$\mathbb{D}: C^{op} \times C \to Set$

$\int_C \mathbb{D}$ * Objects: (c, d) with $c \in C$, $d \in \mathbb{D}(c, c)$

$\mathcal{D}: C^{op} \times C \to Cat$

$\int^G_C \mathcal{D}$ * Objects: (c, d) with $c \in C$ $d \in \mathcal{D}(c, c)$

$$\begin{tikzcd}
(c,d) \arrow[d, "{(\gamma,\delta)}"] & \mathcal{D}(c,c) \ni d \arrow[d, mapsto] \\
& \bullet \arrow[d, "\delta"'] \quad \mathcal{D}(c,c') \\
(c',d') & \bullet \\
& \mathcal{D}(c',c') \ni d' \arrow[u, mapsto]
\end{tikzcd}$$

$$\begin{tikzcd}
& \int \mathcal{D} \arrow[dl] \arrow[dr] & \\
\mathcal{C} & & \mathcal{C}
\end{tikzcd}$$

$x : A,\ y : B\ z : C \vdash$

$$\begin{tikzcd} \int C \arrow[d] \arrow[dd, bend right] \\ \int B \arrow[d] \\ A \end{tikzcd}$$

$$c \xrightarrow{g} c' \xrightarrow{h} c''$$

$$(c,d) \to (c',d') \to (c'',d'')$$

$d \in D(c,c)$

$fd = d''f \in D(c,c'')$

$d'' \in D(c'',c'')$

$[\mathcal{C}, \mathcal{D}]$

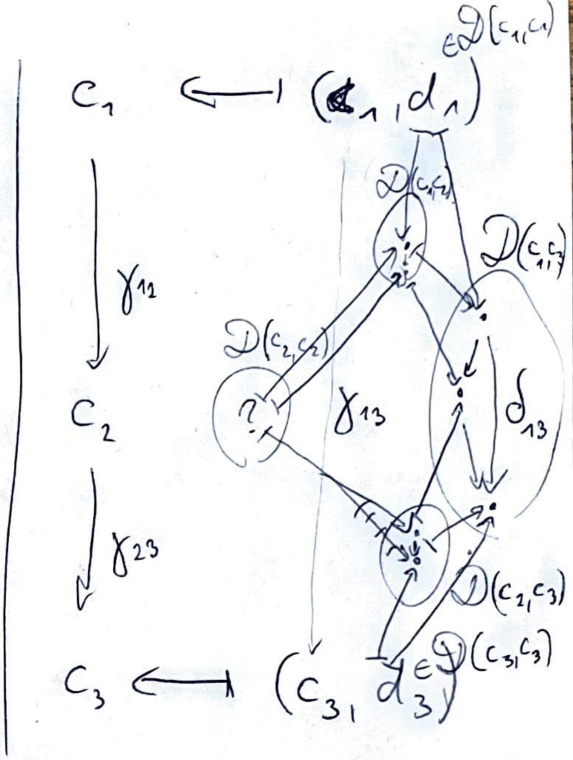
$[\mathcal{C}x, \mathcal{D}] : I^{op} \to \mathcal{C}at$

$[\mathcal{C}x, \mathcal{D}y] : I^{op} \times I \to \mathcal{C}at$

$$\left.\begin{array}{l} \mathcal{C} : I \to \mathcal{C}at \\ \mathcal{D} : I \to \mathcal{C}at \end{array}\right\} \left.\begin{array}{l} [\mathcal{C}, \mathcal{D}] \\ : I^{op} \times I \to \mathcal{C}at \end{array}\right\} \begin{tikzcd} \int_I [\mathcal{C}, \mathcal{D}] \arrow[d] \\ I \arrow[u, bend left] \end{tikzcd}$$

$$\begin{tikzcd}
\mathcal{C}^{\to} \arrow[r, equal] \arrow[d, "\mathrm{cod}"'] & \mathcal{C}^{\to} \arrow[dl, "\mathrm{cod}"] \\
\mathcal{C} &
\end{tikzcd}$$

$$\begin{tikzcd}
\mathcal{C}at^{\to} \arrow[r] \arrow[d, "\cancel{\mathrm{cod}}"'] & \mathcal{C}at^{\to} \arrow[dl, "\mathrm{cod}"] \\
\mathcal{C}at &
\end{tikzcd}$$



$$\begin{tikzcd}
\mathcal{D}^\circ[F] \arrow[d, dashed] & \mathcal{D}^\circ \arrow[d] \\
\mathcal{C} \arrow[r, "F"'] & \mathcal{D}
\end{tikzcd}$$

$$\mathrm{Obj}(c, \mathcal{D}^\circ[F]) = \mathrm{Obj}_{Fc}(\mathcal{D}^\circ)$$

$$\mathrm{Hom}(c \xrightarrow{\gamma} c' \mid d^\circ, d^{\circ\prime})$$

$$= \mathrm{Hom}(Fc \xrightarrow{F\gamma} Fc' \mid d^\circ, d^{\circ\prime})$$

$x : A, y : B\ z : C \vdash$

$$\int C \to \int B \to A$$

$$c \xrightarrow{g} c' \xrightarrow{h} c''$$

$$\cancel{(c,d)}$$

$d \in \mathcal{D}(c,c)$

$$(c,d) \to \cancel{(c', d')} \to (c'', d'')$$

$fd = d''f \in \mathcal{D}(c, c'')$

$d'' \in \mathcal{D}(c'', c'')$

$[\mathcal{C}, \mathcal{D}]$

$[\mathcal{C}x, \mathcal{D}] : I^{op} \to \mathcal{C}at$

$[\mathcal{C}x, \mathcal{D}y] : I^{op} \times I \to \mathcal{C}at$

$\mathcal{C} : I \to \mathcal{C}at$

$\mathcal{D} : I \to \mathcal{C}at$

$[\mathcal{C}, \mathcal{D}] : I^{op} \times I \to \mathcal{C}at$

$$\int_I [\mathcal{C}, \mathcal{D}] \to I$$

$f : \mathcal{V} \to \mathcal{W}$ $\qquad$ $a \in \hat{\mathcal{W}}$ $\qquad$ $c \in \hat{\mathcal{V}}$

$$W \Rightarrow f_!(f^* a \times c)$$
$$= \exists V.\ (W \to fV) \times (V \Rightarrow f^* a \times c)$$
$$= \exists V.\ (W \to fV) \times (fV \Rightarrow a) \times (V \Rightarrow c)$$

$$\begin{tikzcd}
 & & a \\
W \arrow[r] & fV \arrow[ur, Rightarrow] & & V \arrow[dr, Rightarrow] \\
 & & & & c
\end{tikzcd}$$

ok

$$W \Rightarrow a \times f_! c$$
$$= (W \Rightarrow a) \times \exists V.\ (W \to fV) \times (V \Rightarrow c)$$

$$\begin{tikzcd}
 & & a \\
W \arrow[urr, Rightarrow] \arrow[r] \arrow[d, "\text{ess surj } \cong"'] & fV & & V \arrow[dr, Rightarrow] \\
fV' \arrow[ur, "\text{full}", "f\varphi"'] & & V' \arrow[ur, "\varphi"] \arrow[rr] & & c
\end{tikzcd}$$

f ess surj

Plan:

- sort out function types
  - do all types have to be fibrations?
  - can we take ▷ to be the type of cats w/ fin products & fin.-prod pres functors.
    ↳ is the Stack answer correct?

- write out a very different example from SysFω.

$-\times\underline{\Pi}$ $\qquad$ $-\times\underline{\Pi}$

$\hat{D}(B^A)\delta = B^A(D\delta) \overset{\text{nat}}{=} \forall(\gamma' \in \Gamma/D\delta).\, A\gamma \to B\gamma$

$\hom(y_{\delta'}, B^A D -)$

$\cong \hom(y_{D\delta'}, B^A)$

$\cong \hom(y_{D\delta'} \times A, B)$

$(BD)^{(AD)}(\delta) =$

$\overset{\text{nat}}{\forall}(\delta' \in \Delta/\delta).\, AD\delta \to BD\delta$

$\cong \hom(y_{\delta'} \times AD, BD)$

$\overset{?}{\cong} \hom(y_{\delta'}, (BD)^{(\cancel{BA})\,AD})$

$\Gamma \longrightarrow Set^{op}$

$$\frac{\hom(\gamma, B^A)}{\hom(A \times \gamma, B)}$$

$\hom(A, B)$

$\cancel{\hom\, B}$

$\Delta \xrightarrow{D} \Gamma$

$\hat{\Gamma} \xrightarrow{\hat{D}} \hat{\Delta}$

$F \longmapsto \delta \longmapsto \cancel{F}\, \hat{D}(F)(\delta)$

$= F(\cancel{\delta}\, D\delta)$

$$(F_! T)\delta = \exists^{\text{card}}_{\gamma} . \mathrm{Hom}(\delta, F\gamma) \times T_\gamma$$

$$= \mathrm{colim}_{(\gamma, -) \in \int_\Gamma \mathrm{Hom}(\delta, F\_)} T_\gamma$$

$$\int^{\delta \downarrow F} \mathrm{Hom}_{(\delta\downarrow F)^2}((x,y), D\_)$$

$F : \Gamma \to \Delta$

$F_! : \hat{\Gamma} \to \hat{\Delta}$

$T \in \hat{\Gamma}$ $\delta \in \Delta$

Γ

$\Gamma \vdash a : A$

$$KTm_{\Theta}(K_1 ; K_2) \xrightarrow{lam_{\Theta}} KTm_{\Theta}(K_1 \to K_2)$$

$$\frac{\Theta, x : K_1 \vdash f : K_2}{\Theta \vdash \lambda x. f : K_1 \to K_2} \qquad \frac{\Delta, x : K_1 \vdash \delta^* f : K_2}{\Delta \vdash \delta^* \lambda x. f : K_1 \to K_2}$$

$$\begin{tikzcd}
KTm_{\Theta}(K_1 ; K_2) \arrow[r, "\delta"] \arrow[d, "lam_{\Theta}"'] & KTm_{\Delta}(K_1 ; K_2) \arrow[d, "lam_{\Delta}"] \\
KTm_{\Theta}(K_1 \to K_2) \arrow[r, "\delta"] & KTm_{\Delta}(K_1 \to K_2)
\end{tikzcd}$$

$\hom(L$

$LX \otimes LY \cong L(X \otimes Y)$

$$\hom(\cancel{Z}, R(Y^X)) \overset{?}{\cong} \hom(Z, RY^{RX})$$

$$\begin{array}{ccc} \hom(\cancel{Z}, R(Y^X)) & \overset{?}{\cong} & \hom(Z, RY^{RX}) \\ \wr\| & & \wr\| \\ \hom(R, Y^X) & & \hom(Z \times RX, RY) \\ \wr\| & & \\ \hom(L\cancel{Z} \times X, Y) & & \end{array}$$

$$\begin{tikzcd} \Gamma \arrow[r, "P"] \arrow[d, "F"'] \arrow[dr, phantom, "\Downarrow"] & \mathrm{Set} \\ \Delta \arrow[ur, dashed, "F_! P"'] & \end{tikzcd}$$

$\mathrm{Lan}_F P$

$$\hom(\mathrm{Lan}_F P\, R, Y^X) \cong \mathrm{Lan}_F \hom(P-, Y^X)$$

$$\cong \mathrm{Lan}_F \hom(P- \times X, Y)$$

$$\frac{\Gamma^{+} \vdash T : \mathcal{O}}{\Gamma \vdash T \ \text{utype}}$$

↳ unannot

$$\frac{T : (\Gamma, \Gamma^{\text{core}}) \longrightarrow (\text{Set}^{\text{op}}, \text{Set}^{\text{core}})}{T : \Gamma \longrightarrow \text{Set}^{\text{op}} \quad (\Gamma^{\text{pc}} \text{ ignored})}$$

$$\frac{\begin{array}{ll} \Gamma^{+} \vdash A : \mathcal{O} \ \cancel{\text{dtype}} & \\ \Gamma^{+} \vdash B \ \cancel{\text{dtype}} : \mathcal{O} & \end{array} \Big\} \begin{array}{l} \Gamma^{\text{erased}} \vdash A \ \text{utype} \\ \Gamma^{\text{erased}} \vdash B \ \text{''} \end{array} \quad \begin{array}{l} \Gamma \vdash a : A \\ \Gamma \vdash b : B \end{array}}{\Gamma \vdash (a, b) : A \times B}$$

$$\frac{}{\Gamma \vdash \mathcal{O} \ \text{dtype}}$$

▷

$$\frac{\Gamma^{\text{erase}} \vdash A \ \text{dtype}}{\Gamma, x :^{+} A \vdash x : A}$$

$$\Gamma, x :^{\text{nat}} A \vdash x : A$$

$$\frac{\Gamma \vdash A, B \ \cancel{\text{dtype}} \ \text{utype}}{\Gamma \vdash A \to B \ \cancel{\text{dtype}} \ \text{utype}}$$

$$\frac{\Gamma \vdash T : \mathcal{U}}{\Gamma \vdash El\,T \ \text{type}}$$

$$\frac{T \in Tm(\Gamma, \mathcal{U})}{T \in Ty(\Gamma)}$$

$\Gamma \vdash A$ dtype

$\Gamma \vdash A$ utype

$\Gamma \longrightarrow \mathrm{Cat}^{op}$

$\Gamma \longrightarrow \mathrm{Set}^{op}$

$\Delta \downarrow F \qquad F : \Gamma \to \Delta$

$(\delta \downarrow F)^{op} \xrightarrow{D} (\delta \downarrow F)^{op} \times (\delta \downarrow F)^{op}$

$(x, y) \downarrow D$

$\big((\gamma, f : \delta \to F\gamma), (\gamma', f' : \delta \to F\gamma')\big)$

$\uparrow$

$\big((\gamma'', f'' : \delta \to F\gamma''), \quad '' \quad \big)$

$\gamma'' := \gamma \times \gamma'$

$$\begin{tikzcd}
 & F\gamma & \\
\delta \arrow[ur] \arrow[r] & F\gamma' & F\gamma'' \arrow[ul] \arrow[l]
\end{tikzcd}$$

Syntax

$\triangleright$

$\circ$

$A : \triangleright$

$B : \circ$

$\diamond$

$\cancel{\Gamma, A : \circ \;;\; \Gamma, A : \diamond}$

$\Gamma \vdash A : \circ$

$\Gamma, A \overset{nat}{:} \circ \;/\; \Gamma, A \overset{nat}{:} \triangleright$

$\Gamma, A \overset{+}{:} \circ \;/\; \Gamma, A \overset{+}{:} \triangleright$

$\Gamma \vdash A : \triangleright$

$\Gamma \vdash A : \circ$

Semantics

$\mathbf{Cat}^{op}$

$\mathbf{Set}^{op}$

$A \in \mathbf{Cat}^{op}$

$B \in \mathbf{Set}$

$\cancel{\Gamma \times \mathbf{Set}^{op} \;;\; \Gamma \times \mathbf{Cat}^{op}}$

$\left(\int^{el}_{\Gamma} A,\ \{f \mid f \text{ is cartesian in } \Gamma\}\right)$

$\left(\Gamma \times \mathbf{Set}^{op},\ \Gamma^{pc} \times \mathbf{Set}^{op} \text{ cartesian}\right) \;/\; \left(\Gamma \times \mathbf{Cat}^{op},\ \Gamma^{pc} \times \mathbf{Cat}^{op}\right)$

$\left(\Gamma \times \mathbf{Set}^{op},\ \Gamma^{pc} \times \mathbf{Set}^{core}\right) \;/\; \left(\Gamma \times \mathbf{Set}^{op},\ \Gamma^{pc} \times \mathbf{Cat}^{core}\right)$

$$\begin{tikzcd} \Gamma^{pc} \arrow[r] \arrow[d] & \mathbf{Cat}^{core} \arrow[d] \\ \Gamma \arrow[r] & \mathbf{Cat}^{op} \end{tikzcd}$$

$$\begin{tikzcd} \Gamma^{pc} \arrow[r] \arrow[d] & \mathbf{Set}^{core} \arrow[d] \\ \Gamma \arrow[r] & \mathbf{Set}^{op} \end{tikzcd}$$

$KCtx \in \mathcal{C}at$

$\diamond \in KCtx$

$Kind \in Set$

$\langle\rangle : KCtx \to Kind \to KCtx$

- $\eta : Hom(\Theta, \diamond) \cong T$

-

$KTm : (KCtx \to Kind \to Set^{op}$

$\_ \to \_ : Kind \to Kind \to Kind$

$* : Kind$

$A : x,\ B : y,\ C : z \vdash$

$$\begin{array}{c} \int C \\ \downarrow \\ \int B \\ \downarrow \\ A \end{array}$$

Γ