial investigation of whether the same mechanisms might contribute to numerical processing of total information as well as subset information.

Notably, we investigated this with adult participants (i.e., college undergraduates) because adults already have extensive educational experience with whole numbers, fractions, and percentages, allowing us to investigate the nature of this interference at an educational stage in which both types of quantity should be highly familiar.

Experiment 1

In Experiment 1, we explored spontaneous preferences to attend to number and proportion of a display, without probes or explicit instructions. In doing so, we addressed our first question of whether numerical information is more salient than proportional information in discrete nonsymbolic displays.

Method

Participants

The sample consisted of 60 undergraduate college students (M_{age} = 19.35 years; range = 18 years to 22 years; n_{female} = 47, n_{male} = 13) who completed a match-to-sample (MTS) task and an open-ended survey about their strategies (followed by a subjective numeracy scale, reported in the online supplemental material only). Our sample size was sufficient to detect an effect as small as .36 on the primary analysis (a one-sample *t* test vs. chance) with 80% power (sensitivity analysis in G*Power; Faul et al., 2007). Adults participated for partial course credit in our campus laboratory. The Boston College Institutional Review Board approved all study procedures and participants provided informed consent prior to participating.

Stimuli

Each trial in the spontaneous MTS task consisted of three dot arrays: a sample stimulus (the initial stimulus, to which participants needed to identify a match) and two choice stimuli presented simultaneously as response options. All stimuli were comprised of a set of red and blue intermixed dots (size of the dots was constant both within and between arrays at 1.27 cm in diameter) presented inside a white rectangle (22.2-cm high × 26.7-cm wide). Sample stimuli were presented in the center of the screen (47.6-cm wide × 26.7-cm long computer monitor) and the two choice alternatives were presented side by side, surrounded by a 1.1-cm gray border.

The spontaneous MTS task consisted of two types of trials: standard trials and competition trials. On standard trials (see Figure 1, Panel A), one of the choice alternatives matched the sample stimulus on both the number of red dots and the proportion of red dots (i.e., it had the same number of red and blue dots as the sample stimulus and only the arrangement differed). The other choice alternative did not match the sample stimulus on either feature (e.g., if sample stimulus had four red dots and eight blue dots, one comparison stimulus would have four red dots and eight blue dots, and the other comparison stimulus might have six red dots and five blue dots). On competition trials (see Figure 1, Panel B), one of the choice alternatives matched the sample on the number of red dots, but not the proportion of red dots (i.e., it had a different number of blue dots, thereby altering the red-to-blue ratio [number of red match]) and the other choice alternative matched the sample on the proportion of red dots but not in the number of red or blue dots (i.e., the absolute quantity of red and blue dots differed from the sample, but the red-to-blue ratio remained the same [proportion match]). For example, if the sample stimulus had four red dots and eight blue dots, the number match choice alternative might have four red dots and five blue dots (same number of red dots) and the proportion match choice alternative might have seven red dots and 14 blue dots (same proportion of red).

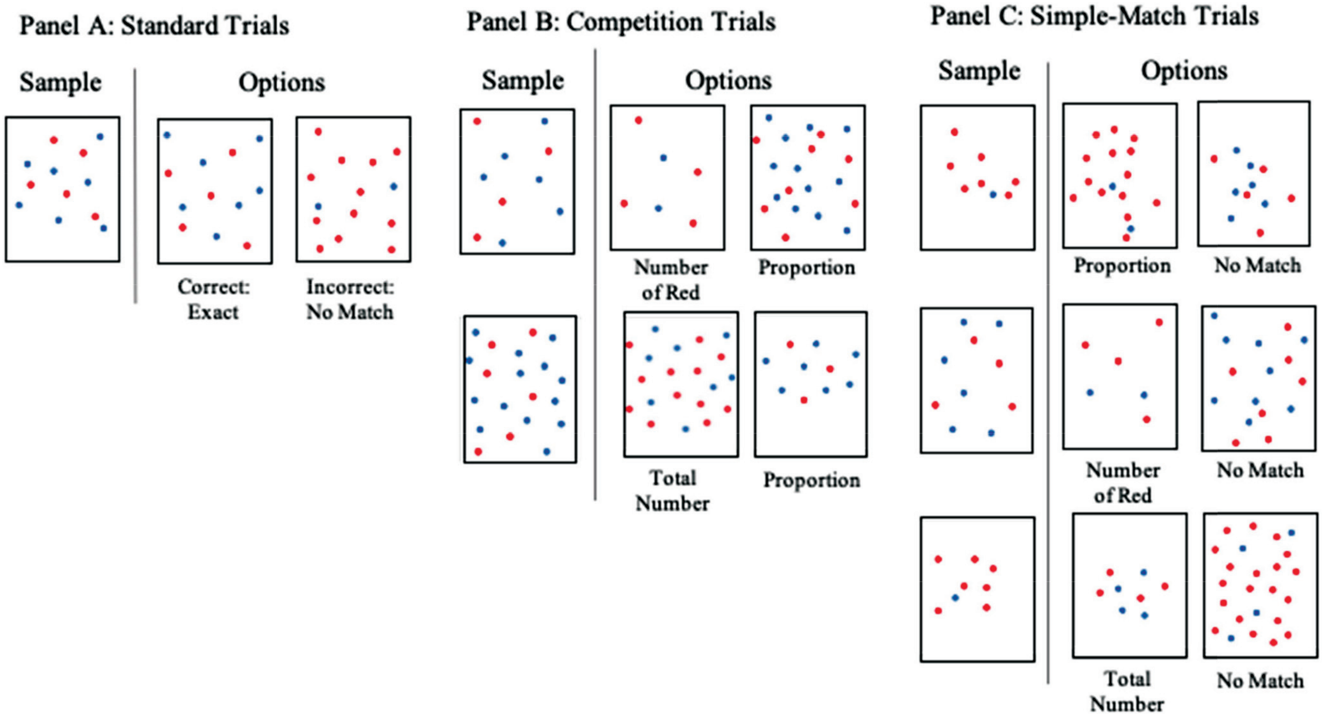
For all stimuli, the number of dots ranged from one to 18 for red dots, one to 24 for blue dots, and three to 30 for total number of dots and the proportion of red dots (out of the total) ranged from .1 to .94. For standard trials, the average ratio between the number of red dots displayed on each of the two choice alternatives was 2.0 (range = 1.2 to 2.7) and the average ratio between the proportion of red dots (i.e., the ratio of ratios) between the two choice alternatives was 2.0 (range = 1.4 to 2.7). For competition trials, the number of red dots in the two choice alternatives differed by an average ratio of two and the proportion of red dots in the two choice alternatives differed by an average ratio of approximately 1.9 (range = 1.3 to 3.2).

Additionally, to provide some control over the spread of the dots, we equated dot density across the correct and incorrect alternatives by changing the size of the background upon which the arrays were placed dependent upon the number of dots in the array (e.g., the background of an array containing 10 dots would be half the size of the background of an array containing 20 dots). We did not control for other continuous aspects of the displays (e.g., cumulative area or contour length) because these features are available for both numerical and proportional information (i.e., the relation between the number of red and the proportion of red out of the total is the same regardless of if that amount/proportion is calculated based on number or cumulative area). We discuss this further in the General Discussion.

Procedure

Participants were told that they would play a matching game in which they would see a single picture followed by two pictures. Instructions did not use the words number, percentage, proportion, quantity, sets of dots, or any other numerical or quantity-based words, but instead simply emphasized matching "pictures." Participants received a total of 56 trials: 12 initial standard trials used as familiarization, followed by 44 test trials (standard and competition trials, intermixed in a random order). Each trial consisted of a visual mask (1,000 ms), a target stimulus (1,000 ms), a visual mask (1,000 ms), the two response options (displayed until response selected), and feedback (1,000 ms). See Panel A of Figure 2 for a visual depiction of this trial procedure. Participants selected their response by pressing the left or right arrow key to correspond to the left or right stimulus, respectively. All trials

Figure 1
Example Stimuli of Each Type From Each Experiment

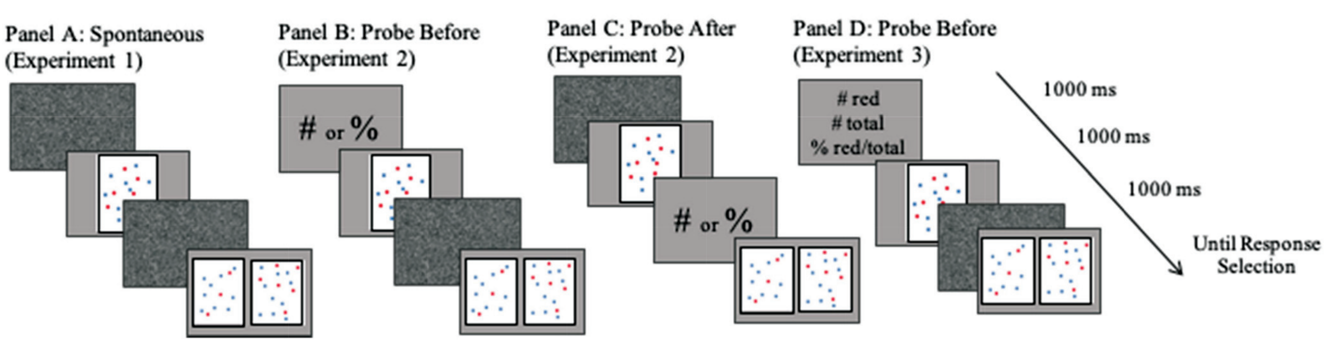


Note. Panel A: Standard trials had the same basic structure in all three experiments. Panel B: Competition trials were used in all three experiments. In Experiments 1 and 2, only proportion versus number of red trials (top) were used. In Experiment 3, both proportion versus number of red (top) and proportion versus total number (bottom) were used. Panel C: Simple-Match trials were used only in Experiments 2 and 3. In Experiment 2, proportion trials (top example) and number of red (middle example) trials were used. In Experiment 3, all three trial types, proportion (top), number of red, (middle), and total number (bottom) were used. See the online article for the color version of this figure.

(familiarization and test) were presented continuously, without a break or pause.

On each trial, participants were given feedback in the form of a green check (to indicate they were correct) or a red X (to indicate they were incorrect), following their response selection. During familiarization (only standard trials), participants were shown they were correct (green check) if they selected the option that matched on both number and proportion of red but were shown they were incorrect (red X) if they selected the option that did not match on either feature. These familiarization trials were intended to teach participants to discriminate the choice alternatives; however, on these trials it was ambiguous whether number or proportion was

Figure 2
Procedure for Each Experiment



Note. Panel A: Spontaneous match-to-sample (MTS) task without a probe from Experiment 1. Panel B: Delayed MTS task from Experiment 2, with the probe occurring before displaying the sample. Panel C: Delayed MTS from Experiment 2, with the probe occurring after displaying the sample. Panel D: Delayed MTS task from Experiment 3, with the probe always before the sample and with three probe options. See the online article for the color version of this figure.

the relevant stimulus dimension. Following familiarization, participants received 44 test trials made up of 22 standard (repeating the 12 used as familiarization, plus 10 new trials) and 22 competition trials, intermixed. On standard trials, feedback was the same as for familiarization. On competition trials, feedback always indicated a correct response (i.e., a green check) regardless of selection (number or proportion of red). Thus, we were critically interested in which option participants selected on competition trials. That is, which feature would they rely upon when number and proportion conflict with each other?

After the spontaneous MTS task, participants were shown an empty text box on the screen and asked to briefly explain what information they used to choose which picture matched. We collected strategy reports to investigate whether participants had explicit awareness of the strategy they were using, given the ambiguity of the task demands. In other words, given that this task did not involve learned or required symbolic strategies, the strategy reports allow us to investigate adults' interpretation of the task demands and attention to different features, rather than their ability to execute a specific learned strategy. For example, it may be that adults do show a preference for selecting one match over the other (e.g., choosing number), but that they are not explicitly aware of that strategy and are not able to articulate it. On the other hand, it may be that numerical information is so salient that adults show both a behavioral preference and an explicit awareness of this behavior.

Finally, given the relation between attention to proportional information and later fraction abilities in children (McMullen et al., 2014, 2016), we included measures of symbolic numeracy in both Experiments 1 and 2 (Fagerlin et al., 2007; Weller et al., 2013) to explore this link in adults. We were interested in whether adults who are more comfortable processing symbolic proportion and numeracy in decision making and real-life scenarios (as measured by the numeracy scales) would also be more likely to show better proportional reasoning abilities in this nonsymbolic context and less likely to show numerical interference. However, these results were inconsistent across studies and small and so we report the results of these measures (including a meta-analytic approach) only in the online supplemental material.

Results and Discussion

Spontaneous MTS Task

Participants completed the familiarization standard trials (M = .85) and the test standard trials (M = .90) with high accuracy. Participants who scored below 75% on the test standard trials (n = 5), suggesting they were not reliably able to isolate either number or proportion as being relevant during familiarization and did not learn either matching rule, were excluded from analyses. Thus, we looked at performance on the competition test trials for the remaining 55 participants. On average, participants chose the option that matched the sample stimulus number on 68% of the competition trials, which was significantly greater than chance (50%), $t(54) = 5.39$, $p < .001$. Furthermore, 40 out of the 55 participants preferred number (choosing number more than 50% of the time), which is significantly more than half of the participants (binomial p = .001). Thus, overall, there was a significant preference to attend to numerical information rather than proportional information, suggesting that in the absence of other quantitative cues or direct instruction, numerical information may be more salient than proportional information.

Explicitly Reported Strategies

Responses were coded based on whether they mentioned strategies of interest. Notably, participant responses could have been coded as falling within more than one relevant category (thus, percentage of participants is included to provide a better sense of the popularity of each response, but these values will add up to more than 100%). Forty-one people (68%) reported at least one strategy that was based on absolute number: Thirty-two (53% of total participants) reported attending to the number of red circles, eight (13%) reported attending to the number of blue circles, six (10%) reported attending to the color that contained the smallest number of circles, and eight (13%) reported attending to the total number of circles. In contrast, 19 people (32%) reported using a strategy that involved the relation between red and blue circles and 16 people (27%) reported using idiosyncratic spatial cues, such as the density of the dots, the shape or outline of the overall set of dots, or the location of a specific dot, although none of these spatial cues were reliable for selecting the correct match. Last, three participants (5%) reported a strategy that could not be coded into one of these categories (e.g., "I picked the one that matched").

We then compared performance on the spontaneous MTS of those classified as having used a numerical strategy, a relational strategy, and a spatial strategy (as a relatively commonly used, but not reliable or useful, strategy). We analyzed each type of strategy by comparing those who did and did not report using a given strategy, separately.[2] Those who reported a number strategy were significantly more likely to select the number response on competition trials (n = 41, M = .74) than were those who did not report using a number strategy (n = 19, M = .52), $t(58) = -3.66$, $p < .001$, Cohen's d = 1.02, though there was not a significant difference in performance on the standard trials ($M_{\text{number strategy}} = .91$, $M_{\text{no number strategy}} = .88$), $t(58) = -.95$, p = .343, Cohen's d = .27. Conversely, those who reported a relational strategy (a different but potentially overlapping subset from above; n = 19) were less likely to select the number response on competition trials (M = .54) than those who did not report using a relational strategy (n = 41, M = .73), $t(58) = 3.1$, p = .003, Cohen's d = .84. There was a small but not significant difference in performance between these two groups on the Standard trials as well ($M_{\text{relational strategy}} = .93$, $M_{\text{no relational strategy}} = .88$), reporting Welch's t test for unequal variances (variance test, p = .02), $t(53.5) = -1.7$, p = .099, Cohen's d = .39. Although not statistically significant, this pattern might suggest that using a relational strategy is a more advanced or more reliable strategy, such that those who reported using it were better able to make the correct match, even when other strategies were possible. Last, those who reported using a spatial strategy (n = 16) did not significantly differ on the competition trials from those who did not (n = 44; $M_{\text{spatial}} = .61$, $M_{\text{not spatial}} = .69$), $t(58) = 1.2$,

[2] Note that because adults could be categorized into multiple strategy categories, they may be included in multiple analyses as using that strategy. Analyses that directly compare those who only reported a numerical strategy, and those who reported only a proportional strategy arrived at very similar conclusions and are reported in the online supplemental material.

$p = .241$, Cohen's $d = .35$. However, they did perform significantly worse on the standard trials ($M_{\text{spatial}} = .79$, $M_{\text{not spatial}} = .94$), reporting Welch's t test because of a significant difference in variances ($p < .001$), $t(16.98) = 3.72$, $p = .002$, Cohen's $d = 1.54$. Thus, those who reported idiosyncratic spatial strategies performed worse overall on noncompetition trials, suggesting (in line with their own self reports) that they were less able to reliably isolate either relevant quantity dimension.

Taken together, these findings suggest that adults tend to spontaneously attend to numerical information rather than proportional information and their explicit self-reports of their strategies seem to correspond with their actual performance. Those who reported a number strategy were more likely to match on number (compared with those who did not report using a number strategy) and those who reported a relational strategy were less likely to match on number (compared with those who did not report using a relational strategy). Consequently, these findings provide support for the hypothesis that numerical information carries greater saliency than proportional information in the absence of a cue to attend to one or the other.

Experiment 2

Experiment 1 provides evidence that when presented with nonsymbolic dot arrays with two features (i.e., red and blue dots) in an ambiguous task, adults are more likely to match on numerical information than proportional information. In Experiment 2, we more directly manipulated the task to investigate differences in saliency, automaticity of encoding, and effortful processing costs by directing adults' attention to numerical or proportional features using a delayed MTS task.[3]

Method

Participants

The sample consisted of 60 undergraduate college students ($M_{\text{age}} = 19.3$; range = 18 to 24 years, $n_{\text{female}} = 40$, $n_{\text{male}} = 20$) who completed a delayed MTS task. Based on relatively large effects found in our prior experiment (see Experiment S1 in the online supplemental material) for the significant probe and trial type effects (i.e., $\eta_p > .2$, reported in the online supplemental material), we selected our sample size to have at least 80% power for detecting medium sized effects on the central within-subject effects (sensitivity analyses in G*Power; Faul et al., 2007). Participant recruitment procedures were the same as in Experiment 1.

Stimuli

As in the spontaneous MTS in Experiment 1, each trial in the delayed MTS task consisted of three dot arrays: a sample stimulus (the initial stimulus, to which participants needed to identify a match) and two choice stimuli presented simultaneously as response options. In addition, on each trial, participants were shown a probe, which indicated whether the relevant dimension to which to attend was the number of red dots (indicated by a 1.9-cm × 3.8-cm pound or number sign [#]), or the proportion of red dots (indicated by a 3.8-cm × 3.8-cm percent sign [%])[4].

There were three different types of trials in the delayed MTS task, presented in the following random order: standard trials, competition trials, and simple-match trials. Standard trials (see Figure 1, Panel A) and competition trials (see Figure 1, Panel B) were similar to those of Experiment 1. On simple-match trials (see Figure 1, Panel C), one choice alternative matched the sample on only the relevant dimension (indicated by the probe), but the other did not (i.e., it matched on either the number of red dots or on the proportion of red dots, but not both), and the other choice alternative did not match the sample on either dimension. For simple-match trials, the average ratio between the number of red dots in the two choice alternatives was approximately 1.9 (range = 1.29 to 3) and the average ratio between the proportion of red dots in the two choice alternatives was approximately 1.9 (range = 1.29 to 3). The standard and competition stimuli were identical to those used in Experiment 1, though, as noted, participants were cued to respond with a probe that prompted attention to number or proportion.

Procedure

The delayed MTS task consisted of two blocks of trials (probe-before and probe-after blocks), with the order counterbalanced across participants. In the probe-before block (see Figure 2, Panel B), participants first saw a gray probe screen with either a number sign or a percent sign in the middle of the screen (1,000 ms), followed by a sample stimulus (1,000 ms), a visual mask (1,000 ms), and two arrays as choice alternatives (visible until response selection). In the probe-after block (see Figure 2, Panel C), the temporal locations of the visual mask and the probe screen (i.e., # or %) were swapped, such that the participant saw the visual mask first, followed by the sample stimulus, followed by the probe screen, followed by the two choice arrays.

In both conditions, participants were instructed that on the probe screen, the number sign indicated they were to select the choice alternative that matched the sample stimulus on the number of red dots and the percent sign indicated they were to pick the choice alternative that matched the sample in terms of the percentage of red dots (out of the total). Participants selected their response by pressing the right or left arrow on the keyboard to select the right or left choice alternative, respectively.

Each block contained 12 practice trials (four of each trial type, intermixed) and 66 test trials (22 of each trial type, with an equal number of each probe, all intermixed). Participants received accuracy feedback during the practice problems (a green check when they selected the response that correctly matched the probe and a red X when they selected the response that did not match the probe, displayed for 1,000 ms) but did not receive any feedback on the test trials.

[3] Experiment 2 is a more careful replication of an initial experiment (see Experiment S1 in the online supplemental material). The pattern of findings is nearly identical, with only one difference where Experiment S1 showed a small interaction that was not significant in Experiment 2. Thus, Experiment S1 is reported only in the online supplemental material.

[4] There is substantially less research on people's understanding of percentages, relative to fractions or whole numbers (see Tian & Siegler, 2018 for a review), and some have suggested that percentages may be an important bridge between whole numbers and fractions (e.g., Moss & Case, 1999; Sidney et al., 2021). Thus, in the current study we rely on percentage symbols to prompt attention to proportional information (in Experiments 2 and 3), but it is an open question of how this symbolic operationalization of proportion would impact adults' performance, an issue we return to in the General Discussion.

Data Treatment

Accuracy (proportion correct) and reaction time (RT) were recorded on the delayed MTS task. Accuracy was the primary dependent variable, but a brief summary of RT analyses is also provided, as they typically showed similar results (although major discrepancies are discussed in the article; a full report of the RT results is available in the online supplemental material). Only RTs from correct trials and those within three standard deviations of that individual's mean RT on that trial type was used. Outliers at the group level were defined as individual means that were outside three standard deviations of the group mean for that cell (i.e., Trial Type × Probe Type × Probe Timing combination), and they were replaced with the next highest observed value within three standard deviations of the mean for all data in that cell (1.7% of accuracy data; 1.3% of RT data). The pattern of results is the same when these outliers are not replaced. Additionally, when a log transformed proportion correct is used instead to adjust for the distribution of proportion correct scores (as discussed in Collett, 2002 and Cox & Snell, 1989), the same pattern of results is found as when untransformed proportion correct is used.

All analyses were done in R (Version 3.5.1; R Core Team, 2020) using RStudio (R Studio Team, 2016) with the packages *dplyr*, *tidyr*, *readxl*, and *ggplot2* from the *tidyverse* (for data organization and visualization; Wickham, 2017), as well as *ez* (for analysis of variance [ANOVA]; Lawrence, 2016), *effsize* (for effect size calculations; Torchiano, 2018), and *psychReport* (for partial eta-square calculations; Mackenzie, 2018).

Results and Discussion

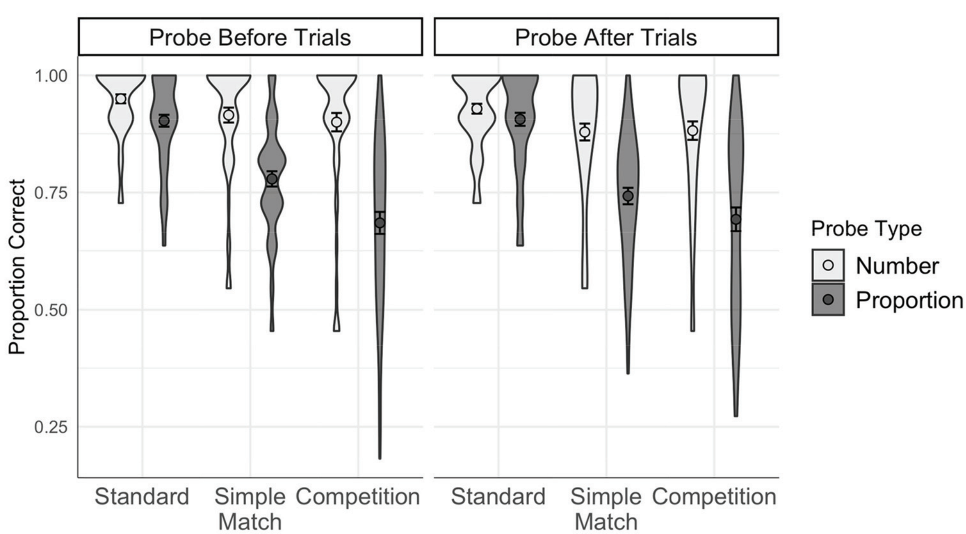
We analyzed performance on the delayed MTS task using a 2 × 3 × 2 × 2 mixed-measures ANOVA, with Probe Type (# or %) × Trial Type (standard, simple-match, competition) × Probe Timing (before or after sample) as repeated measures and block order (probe before block first or probe after block first) as a between-subjects factor, with proportion correct as the primary dependent variable (see Figure 3). When a violation of sphericity was detected, Greenhouse–Geisser correct *p* values are reported.

First, to investigate whether there is evidence of differences in the automaticity of encoding number and proportion, we examined whether probe timing impacted performance. If number and proportion are both automatically encoded, there should not be a cognitive cost for simultaneously encoding both stimulus dimensions. That is, performance on probe-before trials, in which participants know before seeing the sample whether to encode number or proportion, should be comparable to probe-after trials, in which they do not know and thus must encode both. On the other hand, if one (or both) of these quantities is not automatically encoded, then we would expect there to be a cognitive cost to encoding both quantitative dimensions. As such, performance on probe-after trials should suffer relative to probe-before trials. Furthermore, if number and proportion are encoded differently, then we would expect the impact of the temporal location of the probe (probe-before vs. probe-after trials) to differ when matching on number versus proportion.

The ANOVA revealed a small but not significant effect of probe timing, $F(1, 58) = 3.70$, $p = .059$, $\eta_p^2 = .06$, ($M_{\text{probe-before}} = .86$, $M_{\text{probe-after}} = .84$), indicating that participants performed slightly better when they knew ahead of time which quantitative dimension to attend to than when they did not know and were thus required to attend to both dimensions. However, there was not a significant Probe Timing × Probe Type interaction, $F(1, 58) = .98$, $p = .325$, $\eta_p^2 = .02$, suggesting that this small main effect did not vary for number versus proportion. Analyses also revealed a small but not

Figure 3

Proportion Correct From Experiment 2 on Each Trial Type and Each Probe



Note. Trial type is on the *x*-axis, and each probe (left side: light gray bars = number of red; right side: dark gray bars = proportion) on the probe before block (left panel) and the probe after block (right panel) is presented separately. Points are mean performance, error bars are standard error, kernel density violin plots display the distribution of the underlying data.

significant Block Order × Probe Timing interaction, $F(1, 58) = 3.47$, $p = .068$, $\eta_p^2 = .06$; however, because this effect did not involve probe type (i.e., # or %) and likely reflected small practice effects, it was uninformative for the current study. There were no other significant interactions involving probe timing (all $ps > .1$).

Second, to investigate the effect of basic processing differences and the possible added effect of response competition, we consider the effects of trial type and probe type. If number and proportion compete during decision making, then we would expect performance on trials in which the incorrect response option presents competing quantitative information (competition trials) to be worse than on trials in which the incorrect response option does not present competing information (simple-match trials), regardless of the timing of the probe. Additionally, however, if there are general differences in the ease of processing numerical versus proportional information, then we may also see differences between numerical and proportional matching in the absence of competing response options (simple-match and standard trials).

The analysis revealed a main effect of probe type, $F(1, 58) = 121.3$, $p < .001$, $\eta_p^2 = .7$, a main effect of trial type, $F(2, 116) = 85.5$, $p < .001$, $\eta_p^2 = .6$, and a Probe Type × Trial Type interaction, $F(2, 116) = 26.7$, $p < .001$, $\eta_p^2 = .3$. Thus, we examined the effect of Probe Type (# or %) within each trial type separately. For each trial type, accuracy on number trials was higher than that of proportion trials: standard trials ($M_{\text{number}} = .94$, $M_{\text{proportion}} = .90$), $t(59) = 2.6$, $p = .013$, Cohen's $d = .33$; simple-match trials ($M_{\text{number}} = .90$, $M_{\text{proportion}} = .76$), $t(59) = 7.5$, $p < .001$ Cohen's $d = .97$; competition trials ($M_{\text{number}} = .89$, $M_{\text{proportion}} = .69$), $t(59) = 10.4$, $p < .001$, Cohen's $d = 1.3$. The effect of probe type even on simple-match trials suggests there are differences in processing number and proportional information at even a more basic processing level, when neither response competition nor taxing task demands are present. However, beyond this general processing difference, the interaction between probe type and trial type suggests that this numerical advantage varies across trial types. Thus, to investigate the added impact of response competition, we compared performance in the absence of competition (simple-match trials) to performance in the presence of response competition (competition), when matching on both number and proportion. When matching on proportion, people performed significantly worse when a numerical response option was available (competition trials) than when one was not (simple-match trials; $p < .001$, Cohen's $d = .6$). However, the same difference was not significant when matching on number ($p = .542$, Cohen's $d = .08$). Thus, we see evidence for both hypothesized effects. There was a difference in performance when matching on numerical versus proportional information in the absence of response competition, and this was further exacerbated in the presence of response competition, driven by an additional cost to proportional matching in the presence of a numerical option but not numerical matching in the presence of a proportional option. No other effects in the overall ANOVA were significant (all $ps > .1$).

The pattern above is largely replicated when the same repeated-measures ANOVA is run with RT as the dependent variable. As with accuracy, there was a main effect of probe type, $F(1, 58) = 117.7$, $p < .001$, $\eta_p^2 = .7$, trial type, $F(2, 116) = 13.9$, $p < .001$, $\eta_p^2 = .2$, and a Probe Type × Trial Type interaction, $F(2, 116) = 24.1$, $p < .001$. $\eta_p^2 = .29$. In particular, adults were significantly faster on number trials than proportion trials in all trial types: standard trials ($M_{\text{number}} = 1{,}232$ ms, $M_{\text{proportion}} = 1{,}524$ ms), $t(59) = 8.2$, $p < .001$, Cohen's $d = 1.1$; competition trials ($M_{\text{number}} = 1{,}236$ ms, $M_{\text{proportion}} = 1{,}807$ ms), $t(59) = 10.1$, $p < .001$, Cohen's $d = 1.3$; simple-match trials ($M_{\text{number}} = 1{,}195$ ms, $M_{\text{proportion}} = 1{,}803$ ms), $t(59) = 9.5$, $p < .001$, Cohen's $d = 1.2$. However, RT analyses differed from accuracy analyses in that the interaction is not driven by an additional cost to numerical interference during proportional reasoning; neither matching on number nor on proportion showed a difference in RT across the simple-match and competition trial types (number: $p = .083$, $d = .12$; proportion: $p = .914$, $d = .007$). Thus, RT analyses replicate an overall performance difference between number and proportion trials with proportional information requiring more time to process than number across all three trial types, potentially due to a computational cost. However, unlike accuracy there was not an added speed cost introduced by a competing response option, suggesting that the interference or competition during decision-making may not be as evident in RT.[5]

Overall, the results of Experiment 2 provide evidence for at least two of our hypothesized mechanisms: interference from competing numerical information in the subset and more effortful processing of proportional information than subset numerical information. In terms of response competition, when matching proportion, accuracy of performance (but not RT) was worse when a numerical response option was available (competition trials), relative to when a numerical response was not available (simple match), aligning with predictions about competition between number and proportion during the decision-making process. Importantly, the converse was not true; that is, proportional information did not appear to interfere with numerical judgments during decision-making (i.e., performance on competition and simple-match trials was not significantly different when matching on number). Moreover, even in the absence of response competition, we find that responding based on proportional information took longer and was less accurate than matching on the subset numerical information. Thus, it may be that proportional information is more effortfully processed than numerical information, resulting in longer reaction times and more error-prone responding.

On the other hand, we did not find evidence for differences in encoding automaticity between number and proportion. That is, we did not find a differential effect of probe timing on performance across number and proportion trials, such that participants benefited from knowing which dimension to attend to before seeing the sample stimulus equally across the two types of trials. This finding might suggest that both number and proportion are encoded automatically to the same extent. It is also possible, however, that these patterns are attributable to characteristics of our procedure. For example, perhaps the duration of the sample stimulus presentation was too brief to allow an earnest opportunity to encode both number and proportion even when participants knew in advance to which dimension to attend, or, alternatively, was sufficiently long that they were able to actively encode both number and proportion information as successfully as they were able to encode either number or proportion independently. In either case, this would result in little to no difference in the prompt timing conditions.

[5] Reaction time data also revealed several small effects involving block order and probe timing; however, these interactions were small and difficult to interpret and so are not reported here (see the online supplemental material for the full analysis and discussion).

Although we do see evidence of interference from competing numerical information and more effortful processing of proportional information than numerical information, the numerical information was always operationalized as the number of red dots—a numerical subset of the total information. This numerical information differs from proportional information in at least two ways: (a) It is necessarily a smaller number of items, because it is a numerical subset, and (b) it requires only attending to a single feature (the red items) rather than computing over two features (red and blue dots). Thus, in Experiment 3, we investigated whether the two mechanisms found to play a role in the tradeoff between proportion and a numerical subset—the cognitive processing cost of proportional information and numerical interference—also apply relative to the total number of items present (the superset), which requires attending to a larger number and computing over two features.

Experiment 3

Although there were other numerical aspects of the display (e.g., the number of blue dots or the total number of dots) available in Experiments 1 and 2, and some adults in Experiment 1 did report spontaneously using this other numerical information (13% of adults in Experiment 1 reported attending to the total number of dots), both our operationalization of numerical interference and our measure of cognitive processing cost compared proportional information only to the numerical subset, in keeping with prior emphases on numerical subset information (Boyer et al., 2008; Fabbri et al., 2012; Hurst & Cordes, 2018a). Thus, to investigate whether the total number of dots interferes with proportional information to the same extent and via the same mechanisms as the subset-numerical interference found in Experiment 2, we used the same delayed MTS paradigm but included trials that involved matching on the total number of dots as well as the number of red dots and the proportion of red dots. Furthermore, given the small and/or nonsignificant effects of the timing of the probe in Experiment 2, we removed this manipulation and used only probe-before trials.

Method

Participants

The sample consisted of 50 undergraduate college students ($M_{\text{age}} = 18.96$; range = 18–22 years; 30 women, 18 men, and two with missing gender data), who participated for partial course credit in our campus laboratory.[6] This sample provides sensitivity to detect an effect size of around $f = .2$ (considered a medium effect), with 80% power on our central repeated-measures ANOVA (Faul et al., 2007). The rest of our participant recruitment procedures were similar to Experiment 2.

Stimuli

The basic design of the stimuli was the same as Experiment 2, but the specific stimuli were modified to accommodate manipulation of a third feature, the total number of dots, in addition to the number of red dots and the proportion of red dots. The delayed MTS task consisted of the same three types of trials in Experiment 2: standard trials, simple-match trials, and competition trials. On standard trials (see Figure 1, Panel A) one of the choice alternatives matched the sample stimulus on the number of red dots, the total number of dots, and the proportion of red dots (i.e., was identical, except rearranged). The other choice alternative did not match the sample stimulus on any of these three features. On competition trials (see Figure 1, Panel B), there were two kinds of trials: subset versus proportion competition and total versus proportion competition. Subset versus proportion competition trials were identical to competition trials of Experiment 2: one of the choice alternatives matched the sample on the number of red dots (but not the proportion of red dots or the total number of dots) and the other choice alternative matched the sample on the proportion of red dots (but not on the number of red dots or the total number of dots). On the total versus proportion competition trials, one of the choice alternatives matched the sample on the total number of dots (but not on the proportion or number of red dots) and the other choice alternative matched the sample on the proportion of red dots (but not on the total number of dots or the number of red dots). On simple-match trials (see Figure 1, Panel C), one choice alternative matched the sample on only the relevant dimension (i.e., the one being probed on that trial), and the other choice alternative did not match the sample on any of the three dimensions.

For all stimuli, the number of dots ranged from one to 24 for red dots, one to 24 for blue dots, and three to 30 for total number of dots and the proportion of red dots (out of the total) ranged from .1 to .94. The average ratio (larger value/smaller value) between the number of red dots displayed on each of the two choice alternatives ranged from 1.1 to 4.5, with an average ratio of ~2 in each trial type (standard and simple-match trials, $M = 2.1$; competition trials, $M = 2$). The average ratio between the total number of dots in each of the options ranged from 1.2 to 4.5, with an average ratio of ~2 in each trial type (standard trials, $M = 2.1$; simple-match, $M = 1.99$; competition trials, $M = 2$). The average ratio between the proportion of red dots in each of the options ranged from 1.25 to 3.5, with an average ratio of around 2 in each trial type (standard and total vs. proportion competition, $M = 2$; simple-match and subset vs. proportion competition, $M = 1.9$).

Procedure

The delayed MTS task was programmed and administered using PsychoPy3 (Version 3.2.4; Peirce et al., 2019). Given the small and not significant differences based on the location of the probe in Experiment 2, participants in Experiment 3 only completed a single block of trials with the probe always occurring before the sample stimulus (see Figure 2, Panel D) and did not complete the numeracy task. Thus, the procedure matched the probe-before block of Experiment 2. The primary difference was that there were three possible probes: the number of red dots (prompted by "# red"), the total number of dots (prompted by "# total"), and the proportion of red out of the total (prompted by "% red/total").

Participants completed 18 practice trials (three of each type for each probe) with accuracy feedback and 110 test trials

[6] We had planned to collect 60 participants, as in Experiments 1 and 2. However, the COVID-19 pandemic resulted in early termination of data collection. Thus, we report a sensitivity analysis for this new sample size. Although data collection was terminated earlier than expected, it was terminated for nondata-driven reasons and thus should not increase our likelihood of a type 1 error.

without feedback. The test trials were composed of 33 standard trials (11 for each probe), 33 simple-match trials (11 for each probe), and 44 competition trials. For the competition trials, 22 were proportion versus subset (11 with the % probe and 11 with the # red probe), and 22 were proportion versus total (11 with the % probe and 11 with the # total probe). The specific probe that was paired with each trial was randomly determined across participants, within the constraints described above about the number of trials for each probe. The rest of the procedure was the same as Experiment 2.

Data Treatment

As in Experiment 2, accuracy (proportion correct) and RT were recorded. Accuracy was the primary dependent variable and RT analyses are also presented to provide different insight. Only RTs from correct trials and those within three standard deviations of that individual's mean RT on that trial type were used. Additionally, the same pattern of results emerged when a log transformed proportion correct was analyzed to adjust for the distribution of proportion correct scores (as discussed in Collett, 2002 and Cox & Snell, 1989). Outliers at the group level were determined as in Experiment 2 (2% of accuracy data, 1.2% of RT data). The pattern of results is the same when these outliers are not replaced, with one exception discussed in the following text. All analyses were done as in Experiment 2.

Results and Discussion

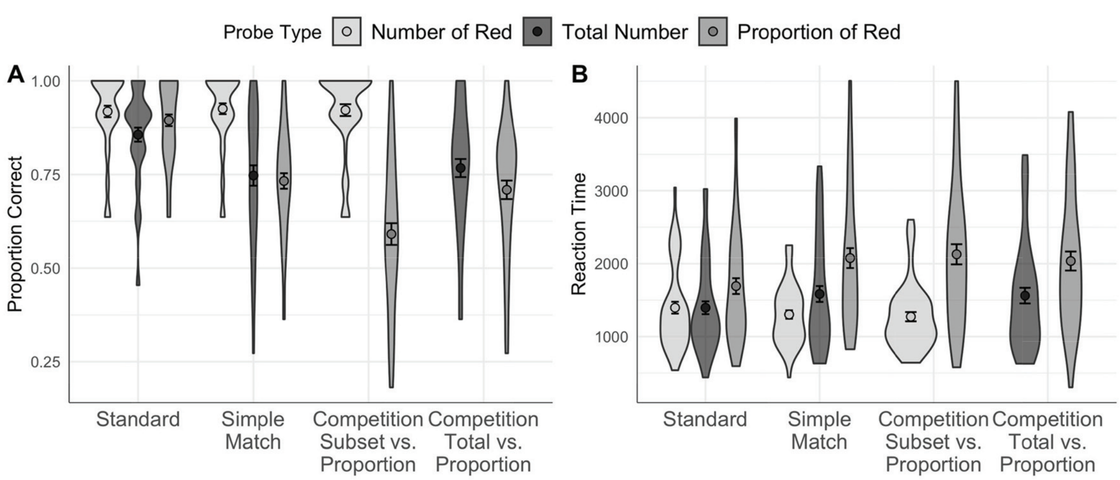
Using the same analytical approach as Experiment 2, we compared performance across probe and trial types. However, all probes and trial types could not be included in the same analysis because of the nature of the stimuli and experiment design (i.e., the standard and simple match trials only have one set of proportion trials, but the competition trials necessarily have two separate sets of proportion trials). Thus, we first compared adults' performance with proportional information to their performance matching on subsets (i.e., the number of red), as a replication of Experiment 2. Then, we compared adults' proportional matching to their performance matching the total number. Thus, the data from the standard and simple-match trials with the proportion probe are identical across the two analyses but are compared with different types of numerical trials. Figure 4 displays accuracy (left panel) and RT (right panel) for all trial types and probes.

Subset Versus Proportion

First, we used a 2 × 3 repeated-measures ANOVA with probe (number red, proportion) and trial type (standard, simple-match, competition) as repeated measures and proportion correct as the dependent variable. Replicating Experiment 2, there was a significant main effect of probe, $F(1, 49) = 165.3$, $p < .001$, $\eta_p^2 = .77$, a main effect of trial type, $F(2, 98) = 51.5$, $p < .001$, $\eta_p^2 = .5$, and a Probe × Trial Type interaction, $F(2, 98) = 38.6$, $p < .001$, $\eta_p^2 = .4$. Accuracy was higher when matching on the number of red than on proportion on the simple-match trials ($M_{\text{number}} = .93$, $M_{\text{proportion}} = .73$) $t(49) = 8.5$, $p < .001$, $d = 1.5$, and competition trials ($M_{\text{number}} = .92$, $M_{\text{proportion}} = .59$), $t(49) = 10.3$, $p < .001$, $d = 1.99$, but not on standard trials ($M_{\text{number}} = .92$, $M_{\text{proportion}} = .89$), $t(49) = 1.4$, $p = .171$, $d = .22$. Thus, as in Experiment 2, the difference in performance on proportion probe Simple-Match trials and number probe Simple-Match trials (in the absence of competition) suggests that proportional matching is generally more error-prone than number matching. Furthermore, to investigate whether the interaction between trial type and Probe type stems from added difficulty with proportional matching in the presence of a competing response option, we compared performance on the simple-match and competition trials for number and proportion separately. Indeed, as in Experiment 2, there was a significant difference between competition and simple-match trials when matching on proportion of red ($p < .001$, $d = .78$) but not when matching on number of red ($p = .805$, $d = .03$).

Figure 4

Proportion Correct and Average Reaction Time Across Trial Types and Three Probes for Experiment 3



Note. Panel A, left: proportion correct. Panel B, right: average reaction time across all trial types. Points are the mean, error bars are standard errors, and kernel density violin plots show the distribution of the underlying data.

Thus, replicating Experiment 2, we find that participants were more accurate when matching based on number in the relevant subset than based on proportion—consistent with proportional information requiring more effortful processing. Notably, unlike Experiment 2, we did not find this difference in the Standard trials, however, suggesting that the effect may be much smaller and less robust when an exact match on multiple features is possible. Moreover, we again found that information about the number of red dots—the numerator—interfered with adults' ability to match proportional information at the time of decision-making, over and above processing differences in the absence of competition, but proportional information did not show a similar interference on numerical judgments.

The same overall pattern was found with response time as the dependent measure, with a significant main effect of probe, $F(1, 49) = 77.9$, $p < .001$, $\eta_p^2 = .61$, trial type, $F(2, 98) = 5.8$, $p = .004$, $\eta_p^2 = .1$, and a Probe Type × Trial Type interaction, $F(2, 98) = 14.6$, $p < .001$, $\eta_p^2 = .22$. Matching on proportion was significantly slower than matching on number across all three trial types: standard trials ($M_{\text{number}} = 1396$ ms, $M_{\text{proportion}} = 1{,}693$ ms), $t(49) = 3.8$, $p < .001$, $d = .4$; simple-match trials ($M_{\text{number}} = 1{,}303$ms, $M_{\text{Proportion}} = 2{,}076$ ms), $t(49) = 7.0$, $p < .001$, $d = .87$; competition trials ($M_{\text{number}} = 1{,}273$ ms, $M_{\text{proportion}} = 2{,}128$ ms), $t(49) = 8.5$, $p < .001$, $d = .88$. However, there was not a significant difference between the Simple-Match and Competition trials when matching on number of red ($p = .429$, $d = .07$) or when matching on proportion of red ($p = .583$, $d = –.05$), suggesting there was not an added time cost to proportional matching in the presence of response competition.

Thus, again replicating RT analyses from Experiment 2, numerical information was processed faster than proportional information, consistent with the proposal that proportional information may require additional processing or computational time than numerical information. However, there was not an added speed cost to the competition trials, meaning that response competition was not evident in terms of RT.

Total Number Versus Proportion

Next, we used the same 2 × 3 repeated-measures ANOVA with probe (total number, proportion) and trial type (standard, simple-match, competition) as repeated measures but on those trials that involved matches on total number (instead of number of red). Note that the standard and simple-match proportion trials were identical to those analyzed in the preceding text but were used here to compare with the total number probe. Descriptive data will be repeated as needed for ease of comparison. There was a significant main effect of trial type, $F(2, 98) = 31.8$, $p < .001$, $\eta_p^2 = .39$, and a significant Trial Type × Probe interaction, $F(1, 49) = 3.67$, $p = .029$, $\eta_p^2 = .07$, but, unlike previous findings involving subsets, not a significant main effect of probe, $F(1, 49) = .53$, $p = .471$, $\eta_p^2 = .01$.[7] In contrast to comparing proportion to the numerical subset, there was not a significant difference in accuracy when matching on proportion versus the total number in any of the three trial types: standard trials ($M_{\text{number}} = .86$, $M_{\text{proportion}} = .89$), $t(49) = 1.9$, $p = .062$, $d = .3$; simple-match trials ($M_{\text{number}} = .75$, $M_{\text{proportion}} = .73$), $t(49) = .56$, $p = .579$, $d = .08$; competition trials ($M_{\text{number}} = .77$, $M_{\text{proportion}} = .71$), $t(49) = 1.89$, $p = .065$, $d = .34$. Thus, in contrast to the number of red items, reasoning about the total number of items was not more accurate than reasoning about proportion. Furthermore, we also did not see evidence of added difficulty from response competition. That is, performance was not significantly different between the simple-match and competition trials when matching on the total number of dots ($p = .452$, $d = -.1$) or the proportion of red dots ($p = .389$, $d = .1$). Thus, in contrast to the pattern found for Subset versus Proportion trials, we did not find evidence to suggest that total number is processed more accurately than proportion, nor that the total number of dots interfered with adults' ability to match on proportional information at the time of decision making.

When looking at RT, there was a main effect of trial type ($M_{\text{standard}} = 1{,}544$ ms, $M_{\text{simple}} = 1{,}830$ ms, $M_{\text{competition}} = 1{,}800$ ms), $F(2, 98) = 10.0$, $p < .001$, $\eta_p^2 = .17$, with standard trials being significantly faster than competition and simple-match trials (ps $< .001$, ds $> .3$), which were not significantly different from each other ($p = .668$, $d = .03$). Additionally, there was a significant main effect of probe, $F(1, 49) = 44.3$, $p < .001$, $\eta_p^2 = .47$, such that matching on proportion ($M = 1{,}935$ ms) took longer than matching on total number ($M = 1{,}514$ ms), again consistent with more effortful processing of proportional information relative to numerical information. However, the interaction between trial type and probe type was not significant, $F(2, 98) = 2.3$, $p = .109$, $\eta_p^2 = .04$.

Taken together, these findings suggest that the total number of dots did not compete with proportional information at the time of decision making (in terms of either accuracy or RT), but proportional information did require more processing time, resulting in longer reaction times, than matching on the total number of dots, consistent with the pattern found for the numerical subset trials as well. Consequently, the findings comparing a numerical subset to proportion replicate Experiment 2, providing support for two of our hypothesized mechanisms: cognitive cost to proportional reasoning (in terms of accuracy and RT) and interference during decision making. However, when comparing total number to proportion we only find support for one hypothesized mechanism, a cognitive processing cost of proportion versus total number, and this cost was only apparent in terms of processing time and did not lead to a cost in performance accuracy.

Across Study Meta-Analysis

When comparing proportion versus a numerical subset, two of our target mechanisms (processing costs and response competition), were directly tested in Experiment 2, 3, and S1 in the online supplemental material. Thus, to give a more robust estimate of the size of these effects we applied meta-analytic methods using the *metafor* v2.4 package in R (Viechtbauer, 2010), using change score standardized mean change as our effect size (given the repeated-measures design) and random effect models with restricted maximum likelihood (although, the resulting estimates are almost identical when a fixed effects model is used, given that these experiments come from a single paradigm and lab environment). Forest plots are available in the online supplemental material.

[7] This interaction was not robust to outlier treatment and is not significant when outliers are not replaced, $F(1, 49) = 3.0$, $p = 0.054$, $\eta_p^2 = 0.06$. However, this does not dramatically change the interpretation, as the interaction does not stem from the pattern of behavior we would expect from response competition.

To estimate the difference in processing cost between the numerical subset and proportion, we compared the simple match proportion trials and simple match numerical subset trials. There was a significant effect of proportion correct, (d_{RM} = .91), 95% confidence interval (CI) [.71, 1.11], $p < .001$, with relatively low heterogeneity across experiments, (I^2 = 27%), $Q(df = 2) = 3.07$, $p = .22$. There was also a significant effect of RT, (d_{RM} = −1.1), 95% CI [−1.3, −.90], $p < .001$, with relatively low heterogeneity across experiments, (I^2 = 0%), $Q(df = 2) = .68$, $p = .71$.

To estimate response competition in proportional matching, we compared the simple match proportion trials to the competition proportion trials. There was a significant interference effect in terms of proportion correct, (d_{RM} = .53), 95% CI [.38, .69], $p <$ .001, with low heterogeneity, (I^2 = 0%), $Q(df = 2) = 1.66$, $p = .44$. However, there was a much smaller and still not significant interference effect in terms of RT, (d_{RM} = −.12), 95% CI [−.26, .02], $p = .10$, with low heterogeneity, (I^2 = 0%), $Q(df = 2) = 1.70$, $p = .43$.

General Discussion

Across three experiments, we investigated potential explanations for the often reported finding of lower behavioral performance on proportional reasoning tasks compared with numerical tasks by investigating how well adults match discrete quantity displays based on either absolute number or relative proportion. Importantly, we manipulated whether and when participants' attention was explicitly directed toward the relevant information and the structure of the choice alternatives to investigate four potential mechanisms: (a) differences in saliency, (b) differences in the automaticity of encoding, (c) differences in the ease or effortfulness of processing, and (d) difficulty with response inhibition during decision-making. Moreover, in Experiment 3, we investigate whether the differences between proportion and the numerical subset found in Experiment 2 also apply to proportion versus the numerical total. Overall, as summarized in Table 2, our findings provide some evidence for three of the four mechanisms, but also suggest that differences between numerical and proportional information found in Experiment 2 and in other prior research (e.g., Fabbri et al., 2012) might be specific to differences between proportion and numerical information about the subset, rather than numerical information more generally. We discuss the first three mechanisms together, as they all consider various ways in which numerical information may be privileged before the response decision. Then, we discuss response competition, as it arises during decision making, after encoding and processing. Throughout, we highlight the theoretical and methodological implications of these findings, given that they may only apply to some numerical information (i.e., the subset) and not others (i.e., the total).

Saliency, Encoding, and Ease of Processing

First, we found that when provided no information about what to attend to, adults exhibited a clear tendency to spontaneously match on numerical information more than any other dimension. Thus, adults were likely to prioritize numerical information about the relevant or salient subset over information about the relative proportion, suggesting that numerical information about a subset is more immediately salient and draws adults' attention.

Second, we experimentally manipulated whether adults knew ahead of time what information to attend to in order to investigate how automatically numerical and proportional information is encoded. Consistent with other work (Fabbri et al., 2012), we did not find large differences in adults' abilities to encode numerical versus proportional information. That is, although performance on probe-before trials was slightly better than performance on probe-after trials, this performance cost was comparable across trials in which participants were asked to match based on number and those in which they were asked to match based on proportion (Experiment 2; although see small differences in Experiment S1 in the online supplemental material).

Third, we further investigated the issue of effortfulness by looking at ease of processing more generally, beyond the encoding

Table 2
Summary of the Relevant Results of Each Experiment

Testable mechanism	Specific question	Experiment 1: Subset versus proportion	Experiment 2: Subset versus proportion	Experiment 3: Subset versus proportion	Experiment 3: Total versus proportion
Saliency	Are adults more likely to match on the number of red than proportion of red in the absence of instruction?	Yes			
Encoding automaticity	Is there a difference in the additional cost to performance when adults must encode both number and proportion versus can focus on the relevant dimension only, for number versus proportion?		No		
Processing costs	Is there a general cost to accuracy or reaction time when matching on proportion, in the absence of encoding or competition demands?		Yes—Accuracy and RT Meta-analysis across Experiments 2, 3, and S1[a]: Yes—Accuracy & RT	Yes—Accuracy and RT	Partial—RT only
Response competition	Is there an additional cost to performance (accuracy) when matching on proportion in the presence of a numerical match (vs. not)?		Yes—Accuracy Meta-analysis across Experiments 2, 3, and S1[a]: Yes—Accuracy	Yes—Accuracy	No

Note. RT = reaction time.
[a] Experiment S1 is reported only in the online supplemental material.

process. We found that matching on number, be it the number of red dots or the total number of dots, was significantly faster than matching on proportion. These findings are consistent with the possibility that processing proportional information requires a computation across two quantity representations in a way that matching on number does not. However, this processing cost of proportion only resulted in lower accuracy compared with numerical subset information, whereas participants were similarly accurate when matching on proportion and on the numerical total.

In summary, results suggest that processing differences between proportion and subset numerical information are evident in terms of overall effort involved in processing, error-prone responding, and in baseline differences in saliency, but are not evident at the stage of encoding (i.e., number does not appear to be more automatically encoded than proportional information). Additionally, proportional information shows some evidence of more effortful processing than the numerical total, but results revealed differences only in RT not in accuracy.[8] Taken together, what do these patterns suggest about how adults engage with numerical versus proportional information? One possibility is that adults encoded multiple bits of numerical information (e.g., the number of red and the number of blue OR the number of red and the total), regardless of what information they were directed to attend to, and always computed proportional information from this numerical information after the initial numerical encoding. This explanation is consistent with the proposal that numerical information is directly abstracted from a display, but proportional information is tracked via a computational process involving numerical representations (e.g., Gallistel, 1990; Gallistel et al., 2006). That is, adults may only be able to represent proportion after tracking number, resulting in a computational processing cost evident in their RT. Notably, however, this is counter to other work suggesting that proportional information is automatically accessed and that computations are not necessary even for symbolic fractions (Binzak & Hubbard, 2020; Lewis et al., 2014, 2016).

A related possibility is that numerical information is more salient and more easily processed because of people's experiences with numerical information. Numerical information is taught earlier than proportional information (e.g., counting is learned before fractions), and adults' daily experiences likely involve more instances of absolute numerical information than relative proportions or fractions. These privileged and frequent experiences with number, relative to proportion, may lead people to pay more attention to numerical information or believe that in an experimental setting with minimal instructions, such as in the spontaneous MTS task, we (as experimenters) are expecting numerical-based matches. Further investigating the extent to which this is a top-down effect due to expectations and cognitive processing versus a bottom-up effect due to differences in attention allocation and perceptual features is an important next step for more deeply understanding the cause of these differences between numerical versus proportional matching performance.

Response Competition and Interference

Last, our results provide support for the role of response competition arising from subset numerical information during decision-making, but not total numerical information. Specifically, numerical information about a relevant subset (i.e., the number of red) interfered with proportion matching at the time of response selection, over and above differences in processing in the absence of response competition. Thus, inhibiting numerical response selection about a salient subset when matching proportion may be particularly difficult.

Notably, we did not find any evidence of response competition when the numerical response option included the same total number of dots rather than the number in the numerator-based subset (in this case, red dots). Interestingly, this suggests that different aspects of the numerical information that comprise proportion may differentially interfere with proportional processing, such that the numerator information interferes with proportion in a way that the total number or denominator information does not. Given that proportional information is derived from the absolute numerical information, investigating how the different component parts interact with proportional reasoning can provide further insight into what computation might be occurring. For example, adults could encode the proportional information as a proportion or fraction (where the numerator is the number of red units, and the denominator is the total number of red plus blue units) or as a ratio (where the red to blue relation is compared), and although the numerical information of one subset (red units) is necessary in both cases, only the former relies explicitly on the total number of dots. Furthermore, the fact that we only see numerical interference from a single subset, and not from the total numerical information, might suggest that the need to compute over two features (red and blue) and more information also contributes to the disadvantage of processing proportional information. That is, inherent to proportional information is the fact that in order to track it, one must track both the subset and the total number and properly integrate across two features, rather than focusing on a single feature. Therefore, it may be that this natural confound, in terms of attention allocation and the amount of information to attend to, explains typical numerical interference effects for numerical subsets, which align with these confounds (e.g., Boyer et al., 2008; Fabbri et al., 2012; Hurst & Cordes, 2018a; see also current study Experiment 2), but not for the numerical total. However, the fact that processing proportion still takes longer than processing the total number (which includes the same number of items and computing over two features), suggests that this cannot be the whole explanation. More work is needed to further investigate this difference in interference from numerator numerical information and denominator numerical information and how this difference may inform our understanding of proportional reasoning.

Critically, the data did not reveal any evidence of proportional information interfering with numerical judgments, either for the number in a subset or the number in the total. Therefore, although several open questions remain about how these pieces of information are integrated, our data make it clear that judgements about the number of red dots, the total number of dots, and the proportion of red dots were not equally weighted in the decision-making process, but instead that numerical information about the salient and relevant subset was prioritized above proportional information. Future research

[8] We must also note, however, that we only analyzed RTs on correct trials, which has been criticized as less informative than using a modeling approach that combines RTs from correct as well as error trials (Ratcliff et al., 2016; Ratcliff & Rouder, 1998). Features of our experimental design limit our ability to pursue this sort of modeling effort here.

should further investigate how all three of these pieces of information are represented, computed, and integrated.

Interestingly, using exploratory correlation analyses, these data also generate hypotheses about how these explanations for the difference in numerical versus proportional reasoning come about by suggesting that individual differences in the strength of these phenomena are related. Adults' with a larger difference in performance matching on number versus proportion, in the absence of competition, also showed larger numerical interference in the presence of competition – and this was true for both the numerical subset (Experiment 2: accuracy, $r[58] = .35$, $p = .006$; RT, $r[58] = .23$, $p = .08$; Experiment 3: accuracy, $r[48] = .37$, $p = .009$; RT, $r[48] = .50$, $p < .001$) and the numerical total information (Experiment 3: accuracy, $r[48] = .42$, $p = .002$; RT, $r[48] = .44$, $p = .001$).[9] Although more experimental work is needed, this finding generates interesting hypotheses about the cause of numerical interference; numerical response competition may be due, at least in part, to differences in the basic processing and representation of numerical versus proportional information. Moreover, these individual differences were found even for total numerical information, which did not show significant group-level interference.

Implications

Although the most direct implication of the current study is to explain the often-reported difficulties seen in both children and adults in terms of nonsymbolic (e.g., Boyer et al., 2008; Hurst & Cordes, 2018a; Jeong et al., 2007) and symbolic (e.g., Alibali & Sidney, 2015; Braithwaite & Siegler, 2018; Hurst & Cordes, 2016, 2018b; Lortie-Forgues et al., 2015; Ni & Zhou, 2005) proportional reasoning, our findings also have implications for the way we conceptualize distinct types of quantity more generally. In particular, our findings do not suggest that the difficulty associated with processing proportional information stems from only one component of these paradigms or one aspect of numerical information that can be immediately targeted and rectified. Instead, our findings suggest that both subset and total numerical information may be privileged over proportional information in ways that impact basic performance differences, in the absence of competing or costly task demands, and that the numerical information about the relevant and salient subset may cause additional interference at the time of response selection.

The multifaceted advantage of numerical information found in the current study is in apparent contrast to other work suggesting that proportional information is automatically encoded and perceptually available (Binzak & Hubbard, 2020; Fabbri et al., 2012; Lewis et al., 2016), is available and may even be preferred earlier in development (Denison et al., 2013; Denison & Xu, 2010; Hurst & Cordes, 2018b; McCrink & Wynn, 2007), and may provide a foundation for other aspects of magnitude representation (Bonn & Cantlon, 2017). Thus, what accounts for the pervasive difficulties with proportional reasoning, even in terms of basic processing costs, found here? Behavioral patterns seen in children who are highly attentive to numerical information have been attributed to an over emphasis on the counting routine and whole number learning more generally (Boyer et al., 2008; Lortie-Forgues et al., 2015; Ni & Zhou, 2005) or from a lack of understanding or precision in representing proportional information (Alibali & Sidney, 2015; Lortie-Forgues et al., 2015). However, the question remains, why does a performance discrepancy remain pervasive in adulthood, well after the instruction of formal fractions, decimals, and percentages?

One potential explanation is that the representation of numerical information is made easier and/or the representation of proportional information is made more difficult by our formal symbolic mathematical systems. That is, whether or not the underlying nonsymbolic representations are part of the same system or show similar representational precision, mapping these systems with symbols and number words may impact how they are interpreted, even for adults. In particular, the very well learned mapping between number words and discrete quantity may make it easier and more automatic to encode this information (e.g., a participant who sees five red dots and 13 blue dots may encode this as "five red dots"). On the other hand, the symbolic number system for proportion (i.e., fractions, decimals, or percentages) is much less straightforward and automatic (Kallai & Tzelgov, 2012, 2014; Obersteiner et al., 2013; e.g., participants may not be able to readily generate a symbol or verbal label to describe the proportion "5/18"). Thus, the symbolic number systems may impact how easily adults are able to access the nonsymbolic representations of number and proportion and provide accurate estimates of these values.

Taken as is, then, it may be tempting to say that the current study suggests that not only are discrete whole number quantities and proportional or ratio quantities represented within distinct systems, but that the discrete whole number representational system may be psychologically privileged. This interpretation would be consistent with arguments that whole numbers are a core knowledge system available in infancy (Spelke & Kinzler, 2007) and that continuous and discrete representations of quantity may be psychologically distinct (e.g., Agrillo et al., 2010; Dormal et al., 2006; Hamamouche & Cordes, 2020; Young & Cordes, 2013). However, this would be counter to other findings suggesting that proportional information is directly processed in some contexts (Binzak & Hubbard, 2020; Fabbri et al., 2012; Lewis et al., 2014, 2016) and to recent arguments suggesting that both whole numbers and fractions are represented within a single rational number system (Clarke & Beck, 2021). Thus, although the current study highlights ways in which discrete whole number information and proportional information are different, they also share commonalities that have been empirically demonstrated, such as the presence of distance effects and the developmental trajectories of discrimination abilities(e.g., Matthews & Chesney, 2015; Park et al., 2021), and in terms of their formal mathematical nature (i.e., whole numbers are rational numbers and can be represented with fractions; $6/2 = 3$). Thus, additional research is needed to better understand the shared and divergent processes that underlie whole number and proportional representation. Importantly, there are several important considerations about the current experiment that highlight where future work is needed to further differentiate these claims.

[9] Individual differences in general processing differences were calculated 7as Number Probes on Simple-Match Trials – Proportion Probes on Simple-Match Trials and numerical interference as [Proportion Probes on Competition Trials – Proportion Probes on Simple-Match Trials].

Limitations

First, it is worth noting that one particularly critical aspect of the current stimuli was the discrete nature of the dot displays. It may be that these intermixed dot displays make numerical information in particular immediately salient, and that relational information would be more salient than absolute number or amount in other visual contexts. For example, despite the availability of both numerical and proportional information, proportional information may be more salient (or at least, as salient as number) in the case of continuous displays that have been made discrete (e.g., dividing a rectangle into pieces) or discrete displays that have been clustered together (e.g., partitioned dot displays with all the red dots grouped together and all the blue dots grouped together). Some work suggests that, at least in the case of continuous representations that are made discrete, it is possible to encourage young children to attend to proportion and ignore number, preventing numerical interference (Boyer & Levine, 2015; Hurst & Cordes, 2018a, 2019). Thus, future work should further investigate how variability across contexts may impact the relative saliency of both number and proportion.

In addition, although our results reveal that matching numbers, be it the number of a subset or the total number, was faster than matching proportion, even on simple-match trials in which neither response option matched on the competing dimension, there was still some element of "competition" in the sense that both number and proportion were available for encoding in the sample stimulus itself and the proportional information must be encoded through the numerical relations (i.e., red out of total or red-to-blue ratio). Furthermore, continuous magnitude dimensions that co-occur with numerical information, such as cumulative area (e.g., Leibovich et al., 2017; Savelkouls & Cordes, 2020) were available in the current study and could have been used to support matching on both number and proportion. That is, it could be that adults relied on the absolute cumulative area and/or the proportion of cumulative area. Although beyond the scope of the current project, an important question for future work is how these two orthogonal dimensions—relative proportion versus absolute amount and continuous magnitude versus discrete magnitude—relate and possibly interact with each other. Work with young children suggests that they are able to think about proportion in the context of continuous displays based on area alone (Boyer et al., 2008; Hurst & Cordes, 2018a; Jeong et al., 2007), potentially even more accurately than with discrete displays. Importantly, however, this has the additional confound of not just comparing different quantity representations (proportion vs. number), but also different perceptual features (discrete vs. continuous). Given the long standing debate about whether continuous area and discrete number are represented via the same or different representational system across development (Clearfield & Mix, 1999; Leibovich et al., 2017; Newcombe et al., 2015; Walsh, 2003), it remains unclear how continuous absolute amount, discrete number, and both continuous and discrete proportion may be represented and understood relative to each other.

Last, it is worth noting that the current study represented proportions with nonsymbolic ratio-based quantities and used a percent sign (%) as the indicator for proportion. It is unclear how the results may have differed had the proportional quantities been directly associated with fractions, decimals, or percentages. In particular, adults show large differences in processing magnitude information represented with different formats, performing better on quantitative reasoning tasks represented with decimals than fractions (e.g., DeWolf et al., 2014; Hurst & Cordes, 2016). They also show distinct alignment tendencies, which parallel the aforementioned discrete versus continuous representations, in that fractions more strongly align with discrete contexts (Rapp et al., 2015). However, it is unclear how percentages (the symbolic reference frame used in the current studies) may fit into these differences, as percentages borrow from both fractions and place-value and substantially less research has investigated percentage based reasoning (see Tian & Siegler, 2018 for a review). Thus, the difficulty of the system for symbolically representing proportion may impact the automaticity and accuracy of the underlying representation of proportional magnitude, leading to differences in the way that this information is encoded and represented and why numerator numerical information may be particularly distracting at the level of decision making. Precisely how this difference may further depend on the specific structure of the proportional number system, and the symbol used to represent it, however, remains an open question.

Conclusions

In summary, we present three experiments investigating how adults process discrete dot displays in terms of both the numerical and proportional information available in those displays. Overall, we provide evidence that numerical information about one subset is more salient, faster to process, less error-prone, and more difficult to inhibit relative to proportional information. Whereas numerical information about the total set is faster to process than proportion, but not less error-prone or more difficult to inhibit. As such, the privileged status of numerical information over proportional information is evident throughout processing, but also dependent on the specific aspect of numerical information being considered. Importantly, the explanations and processes investigated here are also likely interrelated and dependent on each other, and future research must continue to investigate what psychological and educational mechanisms may be leading to pervasive differences between numerical and proportional quantity representations.

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