Bridging the Gap Between Value and Policy Based Reinforcement Learning

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Topic: Q-Value Based RL

Presenter: Michael Pham-Hung

Motivation

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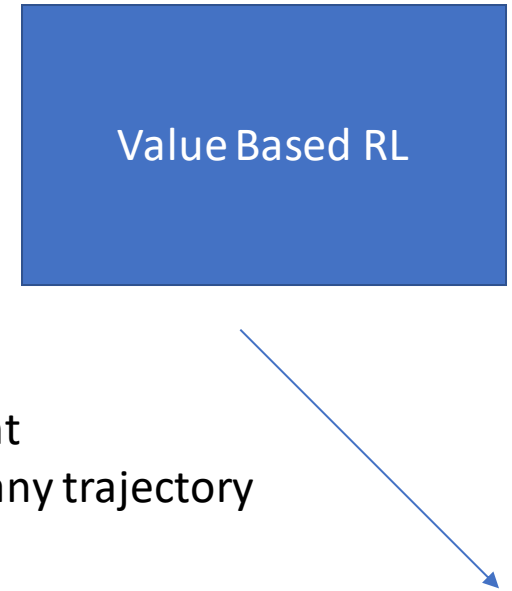
i.e. Q-Learning



+ Data efficient
+ Learn from any trajectory

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i.e. REINFORCE

+ Stable deep function approximators

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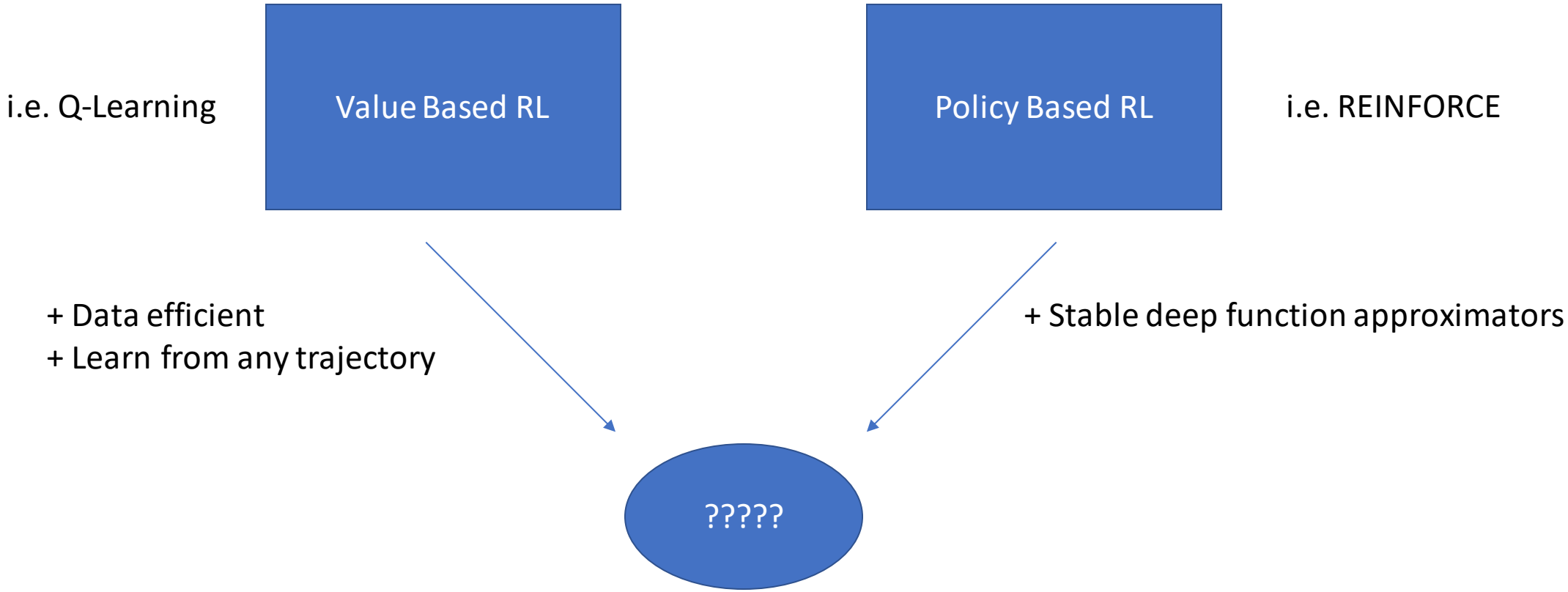
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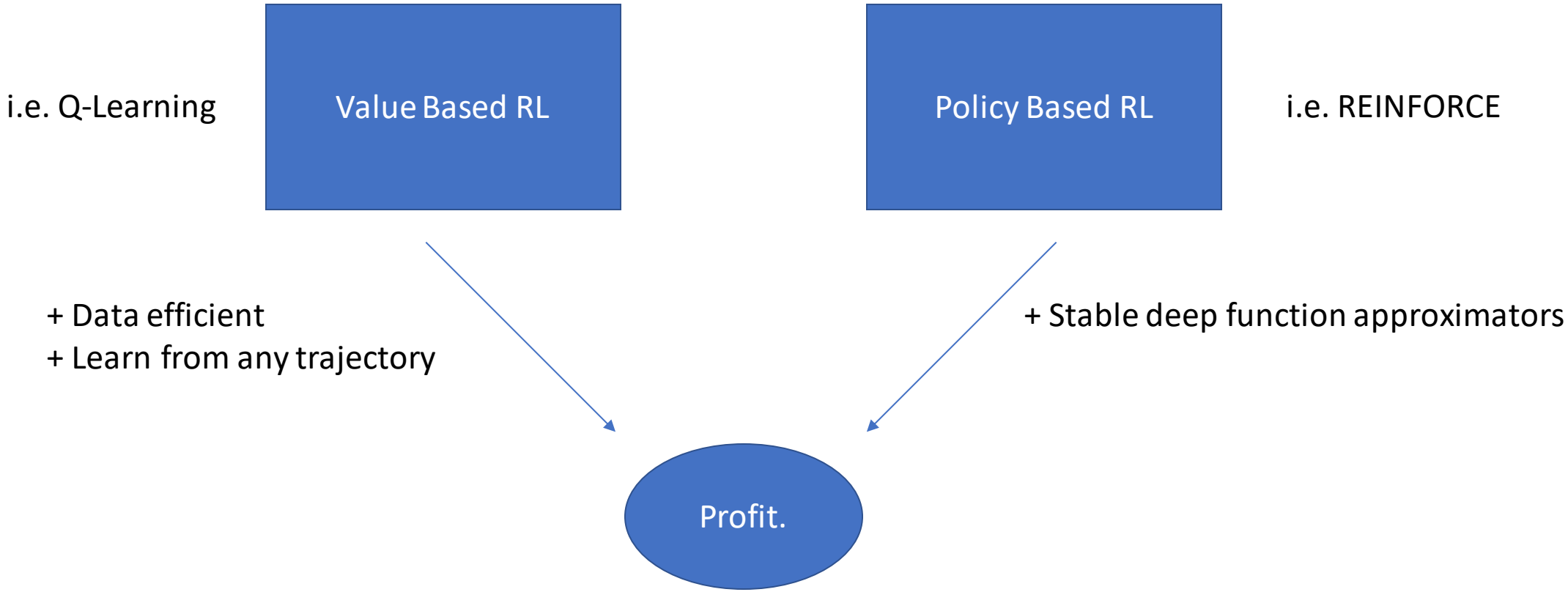
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Profit.

Contributions

Problem: Combining the advantages of on-policy and off-policy learning.

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Limitations of prior work:

- Prior work remain potentially unstable and are not generalizable.

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Limitations of prior work:

- Prior work remain potentially unstable and are not generalizable.

Key Insight: Starting from a theoretical approach rather than naïve approaches can be more fruitful.

Revealed: Results in a quite flexible algorithm.

Outline

Background

- Q-Learning Formulation
- Softmax Temporal Consistency
- Consistency between optimal value and policy

PCL Algorithm

- Basic PCL
- Unified PCL

Results

Limitations

Q-Learning Formulation

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One-hot distribution

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Hard-max Bellman temporal consistency!

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Hard-max Bellman temporal consistency!

Or in terms of optimal action values

$$Q^{\circ}(s,a) = r(s,a) + \max_{a'} Q^{\circ}(s',a')$$

Soft-max Temporal Consistency

- Augment the standard expected reward objective with a discounted entropy regularizer
    - This helps encourages exploration and helps prevent early convergence to sub-optimal policies

$$O_{ENT}(s,\pi) = O_{ER(s,\pi)} + \tau\mathbb{H}(s,\pi)$$

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$$O_{ENT}(s,\pi) = \sum_a \pi(a|s)[r(s,a) - \tau\log\pi(a|s) + \gamma O_{ENT}(s',\pi)]$$

$$\pi^*(a|s) \propto \exp\left\{\frac{r(s,a) + \gamma V^*(s')}{\tau}\right\}$$

Form of a Boltzmann distribution … No longer one hot distribution! Entropy term prefers the use of policies with more uncertainty.

Subbing $\pi^*(a|s)$ into O_{ENT} yields:

$$V^*(s) = O_{ENT}(s, \pi^*) = \tau \log \sum_a \exp\left\{\frac{r(s,a) + \gamma V^*(s')}{\tau}\right\}$$

And

$$Q^*(s,a) = r(s,a) + \gamma V^*(s') = r(s,a) + \gamma\tau \log \sum_{a'} \exp\left(\frac{Q^*(s',a')}{\tau}\right)$$

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Note the log-sum-exp form!

Consistency Between Optimal Value & Policy

- Let $\pi^*(a|s) = \dfrac{\exp\left\{\left(r(s,a)+\gamma V^*(s')\right)/\tau\right\}}{\exp\{V^*(s)/\tau\}}$

Normalization Factor

Consistency Between Optimal Value & Policy

- Let $\pi^*(a|s) = \frac{\exp\{(r(s,a)+\gamma V^*(s'))/\tau\}}{\exp\{V^*(s)/\tau\}}$

$$\log(\pi^*(a|s)) = (r(s,a) + \gamma V^*(s'))/\tau \; - V^*(s)/\tau$$

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Valid for any action a

Consistency Between Optimal Value & Policy

Note: The optimal policy can also be characterized in terms of Q^*:

$$\pi^*(a|s) = \exp\{\frac{Q^*(s,a) - V^*(s)}{\tau}\}$$

Theorem 1: For $\tau > 0$, the policy π^* that maximizes O_{ENT} and state values $V^*(s)$ $= \max_{\pi} O_{ENT}(s, \pi)$ satisfy the following temporal consistency property for any state s and action a (where $s' = f(s, a)$):

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$$V^*(s) - \gamma^{t-1} V^*(s') = \sum_{i=1}^{t-1} \gamma^{i-1} [r(s_i, a_i) - \tau \log \pi^*(a_i|s_i)]$$

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Algorithm - Path Consistency Learning (PCL)

PCL attempts to minimize the squared soft consistency error over a set of sub-trajectories E.

Define a notion of soft consistency for d-length sub-trajectory $s_{i:i+d} \equiv (s_i, a_i, \dots, s_{i+d-1}, a_{i+d-1}, s_{i+d})$. From corollary 2.

$$C(s_{i:i+d}, \theta, \phi) = V_\phi(s_i) - \gamma^d V(s_{i+d}) + \sum_{j=o}^{d-1} \gamma^j [r(s_{i+j}, a_{i+j}) - \tau \log \pi_\theta(a_{i+j}|s_{i+j})]$$

Goal is to find V_ϕ and π_θ that gets $C(s_{i:i+d}, \theta, \phi)$ close to zero

Define squared soft consistency error:

$$O_{PCL} = \sum_{s_{i:i+d} \in E} \frac{1}{2} C(s_{i:i+d}, \theta, \phi)^2$$

We then get updates for θ and ϕ by taking the gradient:

$$\Delta\theta = \eta_\pi C(s_{i:i+d}, \theta, \phi) \sum_{j=0}^{d-1} \gamma^j \nabla_\theta \log \pi_\theta \left(a_{i+j} \middle| s_{i+j}\right)$$

$$\Delta\phi = \eta_v C(s_{i:i+d}, \theta, \phi) \left[\nabla_\phi V_\phi(s_i) - \gamma^d \nabla_\phi V_\phi(s_{i+d})\right]$$

Where n_π and n_v are the learning rates for policy and value.

Algorithm - Path Consistency Learning (PCL)

Given that the consistency property must hold on **any** path, the PCL algorithm can apply the updates both to trajectories sampled on-policy from $\boldsymbol{\pi_\theta}$ as well as trajectories sampled from a **replay buffer**

Algorithm 1 Path Consistency Learning

Input: Environment ENV, learning rates η_π, η_v, discount factor γ, rollout d, number of steps N, replay buffer capacity B, prioritized replay hyperparameter α.

function Gradients($s_{0:T}$)

// We use $G(s_{t:t+d}, \pi_\theta)$ to denote a discounted sum of log-probabilities from s_t to s_{t+d}.

Compute $\Delta\theta = \sum_{t=0}^{T-d} C_{\theta,\phi}(s_{t:t+d}) \nabla_\theta G(s_{t:t+d}, \pi_\theta)$.

Compute $\Delta\phi = \sum_{t=0}^{T-d} C_{\theta,\phi}(s_{t:t+d}) \left(\nabla_\phi V_\phi(s_t) - \gamma^d \nabla_\phi V_\phi(s_{t+d})\right)$.

Return $\Delta\theta, \Delta\phi$

end function

Initialize θ, ϕ.

Initialize empty replay buffer $RB(\alpha)$.

for $i = 0$ **to** $N - 1$ **do**

Sample $s_{0:T} \sim \pi_\theta(s_{0:})$ on ENV.

$\Delta\theta, \Delta\phi =$ Gradients$(s_{0:T})$.

Update $\theta \leftarrow \theta + \eta_\pi \Delta\theta$.

Update $\phi \leftarrow \phi + \eta_V \Delta\phi$.

Input $s_{0:T}$ into RB with priority $R^1(s_{0:T})$.

If $|RB| > B$, remove episodes uniformly at random.

Sample $s_{0:T}$ from RB.

$\Delta\theta, \Delta\phi =$ Gradients$(s_{0:T})$.

Update $\theta \leftarrow \theta + \eta_\pi \Delta\theta$.

Update $\phi \leftarrow \phi + \eta_v \Delta\phi$.

end for

Algorithm - Unified PCL

Recall:

$$Q^*(s,a) = r(s,a) + \gamma V^*(s') = r(s,a) + \gamma\tau \log \sum_{a'} \exp\left(\frac{Q^*(s',a')}{\tau}\right)$$

$$\Rightarrow\ V_\rho(s) = \tau \log \sum_{a} \exp\left\{\frac{Q_\rho(s,a)}{\tau}\right\}$$

And,

$$\pi_\rho(a|s) = \exp\left\{\frac{\left(Q_\rho(s,a) - V_\rho(s)\right)}{\tau}\right\}$$

Algorithm - Unified PCL

Then the new update rule for ρ is:

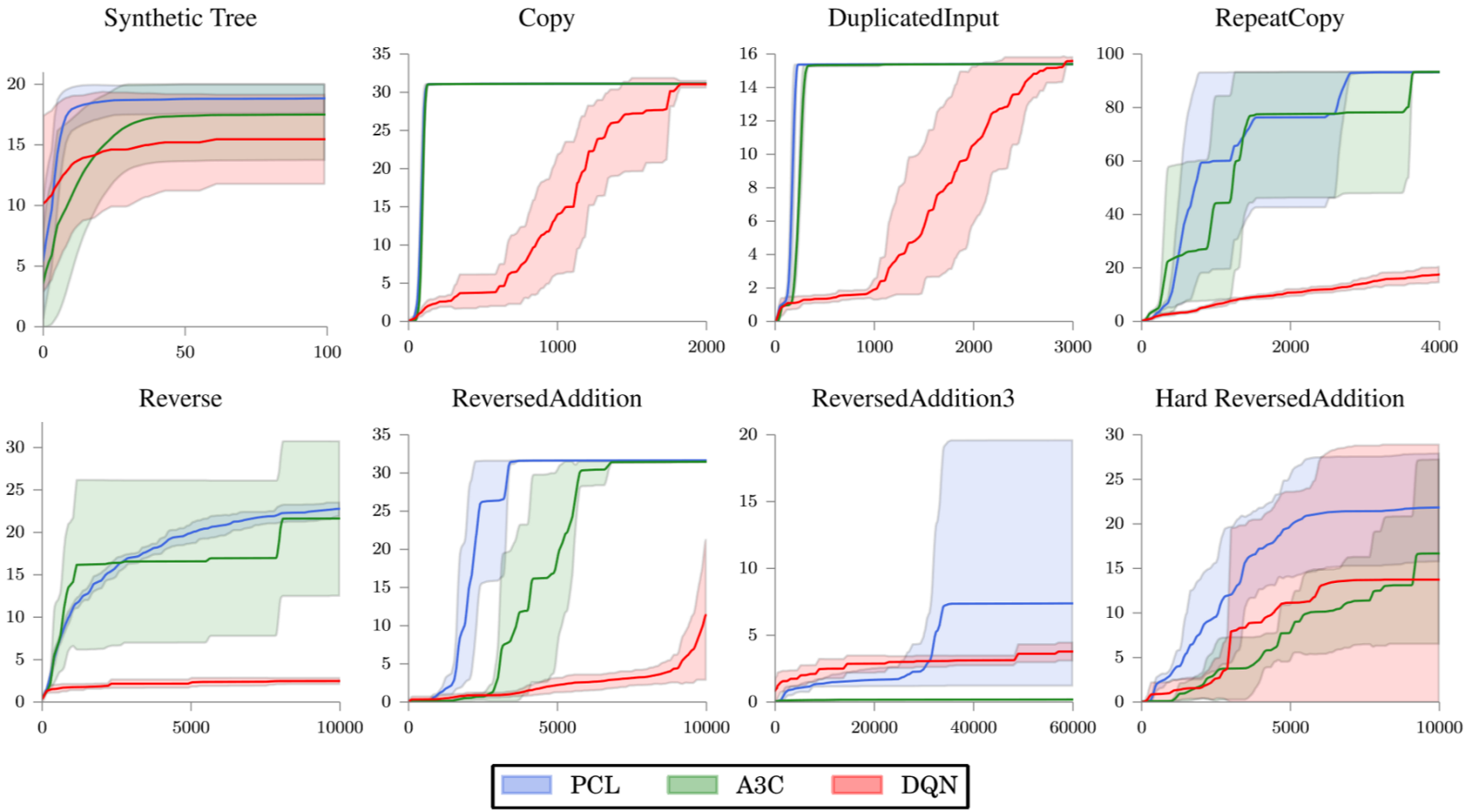
$$\Delta\rho = \eta_\pi C(s_{i:i+d}, \rho) \sum_{j=0}^{d-1} \gamma^j \nabla_\rho \log \pi_\rho\left(a_{i+j} \middle| s_{i+j}\right)$$
$$+ \eta_v C(s_{i:i+d}, \rho) \left(\nabla_\rho V_\rho(s_i) - \gamma^d \nabla_\rho V_\rho(s_{i+d})\right)$$

Merging the policy and value function models in this way is significant because it presents a new actor-critic paradigm where the policy (actor) is not distinct from the values (critic)

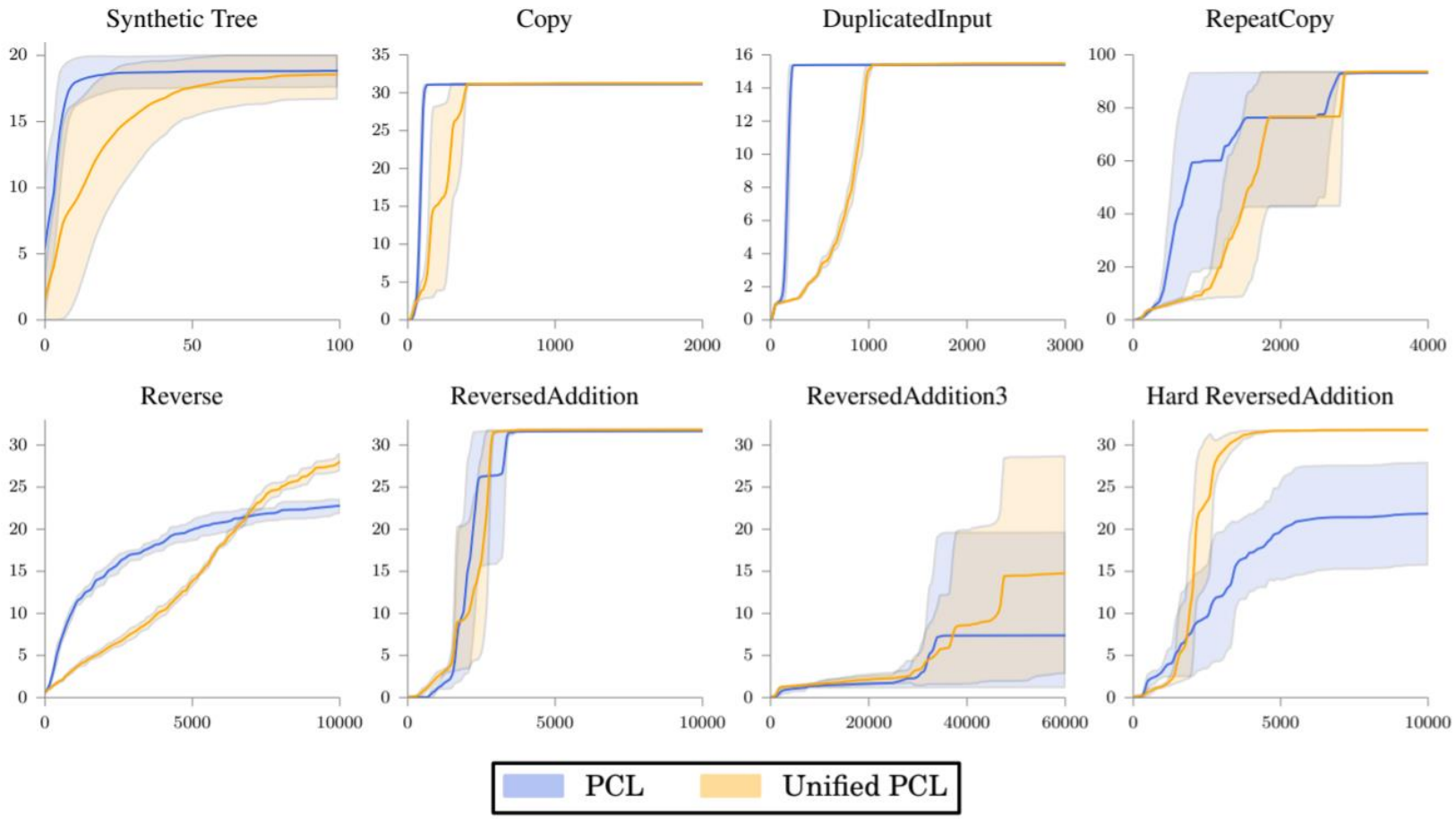
Experimental Results

PCL can consistently match or beat the performance of A3C and double Q-learning.

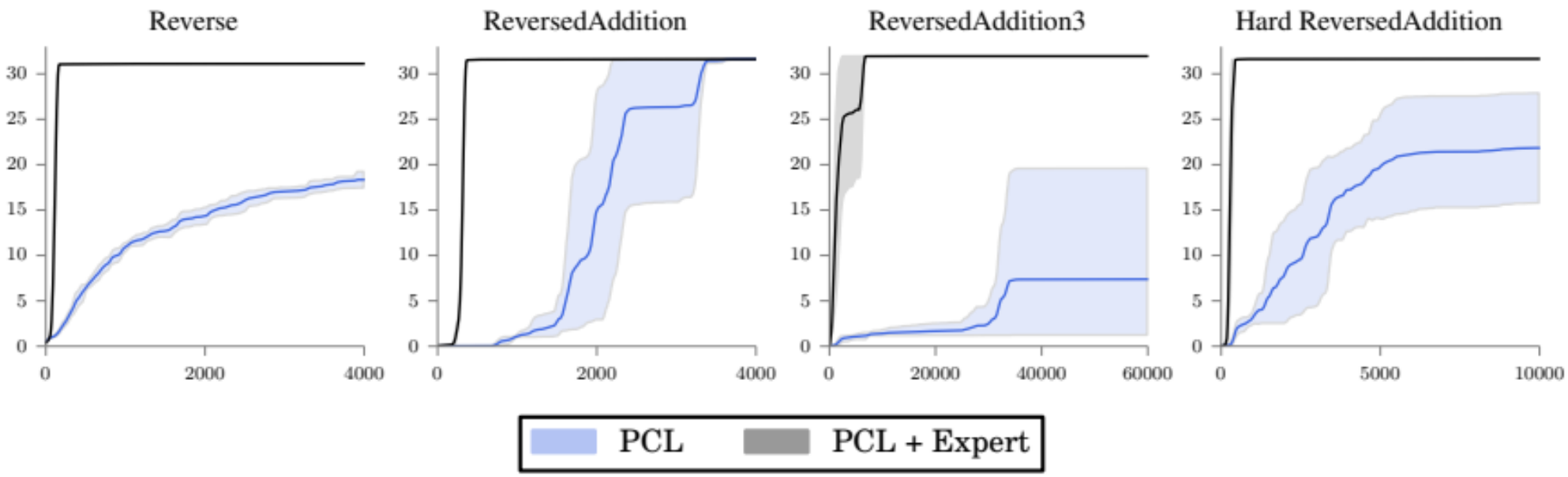
PCL and Unified PCL are easily implementable with expert trajectories. Expert trajectories can be prioritized in the replay buffer as well.



The results of PCL against A3C and DQN baselines. Each plot shows average reward across 5 random training runs (10 for Synthetic Tree) after choosing best hyperparameters. A signal standard deviation bar clipped at the min and max. The x-axis is number of training iterations. PCL exhibits comparable performance to A3C in some tasks, but clearly outperforms A3C on the more challenging tasks. Across all tasks, the performance of DQN is worse than PCL.



The results of PCL vs. Unified PCL. Overall found that using a single model for both values and policy is not detrimental to training. Although in some of the simpler tasks PCL has an edge over Unified PCL, on the more difficult tasks, Unified PCL performs better.



The results of PCL vs. PCL augmented with a small number of expert trajectories on the hardest algorithmic tasks. We find that incorporating expert trajectories greatly improves performance.

Discussion of results

Using a single model for both values and policy is not detrimental to training

The ability for PCL to incorporate expert trajectories without requiring adjustment or correction is a desirable property in real-world applications

Critique / Limitations / Open Issues

- Only implemented on simple tasks
  - Addressed with Trust-PCL, which enables a continuous action space.

- Soft Consistency error seems computationally expensive.

Contributions

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Limitations of prior work:

- Prior work remain potentially unstable and are not generalizable.

Key Insight: Starting from a theoretical approach rather than naïve approaches can be more fruitful.

Revealed: Results in a quite flexible algorithm.

Exercise Questions

1. Why is the distribution for the optimal policy not a one hot distribution?

2. Derive the Softmax consistency from $\pi^* = \frac{\exp\left\{\left(r(s,a)+\gamma V^*(s')\right)/\tau\right\}}{\exp\{V^*(s)/\tau\}}$