SCMA329 Practical Mathematical Financial Modeling

Pairote Satiracoo

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Contents

1 Dates and date functions

Dates and date functions will be discussed in this section. Depending on the system setting, the dates used in this section corresponds to the U.S. setting, i.e. month/day/year. Thus, 7/1/2017 corresponds to 1 July 2017.

The following date formats are recognized by Excel:

- 7/1/2017, 7/1/17
- 7-1-2017, 7-1-17,
- 7-1/2017, 7-1/17,
- July 1, 2017
- 1-Jul-2017

if the date input is without the year, then it is the date on the current year, i.e. 7-1, 7/1, July 1, Jul 1, corresponds to 1 July 2018.

1.1 Inconsistent date entries

Example 1.1. *What will be the output when we enter dates by using the following two digits for the years:*

1. *7/1/01*
2. *7/1/60*

Excel interprets two digit years between 00 and 29 as 21st century dates, and two digit years between 30 and 99 as 20th century dates.

1.2 Date serial number

Excel stores dates as sequential serial numbers so that they can be used in calculations. For example,

- 1 January 1900 is serial number 1,
- 1 January 2018 is serial number 43101 because it is 43101 days after 1 January 1900.

1.3 Date functions

The list of date functions can be seen by choosing

Formulas → **Function Library** → **Function Library** → **Date & Time**.

Function	**Description**
DATE	Returns the serial number of a particular date
DATEVALUE	Converts a date in the form of text to a serial number
DAY	Converts a serial number to a day of the month
DAYS	Returns the number of days between two dates
WEEKDAY	Converts a serial number to a day of the week
NETWORKDAYS	Returns the difference between two dates, excluding weekend days (Saturdays and Sundays)
WORKDAY	Returns a number that represents a date that is the indicated number of working days before or after
EDATE	Returns the serial number that represents the date that is the indicated number of months before or a
EOMONTH	Returns the serial number for the last day of the month that is the indicated number of months before
YEARFRAC	Returns a decimal value that represents fractional years between two dates

For a complete list, see the date and time functions available online. More detailed examples are also given in the Excel lab.

2 Working with Texts

In this section, we will see how Excel handles text strings, and how we use text functions to modify and manipulate text strings. First, we give a note regarding texts in Excel.

- A single cell can hold up to 32,000 characters. In case you need to display a lot of text in a worksheet, then use a text box (Choose Insert ⇒ Text ⇒ Text Box). It will be easier to edit texts in the text box than in cells.
- Sometimes, when you download numerical data from the internet or database, the imported values are treated as text, i.e. when you do a calculation with such data, you will get a #VALUE error. When a number is not treated as a number, there will be an error indicator. By clinking to expand a list of options, you can then convert it to the number.
- Another issue that you may encounter is about currency that uses different characters to separate thousands or decimals. https://en.wikipedia.org/wiki/Decimal_separator

2.1 Text functions

The following Excel functions can be used to modify text strings in the format you need. Alternatively, one may extract data by using the Convert Text To Columns Wizard (choose Data ⇒ Text To Columns).

1. RIGHT(text,[n]) returns the last n characters in a text string.
2. LEFT(text, [n]) returns the first n characters in a text string.
3. MID(text, start_num, num_chars) returns num_char characters from a text string, starting at start_num.
4. TRIM(text) removes all spaces from text except for single spaces between words. Use TRIM when text strings have irregular spacing.
5. LEN(text) returns the number of characters in a text string.
6. FIND(find_text, within_text, [start_num]) return the location at or after character start_num of the first character of find_text in within_text.
7. SEARCH(find_text, within_text, [start_num]) has the same syntax as FIND, but it is not case sensitive.
8. SUBSTITUTE(text, old_text, new_text, [instance_num]) is used to replace new_text for old_text in a text string. Here Instance_num is optional. It specifies which occurrence of old_text you want to replace with new_text. If you specify instance_num, only that instance of old_text is replaced. Otherwise, every occurrence of old_text in text is changed to new_text.

2.2 Character codes

Excel uses the standard ASCII character set. Therefore, each character has its own code. For example, to get the code number of “A”, simply type

=CODE(“A”), which returns the code number 65.

The CHAR function reverses the role of CODE function, i.e.

=CHAR(65), which returns the letter A. The input for the CHAR function should be a value between 1 and 255.

For the complete list of ASCII codes, please visit https://theasciicode.com.ar

Example 2.1. *Create an Excel file to list all the first 255 ASCII codes. It is a good idea to compare the outputs with those from the website above.*

2.3 Determining whether two string are identical

In order to determine whether strings in cell A1 and A2 have the same contents, we use = A1 = A2, which returns either TRUE or FALSE. Note that the comparison is not case-sensitive.

Alternative, the function that provides an exact, case-sensitive comparison is EXACT function.

2.4 Joining two or more cells

To join two or more cells, Excel uses an ampersand &. For example if the string "the effective interest rate per annum" is in A1 and the value of 5% (formatted value) is in B1. Then use the formula

= A1 & " is " & B1, which returns the effective interest rate per annum is 0.05.

A better solution is to use the TEXT function to format the value as text as follows.

= A1 & " is " & TEXT(B1," 0.00%") , which returns the effective interest rate per annum is 0.05.

Use **Home Format Format cells** to obtain the list of various text formats.

For example, if A1 contains the principle of $10,000, then

= "The principle is" & TEXT(A1, " $#,##0"), which returns The principle is $10,000.

The following table gives examples that are format with TEXT function.

```
=TEXT(1234.567,"$#,##0.00")
Currency with a thousands separator and 2 decimals, like $1,234.57. Note that Excel rounds the value to

=TEXT(TODAY(),"MM/DD/YY")
Today's date in MM/DD/YY format, like 03/14/12

=TEXT(TODAY(),"DDDD")
Today's day of the week, like Monday

=TEXT(NOW(),"H:MM AM/PM")
Current time, like 1:29 PM

=TEXT(0.285,"0.0%")
Percentage, like 28.5%

=TEXT(4.34 ,"# ?/?")
Fraction, like 4 1/3

=TRIM(TEXT(0.34,"# ?/?"))
Fraction, like 1/3. Note this uses the TRIM function to remove the leading space with a decimal value.

=TEXT(12200000,"0.00E+00")
Scientific notation, like 1.22E+07
```

```
=TEXT(1234567898,"[<=9999999]###-####;(###) ###-####")
Special (Phone number), like (123) 456-7898

=TEXT(1234,"0000000")
Add leading zeros (0), like 0001234
```

The TEXT function will convert numbers to text. It is best practice to keep your original values in one cell, and formatted number in another cell. When you do calculation, you should refer to the cells containing original values.

3 Interest Rate, Present Values and Cashflows

3.1 Working with a single cashflow in Excel

3.1.1 How to calculate the future value of a single cashflow

Suppose an amount C is deposited in an account that pays a fixed interest at the rate of $i\%$ per time units. Then after t time units, the deposit will have accumulated to

$$C(1+i)^t.$$

In Excel, the function FV calculates the future value of a single investment (and also periodic constant payments) and a constant interest rate.

The syntax of the function is:

$$\text{FV(rate, nper, pmt, [pv], [type])},$$

where

- rate is the interest rate per period
- nper is the number of periods over which the investment is made.
- pmt (used for an annuity type) is the payment made each period and cannot change over the life of the annuity.
- pv (optional) is an additional cash flow now (time 0)
- fv is an additional cash flow nper periods from now.
- type (optional) is an optional argument that defines whether the payment is made at the start or the end of the period:

  0 - the payment is made at the end of the period;

  1 - the payment is made at the start of the period.

  If the [type] argument is omitted, it takes on the default value of 0.

The following timeline illustrates the cashflows used for the FV function (assuming that the payments (pmt) are made in arrears).

Note For a single cash flow, we set pmt argument to be 0, as there are no ongoing payments after the initial investment.

Example 3.1. *You plan to invest today with an interest rate of 3% per year effective. How much money will you accumulate at the end of 2 years?*

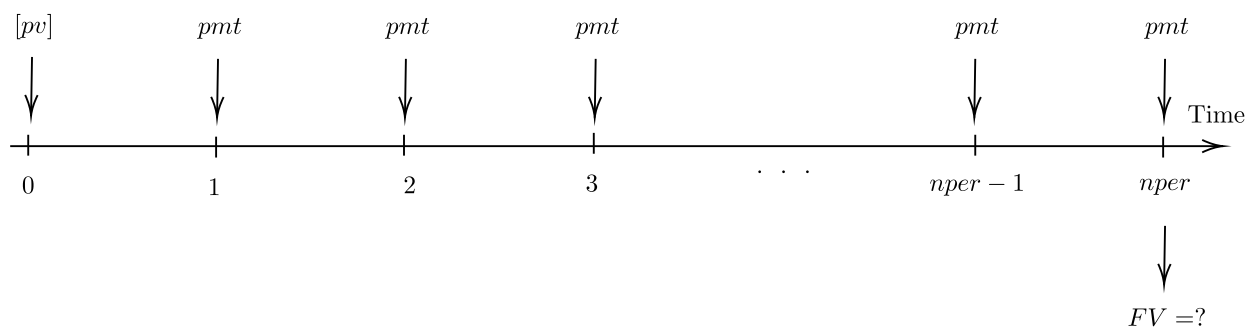


Figure 1: Timeline of cashflows for FV function

Solution: The accumulation of today in 2 years can be calculated by Excel as

$$\text{FV}(3\%, 2, 0, -100).$$

3.1.2 How to calculate the present value of a single cashflow

Similarly, the present value of a future cashflow C required at time t time units at a fixed interest of $i\%$ per time units can be calculated as

$$\frac{C}{(1+i)^t}.$$

In Excel, the function PV calculates the present value of a single investment (and also periodic constant payments) and a constant interest rate. The syntax of the function is:

$$\text{PV(rate, nper, pmt, [fv], [type])}$$

where fv is an additional cash flow nper periods from now.

Example 3.2. *How much should you deposit into the account with an interest of 8% so that 10 years from now its value would be ?*

Solution: The present value of today in 10 years can be calculated by Excel as

$$\text{PV}(8\%, 10, 0, -1000).$$

3.2 Present values of a series of cashflows

Consider a series of cashflows defined by

1. the times of payments (cashflows), denoted by $t_1, t_2, \ldots$, and
2. the amount of payments, denoted by C_r (in short for C_{t_r}), which will be paid at time t_r, for $r = 1, 2, \ldots$. The amounts can be positive or negative

The present value at any time t of this series of cashflow is

$$PV(t) = \sum_{r=1}^{\infty} C_r (1+i)^{t-t_r} = \sum_{r=1}^{\infty} C_r v^{t_r - t}$$

where i is the effective rate of interest and $v = 1/(1+i)$.

Notes 1. At a fixed effective rate of interest, the original series of cashflows is equivalent to a single payment of amount $PV(t)$ at time t.

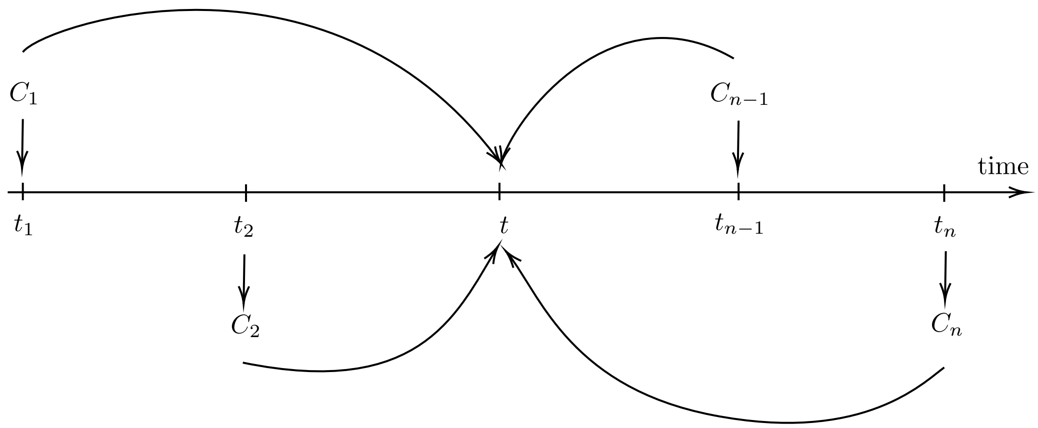


Figure 2: Timeline of a series of cashflows

2. If two different series of cashflows have the same PV at one time at a given effective rate of interest, then they have the same PV at any time at that effective rate of interest.

3.2.1 Level Annuities certain

An *annuity* is a regular series of payments (cashflows). When the payments are certain which are payable for a definite period of time, we call it an *annuity certain.*

- If the payments are made at the end of each time period, they are paid *in arrear.*
- Otherwise, payments are made at the beginning of each time period, they are pain *in advance.*
- An annuity paid in advance is also known as an *annuity due*
- If each payment is for the same amount, this is a *level* annuity.

Example 3.3. *Let i be the constant effective rate of interest per time unit. In Excel, one can calculate the accumulated value of a level annuity certain having cashflow of pmt unit at the end of each of the next n time units by*

$$FV(i\%, n, pmt, 0).$$

The cashflows of this annuity is shown in the timeline below.

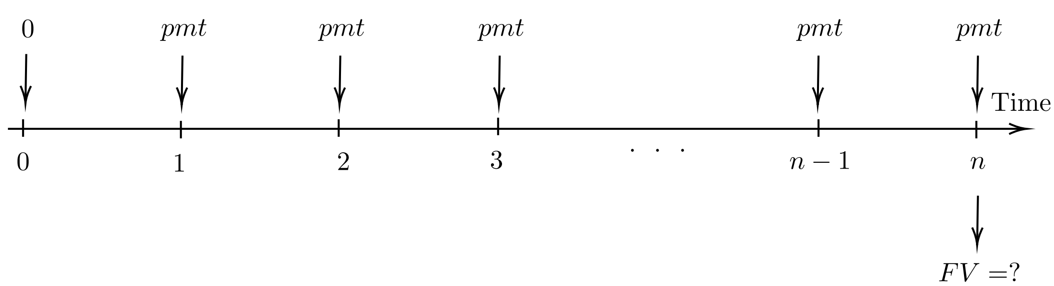


Figure 3: Level Annuity Certain

Note *The last argument of the above syntax, [pv] (optional) which is an additional cash flow now (at time 0), has been set to 0.*

Example 3.4. *Let i be the constant effective rate of interest per time unit. In Excel, one can calculate the present value at time 0 of a level annuity certain having cashflow of pmt unit at the end of each of the next n time units by*

$$PV(i\%, n, pmt, 0).$$

The timeline of these cashflows is shown in the figure below:

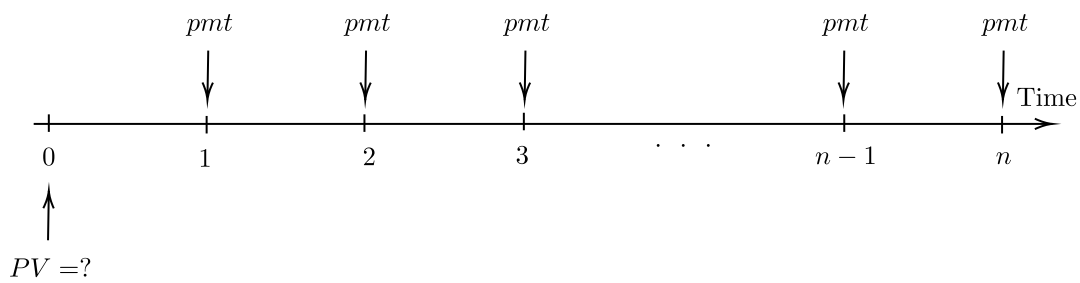


Figure 4: Level Annuity Certain

Note The last argument of the above syntax, [fv] (optional) which is an additional cash flow at time n, has been set to 0.

Example 3.5. *Given the effective rate of interest of* 8% *p.a., use Excel to calculate*

1. *the accumulation at 12 years of payable yearly in arrear for the next 12 years.*
2. *the present value now of ,000 payable yearly in arrear for the next 6 years.*
3. *the present value now of ,000 payable half-yearly in arrear for the next 12.5 years.*

Example 3.6. *Let $i = 4\%$ effective per time unit. Cashflows are given as follows:*

- $C_1 = 200$ *at time* $t_1 = 1$.
- $C_2 = 300$ *at time* $t_2 = 3$.
- $C_3 = -100$ *at time* $t_3 = 5$.
- $C_4 = -50$ *at time* $t_4 = 6$.

Develop the model using Excel to calculate

1. *the accumulation at time* $t = 7$.
2. *the present value at time* $t = 0$.
3. *the present value at time* $t = 4$.

4 Lookup Formulas and Data Tables

4.1 Lookup function

Lookup function can be used to return (or retrieve) a value from a table or a range of data. For example, use lookup function to find some information such as date of birth, address etc. from a customer database. Two useful lookup functions are `VLOOKUP` and `HLOOKUP`.

- The `VLOOKUP` function searches a value in the first column and returns a value in the same row from a column you specify in the table or array.

- The `HLOOKUP` function searches a value in the top row and returns a value in the same column from a row you specify in the table or array.

The syntaxes for the `LOOKUP` functions are

`=VLOOKUP(lookup value, range containing the lookup value, the column number in the range containing the return value, [range_lookup])`.

`=VLOOKUP(lookup_value, table_array, col_index_num, [range_lookup])`.

`=HLOOKUP(lookup_value, table_array, row_index_num, [range_lookup])`.

The `[range_lookup]` is an optional which can be specified as `TRUE` for approximate match or `FALSE` for an exact match.

Notes

1. The EXCEL `LOOKUP` functions treat empty cells in the result range as **zeros**.
2. If the range_lookup is `FALSE` and an exact match is not found, the function returns #N/A.
3. If the range_lookup is `TRUE` or omitted, the first column of the `VLOOKUP` table must be in ascending order. If lookup_value is smaller than the the smallest value in the first column of the table_array, then the function returns #N/A.
4. Use `IFERROR` function to avoid the return #N/A. Use the following syntax

   `=IFERROR(value, value_if_error)`

   (for example, `=IFERROR(VLOOKUP(...),"Value is not availble"`)
5. The wildcard character * and ? can be used when the range_lookup argument is text.
6. The wildcard character ? refers to any single character, for example "analy?e" finds "analyse" and "analyze".
7. The wildcard character * refers to any number of characters, for example "bio*" finds "biology" and "biotechnology".

Example 4.1. Import SET Index historical daily data starting from 1 January 2020 up to now from https://www.investing.com/indices/thailand-set-historical-data. Use `LOOKUP` function to create the table showing

1. the closing price on the first working day of each month (not including holidays).
2. the closing price on the last working day of each month (not including holidays).

Hint: use

`=WORKDAY(DATE(YEAR(A1),MONTH(A1),1)-1,1)` to return the first working day of a month and

`=WORKDAY(DATE(YEAR(A1),MONTH(A1)+1,1),-1)` to return the last working day of a month.

Example 4.2. Use the SET Index historical data above. On your selected cells, find the lowest and highest closing prices and the dates when those occurred (sorting data is not allowed).

4.2 MATCH and INDEX functions

The `MATCH` and `INDEX` functions are often used together to perform lookups.

The `MATCH` function returns the relative position of a specified item in a range of cells. For example, if the range A1:A3 contains the values 2, 4 and 6, then the formula

`=MATCH(4,A1:A3,0)` returns the number 2, i.e. the second item in the range.

The syntax for MATCH is

```
=MATCH(lookup_value, lookup_array, [match_type])
```

The optional argument is match_type consisting of 3 values, -1, 0 and 1, that specify how the match is determined.

- If match_type = 1 or omitted, then `MATCH` finds the largest value less than or equal to lookup_value where lookup_array must be in **ascending order**.
- If match_type = 0, then `MATCH` finds the **first value exactly equal to** lookup_value.
- If match_type = -1, then MATCH finds the smallest value greater than or equal to lookup_value where lookup_array must be in **descending order**.

The `INDEX` function returns a value from within a table or range. The syntax for INDEX is

```
=INDEX(array, row_num, [column_num])
```

If array contains only one row or column, the corresponding row_num or column_num argument is optional.

4.3 Data tables

Data tables allows you to create a table of values from

- a formula as one or two variables from the formula are varied, and
- more formulas for various values of a single input cell.

As a result, you can examine a range of possible values when one or two variables are systematically changed. For example, a data table can be used to display the amount your investment will grow to for various numbers of compounding.

Example 4.3. An amount of P is invested for n years at a nominal interest rate of i%. Create a worksheet to calculate the effective annual interest rate you will earn and the accumulation at time n years for various numbers of compounding periods per year including

1. annual compounding
2. semi-annual compounding
3. quarterly compounding
4. monthly compounding
5. bi-weekly compounding
6. weekly compounding
7. daily compounding

Model formulation

1. **Set up input variables:** Referring to the corresponding excel file, create the labels for the input variables and enter the values in cells B4:B7.
2. **Calculate the output values:** The effective annual interest rate $i\%$ which is equivalent to the given nominal rate $i^{(p)}$ convertible p times a year can be calculated by

$$i = \left(1 + \frac{i^{(p)}}{p}\right)^p - 1.$$

Hence, the value of the investment at time n years is

$$FV = P(1+i)^n.$$

The output values are calculated and placed in cells B10 and B11, respectively.

3. **Set up data table (a one-input data table) :**
   1. Create the labels and values for different compounding frequencies.
   2. In cells C16 and D16, create the references to cells B10 and B11, which are the effective annual interest rate and future value of investment, respectively.

      The positions of the cells C16 and D16, the formulas used to calculate the required values, are **one row above and to the right** of the column of values. The data table is column-oriented because the variable values are in a column, B17:B23 in this example.
   3. Select the range of cells that contains the formulas and values that you want to substitute, i.e. the range B16:D23.
   4. Choose Data What-If Analysis Data table. Then type the cell reference for the input cell, cell B5 in this case, in the **Column input cell** because the data table is column-oriented. Click OK and Excel will complete the data table.

Notes

1. A new formula can be added to the existing one-variable data table by typing the new formula in a blank cell to the right of the existing formula(s) in the top row of the data table. Then repeat Step 3.4 as given above.
2. The contents of the data table are generated with a multicell array formula (details will be discussed in a subsequent section):

   `{=TABLE(,B5)}`.

Example 4.4. Suppose you borrow L from a bank to be repaid by the end of n years at an interest rate of $i\%$ per annum effective. If you agree to repay the loan and the interest in level instalments throughout term of loan.

1. Create a model to calculate the amount of annual instalments X for various values of number of instalments n and interest rate of $i\%$ per annum effective.
2. Set up a two-input data table to show annual instalments X for different values of n and i.
3. Comment on the results obtained.

5 Analysing data using Goal Seeking and Solver

5.1 Analysing data using Goal Seeking

Goal Seek feature allows a user to find the value that is an input for a formula (a single input cell) to produce a required result. For example, given a series of cashflows $C_1, C_2, \ldots C_n$ (positive and negative) payable at times $t_1, t_2, \ldots, t_n$, you can set up your worksheet to calculate the present value at time 0 defined by

$$PV_i(0) = \sum_{i=1}^{n} C_i \left(\frac{1}{1+i}\right)^{t_i}.$$

The solution i of the equation

$$PV_i(0) = 0,$$

which is the yield of the cashflows, can be directly obtained by using Goal Seek. Here the yield i is the value for a worksheet input that makes the value of the formula $PV_i(0)$ (or the present value function expressed in terms of i) match the goal, which is zero in this case.

Note Goal Seek works only with one variable input value. You can use the Solver add-in when more than one input value is needed; for example, both the loan amount and the monthly payment amount for a loan.

We first illustrate how to use Goal Seek to find the interest rate from a loan in the following example.

Example 5.1. *Suppose that you borrow 5,000 for a term of 3 years. The loan is to be repaid by 3 level annual repayments of 2,010.57 at the end of each year. Calculate the interest rate for this loan.*

Model formulation for Example 5.1

1. **Set up input variables** Create the labels for the input variables in cells A4:A6 for loan amount, term of the loan, and payment, respectively. Then enter the values 5,000, 3 and 2,010.57 in cells B4-B6, respectively. Create the label for the output variable (i.e. the interest rate i). You may enter the guessing value for the interest rate, say for e.g. $i = 1\%$ in the corresponding cell (i.e. the cell B9)
2. **Construct year-by-year cashflows** Use the guessing value of i to construct year-by-year cashflow to calculate the present value of the cashflows (in cell E12 in our example)
3. **Use Goal Seek to calculate the interest rate**. Choose

   Data → Data Tools → What-If Analysis → Goal Seek

   1. Then in the **Set cell** box, enter the reference for the cell that contains the formula (or the goal we want to match), i.e. cell E12.
   2. In the **To value** box, enter the result of the the formula, i.e. the value of the present value of the cashflows which is 0.
   3. In the **By changing cell** box, enter the reference for the cell that you want to adjust, i.e. cell B9.
4. **Goal Seek status box** then displays the target value and the value that excel calculated.

Note

1. In case that Goal Seek cannot find a solution but there is a solution to the problem.
   1. You should change the value used in the by changing cell box to a value that should be close to the solution.
   2. Increase the Maximum Iterations setting which allows Excel to try more possible solutions. To set the Maximum Iterations, go to

      File → Excel → Options→ Formula tab
2. In order to increase the accuracy of the (approximated) solution, set the option value for Maximum Change (located in **Excel Options Formula tab**) to a small value. The defaul value is 0.001 which gives only 3 decimals accuracy.

Example 5.2. *Consider a transaction from an investment that offers*

- *to pay an investor of amounts $B_1, B_2, \dots, B_n$ at time $t_1, t_2, \dots, t_n$*
- *in return for outlays (amounts to be paid by the investor) of amounts $A_1, A_2, \dots, A_n$ at these times, respectively.*

Only one of A_i and B_i will be non-zero in general. Make all the important variables of the problem input variables. Build the model to calculate the yield of the investment for the two examples in the lecture note.

5.2 Analysing data using Solver

Goal Seek has a limitation that can solve for only one adjustable cell. Solver extends this Goal Seek function and can do the following:

- work with multiple adjustable cells.
- specify constrains for the values of adjustable cells.
- find an optimum value for a formula.
- generate multiple solutions to a problem.

5.3 A simple Solver example

We will describe how to use Solver to solve same the problem as Goal Seek.

1. **Set up input variables** This is similar to Goal Seek section.
2. **Set up a formula for the output variable** This is similar to Goal Seek section.
3. **Use Goal Seek to calculate the interest rate**. Choose

   Data → **Analysis** → **Solver**

   1. Then in the **Set Objective** box, enter the reference for the cell that contains the formula, i.e. cell B6.
   2. In the **To** box, choose **Value of** box and then enter the result of the the formula.
   3. In the **By Changing Variable cells** box, enter the reference for the cell that you want to adjust, i.e. cell B5.

6 Analysing data with pivot tables

Excel provides a powerful tool namely **pivot table** to summarise, analyse, explore and present data.

- Users can simply create useful summary of data and rearrange the information in any form with a few mouse clicks.
- Pivot tables can create frequency distributions and cross-tabulations of several different data dimensions.
- The users can also create **pivot charts** based on pivot tables, which provide meaningful, graphical data representation.

The following examples illustrate how to create pivot tables in Excel. The dataset used in the examples contains data on sales from a superstore The information on this data includes

- Order and ship dates,
- Ship modes,
- Customer details,
- Product details,
- Sale and profit amounts.

The file can be downloaded from https://community.tableau.com/docs/DOC-1236.

Example 1 Suppose that the manager is interested in profits, broken down by region and sub-category. Instead of sorting the data and creating formulas to answer the question, you can summarise the data by creating a pivot table, which can be done in the following steps:

1. Select any cell or table containing the data.
2. From the **Insert** tab and click **PivotTable**.
3. Specify the data and location for the pivot table in the **PivotTable dialog box**.
4. A blank **PivotTable** and **PivotTable Fields List** will appear.
5. Drag the field names (or column headers) at the top to one of the four boxes at the bottom of the **PivotTable Fields List**.

   For this example,

   - **Drag the Profit field into the Values area**. At this point, the pivot table displays the total of all the values in the Profit column.
   - **Drag the Sub-Category field into the Row Labels area**. The pivot table shows the total profit for each of the sub categories.
   - **Drag the Region field into the Column Labels area**. The pivot table shows the total profit for each of the sub categories, cross-tabulated by region.
6. The pivot table will calculate and summarise the selected fields. The result is shown in the Table 1.

1. The pivot table automatically updates the table with every change you make in the PivotTable Field List.
2. You can sort data in a PivotTable. Just right-click any cell in the column to sort and choose Sort from the short-cut menu.
3. You can apply number formatting, for example changing the Number Format to Currency.
4. Pivot table data is usually summarised using a sum. You can also summarise your data using different summary techniques, such as Sum, Count, Average, etc.

Table 2: Table 1: The pivot table from Example 1.

Sum of Profit	Region				
Sub-Category	Central	East	South	West	Grand Total
Accessories	7251.63	11195.86	7004.54	16484.6	41936.64
Appliances	-2638.62	8391.41	4123.94	8261.27	18138.01
Art	1195.16	1899.94	1058.59	2374.1	6527.79
Binders	-1043.64	11267.93	3900.66	16096.8	30221.76
Bookcases	-1997.9	-1167.63	1339.49	-1646.51	-3472.56
Chairs	6592.72	9357.77	6612.09	4027.58	26590.17
Copiers	15608.84	17022.84	3658.91	19327.24	55617.82
Envelopes	1777.53	1812.41	1465.48	1908.76	6964.18
Fasteners	236.62	263.99	173.72	275.19	949.52
Furnishings	-3906.22	5881.41	3442.68	7641.27	13059.14
Labels	1073.08	1129.28	1040.77	2303.12	5546.25
Machines	-1486.07	6928.64	-1438.89	-618.93	3384.76
Paper	6971.9	9015.37	5947.06	12119.24	34053.57
Phones	12323.03	12314.69	10767.28	9110.74	44515.73
Storage	1969.84	8389.37	2274.3	8645.32	21278.83
Supplies	-661.89	-1155.14	1.88	626.05	-1189.1
Tables	-3559.65	-11025.38	-4623.06	1482.61	-17725.48

Sum of Profit	Region				
Grand Total	39706.36	91522.78	46749.43	108418.45	286397.02

6.1 Filters

Filters can be used to display part of your data that you need. For example, you can determine how various discount rates affect the profits.

1. Drag a field to the **Filters** section. In this example, we will use Discount field.
2. The **filter** will appear above the pivot table. Then use a drop-down list to filter the displayed data by one or more items.

The pivot table filltered by discount field is shown in Table 2.

Table 3: Table 2: The pivot table is filtered by discount.

Discount	0.5				
Sum of Profit	Region				
Sub-Category	Central	East	South	West	Grand Total
Bookcases		-4255.81			-4255.81
Machines			-7635.23		-7635.23
Tables	-4309.74			-4305.64	-8615.39
Grand Total	-4309.74	-4255.81	-7635.23	-4305.64	-20506.43

6.2 Modifying the pivot table

You can add further summary information by using the **PivotTable Fields List**. For example, you can add a second field (category) to the Row Label area. Here you also need to change the order in which the fields are listed, i.e. Category should be on the top level followed by sub-category. Table 3 shows the pivot table when two fields are added in row labels.

Table 4: Table 3: Two fields are used for row labels.

Sum of Profit		Region				
Category	Sub-Category	Central	East	South	West	Grand Total
Furniture	Bookcases	-1997.9	-1167.63	1339.49	-1646.51	-3472.56
	Chairs	6592.72	9357.77	6612.09	4027.58	26590.17
	Furnishings	-3906.22	5881.41	3442.68	7641.27	13059.14
	Tables	-3559.65	-11025.38	-4623.06	1482.61	-17725.48
Furniture Total		-2871.05	3046.17	6771.21	11504.95	18451.27
Office Supplies	Appliances	-2638.62	8391.41	4123.94	8261.27	18138.01
	Art	1195.16	1899.94	1058.59	2374.1	6527.79
	Binders	-1043.64	11267.93	3900.66	16096.8	30221.76
	Envelopes	1777.53	1812.41	1465.48	1908.76	6964.18
	Fasteners	236.62	263.99	173.72	275.19	949.52
	Labels	1073.08	1129.28	1040.77	2303.12	5546.25
	Paper	6971.9	9015.37	5947.06	12119.24	34053.57
	Storage	1969.84	8389.37	2274.3	8645.32	21278.83
	Supplies	-661.89	-1155.14	1.88	626.05	-1189.1
Office Supplies Total		8879.98	41014.58	19986.39	52609.85	122490.8
Technology	Accessories	7251.63	11195.86	7004.54	16484.6	41936.64

Sum of Profit		Region				
	Copiers	15608.84	17022.84	3658.91	19327.24	55617.82
	Machines	-1486.07	6928.64	-1438.89	-618.93	3384.76
	Phones	12323.03	12314.69	10767.28	9110.74	44515.73
Technology Total		33697.43	47462.04	19991.83	44303.65	145454.95
Grand Total		39706.36	91522.78	46749.43	108418.45	286397.02

Example 2 Create a pivot table to answer the following questions

1. What are the total sales obtained from the top 5 customer who have spent the most, broken down by sub-category? How many orders did each customer have for each product?
2. How does the sales in the Central region compared with the other regions combined?
3. In which region does the superstore obtain the highest profit for each year?
4. Create a frequency distribution of the profit, filtered by year.

6.3 Working with Nonnumeric Data

Pivot tables can be used for nonnumeric data. In this case, a useful pivot table can be created that counts the items rather than sums of them.

The following Pivot table cross-tabulates the segment for each combination of segment and region. Here, we specify each of the Pivot table fields as follows:

- The Segment field is used for the Columns.
- The Region field is used for the Rows.
- The Segment field is also used for the Values and is summarised by Count.
- A second instance of the Segment field is added to the Values section. To display percentages, in Field Setting, choose Show Values As Percent of Column Total.

width=1

Table 5: Table 4: The Pivot Table for Nonnu

	Segment			
	Consumer		Corporate	
	Count of Segment	Count of Segment by Percentage	Count of Segment	Count of Segment by Percentag
Central	1212	23.35%	673	22.28%
East	1469	28.30%	877	29.04%
South	838	16.14%	510	16.89%
West	1672	32.21%	960	31.79%
Grand Total	5191	100.00%	3020	100.00%

7 Bonds and Inflation

7.1 Zero-Coupon Bonds

The simplest type of bond is the one without coupons. Such bonds are referred to as **zero-coupon bonds**. More precisely, An n**-year zero-coupon bond** is a fixed interest security, redeemed at par with term n and no coupon payments.

Example 7.1. *What cash flows will an investor receive if the investor holds an n-year zero-coupon per 100 THB face value until maturity? (Here we let P_n denote the price now of the n-year zero-coupon)*

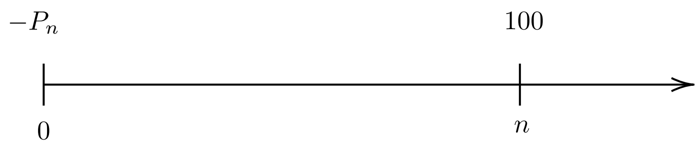


Figure 5: cashflows of a zero-coupon bond

To calculate the rate of return of buying a bond and holding it until maturity, it is simply the discount rate that equate the present value of the investment to its costs (or bond price). Recall that P_n denotes the price now of the n-year zero-coupon with FV face value and y_n is the **yield to maturity**. The yield to maturity satisfies the following equation

$$P_n = \frac{FV}{(1+y_n)^n}.$$

Hence,

$$y_n = \left(\frac{FV}{P_n}\right)^{1/n} - 1.$$

Example 7.2. *(Excel) Download the yields to maturity of zero-coupon bonds per THB 100 face value with the terms ranging from 0.08 to 47 years from the following website (as of 11/03/2020)*

http://www.thaibma.or.th/EN/Market/YieldCurve/Zero.aspx

1. *Using the zero-coupon rates provided from the given website, plot the interest rates (i.e. yield to maturity) as a function of their maturities at any point in time. Give a brief explanation regarding to the shape of the plot.*
2. *Determine the price now of the zero-coupon bond corresponding to the given yields to maturity.*

Notes 1. We will often refer to the yield to maturity of the appropriate maturity, zero-coupon risk-free bond as the **risk-free interest rate**.

2. The term **spot interest rate** is also used to refer to these risk-free interest rate.
3. All n-year risk-free investments must earn the same return as the risk-free interest rate.
4. The plot in Example 7.2 is known as the **yield curve**, which plots the risk-free interest rate for different maturities.
5. The yield curve is also called the **term structure of interest rates**.
6. Yield curves very from day to day, and could be increasing, or decreasing in shape, or even humped, depending on market expectations.

7.2 Bond pricing using yield to maturity

Recall that a tax-exempt investor receives the full amount of the coupon and redemption payments. The price of an n year fixed interest bond which pays coupons of D per annum payable p thly in arrears and has redemption amount R is

$$P = Da_{\overline{n}|}^{(p)} + Rv^n$$

at rate i per annum.

In practice, we can calculate by using a suitable time period, for example a period of half a year as shown in the previous examples. Then the formula can be written as

$$P = \frac{D}{2} a_{\overline{2n|}} + Rv^{2n}$$

Example 7.3. *(Excel) (Modified from Financial Modeling Using Excel and VBA) The bond price is the present value of its expected future cashflows discounted at a rate appropriate for its risk. There are five key bond variables including price, redemption rate, coupon rate, remaining life, and discount rate, and any four of them determine the fifth. Add suitable spinners to the annual coupon rate and yield to maturity so that you can study how these variables affect the price of a bond. (Hint : https://support.office.com/en-us/article/add-a-scroll-bar-or-spin-button*

1. *Create an Excel model to calculate the price of a bond given its yield to maturity, redemption rate, annual coupon rate, frequency of coupon payments, and remaining life (and also face value). Make all of these input variables so that a user can use the model to value any bond for which these data are available.*
2. *Create a chart to show how the price of a bond grows or declines to redemption amount as it approaches maturity.*

Example 7.4. *Make a copy of your Excel worksheet for bond pricing calculation. The Excel model can be used to investigate how the value of a bond changes with changes in the values of the other variables.*

Use the spinner to change the yield to maturity from a value less than the coupon rate to a value greater than the coupon rate to see how the valuation curve changes. What conclusion can you make regarding the change in the values of the bond in respect of the change of the yield to maturity? Write your conclusion in cell D1.

Example 7.5. *Make a copy of your Excel worksheet for bond pricing calculation. The Excel model can be used to investigate how the value of a bond changes with changes in the values of the other variables.*

Change the coupon rate to zero for a zero-coupon bond and notice that the longer the remaining life, the lower the price of the bond. Then use the spinner to vary the yields to maturity to see how valuation curve changes. What conclusion can you make regarding the changes occurring? Write your conclusion in cell D1.

7.3 Interest rate risk and bond prices

While the effect of time on bond prices is predictable, unpredictable changes in interest rates will also affect bond prices. Further, bonds with different characteristics will respond differently to changes in interest rates—some bonds will react more strongly than others.

Example 7.6. *Consider a 10-year coupon bond and a 30-year coupon bond, both with 10% annual coupons and redeemable at par.*

1. *By what percentage will the price of each bond change if its yield to maturity increases from 5% to 6%? Answer in cells D1 and D2 respectively.*
2. *Plot the values of the 10-year and 30-year coupon bonds as the function of yield to maturity. Which of the bonds is more sensitive to a change in the yield? Why?*

7.4 Inflation

Create an Excel model to calculate the required quantities as given in the following questions. Look for the sheet CPI containing the CPI index from the Bureau of Trade and Economic Indices.

Example 7.7. *A 5-year bond with nominal value of ฿100 was issued in 1/1/2013. The coupon rate was 8% p.a. payable yearly in arrears. The redemption was 100% after 5 years. The bond was issued at 100%.*

1. *Calculate the monetary yield to a non-tax paying investor.*
2. *With reference to the monthly CPI, calculate the real yield to a non-tax paying investor.*
3. *Assuming that over the period of the transaction the annual rate of inflation will be constant and equal to 3.01%, calculate the investor?s real yield on the transaction.*

8 Introducing Visual Basic

The notes are obtained from the textbook **"Excel VBA Programming For Dummies"** written by **Michael Alexander and John Walkenbach**

A macro is a sequence of instructions that automates some aspect of Excel so that you can work more efficiently and with fewer errors.

- Excel records your actions and converts them into a VBA macro.
- When you execute this macro, Excel performs the actions again.
- More advanced users, though, can write code that tells Excel to perform tasks that can't be recorded.

8.1 Components of VBA editor (VBE)

The VBE consists of many parts including

Project (Explorer) window displays a hierarchical list of the projects and all of the items contained and referenced by each project.

If the project has any VBA modules, the project listing also shows a **Modules node**. A project may also contain a node called **Forms**, which contains UserForm objects (which hold custom dialog boxes).

With the project window, you can also add and remove a VBA module, export and import objects.

Properties window lists the properties for selected objects and their current settings. You can change these properties at the time during which you build an application in the development environment.

Immediate window displays information resulting from debugging statements in your code or from commands typed directly into the window.

Locals window displays all of the declared variables in the current procedure and their values.

Example 8.1. (Project (Explorer) window). *In the VBE, select the project, namely Book1 and*

1. *add a new VBA module,*
2. *then remove the VBA module you have just created from the project.*

Note User can also add a new UserForm (try it yourself). The UserFrom will be discussed later.

Example 8.2. *(Properties window) In workbook Book1, add at least 2 new worksheets into it.*

Then in the VBE, perform the following tasks in the project Book1 by using the Properties window :

1. *Select Sheet1 from the project Book1 and change the name of Sheet1 to Sheet2. What are the obtained results?*
2. *Change the visible property from -1 to 0 and observe the outputs. What will happen if the visible property value is set to be 2?*
3. *Insert a new Module and rename it as MyFirstModule.*

8.2 Two Types of VBA Macros

A VBA macro (also known as a VBA procedure) can be one of two types: a Sub or a Function.

- VBA Sub procedures. You can think of a Sub procedure as a new command that either the user or another macro can execute.
- VBA functions. A function always returns a single value (just as a worksheet function always returns a single value). A VBA function can be executed by other VBA procedures or used in worksheet formulas, just as you would use Excel's built-in worksheet functions.

8.3 Creating VBA Macros

Excel provides two ways to create macros:

- Turn on the macro recorder and record your actions. To start the macro recorder, choose

  Developer → Code → Record Macro
- Enter the code directly into a VBA module. The easiest way to create a simple macro is to record your actions. To develop more complex macros, however, you have to enter the VBA code manually, or write a program. To save time, you can often combine recording with manual code entry.

Example 8.3. *Write a simple macro which uses the VBA builtin MsgBox function to display a message in a dialog box. Name it as MyFirstMacro.*

Here is a possible answer.

```
Sub MyFirstMacro()
    'This is my first macro.
MsgBox "This is my first macro"
End Sub
```

The following procedure shows how to run a procedure from another procedure.

Example 8.4. *(Executing the procedure from another procedure) Enter this new procedure (either above or below MyFirstMacro code, it makes no difference) to execute a procedure from another procedure.*

```
Sub MySecondMacro()
    'Use the Call keyword to execute a procedure from another procedure.
    MyFirstMacro
End Sub
```

8.4 How VBA works

A concise summary of how VBA works:

- You perform actions in VBA by writing (or recording) code in a VBA module sheet and then executing the macro in any one of various ways. VBA modules are stored in an Excel workbook, and a workbook can hold any number of VBA modules. To view or edit a VBA module, you must activate the VB Editor window. (Press Alt+F11 to toggle between Excel and the VB Editor window.)
- A VBA module consists of procedures. A procedure is basically computer code that performs some action.
- A VBA module also can store function procedures. A function procedure performs calculations and returns a single value. A function can be called from another VBA procedure or can even be used in a worksheet formula.

- VBA manipulates objects. Excel provides more than 100 classes of objects that you can manipulate. Examples of objects include a workbook, a worksheet, a range on a worksheet, a chart, and a rectangle shape.
- Objects are arranged in a hierarchy and can act as containers for other objects.

  For example,

  - Excel itself is an object called Application, and it contains other objects, such as Workbook objects.
  - The Workbook object can contain other objects, such as Worksheet objects and Chart objects.
  - A Worksheet object can contain objects such as Range objects, PivotTable objects, and so on. The arrangement of all these objects is referred to as an object model.
- Objects that are alike form a collection. For example,
  - the Worksheets collection consists of all worksheets in a particular workbook.
  - The ChartObjects collection consists of all ChartObjects on a worksheet.
  - Collections are objects in themselves.
- You refer to an object in your VBA code by specifying its position in the object hierarchy, using a period as a separator.

  For example, you can refer to a workbook named Book1.xlsx as

  ```
  Application.Workbooks("Book1.xlsx")
  ```

  This expression refers to the Book1.xlsx workbook in the Workbooks collection. The Workbooks collection is contained in the Application object (that is, Excel). Extending this to another level, you can refer to Sheet1 in Book1 as follows:

  ```
  Application.Workbooks("Book1.xlsx").Worksheets("Sheet1")
  ```

  You can take it to still another level and refer to a specific cell as follows:

  ```
  Application.Workbooks("Book1.xlsx").Worksheets("Sheet1"). \_

      Range("A1")
  ```
- If you omit specific references, Excel uses the active objects.

  If Book1.xlsx is the active workbook, the preceding reference can be simplified as follows:

  ```
  Worksheets("Sheet1").Range("A1")
  ```

  If you know that Sheet1 is the active sheet, you can simplify the reference even more:

  ```
  Range("A1")
  ```
- Objects have properties. A property can be thought of as a setting for an object. For example,
  - a Range object has properties, such as Value and Address.
  - A Chart object has properties such as HasTitle and Type.

  You can use VBA both to determine object properties and to change them.
- You refer to properties by combining the object with the property, separated by a period. For example, you can refer to the value in cell A1 on Sheet1 as follows:

  ```
  Worksheets("Sheet1").Range("A1").Value
  ```
- You can assign values to variables. To assign the value in cell A1 on Sheet1 to a variable called Interest, use the following VBA statement:

```
Interest = Worksheets("Sheet1").Range("A1").Value
```

- Objects have methods. A method is an action that is performed with the object. For example, one of the methods for a Range object is ClearContents. This method clears the contents of the range.
- You specify methods by combining the object with the method, separated by a period. For example, to clear the contents of range A1:C12, use the following statement:

  ```
  Worksheets("Sheet1").Range("A1:C12").ClearContents
  ```

- VBA also includes all the constructs of modern programming languages, including variables, arrays, looping, and so on.

8.5 Essential VBA Language Elements

8.5.1 Working with Variables

VBA's main purpose is to manipulate data. VBA stores the data in your computer's memory. Some data, such as worksheet ranges, resides in objects. Other data is stored in variables that you create.

A variable is simply a named storage location in your computer's memory that's used by a program. Here are examples of variables

```
x = 1
InterestRate = 0.025
FaceValue = 1000
DataEntered = False
x = x + 1
UserName = "Peter Rabbit"
Date_Started = #1/15/2015#
```

8.5.2 Data types and declaring variables

In this section, you discover how to declare a variable as a certain data type. A variable's scope determines which modules and procedures can use the variable.

8.5.2.1 Data types A data type is the characteristic of a variable that determines what kind of data it can hold. Data types include numerical and non-numerical as well as user-defined types and specific types of objects. For details about the supported data types, storage sizes and ranges, please see

https://docs.microsoft.com/en-us/office/vba/language/reference/user-interface-help/data-type-summary

1. Numeric Data Types are types of data that consist of numbers, which can be computed mathematically with various standard operators such as addition, subtraction, multiplication, division and more. Numeric data types include **Byte, Integer, Long, Single, Double, Currency and Decimal**.
2. Non-Numeric Data Types are data that cannot be manipulated mathematically using standard arithmetic operators. The non-numeric data comprises text or **string** data types, the **Date** data types, the **Boolean** data types that store only two values (true or false), **Object** data type and **Variant** data type.

Note The **Variant** data type is the data type for all variables that are not explicitly declared as some other type (using statements such as Dim, Private, Public, or Static).

8.5.2.2 Procedure-level variables The lowest level of scope for a variable is at the procedure level. (A procedure is either a Sub or a Function procedure.) Variables declared with this scope can be used only in the procedure in which they're declared. When the procedure ends, the variable no longer exists, and Excel frees up its memory. If you execute the procedure again, the variable comes back to life, but its previous value is lost.

The following example shows some procedure-only variables declared by using Dim statements:

```
Sub declaringVariables()
    Dim x As Integer
    Dim First As Long
    Dim InterestRate As Single
    Dim IssueDate As Date
    Dim UserName As String
    Dim MyValue
    '... [The procedure's code goes here] ...
End Sub
```

Note

1. A declaration statement can be placed within a procedure to create a **procedure-level variable**. Or it may be placed at the top of a module, in the Declarations section, to create a **module-level variable**.
2. If a declaration statement is given within a procedure, the variable can be used only in that procedure. If the statement appears in the **Declarations section** of the module, the variable is available to all procedures within the module, but not to procedures in other modules in the project.

You can declare several variables in one statement. To specify a data type, you **must include the data type for each variable**.

The following statement declares intX, intY and intZ as integer.

```
Dim intX As Integer, intY As Integer, intZ As Integer
```

However, the following statement declares intZ as integer, while intX, intY are as Variant type.

```
Dim intX, intY, intZ As Integer
```

Note VBA has three other keywords that are used to declare variables: **Static, Public and Private**. For more details, follow from the following link:

https://docs.microsoft.com/en-us/office/vba/language/ concepts/getting-started/declaring-variables

Recall that the Locals window is very useful when we want to display all of the declared variables in the current procedure and their values.

Example 8.5. *(Locals window) Place a cursor anyway in the declaringVariables procedure. On the debug menu, click Step Into command in the debug menu once. Report the outputs obtained from the Locals window.*

Example 8.6. *Copy the following code and paste it before any procedures in a module. What happens if you run the procedure?*

```
Option Explicit

Sub WithOptionExplicit()
    Dim x As Integer
    x = 20
    y = 5
End Sub
```

Note The **Option Explicit** statement is used at the module level to force explicit declaration of all variables in that module. When this statement is present, you will not be able to run your code if it contains any undeclared variables.

Example 8.7. *Procedure-level variables) Step into (by pressing F8) the following macro to observe the values of the variables in the Locals window. What are the values of x and y when the procedure is completely run?*

```
Option Explicit

Sub VariableScope()
    Dim x As Integer
    Dim y As Integer
    x = x + 1
    y = y + 1
End Sub
```

Note Variables declared at the procedure level are the most efficient because VBA frees up the memory they use when the procedure ends.

8.5.2.3 Module-level variables

Sometimes, you want a variable to be available to all procedures in a module. If so, just declare the variable (using Dim or Private) before the module's first Sub or Function statement, outside any procedures. You do this in the Declarations section, at the beginning of your module. (This is also where the Option Explicit statement is located.)

8.5.2.4 Public variables

If you need to make a variable available to all the procedures in all your VBA modules in a workbook, declare the variable at the module level (in the Declarations section) by using the Public keyword. Here's an example:

Example 8.8. *(Public variables)*

1. *Run the first macro three times. What are the values of x and y when the procedure is completely run?*
2. *Then run the second macro. Is the value of x available to the second macro and what is the output observed from the Message box?*

```
Option Explicit
Public y As Integer


Sub VariableScope2()
    Dim x As Integer

    x = x + 1
    y = y + 1
    MsgBox "The value of y is " & y

End Sub

Sub yValue()
    MsgBox "The value of y is " & y
End Sub
```

8.6 Working with constants

Sometimes you need to refer to a value or string that never changes. In such a case, you need a constant - a named element whose value doesn't change.

As shown in the following examples, you declare constants by using the Const statement.

```
Const NumQuarters As Integer = 4
Const Rate = .0725, Period = 12
```

8.7 Working with dates

In VBA, place dates and times between two hash marks, as shown in the preceding examples.

```
Dim Today As Date
Dim StartTime As Date
Const FirstDay As Date = #1/1/2013#
Const Noon = #12:00:00#
```

8.8 Working with Arrays

An array is a group of variables that share a common name. You refer to a specific variable in the array by using the array name and an index number in parentheses.

The following example shows how to declare an array of 100 integers:

```
Dim MyArray(1 To 100) As Integer
```

When you declare an array, you can choose to specify only the upper index. If you omit the lower index, VBA assumes that it's 0. Therefore, both of the following statements declare the same 101-element array:

```
Dim MyArray (0 To 100) As Integer
Dim MyArray (100) As Integer
```

If you want VBA to assume that 1 (rather than 0) is the lower index for your arrays, include the following statement in the Declarations section of your module:

```
Option Base 1
```

This statement forces VBA to use 1 as the first index number for arrays that declare only the upper index. If this statement is present, the following statements are identical, both declaring a 100-element array:

```
Dim MyArray (1 To 100) As Integer
Dim MyArray (100) As Integer
```

8.8.1 Multidimensional arrays

The arrays created in the previous examples are all one-dimensional arrays. Think of one-dimensional arrays as a single line of values. Arrays you create in VBA can have as many as 60 dimensions - although you rarely need more than two or three dimensions in an array. The following example declares an 81-integer array with two dimensions:

```
Dim MyArray (1 To 9, 1 To 9) As Integer
```

You can think of this array as occupying a 9 x 9 matrix. To refer to a specific element in this array, you need to specify two index numbers (similar to its "row" and its "column" in the matrix). The following example shows how you can assign a value to an element in this array:

```
MyArray (3, 4)= 100
```

8.8.2 Dynamic arrays

You can also create dynamic arrays. A dynamic array doesn't have a preset number of elements. Declare a dynamic array with an empty set of parentheses:

```
Dim MyArray () As Integer
```

Before you can use this array, you must use the ReDim statement to tell VBA how many elements the array has. Usually, the number of elements in the array is determined while your code is running. You can use the ReDim statement any number of times, changing the array's size as often as needed. The following example demonstrates how to change the number of elements in a dynamic array. It assumes that the NumElements variable contains a value, which your code calculated.

```
ReDim MyArray (1 To NumElements)
```

When you redimension an array by using ReDim, you wipe out any values currently stored in the array elements. You can avoid destroying the old values by using the Preserve keyword. The following example shows how you can preserve an array's values when you redimension the array:

```
ReDim Preserve MyArray(1 To NumElements)
```

8.9 Working with Range Objects

A Range object represents a range contained in a Worksheet object. Range objects, like all other objects, have properties (which you can examine and sometimes change) and methods (which perform actions on the object).

When you refer to a Range object, the address is always surrounded by double quotes, like this:

```
Range("A1:C5")
```

If the range consists of one cell, you still need the quotes:

```
Range("K9")
```

A Range object can consist of one or more entire rows or columns. You can refer to an entire row (in this case, row 3) by using syntax like this:

```
Range("3:3")
```

You can refer to an entire column (the fourth column in this example) like this:

```
Range("D:D")
```

The following expression refers to a two-area noncontiguous range. Notice that a comma separates the two areas.

```
Range("A1:B8,D9:G16")
```

8.10 The Cells property

Rather than use the VBA Range keyword, you can refer to a range via the Cells property.

The Cells property takes two arguments: a row number and a column number. Both of these arguments are numbers, even though we usually refer to columns by using letters. For example, the following expression refers to cell C2 on Sheet2:

```
Worksheets("Sheet2").Cells(2, 3)
```

You can also use the Cells property to refer to a multi-cell range. The following example demonstrates the syntax you use:

```
Range(Cells(1, 1), Cells(10, 8))
```

This expression refers to an 80-cell range that extends from cell A1 (row 1, column 1) to cell H10 (row 10, column 8). The following statements both produce the same result; they enter a value of 99 into a 10-by-8 range of cells. More specifically, these statements set the Value property of the Range object:

```
Range("A1:H10").Value = 99
Range(Cells(1, 1), Cells(10, 8)).Value = 99
```

The advantage of using the Cells property to refer to ranges becomes apparent when you use variables rather than actual numbers as the Cells arguments. And things really start to click when you understand looping.

8.11 The Offset property

The Offset property provides another handy means for referring to ranges. This property, which operates on a Range object and returns another Range object, lets you refer to a cell that is a particular number of rows and columns away from another cell.

Like the Cells property, the Offset property takes two arguments. The first argument represents the number of rows to offset; the second represents the number of columns to offset.

The following expression refers to a cell one row below cell A1 and two columns to the right of cell A1. In other words, this refers to the cell commonly known as C2:

```
Range("A1").Offset(1, 2)
```

The Offset property can also use negative arguments. A negative row offset refers to a row above the range. A negative column offset refers to a column to the left of the range. The following example refers to cell A1:

```
Range("C2").Offset(-1, -2)
```

And, as you may expect, you can use 0 as one or both of the arguments for Offset. The following expression refers to cell A1:

```
Range("A1").Offset(0, 0)
```

8.12 Some Useful Range Object Properties

The Value property The Value property represents the value contained in a cell. It's a read-write property, so your VBA code can either read or change the value. The following statement displays a message box that shows the value in cell A1 on Sheet1:

```
MsgBox Worksheets("Sheet1").Range("A1").Value
```

It stands to reason that you can read the Value property only for a single-cell Range object. For example, the following statement generates an error:

```
MsgBox Worksheets("Sheet1").Range("A1:C3").Value
```

You can, however, change the Value property for a range of any size. The following statement enters the number 123 into each cell in a range:

```
Worksheets("Sheet1").Range("A1:C3").Value = 123
```

8.13 The Text property

The Text property returns a string that represents the text as it's displayed in a cell - the formatted value. The Text property is read-only. For example, suppose that cell A1 contains the value 12.3 and is formatted to display two decimals and a dollar sign ($12.30). The following statement displays a message box containing $12.30:

```
MsgBox Worksheets("Sheet1").Range("A1").Text
```

But the next statement displays a message box containing 12.3:

```
MsgBox Worksheets("Sheet1").Range("A1").Value
```

If the cell contains a formula, the Text property returns the result of the formula. If a cell contains text, then the Text property and the Value property will always return the same thing, because text (unlike a number) can't be formatted to display differently.

8.14 The Count property

The Count property returns the number of cells in a range. It counts all cells, not just the nonblank cells. Count is a read-only property, just as you would expect. The following statement accesses a range's Count property and displays the result (9) in a message box:

```
MsgBox Range("A1:C3").Count
```

8.15 The Column and Row properties

The Column property returns the column number of a single-cell range. Its sidekick, the Row property, returns the row number of a single-cell range. Both are read-only properties. For example, the following statement displays 6 because cell F3 is in the sixth column:

```
MsgBox Sheets("Sheet1").Range("F3").Column
```

The next expression displays 3 because cell F3 is in the third row:

```
MsgBox Sheets("Sheet1").Range("F3").Row
```

If the Range object consists of more than one cell, the Column property returns the column number of the first column in the range, and the Row property returns the row number of the first row in the range.

8.16 The Address property

Address, a read-only property, displays the cell address for a Range object as an absolute reference (a dollar sign before the column letter and before the row number).

```
MsgBox Range(Cells(1, 1), Cells(5, 5)).Address
```

8.17 The Formula property

The Formula property represents the formula in a cell. This is a read-write property, so you can access it to either view the formula in a cell or insert a formula into a cell. For example, the following statement enters a SUM formula into cell A13:

```
Range("A13").Formula = "=SUM(A1:A12)"
```

Notice that the formula is a text string and is enclosed.

Here"s a statement that will enter a formula that contains quotes:

```
Range("A13").Formula = "=SUM(A1:A12)&"" Stores"""
```

8.18 The NumberFormat property

The NumberFormat property represents the number format (expressed as a text string) of the Range object. This is a read-write property, so your VBA code can either examine the number format or change it. The following statement changes the number format of column A to a percent with two decimal places:

```
Columns("A:A").NumberFormat = "0.00%"
```

8.19 Some Useful Range Object Methods

As you know, a VBA method performs an action. A Range object has dozens of methods but, again, you won't need most of these. In this section, I point out some of the more commonly used Range object methods. The Select method Use the Select method to select a range of cells. The following statement selects a range on the active worksheet:

```
Range("A1:C12").Select
```

Before selecting a range, it"s often a good idea to use one additional statement to ensure that the correct worksheet is active. For example, if Sheet1 contains the range you want to select, use the following statements to select the range:

```
Sheets("Sheet1").Activate
Range("A1:C12").Select
```

Contrary to what you may expect, the following statement generates an error if Sheet1 is not already the active sheet. In other words, you must use two statements rather than just one: one to activate the sheet and another to select the range.

```
Sheets("Sheet1").Range("A1:C12").Select
```

If you use the GoTo method of the Application object to select a range, you can forget about selecting the correct worksheet first. This tatement activates Sheet1 and then selects the range:

```
Application.Goto Sheets("Sheet1").Range("A1:C12")
```

The GoTo method is the VBA equivalent of pressing F5 in Excel, which displays the GoTo dialog box.

8.20 The Copy and Paste methods

You can perform copy and paste operations in VBA by using the Copy and Paste methods. Note that two different objects come into play. The Copy method is applicable to the Range object, but the Paste method applies to the Worksheet object. It actually makes sense: You copy a range and paste it to a worksheet. This short macro (courtesy of the macro recorder) copies range A1:A12 and pastes it to the same worksheet, beginning at cell C1:

```
Sub CopyRange()
    Range("A1:A12").Select
    Selection.Copy
    Range("C1").Select
    ActiveSheet.Paste
End Sub
```

Notice that in the preceding example, the ActiveSheet object is used with the Paste method. This is a special version of the Worksheet object that refers to the currently active worksheet. Also notice that the macro selects the range before copying it. However, you don't have to select a range before doing something with it. In fact, the following procedure accomplishes the same task as the preceding example by using a single statement:

```
Sub CopyRange2()
    Range("A1:A12").Copy Range("C1")
End Sub
```

8.21 The Clear method

The Clear method deletes the contents of a range, plus all the cell formatting. For example, if you want to zap everything in column D, the following statement does the trick:

```
Columns("D:D").Clear
```

8.22 Using Built-In VBA Functions

VBA provides numerous built-in functions. Some of these functions take arguments, and some do not.

The first example uses VBA?s Date function to display the current system date in a message box:

```
Sub ShowDate()
MsgBox "Today is: " & Date
End Sub
```

The following procedure uses the VBA Len function, which returns the length of a text string.

```
Sub GetLength()
Dim MyName As String
Dim StringLength As Long
MyName = Application.UserName
StringLength = Len(MyName)
MsgBox MyName & " has " & StringLength & " characters."
End Sub
```

8.23 Discovering VBA functions

How do you find out which functions VBA provides? You can type VBA, followed by a period. You get a list of items. Those with a green icon are functions. If this feature isn?t working, choose VBE's Tools -> Options, click the Editor tab, and place a check next to Auto List Members.

8.23.1 Using Worksheet Functions in VBA

Although VBA offers a decent assortment of built-in functions, you might not always find exactly what you need. Fortunately, you can also use most of Excel's worksheet functions in your VBA procedures. The only worksheet functions that you cannot use are those that have an equivalent VBA function. VBA makes Excel's worksheet functions available through the WorksheetFunction object, which is contained in the Application object. Here's an example of how you can use Excel's SUM function in a VBA statement:

```
Total = Application.WorksheetFunction.Sum(Range("A1:A12"))
```

You can omit the Application part or the WorksheetFunction part of the expression. In either case, VBA will figure out what you're doing. In other words, these three expressions all work exactly the same:

```
Total = Application.WorksheetFunction.Sum(Range("A1:A12"))
Total = WorksheetFunction.Sum(Range("A1:A12"))
Total = Application.Sum(Range("A1:A12"))
```

My personal preference is to use the WorksheetFunction part just to make it perfectly clear that the code is using an Excel function.

8.24 Controlling execution

VBA uses many constructs that are found in most other programming languages. These constructs are used to control the flow of execution. This section introduces a few of the more common programming constructs.

8.24.1 The If-Then construct

One of the most important control structures in VBA is the If-Then construct, which gives your applications decision-making capability. The basic syntax of the If-Then structure is as follows:

```
If condition Then statements [Else else statements]
```

In plain English, if a condition is true, then a group of statement will be executed. If you include the Else clause, then another group of statements will be executed if the condition is not true.

This procedure checks the active cell. If it contains a negative value, the cell's colour is changed to red. Otherwise, nothing happens.

```
Sub CheckCell()
If ActiveCell.Value < 0 Then ActiveCell.Font.ColorIndex = 3
End Sub
```

Note See https://docs.microsoft.com/en-us/office/vba/api/excel.colorindex for colorindex property.

8.24.2 For-Next loops

You can use a For-Next loop to execute one or more statements a number of times. Here's an example of a For-Next loop:

```
Sub SumSquared()
    Total = 0
    For Num = 1 To 10
        Total = Total + (Num ^ 2)
    Next Num
    MsgBox Total
End Sub
```

8.24.3 The With-End With construct

Another construct that you encounter if you record macros is the With-End With construct. This is a useful when dealing with several properties or methods of the same object. The following is an example:

```
Sub AlignCells()
    With Selection
        .HorizontalAlignment = xlCenter
        .VerticalAlignment = xlCenter
        .WrapText = False
        .Orientation = xlHorizontal
    End With
End Sub
```

8.24.4 The Select Case construct

The Select Case construct is useful for choosing among two or more options. The following example demonstrates the use of a Select Case construct. In this example, the active cell is checked. If its value is less than 0, it's coloured red. If it's equal to 0, it's coloured blue. If the value is greater than 0, it's coloured black.

```
Sub CheckCell()
    Select Case ActiveCell.Value
        Case Is < 0
            ActiveCell.Font.Color = vbRed
        Case 0
            ActiveCell.Font.Color = vbBlue
        Case Is > 0
            ActiveCell.Font.Color = vbBlack
        End Select
End Sub
```

Any number of statements can go below each Case statement, and they all get executed if the case is true.

Note See Color constants for example vbRed, vbBlue etc. for more details:

https://docs.microsoft.com/en-us/office/vba/language/reference/user-interface-help/color-constants

8.25 VBA Examples

Example 8.9. *The NumSign function uses the Select Case construct to take a different action, depending on the value of num. If num is less than 0, NumSign is assigned the text Negative. If num is equal to 0, NumSign is Zero. If num is greater than 0, NumSign is Positive. The value returned by a function is always assigned to the function's name.*

```
Function NumSign(num)
    Select Case num
        Case Is < 0
            NumSign = "Negative"
        Case 0
            NumSign = "Zero"
        Case Is > 0
            NumSign = "Positive"
        End Select
End Function
```

If you work with this function, you might notice a problem if the argument is non-numeric. In such a case, the function returns Positive. In other words, the function has a bug. Following is a revised version that returns an empty string if the argument is non-numeric. This code uses the VBA IsNumeric function to check the argument. If it's numeric, the code checks the sign. If the argument is not numeric, the Else part of the If-Then-Else structure is executed.

```
Function NumSign(num)
    If IsNumeric(num) Then
        Select Case num
            Case Is < 0
                NumSign = "Negative"
            Case 0
                NumSign = "Zero"
            Case Is > 0
                NumSign = "Positive"
```

```
        End Select
    Else
        NumSign = ""
    End If
End Function
```

Example 8.10. *Create a macro that copies a variable-sized range (you don't know the exact row and column dimension).*

The following macro demonstrates how to copy this range from Sheet1 to Sheet2 (beginning at cell A1). It uses the CurrentRegion property, which returns a Range object that corresponds to the block of cells around a particular cell. In this case, that cell is A1.*

```
Sub CopyCurrentRegion()
    Range("A1").CurrentRegion.Copy
    Sheets("Sheet2").Select
    Range("A1").Select
    ActiveSheet.Paste
    Sheets("Sheet1").Select
    Application.CutCopyMode = False
End Sub
```

You can make this macro even more efficient by not selecting the destination. The following macro takes advantage of the fact that the **Copy method can use an argument for the destination range**:

```
Sub CopyCurrentRegion2()
    Range("A1").CurrentRegion.Copy _
    Sheets("Sheet2").Range("A1")
End Sub
```

It's even a bit easier if the data is in the form of a table (created in Excel using Insert → Tables → Table). The table has a name (such as Table1) and expands automatically when new data is added.

```
Sub CopyTable()
    Range("Table1").Copy  Sheets("Sheet2").Range("A1")
End Sub
```

Example 8.11. *The following VBA procedure selects the range beginning at the active cell and extending down to the cell just above the first blank cell in the column. After selecting the range, you can do whatever you want with it*

```
Sub SelectDown()
    Range(ActiveCell,ActiveCell.End(xlDown)).Select
End Sub
```

This example uses the End method of the ActiveCell object, which returns a Range object. The End method takes one argument, which can be any of the following constants:

- xlUp
- xlDown
- xlToLeft
- xlToRight

It is not necessary to select a range before doing something with it. The following macro applies bold formatting to a variable-sized (single column) range without selecting the range:

```
Sub MakeBold()
    Range(ActiveCell, ActiveCell.End(xlDown)) _
```

```
    .Font.Bold = True
End Sub
```

Example 8.12. *The following procedure demonstrates how to select the column that contains the active cell. It uses the **EntireColumn property, which returns a Range object that consists of a full column**:*

```
Sub SelectColumn()
    ActiveCell.EntireColumn.Select
End Sub
```

Example 8.13. *Many macros perform an operation on each cell in a range, or they might perform selected actions based on each cell's content. These macros usually include a For-Next loop that processes each cell in the range.*

The following example demonstrates how to loop through a range of cells. In this case, the range is the current selection. An object variable named Cell refers to the cell being processed. Within the For Each-Next loop, the single statement evaluates the cell and applies bold formatting if the cell contains a positive value.

```
Sub ProcessCells()
    Dim Cell As Range
    For Each Cell In Selection
        If Cell.Value > 0 Then Cell.Font.Bold = True
    Next Cell
End Sub
```

This example works, but what if the user's selection consists of an entire column or row?

Example 8.14. (Prompting for a cell value) You can use VBA's InputBox function to get a value from the user. Then you can insert that value into a cell. The following procedure demonstrates how to ask the user for a value and place the value in cell A1 of the active worksheet, using only one statement:

```
Sub GetValue()
    Range("A1").Value = InputBox( _
    "Enter the value for cell A1")
End Sub
```

Note See https://docs.microsoft.com/en-us/office/vba/api/excel.application.inputbox for details about InputBox.

Example 8.15. *Create a model with VBA to produce a (year-by-year) loan schedule for a fixed rate loan. The loan is to be repaid in equal annual instalments over its life and the first payment is to be made at the end of the first year. Format the output data in the table with a dollar sign and two decimal points, i.e "$#,##0.00".*

---

9 Tutorial 1

1. Calculate the following accumulation:
    1. Accumulate \$5,000 for 4 years at 7.5% per annum effective.
    2. Accumulate \$800 for 2.7 years at 3% per quarter-year effective.
    3. Accumulate \$10,000 for 27 months at 4.25% per half-year effective.
2. Calculate the present values on 1 January 2015 of the following payments at the given rates of interest:
    1. \$1,000 on 1 January 2016, at 7.5% per annum effective.

2. \$100 on 1 October 2016, at 3% per quarter-year effective.

3. \$10,000 on 1 April 2016, at 4.25% per half-year effective.

3. 1. If the effective rate of interest is 4% per annum, calculate the effective rate of interest per month?

   2. If the effective rate of interest is 6.5% per half-year, calculate the effective rate of interest per quarter-year?

4. The effective rate of interest per annum was 4% during 2015, 5% during 2016 and 6% thereafter.

   1. Calculate the accumulation of \$500 from 1 January 2015 to 1 January 2018.

   2. Calculate the accumulation of \$2000 from 1 April 2015 to 1 October 2017.

   3. Calculate the accumulation factor from 1 January 2015 to 1 January 2018.

5. You deposit \$ 3000 to an account that earn 2.5% compounded annually. How much will you have in three years?

6. A person borrows a sum of \$5,000 and agrees to pay this back at the end of 1 year with interest calculated at an effective rate of 10% per annum. Calculate the amount to be repaid for the loan.

7. You want to have \$1000 in 2 years and \$2000 in 4 years. How much should you deposit now into an account earning the effective rate of 5.75% semiannually?

8. Katy deposits 100 into a saving account which pays interest at i **per quarter** effective.

   At the same time, Taylor deposits 500 into a different saving account which pays a simple interest at an annual rate of i.

   During the last 3 months of the 4th year, they both earn the same amount of interest. Calculate i.

9. An ordinary annuity is a series of equal payments made at the end of consecutive periods over a fixed length of time. Draw a timeline for the following annuity having cashflow of 1 unit at the end of each of the next n time units.

10. Draw a timeline to illustrate this insurance benefit: Whole Life Insurance - payable immediately on death - has following conditions:

    - death benefit (sum insured) of 1
    - payable immediately on the death
    - of an individual currently aged x
    - for death occurring any time in the future.

11. (Excel) It is a good exercise to check whether the Excel worksheet you have developed so far for calculating the present value and future value can be applied to the questions in this Tutorial. What would you do to improve the Excel worksheet that can be applied to a more general scenario?

10 Tutorial 2

1. Starting at 1 January 2015, the effective rate of interest per annum was 3% per quarter-year for 9 months, 4% per half-year for 15 months and and 2% per month thereafter.

   1. Calculate the accumulation factor from 1 January 2015 to 1 January 2018.

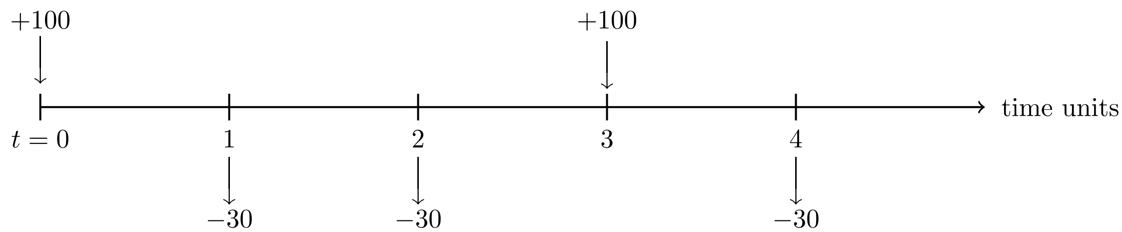
   2. Calculate the accumulation of \$5,000 from 1 July 2015 to 1 October 2017.

   3. Calculate the accumulation of \$100 from 1 March 2016 to 1 August 2018.

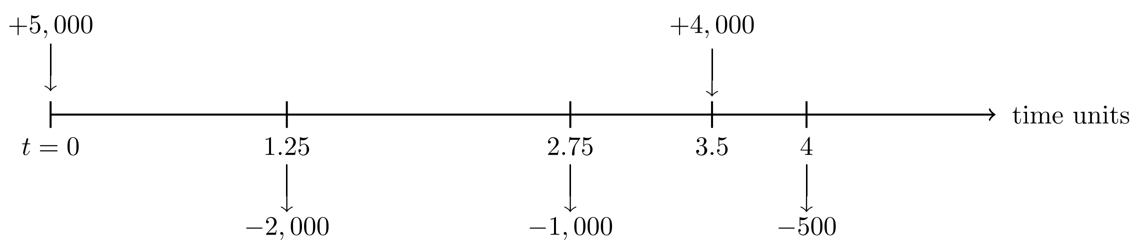
   4. Calculate the present value at 1 January 2015 of \$ 25,000 receivable on 1 July 2016.

   5. Calculate the present value at 1 April 2015 of $ 8,000 receivable on 1 October 2017.
   6. Calculate the discount factor from 1 July 2015 to 1 October 2016.

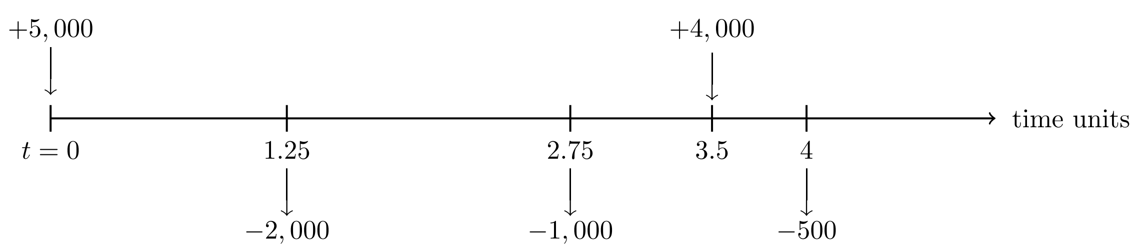
2. The effective rate of interest is 7.25% per time unit. Cashflows are shown in the following time line.
   1. Calculate the accumulation at time time t = 4 units of these cashflows.
   2. Calculate the accumulation at time time t = 8 units of these cashflows.
   3. Calculate the present value at time time t = 0 units of these cashflows.



3. The effective rate of interest is 6% per time unit. Cashflows are shown in the following time line.
   1. Calculate the accumulation at time time t = 5 units of these cashflows.
   2. Calculate the value at time time t = 2 units of these cashflows.
   3. Calculate the present value at time time t = 0 units of these cashflows.



4. The effective rate of interest per annum was 4% during 2015, 3% per half-year until 1 October 2017 and 1.5% per month thereafter. Cashflows are shown in the following time line.
   1. Calculate the accumulation on 1/1/2019 of these cashflows.
   2. Calculate the present value on 1/1/2015 of these cashflows.
   3. Calculate the value at time time 1/7/2017 of these cashflows.



5. (Excel) It is a good exercise to check whether the Excel worksheet you have developed so far for calculating the present value and future value can be applied to the questions in this Tutorial. What would you do to improve the Excel worksheet for a more general scenario?

11 Tutorial 3

1. Calculate the present value now of an annuity payable monthly in advance. The annual amount of the annuity will be \$ 2,400 for the first 10 years and \$ 3,600 for the next 15 years, after which payment will cease. Assume that the effective rate of interest is 2% per annum.

2. Assume that the effective rate of interest will be 3% for 5 years from now, 4% for the next 5 years and 5% thereafter. Calculate the following values:

    1. The present value of an annuity of \$ 1,000 per annum, payable in arrear for 15 years.

    2. The present value of an annuity due of \$ 500 per annum, payable at the beginning of the year for 20 years.

    3. The accumulation value of an increasing annuity payable yearly in arrear for 30 years. The first annual payment is \$ 100, and payments will be increase by \$ 100 each year.

    4. The accumulation value of an increasing annuity payable yearly in advance for 18 years. The first annual payment is \$ 1,000, and payments will be increase by 2% each year (compound).

    5. The present value of an annuity of \$ 200, payable in arrear for 10 years and deferred for 3 years.

3. You borrow \$ 240,000 from a bank to be repaid by the end of 5 years. Assume that the interest rate is 4% per annum. Consider the following four possible options for the loan to be repaid.

    1. Calculate the amount of the repayments to repay if you choose to repay the loan as late as possible.

    2. You may choose to repay interest only during the 5 years term of loan and repay the capital at the end of the term. Calculate interest to be repaid and draw the timeline to illustrate the cashflows for the repayment of the loan.

    3. Calculate the amount X of level instalments to repay the loan which will be paid at the end of each year for 5 years and draw the timeline to illustrate the cashflows for the repayment of the loan.

    4. Calculate the amount Y of level instalments to repay the loan which will be paid at the end of each month for 5 years and draw the timeline to illustrate the cashflows for the repayment of the loan. **Instalment** is a sum of money due as one of several equal payments for something, spread over an agreed period of time.

4. A person now age 30 has received a pension from a company. When he retires at age 60, he will be paid on each birthday from the 60 to the 85th inclusive. The first annual payment will be half of his salary when he retires, and payments will then increase by 2% compounding each year. Currently, he receive a salary of \$ 20,000 and will increase by 3% each year compounding in line with inflation. Assume that the effective rate of interest will be 4% for the next 20 years and 5% thereafter. Calculate the present value now of this pension.

5. (Excel) Use Excel worksheet you have developed so far to calculate the results from the questions in this Tutorial.

12 Tutorial 4

1. Show that the following series of cashflows are equivalent given that an interest rate is 4% per annum effective.

    1. One single payment of amount 14,802.44 at year 10.

    2. a level annuity of 400 payable yearly in arrear for the next 10 years plus a lump sum of 10,000.

    3. a level annuity of 1,232.91 payable yearly in arrear for the next 10 years.

2. You invest in a project which requires you to pay 2,000 and receive back 300 at the end of each of the next 8 years. Calculate the yield of this investment. ANS = 4.2394551%

3. You pay a price of 5,000 for an investment that will repay you 600 per annum payable half-yearly in arrear for the next 12 years. Calculate the yield of this investment. ANS = 3.1491266%

4. An investor pays 100,000 in order to receive 20,000 back at the end of the first 3 years and 25,000 back at the end in the next 4 years. Calculate the yield of this investment. ANS = 12.6209232%

5. You invest in a project which requires you to pay 500,000 at the start of each of the calendar years 2018, 2019 and 2020. The project is expected to return profits of 400,000 for 6 years a the end of each calendar year 2024 to 2029 inclusive. Calculate the yield of this investment. ANS = 5.7285486%

6. (Modified from CT1 2012 IFoA Exam)

   An investor is considering two projects, Project A and Project B. Project A involves the investment of 2,000,000 in a retail outlet. Rent is received quarterly in arrear for 25 years, at an initial rate of 100,000 per annum. It is assumed that the rent will increase at a rate of 5% per annum compound, but with increases taking place every five years. Maintenance and other expenses are incurred quarterly in arrear, at a rate of 12,000 per annum. The retail outlet reverts to its original owner after 25 years for no payment.

   Project B involves the purchase of an office building for 1,000,000. The rent is to be received quarterly in advance at an initial rate of 85,000 per annum. It is assumed that the rent will increase to 90,000 per annum after 20 years. There are no maintenance or other expenses. After 25 years the property reverts to its original owner for no payment.

   Calculate the annual effective internal rate of return for both Projects A and B. Which project is preferable?

7. (Excel) Use Excel worksheet you have developed to calculate the results from the questions in this Tutorial.

13 Tutorial 5

1. You borrow 30,000 for a term of 6 months to be repaid in arrear by level monthly instalments. The rate of interest will be 4% pa effective.

   1. Calculate the monthly repayment.

   2. Construct the complete loan schedule

2. A loan of 800,000 is repayable by equal monthly repayments for 10 years, with interest rate payable at 6.5% pa effective.

   1. Calculate the amount of each monthly payment.

   2. Calculate the interest and capital contents of the 96th repayment.

3.
   1. An investor takes out a loan of 100,000 from a bank to be repaid by level annual instalments in arrear over 12 years where the bank charges an effective annual rate of interest of 7%. Immediately after the 6th repayment has been made, the investor may

      1. extend the term of the loan by extra 2 year, or

      2. miss the next two repayments.

      Calculate the revised repayment amount in each case.

2. Suppose the bank allows the investor to miss the next two repayments but the capital outstanding will be charged interest at 10% pa effective while the investor is not making repayments. Calculate the revised repayment.

3. Suppose in Question 3.2 that the investor will miss the next two repayment and extend the term of the loan by extra 4 years. Calculate the revised repayment.

4. An investor borrows 50,000 for a term of 12 years. The rate of interest will be 4% pa effective for the first 6 years and 5% pa effective thereafter. The loan will be repaid level annual repayments for the first 6 years, and then increasing to twice the origin level for the last 6 years. Calculate the annual repayment.

5. An investor borrows 40,000 for a term of 10 years. The rate of interest will be 6.5% pa effective The loan will be repaid level annual repayments, increasing at 2% per annum.

   1. Calculate the first annual repayment.

   2. Calculate the capital outstanding after the 7th repayment is made.

   3. Calculate the interest content of the 8th repayment.

6. (Excel) Suppose you borrow L from a bank to be repaid by the end of n years at an interest rate of i% per annum effective. If you agree to repay the loan and the interest in equal annual instalments throughout term of loan and the first payment is made at the end of the first year.

   Create a model to produce a loan amortisation (or loan schedule) table. Make the interest rate, loan life, initial loan, and other necessary variables input variables. The loan amortisation table should include the following columns:

   - The year-beginning balance
   - The annual repayment
   - Interest Component
   - Capital content
   - Capital outstanding (the year-end balance)

14 Solutions to Tutorial 1

1. The solutions to each question are as follows:

   1. $5000(1.075)^4 = 6677.345703$
   2. Let i% be the annual rate effective equivalent to 3% per quarter-effective, $i = (1.03)^4 - 1$. Hence, the accumulation is
   $$800(1+i)^{2.7} = 800(1.03)^{4\times 2.7} = 1100.859802.$$
   3. Let j% be the monthly rate effective equivalent to 4.25% per half-year effective, $j = (1.0425)^{2/12} - 1$. Hence, the accumulation is
   $$10000(1.0425)^{(2/12)\times 27} = 12059.86056.$$

2. The solutions to each question are as follows:

   1. $\frac{1000}{1.075} = 930.232558.$
   2. $\frac{100}{(1.03)^7} = 81.309151.$
   3. Let j% be the quarterly rate effective equivalent to 4.25% per half-year effective, $j = (1.0425)^{2/4} - 1$. Hence, the present value is
   $$10000 \times (1+j)^{-5} = 10000(1.0425)^{-(5/2)} = 9011.764643.$$

3. The solutions to each question are as follows:
    1. 0.3274%
    2. 3.1988%
4. The solutions to each question are as follows:
    1. $500(1.04)(1.05)(1.06)$ =578.76
    2. $2000(1.04)^{3/4}(1.05)(1.06)^{3/4} = 2259.299$
    3. $(1.04)(1.05)(1.06) = 1.15752$
5. The account balance in 3 years is $3000(1.025)^3 = 3230.67$.
6. The amount to be repaid for the loan is $5000(1.1) = 5500$.
7. Let X be the amount to be deposited now.

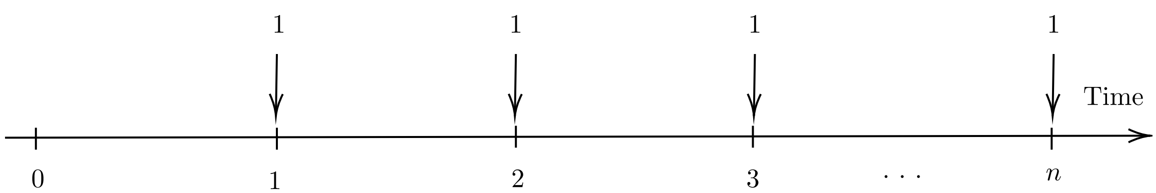
$$X = \frac{1000}{(1.0575)^4} + \frac{2000}{(1.0575)^8} = 2078.36.$$

8. At time 3.75 years, Katy has a balance of $100(1+i)^{15}$. The interest on this balance over the next 3 months is $100(1+i)^{15} \cdot i$. Taylor earns simple interest on the original amount which is equal to $500i \cdot \frac{3}{12}$. Therefore, we solve for i from the following equation:

$$100(1+i)^{15} \cdot i = 500i \cdot \frac{3}{12},$$

which gives $i = 0.014987$.

9. The timeline for the following annuity having cashflow of 1 unit at the end of each of the next n time units is given in the figure below:



10. (More details in the course "Life Contingencies I"). We need to define a random variable T_x = the remaining future life time of a life aged x.

The quantity of interest is the present value of the death benefit assuming the interest rate of i% p.a. effective. It is also a random variable,

$$PV = \frac{1}{(1+i)^{T_x}}.$$

It turns out that the premium rate of this whole life insurance is $E[PV]$, the expected value of the present value, PV.

15 Solutions to Tutorial 2

1. The solutions to each question are as follows:

1. $(1.03)^3(1.04)^{2.5}(1.02)^{12} = 1.528611$
2. $5000A(0.5, 2.75) = 5000(1.03)(1.04)^{2.5}(1.02)^9 = 6788.786068$
3. We first find the rate $j\%$ per month effective that is equivalent to the rate of 4% per half-year effective.
$$j = (1.04)^{1/6} - 1 = 0.00656.$$
The accumulated value is $100A(1 + 2/12, 3 + 7/12) = 100(1 + j)^{10}(1.02)^{19} = 155.522118$.
4. $25000V(0, 1.5) = \frac{25000}{(1.03)^3(1.04)^{1.5}} = 21571.39968$.
5. $8000V(0.25, 2.75) = \frac{8000}{(1.03)^2(1.04)^{2.5}(1.02)^9} = 5720.455921$.
6. $V(0.5, 1.75) = \frac{1}{(1.03)(1.04)^2} = 0.897627$.

2. The solutions to each question are as follows:

   1. With $i = 7.25\%$ per time period, $V(4) = 100(1+i)^4 - 30(1+i)^3 - 30(1+i)^2 + 100(1+i) - 30 = 138.041762$.
   2. $V(8) = V(4) \cdot (1 + i)^4 = 182.641597$.
   3. $PV(0) = V(4) \cdot (1 + i)^{-4} = 104.332903$.

3. The solutions to each question are as follows:

   1. With \$ = 6%\$ per time period, $V(5) = 5000(1+i)^5 - 2000(1+i)^{3.75} - 1000(1+i)^{2.25} + 4000(1+i)^{1.5} - 500(1+i) = 6897.948585$.
   2. $V(2) = V(5)(1 + i)^{-3} = 5791.650645$.
   3. $PV(0) = V(5)(1 + i)^{-5} = 5154.548456$.

4. The solutions to each question are as follows:

   1. $V(1/1/2019) = 100(1.04)(1.03)^{3.5}(1.015)^{15} - 30(1.03)^{3.5}(1.015)^{15} - 30(1.03)^{1.5}(1.015)^{15} + 100(1.015)^{12} - 30 = 152.955693$.
   2. $PV(0) = V(1/1/2015) = \frac{152.955693}{(1.04)(1.03)^{3.5}(1.015)^{15}} = 106.074596$.
   3. $V(1/7/2017) = PV(0)(1.04)(1.03)^3 = 120.546998$.

16 Solutions to Tutorial 3

1. Let j be the effective rate per month equivalent to $i = 2\%$. We have
$$j = (1.02)^{1/12} - 1 = 0.001652.$$
Hence,
$$PV(0) = 200\ddot{a}^{j}_{\overline{120}|} + 300\ddot{a}^{j}_{\overline{180}|}\left(\frac{1}{1.02}\right)^{10} = 60148.03.$$

2. The solutions to each question are as follows:

   1. The cashflows have been splitted into three periods: (a) from time point 0-5, (b) 5-10 and (c) time point 10 onward.
$$PV(0) = 1000(a^{3\%}_{\overline{5}|} + 1.03^{-5}a^{4\%}_{\overline{5}|} + 1.03^{-5}1.04^{-5}a^{5\%}_{\overline{5}|}) = 11489.49$$
   2. We have
$$PV(0) = 500(\ddot{a}^{3\%}_{\overline{5}|} + 1.03^{-5}\ddot{a}^{4\%}_{\overline{5}|} + 1.03^{-5}1.04^{-5}\ddot{a}^{5\%}_{\overline{10}|}) = 7229.67$$
   3. The accumuated value is
$$100(Is)^{3\%}_{\overline{5}|}(1.04)^5(1.05)^{20} + \left[100(Is)^{4\%}_{\overline{5}|} + 500s^{4\%}_{\overline{5}|}\right](1.05)^{20} + \left[100(Is)^{5\%}_{\overline{20}|} + 1000s^{5\%}_{\overline{20}|}\right] = 78929.01$$

4. Let $i_1 = 3\%, i_2 = 4\%$ and $i_3 = 5\%$. The accumuated value is given by

$$
\begin{aligned}
V(18) = &\left[1000(1+i_1)^5 + 1000(1.02)(1+i_1)^4 + 1000(1.02)^2(1+i_1)^3 + ... + 1000(1.02)^4(1+i_1)\right](1.04)^5(1.05)^8 \\
&+ \left[1000(1.02)^5(1+i_2)^5 + 1000(1.02)^6(1+i_2)^4 + 1000(1.02)^7(1+i_2)^3 + ... + 1000(1.02)^9(1+i_2)\right](1.05)^8 \\
&+ \left[1000(1.02)^{10}(1+i_3)^8 + 1000(1.02)^{11}(1+i_3)^7 + 1000(1.02)^{12}(1+i_3)^6 + ... + 1000(1.02)^{17}(1+i_3)\right] \\
&= 1000(1.02)^5\left[\left(\frac{1+i_1}{1.02}\right)^5 + \left(\frac{1+i_1}{1.02}\right)^4 + ... + \left(\frac{1+i_1}{1.02}\right)\right](1.04)^5(1.05)^8 \\
&+ 1000(1.02)^{10}\left[\left(\frac{1+i_2}{1.02}\right)^5 + \left(\frac{1+i_2}{1.02}\right)^4 + ... + \left(\frac{1+i_2}{1.02}\right)\right](1.05)^8 \\
&+ 1000(1.02)^{18}\left[\left(\frac{1+i_3}{1.02}\right)^8 + \left(\frac{1+i_3}{1.02}\right)^7 + ... + \left(\frac{1+i_3}{1.02}\right)\right]
\end{aligned}
$$

Let $1 + j_1 = \frac{1+i_1}{1.02}$. Then, $j_1 = 0.009804$ and

$$\left[\left(\frac{1+i_1}{1.02}\right)^5 + \left(\frac{1+i_1}{1.02}\right)^4 + ... + \left(\frac{1+i_1}{1.02}\right)\right] = \frac{(1+j_1)^5 - 1}{j_1/(1+j_1)} = 5.148995.$$

Let $1 + j_2 = \frac{1+i_2}{1.02}$. Then, $j_2 = 0.019608$ and

$$\left[\left(\frac{1+i_2}{1.02}\right)^5 + \left(\frac{1+i_2}{1.02}\right)^4 + ... + \left(\frac{1+i_2}{1.02}\right)\right] = 5.301921.$$

Let $1 + j_3 = \frac{1+i_3}{1.02}$. Then, $j_3 = 0.029412$ and

$$\left[\left(\frac{1+i_3}{1.02}\right)^8 + \left(\frac{1+i_3}{1.02}\right)^7 + ... + \left(\frac{1+i_3}{1.02}\right)\right] = 9.134790.$$

Therefore, $V(18) = 32814.45$.

5. The present value is

$$
\begin{aligned}
PV(0) &= \left(\frac{200}{(1.03)^4} + \frac{200}{(1.03)^5}\right) + 200a_5^{0.04}(1.03)^{-5} + 200a_3^{0.05}(1.03)^{-5}(1.04)^{-5} \\
&= 350.2192 + 768.0362 + 386.1574 = 1504.413
\end{aligned}
$$

3. The solutions to each question are as follows:

   1. $240000(1.04)^5 = 291996.7$
   2. The interest amounts are $0.04 \times 240000 = 9600$.
   3. By the Principle of Equivalence, we have

   $$240000 = Xa_5^{0.04}.$$

   This gives X = 53910.51.
   4. Level installments are payable monthly, which follows

   $$240000 = Ya_{60}^{j},$$

   where $j = (1.04)^{1/12} - 1$. This gives Y = 4412.23.

4. The person retires in 30 years, when his salary is expected to be $20000 \times (1.03)^{30} = 48545.25$. The first payment will be half of this which is equal to 24272.62. The present value at age 60 of his pension is

$$24272.62 \times \ddot{a}_{26}^{0.029412} = 449717.9$$

(the precise value is 449719.051954). Here we use $\frac{1.05}{1.02} = 1.029412$ and the annuity is paid from the 60th to the 85th birthday inclusive so there are 26 payments made in advance. Therefore, the present value of this at age 30 is

$$449717.9 \times (1.05)^{-10} \times (1.04)^{-20} = 126002.9.$$

(the precise value is 126003.181173)

17 Solutions to Tutorial 4

1. To examine whether the cashflows are equivalent, we compare their present values.

   a. The present value of single payment of amount 14,802.44 at year 10 is

   $$PV(0) = \frac{14,802.44}{1.04^{10}} = 10000.$$

   b. The present value of the level annuity of 400 payable yearly in arrears for the next 10 years plus a lump sum of 10,000 is

   $$PV(0) = 400a_{10}^{0.04} + \frac{10000}{1.04^{10}} = 10000.$$

   c. The present value of the level annuity of 1,232.91 payable yearly in arrears for the next 10 years.

   $$PV(0) = 1,232.91a_{10}^{0.04} = 10000.$$

It follows that the values of these cashflows are the same, i.e. equivalent.

2. The annual yield of this investment i is the solution of the equation of value:

   $$f(i) = -2000(1+i)^8 + 300s_8^i = 0.$$

   If we solve using software, we get $i = 4.2394551$. Instead of using software, you can also use linear interpolation to approximate the solution.

3. Working in time unit of half year, the equation of value is

   $$f(i) = -5000(1+i)^{12\times 2} + 300s_{24}^i = 0.$$

   The yield i per half year is $i = 3.1491266\%$ and hence the annual yield is 6.397423%.

4. The equation of value is

   $$f(i) = -100(1+i)^7 + 20s_3^i(1+i)^4 + 25s_4^i = 0.$$

   The annual yield is 12.6209232%.

5. You are suggested to draw the time line for these cashflows. The equation of value is

   $$f(i) = -5\ddot{s}_3^i(1+i)^9 + 4s_6^i = 0.$$

   The annual yield is 5.7285486%.

18 Solutions to Tutorial 5

1. The monthly repayment can be calculated from this equation

$$X = \frac{30000}{a_6^j} = \frac{30000}{5.931847} = 5057.45,$$

where $j = (1.04)^{1/12} - 1 = 0.003274$.

The complete loan schedule is illustrated below:

Time	Repayment	Interest Content	Capital Content	Capital Outstanding
0	-	-	-	30000
1	X	98.21	4959.23	25040.77
2	X	81.98	4975.47	20065.30
3	X	65.69	4991.76	15073.54
4	X	49.35	5008.10	10065.44
5	X	32.95	5024.50	5040.94
6	X	16.50	5040.94	0

2. The monthly repayment can be calculated from this equation

$$X = \frac{800000}{a_{120}^{j}} = \frac{800000}{88.806749} = 9008.32,$$

where $j = (1.065)^{1/12} - 1 = 0.005262$.

The capital outstanding after 95th repayment (25 payments left) is

$$L_{95} = 9008.32 a_{25}^{j} = 210506.84.$$

Hence, the interest content of the 96th repayment is

$$j \times L_{95} = 0.005262 \times 210506.84 = 1107.62.$$

The capital content of the 96th repayment is

$$X - 1107.62 = 7900.70.$$

3. 1.

   a. **Extending the term of the loan by extra 2 year:** The original repayment is

$$X = \frac{100000}{a_{12}^{0.07}} = \frac{100000}{7.942686} = 12590.20,$$

   The capital outstanding after 6th repayment (6 payments left) is

$$L_6 = X a_{6}^{0.07} = 60011.68.$$

   By extending the term of the loan by extra 2 year, the revised repayment X' can be obtained (for 8 payments) from

$$X' = \frac{L_6}{a_{8}^{0.07}} = 10050.02$$

   b. **Missing the next two repayments:** From the previous result, the capital outstanding after 6th repayment $L_6 = 60011.68$. Then in 2 years, with interest at 7% per annum, this accumulates to

$$L_6 \times (1.07)^2 = 68707.37.$$

   This must now be repaind by only 4 annual repayments, so the new repayment X'' can be obtained from

$$X'' = \frac{68707.37}{a_{4}^{0.07}} = 20284.35.$$

2. The capital outstanding will accumulate (at 10%) to

$$L_6 \times (1.1)^2 = 72614.14.$$

The new repayment amount X''' is

$$X''' = \frac{72614.14}{a_4^{0.07}} = 21437.73.$$

3. The new repayment amount will be

$$\frac{72614.14}{a_8^{0.07}} = 12160.53.$$

4. You are suggested to draw the time line for these cashflows. Let X be the level of repayment of the first 6 years ($2X$ will be repaid after this period for the last 6 years). It can be obtained from

$$50000 = X(a_6^{0.04} + 2a_6^{0.05}(1.04)^{-6}) = 3769.34.$$

5. 1. Let X be the first annual repayment. Then,

$$40000 = X(v + v^2(1.02) + v^3(1.02)^2 + ... + v^{10}(1.02)^9),$$

where $v = 1/(1.065)$. By rewriting the above equation, we have

$$40000 = \frac{X}{1.02}(\frac{1.02}{1.065} + \left(\frac{1.02}{1.065}\right)^2 + \left(\frac{1.02}{1.065}\right)^3 + ... + \left(\frac{1.02}{1.065}\right)^{10}),$$

Let $i' = \left(\frac{1.065}{1.02} - 1\right) = 0.044118$. Hence,

$$40000 = \frac{X}{1.02} \cdot a_{10}^{i'},$$

and $X = 5133.91$.

2. We will calculate the capital outstanding after 7th repayment, L_7 (3 payments left). We first find the amount $X_8$of the 8th repayment,

$$X_8 = X(1.02)^7 = 5897.25.$$

So the capital outstanding after the 7th repayment is equal to the present value of the remaining 3 repayments (see the table below).

Time	7	8	9	10
Payment	$L_7 = ?$	X_8	$X_8(1.02)$	$X_8(1.02)^2$

It follows that

$$\begin{aligned} L_7 &= X_8(v + v^2 + v^3) \\ &= \frac{X_8}{(1.02)} a_3^{i'} \\ &= 15919.94, \end{aligned}$$

where v and i' are the same as above.

3. The interest content of the 8th repayment is

$$L_7 * i = 15919.94 \times 0.065 = 1034.80.$$

19 Tutorial 6

1. A company issues a bond of nominal amount 10,000 with a term of 5 years and a coupon of 6% convertible semiannually, to be redeemed at 110%. Calculate the price of the bond to give a redemption yield of 8% pa effective to an investor who pays no tax.

2. A 10-year bond of nominal amount 1,000 paying a half-yearly coupon of 10% per annum and redeemable at par. Calculate the price of this bond to give a redemption yield of 7% per annum effective after taxes to

   1. an investor who pays no taxes.
   2. an investor who is subject to income tax at 15% but no CGT.
   3. an investor who is subject to income tax at 15% and CGT at 20%.
   4. If the investor question 2.3 would like to secure a redemption yield of 9% pa, calculate the price for the bond.

3. An investor who pays income tax at 20%, but no CGT buys a 15-year bond to be redeemed at 105%, bearing semi-annual coupons of 10% pa.

   1. Calculate the price per 100 THB nominal to give a yield of 9% pa effective.
   2. Just after the 20th coupon payment, the income tax rate changes to 15%. If the investor holds the bond to redemption, calculate the realised yield on the whole transaction.

4. A 15-bond of a nominal amount of 10,000 THB, bearing semi-annual coupons of 6% pa, to be redeemed at 98%.

   1. An investor who is subject to income tax at 30% and CGT at 20% buys this bond for 9,000 THB. Calculate the net yield per annum for this transaction.
   2. If the investor wishes to obtain a net redemption yield of 7% pa, calculate the price that the investor should pay for the bond.

5. A company issues a bond of nominal 100 THB amount with a term of 15 years and a coupon of 7% convertible semiannually, to be redeemed at par.

   1. Calculate the price of the bond if it is priced at issue to give a redemption yield of 8% pa effective to a non-tax paying investor.
   2. Investor A liable to income tax at 25% and capital gain tax at 20% bought the bond on the issue date. Just after the 15th coupon payment, the investor A sold the bond to Investor B. The investor B, subject to income tax at 30% and capital gain tax at 30% paid the price that gives a net redemption yield of 6.5%. Calculate the price that investor B paid.
   3. Calculate the realised yield for Investor A's transaction.

20 Solutions to Tutorial 6

1. Using a time unit of half a year, the effective yield per half year is

$$j = (1.08)^{1/2} - 1 = 0.0392305.$$

   Then,

$$\begin{aligned} P &= 300a_{\overline{10}|}^{j} + 11000(\frac{1}{1.08})^5 \\ &= 9929.03. \end{aligned}$$

2. Using a time unit of half a year, the effect yield per half year is

$$j = (1.07)^{1/2} - 1 = 0.034408.$$

1. Per 1000 nominal,

$$\begin{aligned} P &= 50a^{j}_{\overline{20}|} + 1000(\frac{1}{1.07})^{10} \\ &= 1222.79. \end{aligned}$$

2. With an income tax rate at 15%,

$$\begin{aligned} P &= 42.5a^{j}_{\overline{20}|} + 1000(\frac{1}{1.07})^{10} \\ &= 1115.62. \end{aligned}$$

3. Since

$i^{(2)} = 2 \times j = 0.0688161 < (1 - t_1)\frac{D}{R} = (1 - 0.15)\frac{0.1}{1} = 0.085$. Therefore, no capital gain tax (CGT) is payable.

$$\begin{aligned} P &= 42.5a^{j}_{\overline{20}|} + 1000(\frac{1}{1.07})^{10} \\ &= 1115.62, \end{aligned}$$

which is similar to the previous result.

4. A redepmtion yield of

9% p.a. is equivalent to a yield of

$$k = (1.09)^{1/2} - 1 = 0.0440307.$$

Since $k^{(2)} = 2 \times k = 0.0880613 < (1 - t_1)\frac{D}{R} = (1 - 0.15)\frac{0.1}{1} = 0.085$. Therefore, no capital gain tax (CGT) is payable.

$$\begin{aligned} P &= 42.5a^{k}_{\overline{20}|} + (1000 - 0.2(1000 - P))(\frac{1}{1.09})^{10} \\ P &= 978.07(< 1000)., \end{aligned}$$

3. The solutions are given below:

4. Using a time unit of half a year, the effect yield per half year is

$$j = (1.09)^{1/2} - 1 = 0.0440307.$$

Then,

$$\begin{aligned} P &= 4a^{j}_{\overline{30}|} + 105(\frac{1}{1.09})^{15} \\ &= 94.73. \end{aligned}$$

2. In unit of half-year, the timeline of the transaction is shown in the table below:

Time	0	1	2	⋯	20	21	⋯	30
Payment	-94.73	4	4	⋯	4	$4 + 0.25$	⋯	$4 + 0.25 + 105$

The equation of value is

$$f(i) = -94.73(1+i)^{30} + 4s^{i}_{\overline{30}|} + 0.25s^{i}_{\overline{10}|} + 105 = 0.$$

The change in the tax rate is quite small, so we do not expect a large change in the yield.

By trial and error, in time unit of half a year, we have

$$f(0.044) = 3.2373$$
$$f(0.045) = -2.6999.$$

Therefore, the approximate of i is

$$i \approx 0.044545 \text{ per half-year,}$$

and hence 9.107% effective per year.

4. The solutions are as follows:

    1. CGT is payable because the price paid is $9000 < 9800$ (the redemption amount)

In unit of half-year, the equation of value is

$$f(i) = -9000(1+i)^{30} + 0.7 \times 300 s_{\overline{30|}}^{i} + 9800 - 0.2(9800 - 9000) = 0.$$

By trial and error, we obtain

$$f(0.024) = 380.741$$
$$f(0.025) = -18.541.$$

Therefore, the approximate of i is

$$i \approx 0.024954 \text{ per half-year,}$$

and hence 5.05% effective per year.

2. The investor wishes to obtain a net yield of 7% p.a. effective, which is greater than 5.05%. The price paide will be less than 9000. So CGT is payable.

By the principle of equivalence,

$$P = 210 a_{\overline{30|}}^{j} + (9800 - 0.2(9800 - P))(\frac{1}{1.07})^{15},$$

where

$$j = (1.07)^{1/2} - 1 = 0.034408.$$

Solving for P results in $P = 7258.90\,(< 9800)$.

5. The solutions are as follows:

    1. Using a time unit of half a year, the effect yield per half year is

    $$j = (1.08)^{1/2} - 1 = 0.0392305.$$

    Then,

    $$P = 3.5 a_{\overline{30|}}^{j} + 100(\frac{1}{1.08})^{15}$$
    $$= 92.62.$$

```
2. The bond sold to the investor B has 15 half-years to run.
```

From B's tax position, $i^{(2)} = 2 \times (1.065^{1/2} - 1) = 0.0639767 < (1-t_1)\frac{D}{R} = (1-0.3)\frac{7}{100} = 0.049$. Therefore, capital gain tax (CGT) is payable.

$$P = 2.45 a^{j}_{\overline{15}|} + (100 - 0.3(100 - P))(\frac{1}{1.065})^{7.5},$$

where

$$j = (1.065)^{1/2} - 1 = 0.0319884.$$

This results in $P = 89.16$.

3. Investor A sold the bond for less than price paid for the bond, so A does not pay CGT.

In unit of half-year, the equation of value is

$$f(i) = -96.62(1+i)^{15} + 0.75 \times 3.5 s^{i}_{\overline{15}|} + 89.16 = 0.$$

By trial and error, we obtain

$$f(0.026) = 0.463$$
$$f(0.027) = -1.192.$$

Therefore, the approximate of i is

$$i \approx 0.026280 \text{ per half-year},$$

and hence approximately 5.325% p.a. effective.

21 Tutorial 7

1. Consider a 5-year bond of a nominal amount of 100 THB , bearing annual coupons of 8% pa, to be redeemed at 110%. The bond was issued in July 2012 and its issue price was 100%. With reference to the CPI given in the lecture note, show that the real yield to an investor who is not subject to tax is 8.82%.
2. A particular transaction will provide an effective rate of interest of 6% per annum. The period of the transaction is one year.
   1. If the annual inflation rate over this one year period will be constant and equal to 2.5%, calculate the real yield on this transaction.
   2. Calculate the constant annual rate of inflation at which the investor will obtain a real yield on this transaction of at least 4.25% per annum.
   3. Calculate the real yield on this transaction if inflation will be 3% pa for the first nine months of the year and then 3.25%pa for the remaining three months.
3. Consider a 3-year bond of a nominal amount of 100 THB, bearing annual coupons of 6% pa, to be redeemed at par. The bond was issued in 15 January 2015 and its issue price was 95%.

   An investor who pays no tax purchased this bond on the issue date and held it until redemption.

   1. Show that the yield obtained by this investor is approximately 7.94%.
   2. Show that the real yield on this investment is approximately 1.80% assuming the CPI values are given below:

Year	2015	2016	2017	2018
CPI on 15 January	100	103.3	111.0	119.5

4. An investor who is not liable to tax had the choice of purchasing two investments made on 1 Apr 2018.

   (A) A 10-year bond of a nominal of 100 THB, bearing a half-yearly coupon of 9% per annum and redeemable at par. The issue price was at 110%.

   (B) A 10-year index linked bond at a price of 135 THB per 100 THB nominal, bearing a half-yearly coupon of 4% per annum and redeemable at par. The CPI base figure for indexing is 100.24 and the CPI figure applicable to the next coupon (payable on 1 Oct 2018) is 145.68. (Here 145.68 is the CPI index on 1 Apr 2018).

   Assume that CPI will grow at a rate of 2.5% per annum from its latest known value of 145.68 on 1 Apr 2018.

   1. Calculate the real rate of return (yield) per annum earned on both investments A and B.
   2. Determine which of the two investments yielded the highest real rate of return per annum.

5. An investor who is not liable to tax had the choice of purchasing two investments made on 15 Jan 2013.

   (A) 50,000 was placed in a 5-year term special saving account. The effective rate of interest was 2.5% p.a. for the first year, 3.5 % p.a. for the second year, 4.5 % p.a. for the third year, 5.5% p.a. for the fourth year and 6.5% p.a. for the fifth year.

   (B) 50,000 was used to purchase an annuity payable annually in arrears for 5 years to yield 6% p.a. effective.

   Assume that the values of CPI are as follows:

Year	2013	2014	2015	2016	2017	2018
CPI on 15 January	100	104.17	110.17	111.08	112.67	114.83

   1. Calculate the real rate of return (yield) per annum earned on both investments A and B.
   2. Determine which of the two investments yielded the highest real rate of return per annum.

22 Solutions to Tutorial 7

1. The timeline of the transaction is given in the following table.

Time	0	1	2	3	4	5
Year	2012	2013	2014	2015	2016	2017
Cashflow	-100	8	8	8	8	18 + 100
CPI	97.22	99.17	101.32	100.25	100.36	100.53
Real value of cashflow at $t = 0$	-100	$\frac{(8)(97.22)}{99.17}$	$\frac{(8)(97.22)}{101.32}$	$\frac{(8)(97.22)}{100.25}$	$\frac{(8)(97.22)}{100.36}$	$\frac{(8)(97.22)}{100.53}$
Real value of cashflow at $t = 0$	-100	= 7.84	= 7.68	= 7.76	= 7.75	= 114.11

The real yield i' p.a. effective solve the equation of value as follows:

$$f(i') = -100 + 7.84v + 7.68v^2 + 7.76v^3 + 7.75v^4 + 114.11v^5 = 0,$$

where $v = 1/(1 + i')$.

This gives $i' \approx 8.82\%$.

2. We know that when the annual rate of inflation is constant and equal to q, the real yield i' and the monetary yield i satisfies

$$(1+i) = (1+q)(1+i').$$

1. Given

$i = 0.06$ and $q = 0.025$, we have

$$i' = \frac{1.06}{1.025} - 1 = 0.0341463 = 3.4146341\%.$$

2. Given

$i = 0.06$ and $i' = 0.0425$, we have

$$i' = \frac{1.06}{1.0425} - 1 = 0.0167866 = 1.6786571\%.$$

3. Given

$q = 0.03$ p.a. for 9 months and $q = 0.0325$ p.a. for the next 3 months,

$$i' = \frac{1.06}{(1.03)^{9/12}(1.0325)^{3/12}} - 1 = 0.0285027 = 2.8502689\%.$$

3. The timeline of the transaction is given in the following table.

Time	0	1	2	3
Year	2015	2016	2017	2018
Cashflow	-95	6	6	106
Real value of cashflow at $t = 0$	-95	5.81	5.41	88.70

1. The equation of value is

$$f(i) = -95 + 6a_{\overline{3}|}^{i} + \frac{100}{(1+i)^3} = 0.$$

This gives $i \approx 7.938\%$.

2. The real yield

i' p.a. effective solve the equation of value as follows:

$$f(i') = -95 + 5.81v + 5.41v^2 + 88.70v^3 = 0,$$

where $v = 1/(1+i')$.
This gives $i' \approx 1.80\%$.

4. Question 4 is not examinable.

Investment A:

Date	Real payment (monetary payment $\times \frac{\text{Q(APR 2018)}}{\text{Q(Date)}}$)
1 APR 2018	-110
1 OCT 2018	$4.5 \times \frac{\text{Q(APR 2018)}}{\text{Q(OCT 2018)}} = 4.5 \times \frac{\text{Q(APR 2018)}}{\text{Q(APR 2018)} \times (1.025)^{1/2}} = 4.5 \times \frac{1}{(1.025)^{1/2}}$

Date	Real payment (monetary payment $\times \frac{Q(\text{APR 2018})}{Q(\text{Date})}$)
1 APR 2019	$4.5 \times \frac{Q(\text{APR 2018})}{Q(\text{APR 2019})} = 4.5 \times \frac{Q(\text{APR 2018})}{Q(\text{APR 2018}) \times (1.025)^1} = 4.5 \times \frac{1}{(1.025)^1}$ ⋮
1 APR 2028	$(100+4.5) \times \frac{Q(\text{APR 2018})}{Q(\text{APR 2028})} = (100+4.5) \times \frac{Q(\text{APR 2018})}{Q(\text{APR 2018}) \times (1.025)^{10}} = (100+4.5) \times \frac{1}{(1.025)^{10}}$

The real yield i' p.a. effective solve the equation of value as follows:

$$f(i') = -110 + 4.5\left(\frac{1}{((1+i')(1.025))^{1/2}} + \frac{1}{((1+i')(1.025))^{1}} + \cdots + \frac{1}{((1+i')(1.025))^{10}}\right) = 0,$$

Letting $(1+j) = ((1+i')(1.025))^{1/2}$ gives the equation in terms of j as follows:

$$f(j) = -110 + 4.5 a_{\overline{20}|}^{j} + \frac{100}{(1+j)^{20}} = 0.$$

By linear approximation, we obtain

$$j \approx 0.0378$$

and the real yield per annum is

$$i' \approx 0.0507.$$

Investment B:

The coupon payment is 2 THB increased with inflation, paid every 6 months and the redemption payment is 100 THB also increased with inflation.

We first increase the payments with inflation approximately lagged by 6 months to see how much is acually paid, which gives nominal payments and then calculate their real values in terms of their purchasing power at 1 APR 2018.

Date	CPI (Date − 6/12)	Nominal payment at Date	Real payment
1 OCT 2018	145.68	$2 \times \frac{145.68}{100.24}$	$2 \times \frac{145.68}{100.24} \frac{Q(\text{APR 2018})}{Q(\text{OCT 2018})} = 2 \times \frac{145.68}{100.24} \times \frac{1}{(1.025)^{1/2}}$
1 APR 2019	$145.68 \times 1.025^{1/2}$	$2 \times \frac{145.68 \times 1.025^{1/2}}{100.24}$	$2 \times \frac{145.68 \times 1.025^{1/2}}{100.24} \frac{Q(\text{APR 2018})}{Q(\text{APR 2019})} = 2 \times \frac{145.68}{100.24} \times \frac{1}{(1.025)^{1/2}}$
⋮	⋮	⋮	⋮
1 OCT 2027	145.68×1.025^{9}	$2 \times \frac{145.68 \times 1.025^{9}}{100.24}$	$2 \times \frac{145.68 \times 1.025^{9}}{100.24} \frac{Q(\text{APR 2018})}{Q(\text{OCT 2027})} = 2 \times \frac{145.68}{100.24} \times \frac{1}{(1.025)^{1/2}}$
1 APR 2028	$145.68 \times 1.025^{9.5}$	$(100+2) \times \frac{145.68 \times 1.025^{9.5}}{100.24}$	$(100+2) \times \frac{145.68 \times 1.025^{9.5}}{100.24} \frac{Q(\text{APR 2018})}{Q(\text{APR 2028})} = (100+2) \times \frac{145.68}{100.24} \times \frac{1}{(1.025)^{1/2}}$

The present value at 1 APR 2018 of the real payments at the real yield i' p.a. is

$$f(i') = -135 + 2 \times \frac{145.68}{100.24} \times \frac{1}{(1.025)^{1/2}} \times (2a_{\overline{10}|}^{2(i')}) + 100 \times \frac{145.68}{100.24} \times \frac{1}{(1.025)^{1/2}} \times \frac{1}{(1+i')^{10}} = 0.$$

We solve for i', which results in

$$i' \approx 0.0481$$

per annum.

`2. Investment A yields the higher real rate of return than investment B, hence investment A is preferre`

5. Investment A: The payment received in 5 years is

$$50000(1.025)(1.035)(1.045)(1.055)(1.065) = A.$$

Hence, the real payment in terms of its purchasing power at 15 Jan 2013 is

$$A \times \frac{Q(\text{Jan } 13)}{Q(\text{Jan } 18)}.$$

Therefore, the real yield i% p.a. is

$$50000(1+i')^5 = A \times \frac{Q(\text{Jan } 13)}{Q(\text{Jan } 18)} = A \times \frac{114.83}{100},$$

which gives $i' = 1.64\%$.

Investment B: First we find an annual income of the annuity.

$$\text{Annual income } = \frac{50000}{a_{\overline{5}|}^{0.06}} = 1.186982 \times 10^4.$$

The real yield on investment B is the solution i' p.a. to the equation follows:

$$50000 = \left(\frac{100}{104.17}v + \frac{100}{110.17}v^2 + \cdots + \frac{100}{114.83}v^3\right) \times 1.186982 \times 10^4,$$

At 1.64%, the RHS of the above equation is equal to the price of B at 1.64% = 50614.35.

Therefore, the real yield of B is greater than 1.64%. Hence B gives greater real yield.

23 Tutorial 8

1. 10,000 was placed in a 3-year term special saving account on 15 Jan 2015. The effective rate of interest was 2.5% p.a. for the first year, 3.5 % p.a. for the second year, 4.5 % p.a. for the third year. Assume that the values of CPI are as follows:

Year	2015	2016	2017	2018
CPI on 15 January	100	104.08	106.67	108.83

What is the real rate of return (yield) per annum earned on this investment?

A. 0.618%

B. 0.629%

C. 0.724%

D. 0.762%

E. none of the above

2.

$$Tutorial8-2$$

A company's cash position, measured in million of bahts, follows a generalised Wiener process with a drift rate of 0.25 per quarter and a variance rate of 9 per quarter. The initial cash position is 35. What is the expected cash position at the end of 6 months?

A. 35.25

B. 35.5

C. 35.75

D. 36

E. none of the above

3.

$$Tutorial8-3$$

Suppose that data on a stock price at the end of 63 consecutive trading days gives the sum of the daily returns

$$\sum_{i=1}^{62} \ln(S_i/S_{i-1}) = 0.25$$

and the sum of the daily returns squared

$$\sum_{i=1}^{62} \left(\ln(S_i/S_{i-1})\right)^2 = 0.0042.$$

Assume that there are 252 trading days per year. What is the estimated value of the stock price volatility per annum?

A. 9.28%

B. 10.72%

C. 11.42%

D. 12.37%

E. none of the above

4. What is the standard error (per annum) of the estimate obtained in Question

$$Tutorial8-3$$

?

A. 0.72%

B. 0.85%

C. 0.93%

D. 1.02%

E. none of the above

5. With the drift rate and the variance rate as given in the Question

$$Tutorial8-2$$

, What is the company's initial cash position so that the company has a less than 5% chance of a negative cash position by the end of 1 year? Note that $\Pr(Z \leq 1.645) = 0.95$ for a standard normal random number Z.

   A. 6.48

   B. 6.84

   C. 7.29

   D. 7.92

   E. none of the above

6. Suppose that a stock price follow geometric Brownian motion with an initial price of 40 TBH, an expected return of 8% per annum and a volatility of 30% per annum. Using monthly time steps and the random samples from a normal distribution given in the table below, what is the simulated value of the stock price path at time 3 months?

Period (n)	1	2	3
Random sample from $N(0,1)$ for period n	0.62	1.34	-0.76

   A. 45.92

   B. 46.78

   C. 47.40

   D. 48.80

   E. none of the above

7. An investor who was not subject to tax purchased an index-linked bond issued on 15 January 2016 with a term of 2 years. The annual coupon rate was 2% p.a. payable half-yearly in arrears and the redemption rate was 100%. The coupons and redemption payments were adjusted with reference to the CPI value of 3 months before the payments were made.

   The value of the inflation index at particular dates are as follows:

Date	Oct 15	Jan 16	Apr 16	Jul 16	Oct 16	Jan 17	Apr 17	Jul 17	Oct 17	Jan 18
CPI (Date)	96	97	99	100	102	104	105	109	111	112

   1. Write down the value of the base CPI figure.
   2. Calculate the actual payments per 100 THB nominal received by the investor. Clearly state the date on which each payment was received.
   3. Calculate the real payments per 100 THB nominal in terms of their purchasing power at 15 January 2016.
   4. Calculate the purchase price of the bond per 100 THB nominal if the investor obtained a **real** redemption yield of 0.79% p.a. effective on the bond.

8. Consider a stock that pays no dividends, provides an expected return of 10% per annum with continuous compounding and has a volatility of 25% per annum. Assume that the stock price follows geometric Brownian motion and its current stock price is 40 THB.

1. Find the probability distribution of the logarithm of the stock price S_T in 6 months' time.
2. Calculate the mean and the standard deviation of $\ln S_T$ in 6 months' time.
3. Find the 95% confidence interval of S_T in 6 months' time.

24 Solutions to Tutorial 8

1. Let $i'\%$ be the real rate of return (yield) per annum earned on this investment. It follows that

$$(1+i')^3 = (1.025)(1.035)(1.045)\frac{100}{108.83} = 0.00618.$$

2. The drift rate is $(0.24)(4) = 1$ per year and the variance rate is $(9)(4) = 36$ per year.

At the end of 6 months, the probability distribution of the cash position is normally distributed with mean

$$35 + 1(\frac{1}{2}) = 35.5$$

and variance

$$36(\frac{1}{2}) = 18.$$

Therefore, the expected cash position at the end of 6 months is 35.5.

3. The estimates of the standard deviation of the **daily returns** are given by

$$\begin{aligned} s &= \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(u_i - \bar{u})^2} \\ &= \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}u_i^2 - \frac{1}{n(n-1)}\left(\sum_{i=1}^{n}u_i\right)^2} \\ &= \sqrt{\frac{1}{62-1}(0.0042) - \frac{1}{62(62-1)}(0.25)} \\ &= 0.0072337. \end{aligned}$$

Because we are using observations at intervals of τ measured in years, the estimate of the **annualised volatility** is given by

$$\hat{\sigma} = \frac{s}{\sqrt{\tau}}. = \sqrt{252}s = 0.1148319 = 11.48\%.$$

4. The **standard error of this estimate** is approximately

$$\hat{\sigma}/\sqrt{2n} = \frac{0.1148319}{\sqrt{2(62)}} = 0.0103122 = 1.03\%.$$

5. Recall that the drift rate is $(0.24)(4) = 1$ per year and the variance rate is $(9)(4) = 36$ per year. It follows that

$$X(1) \sim N(X(0)+1, 36) = (X(0)+1) + 6N(0,1),$$

where $X(0)$ is the company's initial cash position.

The company's initial cash position so that the company has a less than 5% chance of a negative cash position by the end of 1 year satisfies

$$\begin{aligned} \Pr((X(0)+1) + 6Z < 0) &= 0.05 \\ \Pr(Z < \frac{-(X(0)+1)}{6}) &= 0.05 \\ \frac{X(0)+1}{6} &= 1.645, \end{aligned}$$

where $Z \sim N(0,1)$. This implies that $X(0) = 8.87$.

7. Following that the proportional return on stocks are normally distributed, the discrete-time version of the model is

$$\frac{\Delta S}{S} = \mu\Delta t + \sigma\sqrt{\Delta t}\epsilon$$

where ϵ has a standard normal distribution. Hence,

$$\begin{aligned} S_t &= S_{t-1} + \Delta S \\ &= S_{t-1} + S_{t-1}(\mu\Delta t + \sigma\sqrt{\Delta t}\epsilon) \\ &= S_{t-1} \cdot (1 + \mu\Delta t + \sigma\sqrt{\Delta t}\epsilon) \end{aligned}$$

where in this case $\Delta t = 1/12, \mu = 0.08$ and $\sigma = 0.3$.

The simulated values of the stock price path at time 3 months are shown in the following table.

Time (t)	S_t
Δt	$40(1 + (0.08)(1/12) + (0.3)\sqrt{(1/12)}(0.62)) = 42.414$
$2\Delta t$	$42.414(1 + (0.08)(1/12) + (0.3)\sqrt{(1/12)}(1.34)) = 47.619$
$3\Delta t$	$42.414(1 + (0.08)(1/12) + (0.3)\sqrt{(1/12)}(-0.76)) = 44.803$

8. The questions are similar to the previous tutorial. Only answers are given here.

   1. CPI(OCT 2015) = 96.

   2.

Date	(Monetary) Payment
Jul 2016	$\frac{2\%}{2}\frac{\text{Q(Date - 3/12)}}{\text{Q(Jan 16 - 3/12)}} = 1.03125 = M_1$
Jan 2017	$\frac{2\%}{2}\frac{\text{Q(Oct 16)}}{\text{Q(Oct 15)}} = 1.0625 = M_2$
Jul 2017	$\frac{2\%}{2}\frac{\text{Q(Apr 17)}}{\text{Q(Oct 15)}} = 1.09375 = M_3$
Jan 2018	$(100+1)\frac{\text{Q(Oct 17)}}{\text{Q(Oct 15)}} = 116.7812 = M_4$

3.

Date	Real Payment = Monetary Payment $\times \frac{\text{Q(Jan 16)}}{\text{Q(Date)}}$
Jul 2016	$1.03125\frac{97}{100} = 1.0003125$
Jan 2017	$1.0625\frac{97}{104} = 0.9909856$
Jul 2017	$1.09375\frac{97}{109} = 0.9733372$
Jan 2018	$116.7812\frac{97}{112} = 101.1408607$

4. The purchase price of the bond per 100 THB nominal if the investor obtained a real redemption yield o

$$\begin{aligned} P &= \frac{1.0003125}{(1+0.0079)^{1/2}} + \frac{0.9909856}{(1+0.0079)^{2/2}} + \frac{0.9733372}{(1+0.0079)^{3/2}} + \frac{101.1408607}{(1+0.0079)^{4/2}} \\ &= 102.50. \end{aligned}$$