Lab 1: The Apollo Missions

Maria Nolan

University of Maryland, College Park

mnolan1@umd.edu

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I. *Introduction*

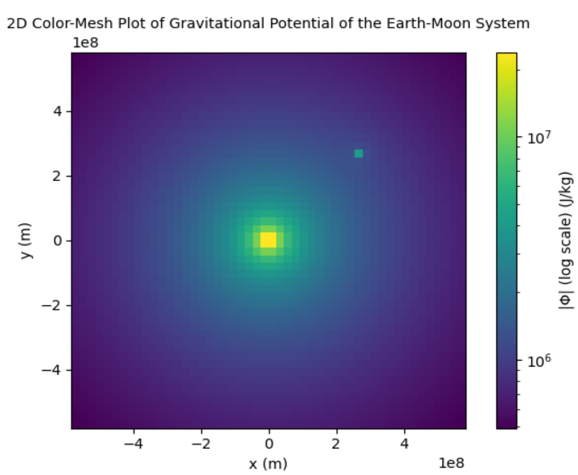
The research and development of the Apollo program is underway, and it is necessary to have a basic understanding of the gravitational potential and forces the mission will experience to carry it out. Furthermore, it is necessary to begin quantifying the performance of the new rocket that will carry the capsule: the Saturn V. Throughout the lab, we research the gravitational potential of the Earth, the gravitational potential of the Earth-Moon system, the gravitational force field of the Earth-Moon system, and the altitude of the Saturn V Rocket after the fuel finishes burning through the use of the Python programming language. The NumPy library (used for mathematical purposes), Matplotlib library (used for creating interactive visualizations), and SciPy library (an extension for NumPy) are all used to perform this research.

Relevant physical constants include:

Quantity	Value
Gravitational Constant (G)	$6.67 \times 10{-}11$ $m^3/kg/s^2$
Gravitational Acceleration (g)	9.81 m/s^2
Mass of the Earth ($M_{\oplus}$)	5.9×10^{24} kg
Mass of the Moon ($M_{\$}$)	7.3×10^{22} kg
Radius of the Earth ($R_{\oplus}$)	6378 km
Radius of the Moon ($R_{\$}$)	1737 km
Distance from Earth to Moon ($d_{\oplus\$}$)	3.8×10^{8} m
Exhaust Velocity of Saturn V Stage 1 (S-1C) (v_e)	2.4×10^{3} m/s
Burn Rate of S-1C ($\dot{m}$)	1.3×10^{4} kg/s
Wet Mass of S-1C (m_0)	2.8×10^{6} kg
Dry Mass of S-1C (m_f)	7.5×10^{5} kg

II. *The Gravitational Potential of the Earth-Moon System*

Figure 1



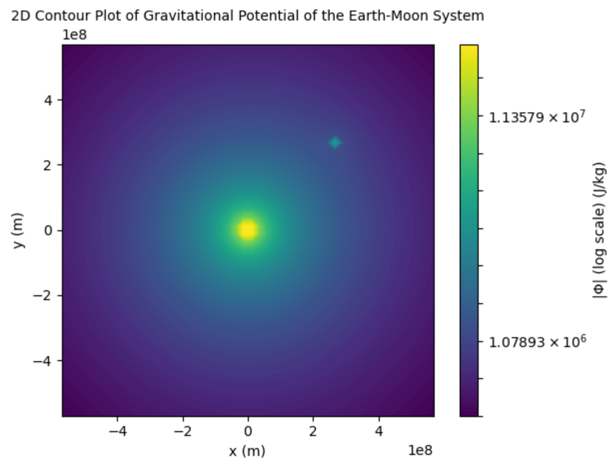
Gravitational potential is the work done per unit mass (joules per kilogram) by some externally applied force. The gravitational potential at a distance r from a mass M is given by:

$$\Phi(r) = -\frac{GM}{r}$$

$$r = \sqrt{(x_m - x)^2 + (y_m - y)^2}$$

where G is the gravitational constant. We can assume that the Earth is at the origin (0,0), and the moon is located at a location $(\frac{d\oplus\$}{\sqrt{2}}, \frac{d\oplus\$}{\sqrt{2}})$. The equations for the gravitational potential of both the Earth and the Moon must be found, then summed to find the equation for the gravitational potential of the Earth-Moon system. Matplotlib's pcolormesh feature and NumPy's library in Python were utilized to create the 2D color-mesh plot in Figure 1. Figure 1 is a visualization of the gravitational potential of the Earth-Moon system, with the Earth being at the origin of the plot. Matplotlib's contour feature and NumPy were also used to create the contour plot in Figure 2, which is just a smoother version of the color-mesh plot in Figure 1. You can see that gravitational potential is the strongest closest to Earth (at the origin), but there is also a spike in gravitational potential around (3E8, 3E8) where we can assume the Moon is located.

Figure 2



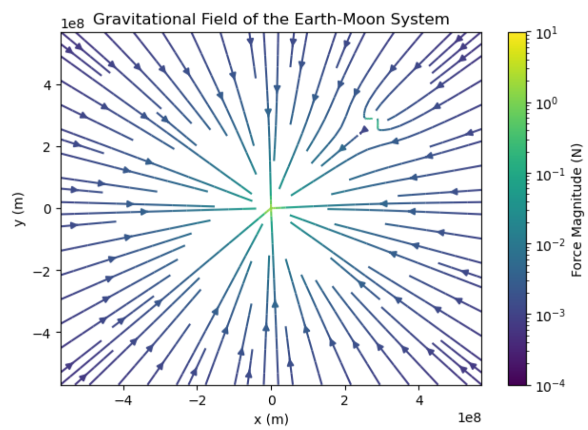
III. The Gravitational Force of the Earth-Moon System

Gravity causes there to be an attractive force from an object with a larger mass to an object with a smaller mass. The gravitational force $\vec{\boldsymbol{F}}$ that a mass M_1 exerts on a mass m_2 is:

$$\vec{\boldsymbol{F}}_{\mathbf{21}} = -G\frac{M_1 m_2}{|\vec{r}_{21}|^2}\hat{\boldsymbol{r}}_{\mathbf{21}}$$

where $\vec{\boldsymbol{r}}_{\mathbf{21}}$ is the displacement vector from M_1 to m_2. Figure 3 was made using NumPy's library and Matplotlib's streamplot feature to help visualize the gravitational field of the Earth-Moon system.

Figure 3



IV. Projected Performance of the Saturn V Stage 1

Rockets function by conservation of momentum. The ejection of fuel backwards propels rockets forward. The change in a rocket's velocity Δv as a function of time t is given by the Tsiolkovsky rocket equation:

$$\Delta v(t) = v_e \ln\left(\frac{m_0}{m(t)}\right) - gt$$

where m_0 is the initial "wet" mass (fuel + rocket parts + payload), $m(t) = m_0 - \dot{m}t$ is the mass at time t, $\dot{m}$ is the fuel burn rate, v_e is the fuel exhaust velocity, and g is the gravitational acceleration. When defining this function using the NumPy library in Python, I made sure to design it carefully so that Δv eventually becomes zero after all the fuel is spent. You can find the altitude h of the rocket at "burnout" (when the fuel is used up) by integrating the velocity over time:

$$h = \int_0^T \Delta v(t) dt$$

where T is the total burn time of the rocket, which by conservation of momentum is:

$$T = \frac{m_0 - m_f}{\dot{m}}$$

where m_f is the final "dry mass" of the rocket once all the fuel is burned. By using NumPy and SciPy in Python, I was able to determine that the burn time is $T = 157.69$ seconds, and the altitude at burnout is $h = 74{,}093.98$ meters ≈ 74.09 kilometers.

V. Discussion and Future Work

Through my research, I was able to contribute towards the development of the Apollo program. I was able to visualize the gravitational potential and the gravitational field of the Earth-Moon system using the programming language Python, which allowed me a better understanding of how gravity works. Furthermore, I defined multiple functions in Python that helped me calculate the altitude of the rocket at "burnout" and the total burn time of the rocket. I was able to determine that the burn time is $T = 157.69$ seconds, and the altitude at burnout is $h = 74{,}093.98$ meters ≈ 74.09 kilometers. Compared to the testing data in which $T = 160$ seconds and $h = 70$ kilometers, my data is slightly off. This could be due to neglecting certain factors such as air resistance, variations in fuel burn rate, or temperature effects. These factors could cause fuel burning to be less efficient than we initially calculated, which would account for the loss of altitude and the increase in burn time compared to the calculations.