Lookup Formulas and Data Tables

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Contents

1 Dates and date functions

Dates and date functions will be discussed in this section. Depending on the system setting, the dates used in this section corresponds to the U.S. setting, i.e. month/day/year. Thus, 7/1/2017 corresponds to 1 July 2017.

The following date formats are recognized by Excel:

- 7/1/2017, 7/1/17
- 7-1-2017, 7-1-17,
- 7-1/2017, 7-1/17,
- July 1, 2017
- 1-Jul-2017

if the date input is without the year, then it is the date on the current year, i.e. 7-1, 7/1, July 1, Jul 1, corresponds to 1 July 2018.

1.1 Inconsistent date entries

Example 1.1. *What will be the output when we enter dates by using the following two digits for the years:*

1. *7/1/01*
2. *7/1/60*

Excel interprets two digit years between 00 and 29 as 21st century dates, and two digit years between 30 and 99 as 20th century dates.

1.2 Date serial number

Excel stores dates as sequential serial numbers so that they can be used in calculations. For example,

- 1 January 1900 is serial number 1,
- 1 January 2018 is serial number 43101 because it is 43101 days after 1 January 1900.

1.3 Date functions

The list of date functions can be seen by choosing

Formulas → **Function Library** → **Function Library** → **Date & Time**.

Function	**Description**
DATE	Returns the serial number of a particular date
DATEVALUE	Converts a date in the form of text to a serial number
DAY	Converts a serial number to a day of the month
DAYS	Returns the number of days between two dates
WEEKDAY	Converts a serial number to a day of the week
NETWORKDAYS	Returns the difference between two dates, excluding weekend days (Saturdays and Sundays)
WORKDAY	Returns a number that represents a date that is the indicated number of working days before or after
EDATE	Returns the serial number that represents the date that is the indicated number of months before or a
EOMONTH	Returns the serial number for the last day of the month that is the indicated number of months before
YEARFRAC	Returns a decimal value that represents fractional years between two dates

For a complete list, see the date and time functions available online. More detailed examples are also given in the Excel lab.

2 Working with Texts

In this section, we will see how Excel handles text strings, and how we use text functions to modify and manipulate text strings. First, we give a note regarding texts in Excel.

- A single cell can hold up to 32,000 characters. In case you need to display a lot of text in a worksheet, then use a text box (Choose Insert ⇒ Text ⇒ Text Box). It will be easier to edit texts in the text box than in cells.
- Sometimes, when you download numerical data from the internet or database, the imported values are treated as text, i.e. when you do a calculation with such data, you will get a #VALUE error. When a number is not treated as a number, there will be an error indicator. By clinking to expand a list of options, you can then convert it to the number.
- Another issue that you may encounter is about currency that uses different characters to separate thousands or decimals. https://en.wikipedia.org/wiki/Decimal_separator

2.1 Text functions

The following Excel functions can be used to modify text strings in the format you need. Alternatively, one may extract data by using the Convert Text To Columns Wizard (choose Data ⇒ Text To Columns).

1. RIGHT(text,[n]) returns the last n characters in a text string.
2. LEFT(text, [n]) returns the first n characters in a text string.
3. MID(text, start_num, num_chars) returns num_char characters from a text string, starting at start_num.
4. TRIM(text) removes all spaces from text except for single spaces between words. Use TRIM when text strings have irregular spacing.
5. LEN(text) returns the number of characters in a text string.
6. FIND(find_text, within_text, [start_num]) return the location at or after character start_num of the first character of find_text in within_text.
7. SEARCH(find_text, within_text, [start_num]) has the same syntax as FIND, but it is not case sensitive.
8. SUBSTITUTE(text, old_text, new_text, [instance_num]) is used to replace new_text for old_text in a text string. Here Instance_num is optional. It specifies which occurrence of old_text you want to replace with new_text. If you specify instance_num, only that instance of old_text is replaced. Otherwise, every occurrence of old_text in text is changed to new_text.

2.2 Character codes

Excel uses the standard ASCII character set. Therefore, each character has its own code. For example, to get the code number of “A”, simply type

=CODE(“A”), which returns the code number 65.

The CHAR function reverses the role of CODE function, i.e.

=CHAR(65), which returns the letter A. The input for the CHAR function should be a value between 1 and 255.

For the complete list of ASCII codes, please visit https://theasciicode.com.ar

Example 2.1. *Create an Excel file to list all the first 255 ASCII codes. It is a good idea to compare the outputs with those from the website above.*

2.3 Determining whether two string are identical

In order to determine whether strings in cell A1 and A2 have the same contents, we use = A1 = A2, which returns either TRUE or FALSE. Note that the comparison is not case-sensitive.

Alternative, the function that provides an exact, case-sensitive comparison is EXACT function.

2.4 Joining two or more cells

To join two or more cells, Excel uses an ampersand &. For example if the string "the effective interest rate per annum" is in A1 and the value of 5% (formatted value) is in B1. Then use the formula

= A1 & " is " & B1, which returns the effective interest rate per annum is 0.05.

A better solution is to use the TEXT function to format the value as text as follows.

= A1 & " is " & TEXT(B1," 0.00%") , which returns the effective interest rate per annum is 0.05.

Use **Home Format Format cells** to obtain the list of various text formats.

For example, if A1 contains the principle of $10,000, then

= "The principle is" & TEXT(A1, " $#,##0"), which returns The principle is $10,000.

The following table gives examples that are format with TEXT function.

```
=TEXT(1234.567,"$#,##0.00")
Currency with a thousands separator and 2 decimals, like $1,234.57. Note that Excel rounds the value to

=TEXT(TODAY(),"MM/DD/YY")
Today's date in MM/DD/YY format, like 03/14/12

=TEXT(TODAY(),"DDDD")
Today's day of the week, like Monday

=TEXT(NOW(),"H:MM AM/PM")
Current time, like 1:29 PM

=TEXT(0.285,"0.0%")
Percentage, like 28.5%

=TEXT(4.34 ,"# ?/?")
Fraction, like 4 1/3

=TRIM(TEXT(0.34,"# ?/?"))
Fraction, like 1/3. Note this uses the TRIM function to remove the leading space with a decimal value.

=TEXT(12200000,"0.00E+00")
Scientific notation, like 1.22E+07
```

```
=TEXT(1234567898,"[<=9999999]###-####;(###) ###-####")
Special (Phone number), like (123) 456-7898

=TEXT(1234,"0000000")
Add leading zeros (0), like 0001234
```

The TEXT function will convert numbers to text. It is best practice to keep your original values in one cell, and formatted number in another cell. When you do calculation, you should refer to the cells containing original values.

3 Interest Rate, Present Values and Cashflows

3.1 Working with a single cashflow in Excel

3.1.1 How to calculate the future value of a single cashflow

Suppose an amount C is deposited in an account that pays a fixed interest at the rate of $i\%$ per time units. Then after t time units, the deposit will have accumulated to

$$C(1+i)^t.$$

In Excel, the function FV calculates the future value of a single investment (and also periodic constant payments) and a constant interest rate.

The syntax of the function is:

$$\text{FV(rate, nper, pmt, [pv], [type])},$$

where

- rate is the interest rate per period
- nper is the number of periods over which the investment is made.
- pmt (used for an annuity type) is the payment made each period and cannot change over the life of the annuity.
- pv (optional) is an additional cash flow now (time 0)
- fv is an additional cash flow nper periods from now.
- type (optional) is an optional argument that defines whether the payment is made at the start or the end of the period:

  0 - the payment is made at the end of the period;

  1 - the payment is made at the start of the period.

  If the [type] argument is omitted, it takes on the default value of 0.

The following timeline illustrates the cashflows used for the FV function (assuming that the payments (pmt) are made in arrears).

Note For a single cash flow, we set pmt argument to be 0, as there are no ongoing payments after the initial investment.

Example 3.1. *You plan to invest today with an interest rate of 3% per year effective. How much money will you accumulate at the end of 2 years?*

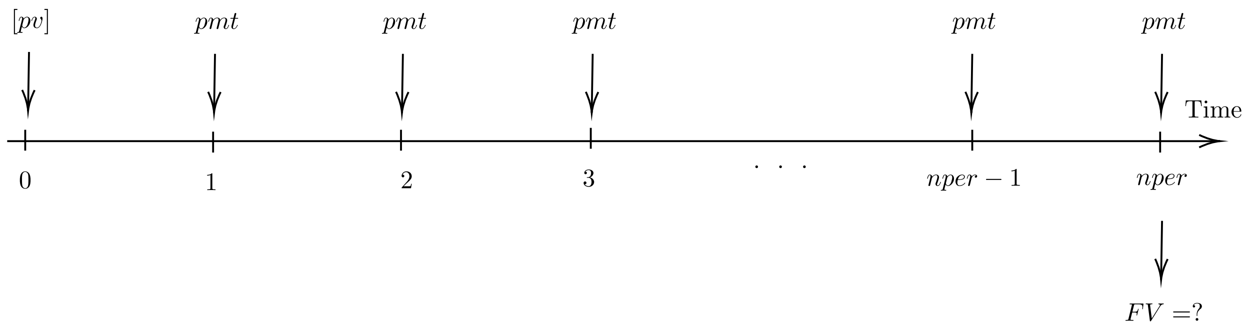

Figure 1: Timeline of cashflows for FV function

Solution: The accumulation of today in 2 years can be calculated by Excel as

$$\text{FV}(3\%, 2, 0, -100).$$

3.1.2 How to calculate the present value of a single cashflow

Similarly, the present value of a future cashflow C required at time t time units at a fixed interest of $i\%$ per time units can be calculated as

$$\frac{C}{(1+i)^t}.$$

In Excel, the function PV calculates the present value of a single investment (and also periodic constant payments) and a constant interest rate. The syntax of the function is:

PV(rate, nper, pmt, [fv], [type])

where fv is an additional cash flow nper periods from now.

Example 3.2. *How much should you deposit into the account with an interest of 8% so that 10 years from now its value would be ?*

Solution: The present value of today in 10 years can be calculated by Excel as

$$\text{PV}(8\%, 10, 0, -1000).$$

3.2 Present values of a series of cashflows

Consider a series of cashflows defined by

1. the times of payments (cashflows), denoted by $t_1, t_2, \ldots$, and
2. the amount of payments, denoted by C_r (in short for C_{t_r}), which will be paid at time t_r, for $r = 1, 2, \ldots$. The amounts can be positive or negative

The present value at any time t of this series of cashflow is

$$PV(t) = \sum_{r=1}^{\infty} C_r(1+i)^{t-t_r} = \sum_{r=1}^{\infty} C_r v^{t_r - t}$$

where i is the effective rate of interest and $v = 1/(1+i)$.

Notes 1. At a fixed effective rate of interest, the original series of cashflows is equivalent to a single payment of amount $PV(t)$ at time t.

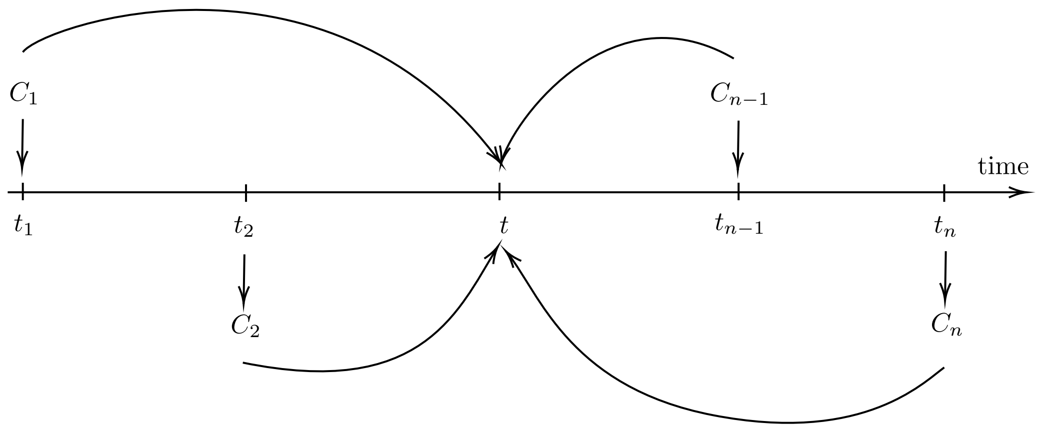

Figure 2: Timeline of a series of cashflows

2. If two different series of cashflows have the same PV at one time at a given effective rate of interest, then they have the same PV at any time at that effective rate of interest.

3.2.1 Level Annuities certain

An *annuity* is a regular series of payments (cashflows). When the payments are certain which are payable for a definite period of time, we call it an *annuity certain.*

- If the payments are made at the end of each time period, they are paid *in arrear.*
- Otherwise, payments are made at the beginning of each time period, they are pain *in advance.*
- An annuity paid in advance is also known as an *annuity due*
- If each payment is for the same amount, this is a *level* annuity.

Example 3.3. *Let i be the constant effective rate of interest per time unit. In Excel, one can calculate the accumulated value of a level annuity certain having cashflow of pmt unit at the end of each of the next n time units by*

$$FV(i\%, n, pmt, 0).$$

The cashflows of this annuity is shown in the timeline below.

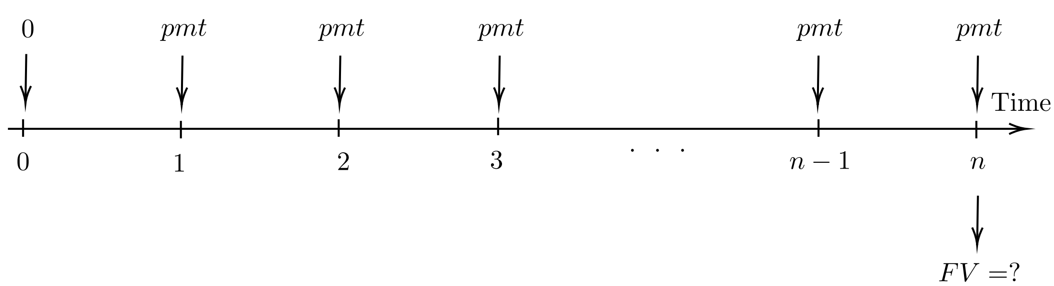

Figure 3: Level Annuity Certain

Note *The last argument of the above syntax, [pv] (optional) which is an additional cash flow now (at time 0), has been set to 0.*

Example 3.4. *Let i be the constant effective rate of interest per time unit. In Excel, one can calculate the present value at time 0 of a level annuity certain having cashflow of pmt unit at the end of each of the next n time units by*

$$PV(i\%, n, pmt, 0).$$

The timeline of these cashflows is shown in the figure below:

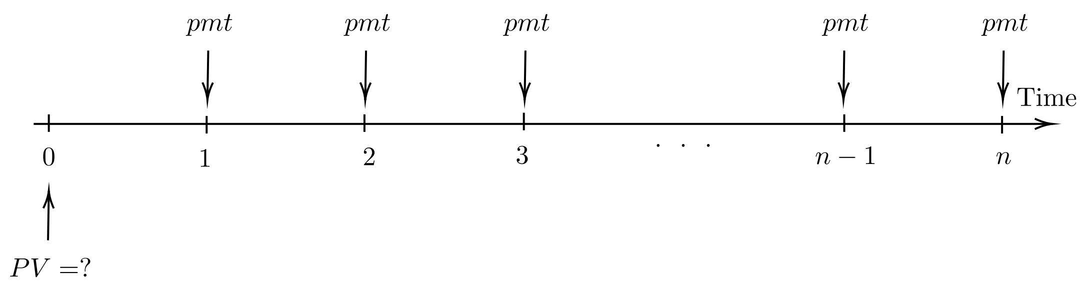

Figure 4: Level Annuity Certain

Note The last argument of the above syntax, [fv] (optional) which is an additional cash flow at time n, has been set to 0.

Example 3.5. *Given the effective rate of interest of* 8% *p.a., use Excel to calculate*

1. *the accumulation at 12 years of payable yearly in arrear for the next 12 years.*
2. *the present value now of ,000 payable yearly in arrear for the next 6 years.*
3. *the present value now of ,000 payable half-yearly in arrear for the next 12.5 years.*

Example 3.6. *Let $i = 4\%$ effective per time unit. Cashflows are given as follows:*

- $C_1 = 200$ *at time* $t_1 = 1$.
- $C_2 = 300$ *at time* $t_2 = 3$.
- $C_3 = -100$ *at time* $t_3 = 5$.
- $C_4 = -50$ *at time* $t_4 = 6$.

Develop the model using Excel to calculate

1. *the accumulation at time $t = 7$.*
2. *the present value at time $t = 0$.*
3. *the present value at time $t = 4$.*

4 Lookup Formulas and Data Tables

4.1 Lookup function

Lookup function can be used to return (or retrieve) a value from a table or a range of data. For example, use lookup function to find some information such as date of birth, address etc. from a customer database. Two useful lookup functions are `VLOOKUP` and `HLOOKUP`.

- The `VLOOKUP` function searches a value in the first column and returns a value in the same row from a column you specify in the table or array.

- The `HLOOKUP` function searches a value in the top row and returns a value in the same column from a row you specify in the table or array.

The syntaxes for the `LOOKUP` functions are

`=VLOOKUP(lookup value, range containing the lookup value, the column number in the range containing the return value, [range_lookup])`.

`=VLOOKUP(lookup_value, table_array, col_index_num, [range_lookup])`.

`=HLOOKUP(lookup_value, table_array, row_index_num, [range_lookup])`.

The `[range_lookup]` is an optional which can be specified as `TRUE` for approximate match or `FALSE` for an exact match.

Notes

1. The EXCEL `LOOKUP` functions treat empty cells in the result range as **zeros**.
2. If the range_lookup is `FALSE` and an exact match is not found, the function returns #N/A.
3. If the range_lookup is `TRUE` or omitted, the first column of the `VLOOKUP` table must be in ascending order. If lookup_value is smaller than the the smallest value in the first column of the table_array, then the function returns #N/A.
4. Use `IFERROR` function to avoid the return #N/A. Use the following syntax

   `=IFERROR(value, value_if_error)`

   (for example, `=IFERROR(VLOOKUP(...),"Value is not availble"`)
5. The wildcard character * and ? can be used when the range_lookup argument is text.
6. The wildcard character ? refers to any single character, for example "analy?e" finds "analyse" and "analyze".
7. The wildcard character * refers to any number of characters, for example "bio*" finds "biology" and "biotechnology".

Example 4.1. Import SET Index historical daily data starting from 1 January 2020 up to now from https://www.investing.com/indices/thailand-set-historical-data. Use `LOOKUP` function to create the table showing

1. the closing price on the first working day of each month (not including holidays).
2. the closing price on the last working day of each month (not including holidays).

Hint: use

`=WORKDAY(DATE(YEAR(A1),MONTH(A1),1)-1,1)` to return the first working day of a month and

`=WORKDAY(DATE(YEAR(A1),MONTH(A1)+1,1),-1)` to return the last working day of a month.

Example 4.2. Use the SET Index historical data above. On your selected cells, find the lowest and highest closing prices and the dates when those occurred (sorting data is not allowed).

4.2 MATCH and INDEX functions

The `MATCH` and `INDEX` functions are often used together to perform lookups.

The `MATCH` function returns the relative position of a specified item in a range of cells. For example, if the range A1:A3 contains the values 2, 4 and 6, then the formula

`=MATCH(4,A1:A3,0)` returns the number 2, i.e. the second item in the range.

The syntax for MATCH is

```
=MATCH(lookup_value, lookup_array, [match_type])
```

The optional argument is match_type consisting of 3 values, -1, 0 and 1, that specify how the match is determined.

- If match_type = 1 or omitted, then `MATCH` finds the largest value less than or equal to lookup_value where lookup_array must be in **ascending order**.
- If match_type = 0, then `MATCH` finds the **first value exactly equal to** lookup_value.
- If match_type = -1, then MATCH finds the smallest value greater than or equal to lookup_value where lookup_array must be in **descending order**.

The `INDEX` function returns a value from within a table or range. The syntax for INDEX is

```
=INDEX(array, row_num, [column_num])
```

If array contains only one row or column, the corresponding row_num or column_num argument is optional.

4.3 Data tables

Data tables allows you to create a table of values from

- a formula as one or two variables from the formula are varied, and
- more formulas for various values of a single input cell.

As a result, you can examine a range of possible values when one or two variables are systematically changed. For example, a data table can be used to display the amount your investment will grow to for various numbers of compounding.

Example 4.3. An amount of P is invested for n years at a nominal interest rate of i%. Create a worksheet to calculate the effective annual interest rate you will earn and the accumulation at time n years for various numbers of compounding periods per year including

1. annual compounding
2. semi-annual compounding
3. quarterly compounding
4. monthly compounding
5. bi-weekly compounding
6. weekly compounding
7. daily compounding

Model formulation

1. **Set up input variables:** Referring to the corresponding excel file, create the labels for the input variables and enter the values in cells B4:B7.
2. **Calculate the output values:** The effective annual interest rate $i\%$ which is equivalent to the given nominal rate $i^{(p)}$ convertible p times a year can be calculated by

$$i = \left(1 + \frac{i^{(p)}}{p}\right)^p - 1.$$

Hence, the value of the investment at time n years is

$$FV = P(1+i)^n.$$

The output values are calculated and placed in cells B10 and B11, respectively.

3. **Set up data table (a one-input data table) :**

    1. Create the labels and values for different compounding frequencies.

    2. In cells C16 and D16, create the references to cells B10 and B11, which are the effective annual interest rate and future value of investment, respectively.

        The positions of the cells C16 and D16, the formulas used to calculate the required values, are **one row above and to the right** of the column of values. The data table is column-oriented because the variable values are in a column, B17:B23 in this example.

    3. Select the range of cells that contains the formulas and values that you want to substitute, i.e. the range B16:D23.

    4. Choose Data What-If Analysis Data table. Then type the cell reference for the input cell, cell B5 in this case, in the **Column input cell** because the data table is column-oriented. Click OK and Excel will complete the data table.

Notes

1. A new formula can be added to the existing one-variable data table by typing the new formula in a blank cell to the right of the existing formula(s) in the top row of the data table. Then repeat Step 3.4 as given above.

2. The contents of the data table are generated with a multicell array formula (details will be discussed in a subsequent section):

    `{=TABLE(,B5)}`.

Example 4.4. Suppose you borrow L from a bank to be repaid by the end of n years at an interest rate of $i\%$ per annum effective. If you agree to repay the loan and the interest in level instalments throughout term of loan.

1. Create a model to calculate the amount of annual instalments X for various values of number of instalments n and interest rate of $i\%$ per annum effective.

2. Set up a two-input data table to show annual instalments X for different values of n and i.

3. Comment on the results obtained.

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5 Tutorial 1

1. Calculate the following accumulation:

    1. Accumulate \$5,000 for 4 years at 7.5% per annum effective.

    2. Accumulate \$800 for 2.7 years at 3% per quarter-year effective.

    3. Accumulate \$10,000 for 27 months at 4.25% per half-year effective.

2. Calculate the present values on 1 January 2015 of the following payments at the given rates of interest:

    1. \$1,000 on 1 January 2016, at 7.5% per annum effective.

    2. \$100 on 1 October 2016, at 3% per quarter-year effective.

3. \$10,000 on 1 April 2016, at 4.25% per half-year effective.

3.
   1. If the effective rate of interest is 4% per annum, calculate the effective rate of interest per month?
   2. If the effective rate of interest is 6.5% per half-year, calculate the effective rate of interest per quarter-year?

4. The effective rate of interest per annum was 4% during 2015, 5% during 2016 and 6% thereafter.
   1. Calculate the accumulation of \$500 from 1 January 2015 to 1 January 2018.
   2. Calculate the accumulation of \$2000 from 1 April 2015 to 1 October 2017.
   3. Calculate the accumulation factor from 1 January 2015 to 1 January 2018.

5. You deposit \$ 3000 to an account that earn 2.5% compounded annually. How much will you have in three years?

6. A person borrows a sum of \$5,000 and agrees to pay this back at the end of 1 year with interest calculated at an effective rate of 10% per annum. Calculate the amount to be repaid for the loan.

7. You want to have \$1000 in 2 years and \$2000 in 4 years. How much should you deposit now into an account earning the effective rate of 5.75% semiannually?

8. Katy deposits 100 into a saving account which pays interest at i **per quarter** effective.

   At the same time, Taylor deposits 500 into a different saving account which pays a simple interest at an annual rate of i.

   During the last 3 months of the 4th year, they both earn the same amount of interest. Calculate i.

9. An ordinary annuity is a series of equal payments made at the end of consecutive periods over a fixed length of time. Draw a timeline for the following annuity having cashflow of 1 unit at the end of each of the next n time units.

10. Draw a timeline to illustrate this insurance benefit: Whole Life Insurance - payable immediately on death - has following conditions:
    - death benefit (sum insured) of 1
    - payable immediately on the death
    - of an individual currently aged x
    - for death occurring any time in the future.

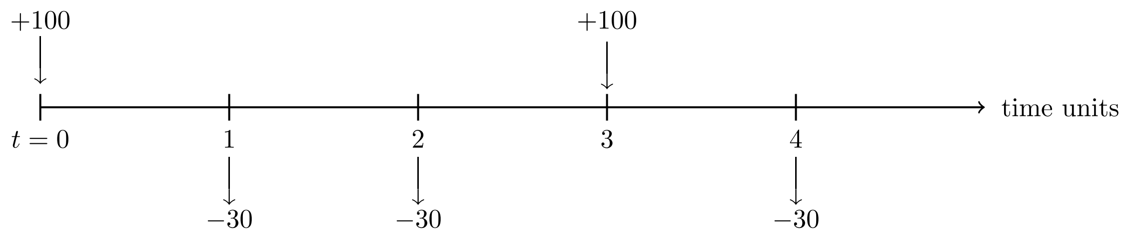
11. (Excel) It is a good exercise to check whether the Excel worksheet you have developed so far for calculating the present value and future value can be applied to the questions in this Tutorial. What would you do to improve the Excel worksheet that can be applied to a more general scenario?

6 Tutorial 2

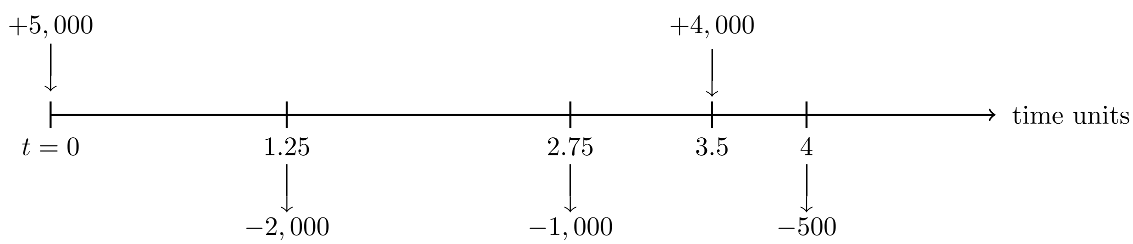
1. Starting at 1 January 2015, the effective rate of interest per annum was 3% per quarter-year for 9 months, 4% per half-year for 15 months and and 2% per month thereafter.
   1. Calculate the accumulation factor from 1 January 2015 to 1 January 2018.
   2. Calculate the accumulation of \$5,000 from 1 July 2015 to 1 October 2017.
   3. Calculate the accumulation of \$100 from 1 March 2016 to 1 August 2018.
   4. Calculate the present value at 1 January 2015 of \$ 25,000 receivable on 1 July 2016.
   5. Calculate the present value at 1 April 2015 of \$ 8,000 receivable on 1 October 2017.

    6. Calculate the discount factor from 1 July 2015 to 1 October 2016.

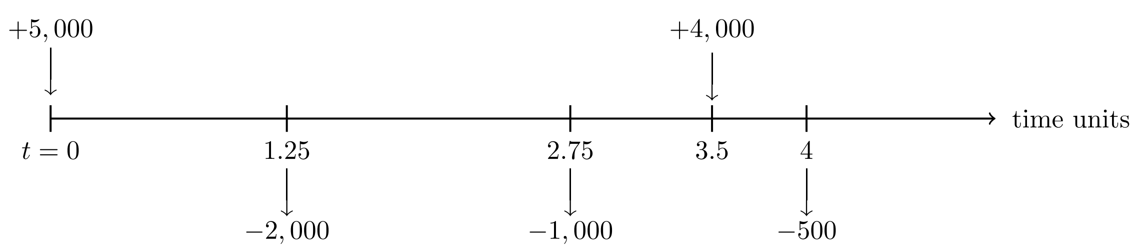
2. The effective rate of interest is 7.25% per time unit. Cashflows are shown in the following time line.
    1. Calculate the accumulation at time time t = 4 units of these cashflows.
    2. Calculate the accumulation at time time t = 8 units of these cashflows.
    3. Calculate the present value at time time t = 0 units of these cashflows.



3. The effective rate of interest is 6% per time unit. Cashflows are shown in the following time line.
    1. Calculate the accumulation at time time t = 5 units of these cashflows.
    2. Calculate the value at time time t = 2 units of these cashflows.
    3. Calculate the present value at time time t = 0 units of these cashflows.



4. The effective rate of interest per annum was 4% during 2015, 3% per half-year until 1 October 2017 and 1.5% per month thereafter. Cashflows are shown in the following time line.
    1. Calculate the accumulation on 1/1/2019 of these cashflows.
    2. Calculate the present value on 1/1/2015 of these cashflows.
    3. Calculate the value at time time 1/7/2017 of these cashflows.



5. (Excel) It is a good exercise to check whether the Excel worksheet you have developed so far for calculating the present value and future value can be applied to the questions in this Tutorial. What would you do to improve the Excel worksheet for a more general scenario?

7 Tutorial 3

1. Calculate the present value now of an annuity payable monthly in advance. The annual amount of the annuity will be \$ 2,400 for the first 10 years and \$ 3,600 for the next 15 years, after which payment will cease. Assume that the effective rate of interest is 2% per annum.

2. Assume that the effective rate of interest will be 3% for 5 years from now, 4% for the next 5 years and 5% thereafter. Calculate the following values:

    1. The present value of an annuity of \$ 1,000 per annum, payable in arrear for 15 years.

    2. The present value of an annuity due of \$ 500 per annum, payable at the beginning of the year for 20 years.

    3. The accumulation value of an increasing annuity payable yearly in arrear for 30 years. The first annual payment is \$ 100, and payments will be increase by \$ 100 each year.

    4. The accumulation value of an increasing annuity payable yearly in advance for 18 years. The first annual payment is \$ 1,000, and payments will be increase by 2% each year (compound).

    5. The present value of an annuity of \$ 200, payable in arrear for 10 years and deferred for 3 years.

3. You borrow \$ 240,000 from a bank to be repaid by the end of 5 years. Assume that the interest rate is 4% per annum. Consider the following four possible options for the loan to be repaid.

    1. Calculate the amount of the repayments to repay if you choose to repay the loan as late as possible.

    2. You may choose to repay interest only during the 5 years term of loan and repay the capital at the end of the term. Calculate interest to be repaid and draw the timeline to illustrate the cashflows for the repayment of the loan.

    3. Calculate the amount X of level instalments to repay the loan which will be paid at the end of each year for 5 years and draw the timeline to illustrate the cashflows for the repayment of the loan.

    4. Calculate the amount Y of level instalments to repay the loan which will be paid at the end of each month for 5 years and draw the timeline to illustrate the cashflows for the repayment of the loan. **Instalment** is a sum of money due as one of several equal payments for something, spread over an agreed period of time.

4. A person now age 30 has received a pension from a company. When he retires at age 60, he will be paid on each birthday from the 60 to the 85th inclusive. The first annual payment will be half of his salary when he retires, and payments will then increase by 2% compounding each year. Currently, he receive a salary of \$ 20,000 and will increase by 3% each year compounding in line with inflation. Assume that the effective rate of interest will be 4% for the next 20 years and 5% thereafter. Calculate the present value now of this pension.

5. (Excel) Use Excel worksheet you have developed so far to calculate the results from the questions in this Tutorial.

8 Tutorial 4

1. Show that the following series of cashflows are equivalent given that an interest rate is 4% per annum effective.

    1. One single payment of amount 14,802.44 at year 10.

    2. a level annuity of 400 payable yearly in arrear for the next 10 years plus a lump sum of 10,000.

    3. a level annuity of 1,232.91 payable yearly in arrear for the next 10 years.

2. You invest in a project which requires you to pay 2,000 and receive back 300 at the end of each of the next 8 years. Calculate the yield of this investment. ANS = 4.2394551%
3. You pay a price of 5,000 for an investment that will repay you 600 per annum payable half-yearly in arrear for the next 12 years. Calculate the yield of this investment. ANS = 3.1491266%
4. An investor pays 100,000 in order to receive 20,000 back at the end of the first 3 years and 25,000 back at the end in the next 4 years. Calculate the yield of this investment. ANS = 12.6209232%
5. You invest in a project which requires you to pay 500,000 at the start of each of the calendar years 2018, 2019 and 2020. The project is expected to return profits of 400,000 for 6 years a the end of each calendar year 2024 to 2029 inclusive. Calculate the yield of this investment. ANS = 5.7285486%
6. (Modified from CT1 2014 IFoA Exam)

   An investor is considering two projects, Project A and Project B. Project A involves the investment of 2,000,000 in a retail outlet. Rent is received quarterly in arrear for 25 years, at an initial rate of 100,000 per annum. It is assumed that the rent will increase at a rate of 5% per annum compound, but with increases taking place every five years. Maintenance and other expenses are incurred quarterly in arrear, at a rate of 12,000 per annum. The retail outlet reverts to its original owner after 25 years for no payment.

   Project B involves the purchase of an office building for 1,000,000. The rent is to be received quarterly in advance at an initial rate of 85,000 per annum. It is assumed that the rent will increase to 90,000 per annum after 20 years. There are no maintenance or other expenses. After 25 years the property reverts to its original owner for no payment.

   Calculate the annual effective internal reate of return for both Projects A and B. Which project is preferable?
7. (Excel) Use Excel worksheet you have developed to calculate the results from the questions in this Tutorial.

9 Tutorial 5

1. You borrow 30,000 for a term of 6 months to be repaid in arrear by level monthly instalments. The rate of interest will be 4% pa effective.
   1. Calculate the monthly repayment.
   2. Construct the complete loan schedule
2. A loan of 800,000 is repayable by equal monthly repayments for 10 years, with interest rate payable at 6.5% pa effective.
   1. Calculate the amount of each monthly payment.
   2. Calculate the interest and capital contents of the 96th repayment.
3. 1. An investor takes out a loan of 100,000 from a bank to be repaid by level annual instalments in arrear over 12 years where the bank charges an effective annual rate of interest of 7%. Immediately after the 6th repayment has been made, the investor may
      1. extend the term of the loan by extra 2 year, or
      2. miss the next two repayments.

      Calculate the revised repayment amount in each case.

2. Suppose the bank allows the investor to miss the next two repayments but the capital outstanding will be charged interest at 10% pa effective while the investor is not making repayments. Calculate the revised repayment.

3. Suppose in Question 3.2 that the investor will miss the next two repayment and extend the term of the loan by extra 4 years. Calculate the revised repayment.

4. An investor borrows 50,000 for a term of 12 years. The rate of interest will be 4% pa effective for the first 6 years and 5% pa effective thereafter. The loan will be repaid level annual repayments for the first 6 years, and then increasing to twice the origin level for the last 6 years. Calculate the annual repayment.

5. An investor borrows 40,000 for a term of 10 years. The rate of interest will be 6.5% pa effective The loan will be repaid level annual repayments, increasing at 2% per annum.

   1. Calculate the first annual repayment.
   2. Calculate the capital outstanding after the 7th repayment is made.
   3. Calculate the interest content of the 8th repayment.

6. (Excel) Suppose you borrow L from a bank to be repaid by the end of n years at an interest rate of i% per annum effective. If you agree to repay the loan and the interest in equal annual instalments throughout term of loan and the first payment is made at the end of the first year.

   Create a model to produce a loan amortisation (or loan schedule) table. Make the interest rate, loan life, initial loan, and other necessary variables input variables. The loan amortisation table should include the following columns:

   - The year-beginning balance
   - The annual repayment
   - Interest Component
   - Capital content
   - Capital outstanding (the year-end balance)

10 Solutions to Tutorial 1

1. The solutions to each question are as follows:

   1. $5000(1.075)^4 = 6677.345703$
   2. Let i% be the annual rate effective equivalent to 3% per quarter-effective, $i = (1.03)^4 - 1$. Hence, the accumulation is
   $$800(1+i)^{2.7} = 800(1.03)^{4\times 2.7} = 1100.859802.$$
   3. Let j% be the monthly rate effective equivalent to 4.25% per half-year effective, $j = (1.0425)^{2/12} - 1$. Hence, the accumulation is
   $$10000(1.0425)^{(2/12)\times 27} = 12059.86056.$$

2. The solutions to each question are as follows:

   1. $\frac{1000}{1.075} = 930.232558.$
   2. $\frac{100}{(1.03)^7} = 81.309151.$
   3. Let j% be the quarterly rate effective equivalent to 4.25% per half-year effective, $j = (1.0425)^{2/4} - 1$. Hence, the present value is
   $$10000 \times (1+j)^{-5} = 10000(1.0425)^{-(5/2)} = 9011.764643.$$

3. The solutions to each question are as follows:
    1. 0.3274%
    2. 3.1988%
4. The solutions to each question are as follows:
    1. $500(1.04)(1.05)(1.06) = 578.76$
    2. $2000(1.04)^{3/4}(1.05)(1.06)^{3/4} = 2259.299$
    3. $(1.04)(1.05)(1.06) = 1.15752$
5. The account balance in 3 years is $3000(1.025)^3 = 3230.67$.
6. The amount to be repaid for the loan is $5000(1.1) = 5500$.
7. Let X be the amount to be deposited now.

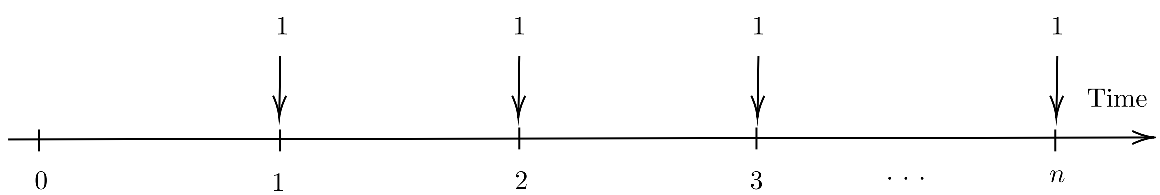
$$X = \frac{1000}{(1.0575)^4} + \frac{2000}{(1.0575)^8} = 2078.36.$$

8. At time 3.75 years, Katy has a balance of $100(1+i)^{15}$. The interest on this balance over the next 3 months is $100(1+i)^{15} \cdot i$. Taylor earns simple interest on the original amount which is equal to $500i \cdot \frac{3}{12}$. Therefore, we solve for i from the following equation:

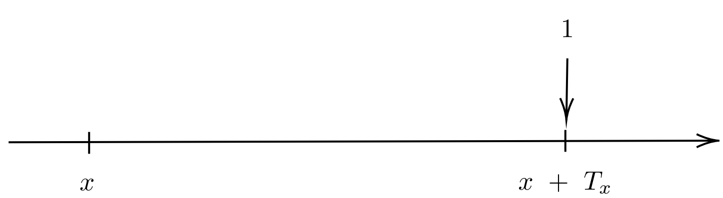
$$100(1+i)^{15} \cdot i = 500i \cdot \frac{3}{12},$$

which gives $i = 0.014987$.

9. The timeline for the following annuity having cashflow of 1 unit at the end of each of the next n time units is given in the figure below:



10. (More details in the course "Life Contingencies I"). We need to define a random variable T_x = the remaining future life time of a life aged x.



The quantity of interest is the present value of the death benefit assuming the interest rate of $i\%$ p.a. effective. It is also a random variable,

$$PV = \frac{1}{(1+i)^{T_x}}.$$

It turns out that the premium rate of this whole life insurance is $E[PV]$, the expected value of the present value, PV.

11 Solutions to Tutorial 2

1. The solutions to each question are as follows:

1. $(1.03)^3(1.04)^{2.5}(1.02)^{12} = 1.528611$
2. $5000A(0.5, 2.75) = 5000(1.03)(1.04)^{2.5}(1.02)^9 = 6788.786068$
3. We first find the rate $j\%$ per month effective that is equivalent to the rate of 4% per half-year effective.

$$j = (1.04)^{1/6} - 1 = 0.00656.$$

The accumulated value is $100A(1 + 2/12, 3 + 7/12) = 100(1 + j)^{10}(1.02)^{19} = 155.522118$.
4. $25000V(0, 1.5) = \frac{25000}{(1.03)^3(1.04)^{1.5}} = 21571.39968$.
5. $8000V(0.25, 2.75) = \frac{8000}{(1.03)^2(1.04)^{2.5}(1.02)^9} = 5720.455921$.
6. $V(0.5, 1.75) = \frac{1}{(1.03)(1.04)^2} = 0.897627$.

2. The solutions to each question are as follows:

1. With $i = 7.25\%$ per time period, $V(4) = 100(1 + i)^4 - 30(1 + i)^3 - 30(1 + i)^2 + 100(1 + i) - 30 = 138.041762$.
2. $V(8) = V(4) \cdot (1 + i)^4 = 182.641597$.
3. $PV(0) = V(4) \cdot (1 + i)^{-4} = 104.332903$.

3. The solutions to each question are as follows:

1. With \$ = 6%\$ per time period, $V(5) = 5000(1 + i)^5 - 2000(1 + i)^{3.75} - 1000(1 + i)^{2.25} + 4000(1 + i)^{1.5} - 500(1 + i) = 6897.948585$.
2. $V(2) = V(5)(1 + i)^{-3} = 5791.650645$.
3. $PV(0) = V(5)(1 + i)^{-5} = 5154.548456$.

4. The solutions to each question are as follows:

1. $V(1/1/2019) = 100(1.04)(1.03)^{3.5}(1.015)^{15} - 30(1.03)^{3.5}(1.015)^{15} - 30(1.03)^{1.5}(1.015)^{15} + 100(1.015)^{12} - 30 = 152.955693$.
2. $PV(0) = V(1/1/2015) = \frac{152.955693}{(1.04)(1.03)^{3.5}(1.015)^{15}} = 106.074596$.
3. $V(1/7/2017) = PV(0)(1.04)(1.03)^3 = 120.546998$.

12 Solutions to Tutorial 3

1. Let j be the effective rate per month equivalent to $i = 2\%$. We have

$$j = (1.02)^{1/12} - 1 = 0.001652.$$

Hence,

$$PV(0) = 200\ddot{a}^j_{\overline{120}|} + 300\ddot{a}^j_{\overline{180}|}\left(\frac{1}{1.02}\right)^{10} = 60148.03.$$

2. The solutions to each question are as follows:

1. The cashflows have been splitted into three periods: (a) from time point 0-5, (b) 5-10 and (c) time point 10 onward.

$$PV(0) = 1000(a^{3\%}_{\overline{5}|} + 1.03^{-5}a^{4\%}_{\overline{5}|} + 1.03^{-5}1.04^{-5}a^{5\%}_{\overline{5}|}) = 11489.49$$

2. We have

$$PV(0) = 500(\ddot{a}^{3\%}_{\overline{5}|} + 1.03^{-5}\ddot{a}^{4\%}_{\overline{5}|} + 1.03^{-5}1.04^{-5}\ddot{a}^{5\%}_{\overline{10}|}) = 7229.67$$

3. The accumuated value is

$$100(Is)^{3\%}_{\overline{5}|}(1.04)^5(1.05)^{20} + \left[100(Is)^{4\%}_{\overline{5}|} + 500s^{4\%}_{\overline{5}|}\right](1.05)^{20} + \left[100(Is)^{5\%}_{\overline{20}|} + 1000s^{5\%}_{\overline{20}|}\right] = 78929.01$$

4. Let $i_1 = 3\%, i_2 = 4\%$ and $i_3 = 5\%$. The accumuated value is given by

$$\begin{aligned}
V(18) = &\left[1000(1+i_1)^5 + 1000(1.02)(1+i_1)^4 + 1000(1.02)^2(1+i_1)^3 + ... + 1000(1.02)^4(1+i_1)\right](1.04)^5(1.05)^8 \\
&+ \left[1000(1.02)^5(1+i_2)^5 + 1000(1.02)^6(1+i_2)^4 + 1000(1.02)^7(1+i_2)^3 + ... + 1000(1.02)^9(1+i_2)\right](1.05)^8 \\
&+ \left[1000(1.02)^{10}(1+i_3)^8 + 1000(1.02)^{11}(1+i_3)^7 + 1000(1.02)^{12}(1+i_3)^6 + ... + 1000(1.02)^{17}(1+i_3)\right] \\
&= 1000(1.02)^5\left[\left(\frac{1+i_1}{1.02}\right)^5 + \left(\frac{1+i_1}{1.02}\right)^4 + ... + \left(\frac{1+i_1}{1.02}\right)\right](1.04)^5(1.05)^8 \\
&+ 1000(1.02)^{10}\left[\left(\frac{1+i_2}{1.02}\right)^5 + \left(\frac{1+i_2}{1.02}\right)^4 + ... + \left(\frac{1+i_2}{1.02}\right)\right](1.05)^8 \\
&+ 1000(1.02)^{18}\left[\left(\frac{1+i_3}{1.02}\right)^8 + \left(\frac{1+i_3}{1.02}\right)^7 + ... + \left(\frac{1+i_3}{1.02}\right)\right]
\end{aligned}$$

Let $1 + j_1 = \frac{1+i_1}{1.02}$. Then, $j_1 = 0.009804$ and

$$\left[\left(\frac{1+i_1}{1.02}\right)^5 + \left(\frac{1+i_1}{1.02}\right)^4 + ... + \left(\frac{1+i_1}{1.02}\right)\right] = \frac{(1+j_1)^5 - 1}{j_1/(1+j_1)} = 5.148995.$$

Let $1 + j_2 = \frac{1+i_2}{1.02}$. Then, $j_2 = 0.019608$ and

$$\left[\left(\frac{1+i_2}{1.02}\right)^5 + \left(\frac{1+i_2}{1.02}\right)^4 + ... + \left(\frac{1+i_2}{1.02}\right)\right] = 5.301921.$$

Let $1 + j_3 = \frac{1+i_3}{1.02}$. Then, $j_3 = 0.029412$ and

$$\left[\left(\frac{1+i_3}{1.02}\right)^8 + \left(\frac{1+i_3}{1.02}\right)^7 + ... + \left(\frac{1+i_3}{1.02}\right)\right] = 9.134790.$$

Therefore, $V(18) = 32814.45$.

5. The present value is

$$\begin{aligned}
PV(0) &= \left(\frac{200}{(1.03)^4} + \frac{200}{(1.03)^5}\right) + 200a_{5}^{0.04}(1.03)^{-5} + 200a_{3}^{0.05}(1.03)^{-5}(1.04)^{-5} \\
&= 350.2192 + 768.0362 + 386.1574 = 1504.413
\end{aligned}$$

3. The solutions to each question are as follows:

   1. $240000(1.04)^5 = 291996.7$
   2. The interest amounts are $0.04 \times 240000 = 9600$.
   3. By the Principle of Equivalence, we have

   $$240000 = Xa_{5}^{0.04}.$$

   This gives X = 53910.51.
   4. Level installments are payable monthly, which follows

   $$240000 = Ya_{60}^{j},$$

   where $j = (1.04)^{1/12} - 1$. This gives Y = 4412.23.

4. The person retires in 30 years, when his salary is expected to be $20000 \times (1.03)^{30} = 48545.25$. The first payment will be half of this which is equal to 24272.62. The present value at age 60 of his pension is

$$24272.62 \times \ddot{a}_{26}^{0.029412} = 449717.9$$

(the precise value is 449719.051954). Here we use $\frac{1.05}{1.02} = 1.029412$ and the annuity is paid from the 60th to the 85th birthday inclusive so there are 26 payments made in advance. Therefore, the present value of this at age 30 is

$$449717.9 \times (1.05)^{-10} \times (1.04)^{-20} = 126002.9.$$

(the precise value is 126003.181173)

13 Solutions to Tutorial 4

1. To examine whether the cashflows are equivalent, we compare their present values.

   a. The present value of single payment of amount 14,802.44 at year 10 is

   $$PV(0) = \frac{14,802.44}{1.04^{10}} = 10000.$$

   b. The present value of the level annuity of 400 payable yearly in arrears for the next 10 years plus a lump sum of 10,000 is

   $$PV(0) = 400a_{10}^{0.04} + \frac{10000}{1.04^{10}} = 10000.$$

   c. The present value of the level annuity of 1,232.91 payable yearly in arrears for the next 10 years.

   $$PV(0) = 1,232.91a_{10}^{0.04} = 10000.$$

It follows that the values of these cashflows are the same, i.e. equivalent.

2. The annual yield of this investment i is the solution of the equation of value:

   $$f(i) = -2000(1+i)^8 + 300s_8^i = 0.$$

   If we solve using software, we get $i = 4.2394551$. Instead of using software, you can also use linear interpolation to approximate the solution.

3. Working in time unit of half year, the equation of value is

   $$f(i) = -5000(1+i)^{12\times 2} + 300s_{24}^i = 0.$$

   The yield i per half year is $i = 3.1491266\%$ and hence the annual yield is 6.397423%.

4. The equation of value is

   $$f(i) = -100(1+i)^7 + 20s_7^i + 25s_4^i = 0.$$

   The annual yield is 12.6209232%.

5. You are suggested to draw the time line for these cashflows. The equation of value is

   $$f(i) = -5\ddot{s}_3^i(1+i)^9 + 4s_6^i = 0.$$

   The annual yield is 5.7285486%.

14 Solutions to Tutorial 5

1. The monthly repayment can be calculated from this equation

   $$X = \frac{30000}{a_6^j} = \frac{30000}{5.931847} = 5057.45,$$

   where $j = (1.04)^{1/12} - 1 = 0.003274$.

The complete loan schedule is illustrated below:

Time	Repayment	Interest Content	Capital Content	Capital Outstanding
0	-	-	-	30000
1	X	98.21	4959.23	25040.77
2	X	81.98	4975.47	20065.30
3	X	65.69	4991.76	15073.54
4	X	49.35	5008.10	10065.44
5	X	32.95	5024.50	5040.94
6	X	16.50	5040.94	0

2. The monthly repayment can be calculated from this equation

$$X = \frac{800000}{a_{120}^{j}} = \frac{800000}{88.806749} = 9008.32,$$

where $j = (1.065)^{1/12} - 1 = 0.005262$.

The capital outstanding after 95th repayment (25 payments left) is

$$L_{95} = 9008.32 a_{25}^{j} = 210506.84.$$

Hence, the interest content of the 96th repayment is

$$j \times L_{95} = 0.005262 \times 210506.84 = 1107.62.$$

The capital content of the 96th repayment is

$$X - 1107.62 = 7900.70.$$

3. 1.

   a. **Extending the term of the loan by extra 2 year:** The original repayment is

$$X = \frac{100000}{a_{12}^{0.07}} = \frac{100000}{7.942686} = 12590.20,$$

The capital outstanding after 6th repayment (6 payments left) is

$$L_6 = X a_6^{0.07} = 60011.68.$$

By extending the term of the loan by extra 2 year, the revised repayment X' can be obtained (for 8 payments) from

$$X' = \frac{L_6}{a_8^{0.07}} = 10050.02$$

   b. **Missing the next two repayments:** From the previous result, the capital outstanding after 6th repayment $L_6 = 60011.68$. Then in 2 years, with interest at 7% per annum, this accumulates to

$$L_6 \times (1.07)^2 = 68707.37.$$

This must now be repaind by only 4 annual repayments, so the new repayment X'' can be obtained from

$$X'' = \frac{68707.37}{a_4^{0.07}} = 20284.35.$$

2. The capital outstanding will accumulate (at 10%) to

$$L_6 \times (1.1)^2 = 72614.14.$$

The new repayment amount X''' is

$$X''' = \frac{72614.14}{a_4^{0.07}} = 21437.73.$$

3. The new repayment amount will be

$$\frac{72614.14}{a_8^{0.07}} = 12160.53.$$

4. You are suggested to draw the time line for these cashflows. Let X be the level of repayment of the first 6 years ($2X$ will be repaid after this period for the last 6 years). It can be obtained from

$$50000 = X(a_6^{0.04} + 2a_6^{0.05}(1.04)^{-6}) = 3769.34.$$

5. 1. Let X be the first annual repayment. Then,

$$40000 = X(v + v^2(1.02) + v^3(1.02)^2 + ... + v^{10}(1.02)^9),$$

where $v = 1/(1.065)$. By rewriting the above equation, we have

$$40000 = \frac{X}{1.02}(\frac{1.02}{1.065} + \left(\frac{1.02}{1.065}\right)^2 + \left(\frac{1.02}{1.065}\right)^3 + ... + \left(\frac{1.02}{1.065}\right)^{10}),$$

Let $i' = \left(\frac{1.065}{1.02} - 1\right) = 0.044118$. Hence,

$$40000 = \frac{X}{1.02} \cdot a_{10}^{i'},$$

and $X = 5133.91$.

2. We will calculate the capital outstanding after 7th repayment, L_7 (3 payments left). We first find the amount $X_8$of the 8th repayment,

$$X_8 = X(1.02)^7 = 5897.25.$$

So the capital outstanding after the 7th repayment is equal to the present value of the remaining 3 repayments (see the table below).

Time	7	8	9	10
Payment	$L_7 = ?$	X_8	$X_8(1.02)$	$X_8(1.02)^2$

It follows that

$$\begin{aligned} L_7 &= X_8(v + v^2 + v^3) \\ &= \frac{X_8}{(1.02)} a_3^{i'} \\ &= 15919.94, \end{aligned}$$

where v and i' are the same as above.

3. The interest content of the 8th repayment is

$$L_7 * i = 15919.94 \times 0.065 = 1034.80.$$