$t : (A : \mathcal{C}) \to \mathcal{D}(A)$

$$\begin{tikzcd}
\bullet \arrow[r] \arrow[d] \arrow[dr, phantom, "\lrcorner", very near start] & \int_{\mathcal{C}} \mathcal{D} \arrow[d] \\
\mathcal{C}' \arrow[r] \arrow[u, bend left] & \mathcal{C} \arrow[u, bend left]
\end{tikzcd}$$

$\mathcal{C}' \to \mathcal{C} \xrightarrow{\mathcal{D}} \mathcal{C}at$

Cowedge from $T : \mathcal{C}^{op} \times \mathcal{C} \to \mathrm{Cat}$ to $\mathcal{C}$

$k : \forall a : T(a,a) \to \mathcal{C} : \_ \mapsto a$

$$\begin{tikzcd}
a \arrow[dd] & & T(a,a) \arrow[dr, "k_a"] \arrow[dd, phantom, "\Downarrow"] & \\
& T(b,a) \arrow[ur] \arrow[dr] & & \mathcal{C} \\
b & & T(b,b) \arrow[ur, "k_b"'] &
\end{tikzcd}$$

$\int$

If $\mathcal{D} = \mathrm{Cat}$

$\mathrm{Hom} : KCtx \Rightarrow KCtx^{\pm} \to O$

$Ty_O(\mathbb{I}) = \cancel{O}$? yes

$Ty_O(\mathcal{C}) = Psh(\mathcal{C})$
compr = cat of elements

$Ty_D(\mathbb{I}) = \mathcal{D}$? yes.

$Ty_D(\mathcal{C}) = [\mathcal{C}^{op}, \mathrm{Cat}]$
compr = Groth const

$\mathcal{D}$ is a category with:

$Ty_O : \mathcal{D}^{op} \to \cancel{Set}\ \mathrm{Cat}$ with compr

$Tm_O : \int_{\mathcal{D}} Ty_O \to Set$

$Ty_D : \mathcal{D}^{op} \to \mathrm{Cat}$ with compr

$Tm_D : \int_{\mathcal{D}} Ty_D \to \mathcal{D}$

~~$\mathrm{Hom} : Prof_O : \mathcal{D}^{op} \times \mathcal{D} \to \mathrm{Cat}$~~

$op : \mathcal{D} \to \mathcal{D}$

~~$Prof_O : \mathcal{D} \to \mathcal{D} \to \mathrm{Cat}$~~

~~$Ty_O(x) \to Prof_O(x,y)$~~

$\mathrm{Hom} : \cancel{Prof_O}$

$\prod_{x \in \mathcal{D}} Prof_O(x, x^{op}) \quad Ty_O(x^{op} \otimes x)$

Obj : ○

Hom : Obj → Obj → ○ □.

id : (x : Obj) → Hom x x

$- \circ -$: (x y z : Obj) → Hom y z → Hom x y → Hom x z

limit : (x y : Obj) → ~~id~~ (φ : Hom x y) → $id_y \circ \varphi = \varphi$.

Obj : Set

Hom : ~~Obj → Obj → Obj(V)~~

Obj → Obj → Obj (□)

id : $I_\square \to \prod_{x \in Obj} \text{Hom}(x,x)$ $\quad \left(\coprod_{x \in Obj} I_\square\right) \to \text{Hom}$

$- \circ -$: (x,y,z : Obj) → Hom x y → Hom y z → Hom x z

$$\begin{tikzcd}
\text{Hom} \times_{Obj} \text{Hom} \arrow[dr, dashed] \arrow[dd] & & \text{Hom} \arrow[dd] \\
 & \text{Hom} \arrow[dl] & \\
Obj \times Obj \times Obj & & Obj \times Obj
\end{tikzcd}$$

$- \circ -$: $\prod_{xyz} \text{Hom}(y,z) \otimes \text{Hom}(x,y) \to_\square \text{Hom}(x,z)$

limit : ∀ x y ○

$$\begin{tikzcd}
\text{Hom } x\,y \arrow[r, "(id_y, id_{xy})"] \arrow[dr] & \text{Hom } y\,y \otimes \text{Hom } x\,y \arrow[d, "\circ"] \\
 & \text{Hom } x\,y
\end{tikzcd}$$