

# Pivot Tables

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## 1 Dates and date functions

Dates and date functions will be discussed in this section. Depending on the system setting, the dates used in this section corresponds to the U.S. setting, i.e. month/day/year. Thus, 7/1/2017 corresponds to 1 July 2017.

The following date formats are recognized by Excel:

- 7/1/2017, 7/1/17
- 7-1-2017, 7-1-17,
- 7-1/2017, 7-1/17,
- July 1, 2017
- 1-Jul-2017

if the date input is without the year, then it is the date on the current year, i.e. 7-1, 7/1, July 1, Jul 1, corresponds to 1 July 2018.

### 1.1 Inconsistent date entries

**Example 1.1.** *What will be the output when we enter dates by using the following two digits for the years:*

1. 7/1/01
2. 7/1/60

Excel interprets two digit years between 00 and 29 as 21st century dates, and two digit years between 30 and 99 as 20th century dates.

### 1.2 Date serial number

Excel stores dates as sequential serial numbers so that they can be used in calculations. For example,

- 1 January 1900 is serial number 1,
- 1 January 2018 is serial number 43101 because it is 43101 days after 1 January 1900.

### 1.3 Date functions

The list of date functions can be seen by choosing

**Formulas → Function Library → Function Library → Date & Time.**

Function	Description
DATE	Returns the serial number of a particular date

Function	Description
DATEVALUE	Converts a date in the form of text to a serial number
DAY	Converts a serial number to a day of the month
DAYS	Returns the number of days between two dates
WEEKDAY	Converts a serial number to a day of the week
NETWORKDAYS	Returns the difference between two dates, excluding weekend days (Saturdays and Sundays)
WORKDAY	Returns a number that represents a date that is the indicated number of working days before or after a date
EDATE	Returns the serial number that represents the date that is the indicated number of months before or after a date
EOMONTH	Returns the serial number for the last day of the month that is the indicated number of months before or after a date
YEARFRAC	Returns a decimal value that represents fractional years between two dates

For a complete list, see the date and time functions available online. More detailed examples are also given in the Excel lab.

## 2 Working with Texts

In this section, we will see how Excel handles text strings, and how we use text functions to modify and manipulate text strings. First, we give a note regarding texts in Excel.

- A single cell can hold up to 32,000 characters. In case you need to display a lot of text in a worksheet, then use a text box (Choose Insert  $\Rightarrow$  Text  $\Rightarrow$  Text Box). It will be easier to edit texts in the text box than in cells.
- Sometimes, when you download numerical data from the internet or database, the imported values are treated as text, i.e. when you do a calculation with such data, you will get a #VALUE error. When a number is not treated as a number, there will be an error indicator. By clicking to expand a list of options, you can then convert it to the number.
- Another issue that you may encounter is about currency that uses different characters to separate thousands or decimals. [https://en.wikipedia.org/wiki/Decimal\\_separator](https://en.wikipedia.org/wiki/Decimal_separator)

### 2.1 Text functions

The following Excel functions can be used to modify text strings in the format you need. Alternatively, one may extract data by using the Convert Text To Columns Wizard (choose Data  $\Rightarrow$  Text To Columns).

1. RIGHT(text,[n]) returns the last  $n$  characters in a text string.
2. LEFT(text, [n]) returns the first  $n$  characters in a text string.
3. MID(text, start\_num, num\_chars) returns num\_char characters from a text string, starting at start\_num.
4. TRIM(text) removes all spaces from text except for single spaces between words. Use TRIM when text strings have irregular spacing.
5. LEN(text) returns the number of characters in a text string.
6. FIND(find\_text, within\_text, [start\_num]) return the location at or after character start\_num of the first character of find\_text in within\_text.
7. SEARCH(find\_text, within\_text, [start\_num]) has the same syntax as FIND, but it is not case sensitive.

8. SUBSTITUTE(text, old\_text, new\_text, [instance\_num]) is used to replace new\_text for old\_text in a text string. Here Instance\_num is optional. It specifies which occurrence of old\_text you want to replace with new\_text. If you specify instance\_num, only that instance of old\_text is replaced. Otherwise, every occurrence of old\_text in text is changed to new\_text.

## 2.2 Character codes

Excel uses the standard ASCII character set. Therefore, each character has its own code. For example, to get the code number of “A”, simply type

=CODE(“A”), which returns the code number 65.

The CHAR function reverses the role of CODE function, i.e.

=CHAR(65), which returns the letter A. The input for the CHAR function should be a value between 1 and 255.

For the complete list of ASCII codes, please visit <https://theasciicode.com.ar>

**Example 2.1.** *Create an Excel file to list all the first 255 ASCII codes. It is a good idea to compare the outputs with those from the website above.*

## 2.3 Determining whether two string are identical

In order to determine whether strings in cell A1 and A2 have the same contents, we use = A1 = A2, which returns either TRUE or FALSE. Note that the comparison is not case-sensitive.

Alternative, the function that provides an exact, case-sensitive comparison is EXACT function.

## 2.4 Joining two or more cells

To join two or more cells, Excel uses an ampersand &. For example if the string “the effective interest rate per annum” is in A1 and the value of 5% (formatted value) is in B1. Then use the formula

= A1 & ” is ” & B1, which returns the effective interest rate per annum is 0.05.

A better solution is to use the TEXT function to format the value as text as follows.

= A1 & ” is ” & TEXT(B1,” 0.00%“), which returns the effective interest rate per annum is 0.05.

Use **Home Format Format cells** to obtain the list of various text formats.

For example, if A1 contains the principle of \$10,000, then

= “The principle is” & TEXT(A1, ” \$#,##0”), which returns The principle is \$10,000.

The following table gives examples that are format with TEXT function.

=TEXT(1234.567,”\$#,##0.00“)

Currency with a thousands separator and 2 decimals, like \$1,234.57. Note that Excel rounds the value to

=TEXT(TODAY(),”MM/DD/YY“)

Today's date in MM/DD/YY format, like 03/14/12

=TEXT(TODAY(),”DDDD“)

Today's day of the week, like Monday

=TEXT(NOW(),”H:MM AM/PM“)

Current time, like 1:29 PM

=TEXT(0.285,"0.0%")

Percentage, like 28.5%

=TEXT(4.34 ,"# ?/?")

Fraction, like 4 1/3

=TRIM(TEXT(0.34,"# ?/?"))

Fraction, like 1/3. Note this uses the TRIM function to remove the leading space with a decimal value.

=TEXT(12200000,"0.00E+00")

Scientific notation, like 1.22E+07

=TEXT(1234567898,"[<=9999999]###-####;(###) ###-####")

Special (Phone number), like (123) 456-7898

=TEXT(1234,"0000000")

Add leading zeros (0), like 0001234

The TEXT function will convert numbers to text. It is best practice to keep your original values in one cell, and formatted number in another cell. When you do calculation, you should refer to the cells containing original values.

## 3 Interest Rate, Present Values and Cashflows

### 3.1 Working with a single cashflow in Excel

#### 3.1.1 How to calculate the future value of a single cashflow

Suppose an amount  $C$  is deposited in an account that pays a fixed interest at the rate of  $i\%$  per time units. Then after  $t$  time units, the deposit will have accumulated to

$$C(1+i)^t.$$

In Excel, the function FV calculates the future value of a single investment (and also periodic constant payments) and a constant interest rate.

The syntax of the function is:

$$\text{FV}(\text{rate}, \text{nper}, \text{pmt}, [\text{pv}], [\text{type}]),$$

where

- rate is the interest rate per period
- nper is the number of periods over which the investment is made.
- pmt (used for an annuity type) is the payment made each period and cannot change over the life of the annuity.
- pv (optional) is an additional cash flow now (time 0)
- fv is an additional cash flow nper periods from now.

- `type` (optional) is an optional argument that defines whether the payment is made at the start or the end of the period:

0 - the payment is made at the end of the period;

1 - the payment is made at the start of the period.

If the `[type]` argument is omitted, it takes on the default value of 0.

The following timeline illustrates the cashflows used for the FV function (assuming that the payments (`pmt`) are made in arrears).

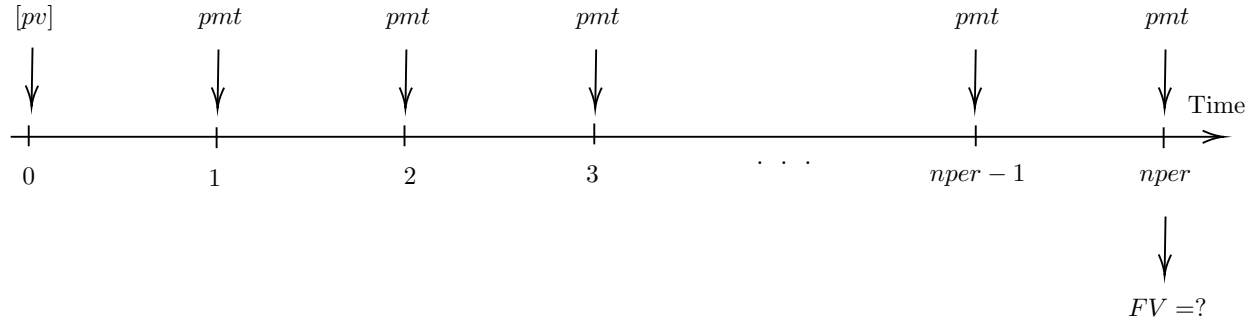


Figure 1: Timeline of cashflows for FV function

**Note** For a single cash flow, we set `pmt` argument to be 0, as there are no ongoing payments after the initial investment.

**Example 3.1.** *You plan to invest today with an interest rate of 3% per year effective. How much money will you accumulate at the end of 2 years?*

**Solution:** The accumulation of today in 2 years can be calculated by Excel as

$$\text{FV}(3\%, 2, 0, -100).$$

### 3.1.2 How to calculate the present value of a single cashflow

Similarly, the present value of a future cashflow  $C$  required at time  $t$  time units at a fixed interest of  $i\%$  per time units can be calculated as

$$\frac{C}{(1 + i)^t}.$$

In Excel, the function `PV` calculates the present value of a single investment (and also periodic constant payments) and a constant interest rate. The syntax of the function is:

$$\text{PV}(\text{rate}, \text{nper}, \text{pmt}, [\text{fv}], [\text{type}])$$

where `fv` is an additional cash flow `nper` periods from now.

**Example 3.2.** *How much should you deposit into the account with an interest of 8% so that 10 years from now its value would be ?*

**Solution:** The present value of today in 10 years can be calculated by Excel as

$$\text{PV}(8\%, 10, 0, -1000).$$

## 3.2 Present values of a series of cashflows

Consider a series of cashflows defined by

1. the times of payments (cashflows), denoted by  $t_1, t_2, \dots$ , and
2. the amount of payments, denoted by  $C_r$  (in short for  $C_{t_r}$ ), which will be paid at time  $t_r$ , for  $r = 1, 2, \dots$ .  
The amounts can be positive or negative

The present value at any time  $t$  of this series of cashflow is

$$PV(t) = \sum_{r=1}^{\infty} C_r(1+i)^{t-t_r} = \sum_{r=1}^{\infty} C_r v^{t_r-t}$$

where  $i$  is the effective rate of interest and  $v = 1/(1+i)$ .

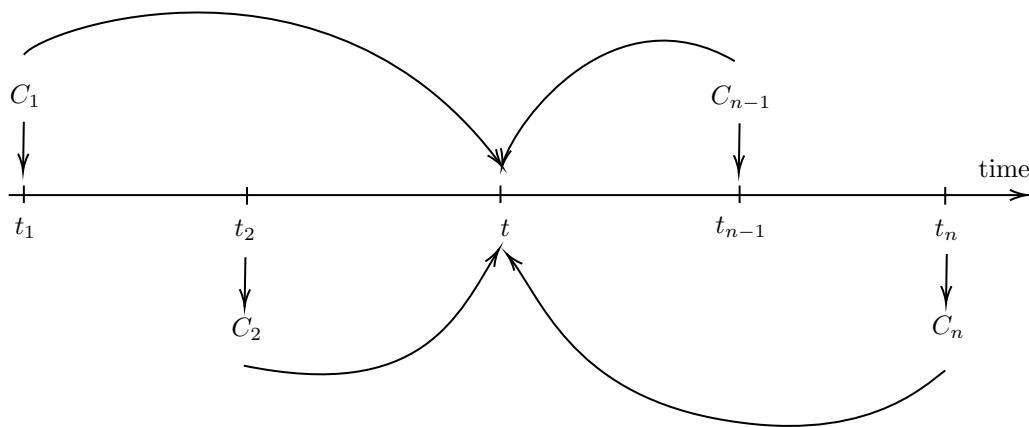


Figure 2: Timeline of a series of cashflows

**Notes** 1. At a fixed effective rate of interest, the original series of cashflows is equivalent to a single payment of amount  $PV(t)$  at time  $t$ .

2. If two different series of cashflows have the same  $PV$  at one time at a given effective rate of interest, then they have the same  $PV$  at any time at that effective rate of interest.

### 3.2.1 Level Annuities certain

An *annuity* is a regular series of payments (cashflows). When the payments are certain which are payable for a definite period of time, we call it an *annuity certain*.

- If the payments are made at the end of each time period, they are paid *in arrear*.
- Otherwise, payments are made at the beginning of each time period, they are paid *in advance*.
- An annuity paid in advance is also known as an *annuity due*.
- If each payment is for the same amount, this is a *level annuity*.

**Example 3.3.** Let  $i$  be the constant effective rate of interest per time unit. In Excel, one can calculate the accumulated value of a level annuity certain having cashflow of  $pmt$  unit at the end of each of the next  $n$  time units by

$$FV(i\%, n, pmt, 0).$$

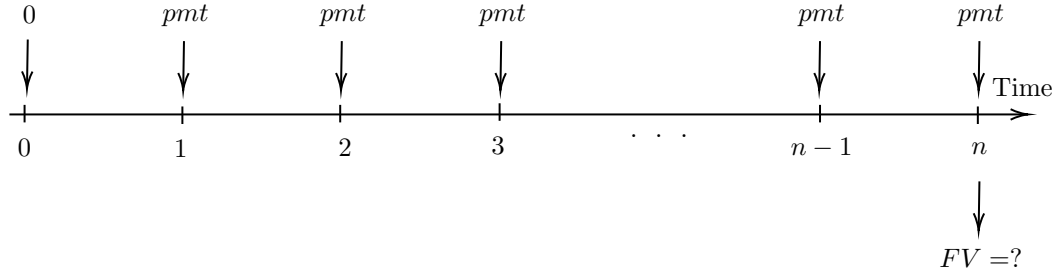


Figure 3: Level Annuity Certain

The cashflows of this annuity is shown in the timeline below.

**Note** The last argument of the above syntax,  $[pv]$  (optional) which is an additional cash flow now (at time 0), has been set to 0.

**Example 3.4.** Let  $i$  be the constant effective rate of interest per time unit. In Excel, one can calculate the present value at time 0 of a level annuity certain having cashflow of  $pmt$  unit at the end of each of the next  $n$  time units by

$$PV(i\%, n, pmt, 0).$$

The timeline of these cashflows is shown in the figure below:

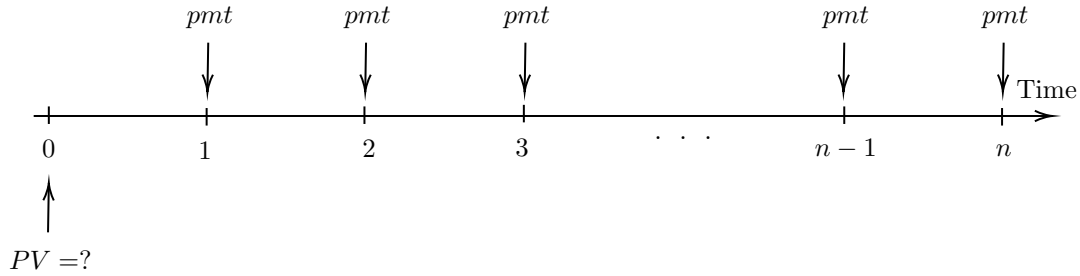


Figure 4: Level Annuity Certain

**Note** The last argument of the above syntax,  $[fv]$  (optional) which is an additional cash flow at time  $n$ , has been set to 0.

**Example 3.5.** Given the effective rate of interest of 8% p.a., use Excel to calculate

1. the accumulation at 12 years of payable yearly in arrear for the next 12 years.
2. the present value now of ,000 payable yearly in arrear for the next 6 years.
3. the present value now of ,000 payable half-yearly in arrear for the next 12.5 years.

**Example 3.6.** Let  $i = 4\%$  effective per time unit. Cashflows are given as follows:

- $C_1 = 200$  at time  $t_1 = 1$ .
- $C_2 = 300$  at time  $t_2 = 3$ .
- $C_3 = -100$  at time  $t_3 = 5$ .
- $C_4 = -50$  at time  $t_4 = 6$ .

Develop the model using Excel to calculate



1. the accumulation at time  $t = 7$ .
2. the present value at time  $t = 0$ .
3. the present value at time  $t = 4$ .

## 4 Lookup Formulas and Data Tables

### 4.1 Lookup function

Lookup function can be used to return (or retrieve) a value from a table or a range of data. For example, use lookup function to find some information such as date of birth, address etc. from a customer database. Two useful lookup functions are VLOOKUP and HLOOKUP.

- The VLOOKUP function searches a value in the first column and returns a value in the same row from a column you specify in the table or array.
- The HLOOKUP function searches a value in the top row and returns a value in the same column from a row you specify in the table or array.

The syntaxes for the LOOKUP functions are

`=VLOOKUP(lookup value, range containing the lookup value, the column number in the range containing the return value, [range_lookup]).`

`=VLOOKUP(lookup_value, table_array, col_index_num, [range_lookup]).`

`=HLOOKUP(lookup_value, table_array, row_index_num, [range_lookup]).`

The `[range_lookup]` is an optional which can be specified as `TRUE` for approximate match or `FALSE` for an exact match.

#### Notes

1. The EXCEL LOOKUP functions treat empty cells in the result range as **zeros**.
2. If the `range_lookup` is `FALSE` and an exact match is not found, the function returns `#N/A`.
3. If the `range_lookup` is `TRUE` or omitted, the first column of the VLOOKUP table must be in ascending order. If `lookup_value` is smaller than the the smallest value in the first column of the `table_array`, then the function returns `#N/A`.
4. Use IFERROR function to avoid the return `#N/A`. Use the following syntax  
`=IFERROR(value, value_if_error)`  
(for example, `=IFERROR(VLOOKUP(...), "Value is not availble" )`)
5. The wildcard character `*` and `?` can be used when the `range_lookup` argument is text.
6. The wildcard character `?` refers to any single character, for example `"analy?e"` finds `"analyse"` and `"analyze"`.
7. The wildcard character `*` refers to any number of characters, for example `"bio*"` finds `"biology"` and `"biotechnology"`.

**Example 4.1.** Import SET Index historical daily data starting from 1 January 2020 up to now from <https://www.investing.com/indices/thailand-set-historical-data>. Use LOOKUP function to create the table showing

1. the closing price on the first working day of each month (not including holidays).
2. the closing price on the last working day of each month (not including holidays).

Hint: use

`=WORKDAY(DATE(YEAR(A1),MONTH(A1),1)-1,1)` to return the first working day of a month and

`=WORKDAY(DATE(YEAR(A1),MONTH(A1)+1,1),-1)` to return the last working day of a month.

**Example 4.2.** Use the SET Index historical data above. On your selected cells, find the lowest and highest closing prices and the dates when those occurred (sorting data is not allowed).

## 4.2 MATCH and INDEX functions

The MATCH and INDEX functions are often used together to perform lookups.

The MATCH function returns the relative position of a specified item in a range of cells. For example, if the range A1:A3 contains the values 2, 4 and 6, then the formula

`=MATCH(4,A1:A3,0)` returns the number 2, i.e. the second item in the range.

The syntax for MATCH is

`=MATCH(lookup_value, lookup_array, [match_type])`

The optional argument is match\_type consisting of 3 values, -1, 0 and 1, that specify how the match is determined.

- If match\_type = 1 or omitted, then MATCH finds the largest value less than or equal to lookup\_value where lookup\_array must be in **ascending order**.
- If match\_type = 0, then MATCH finds the **first value exactly equal to** lookup\_value.
- If match\_type = -1, then MATCH finds the smallest value greater than or equal to lookup\_value where lookup\_array must be in **descending order**.

The INDEX function returns a value from within a table or range. The syntax for INDEX is

`=INDEX(array, row_num, [column_num])`

If array contains only one row or column, the corresponding row\_num or column\_num argument is optional.

## 4.3 Data tables

Data tables allows you to create a table of values from

- a formula as one or two variables from the formula are varied, and
- more formulas for various values of a single input cell.

As a result, you can examine a range of possible values when one or two variables are systematically changed. For example, a data table can be used to display the amount your investment will grow to for various numbers of compounding.

**Example 4.3.** An amount of  $P$  is invested for  $n$  years at a nominal interest rate of  $i\%$ . Create a worksheet to calculate the effective annual interest rate you will earn and the accumulation at time  $n$  years for various numbers of compounding periods per year including

1. annual compounding
2. semi-annual compounding
3. quarterly compounding
4. monthly compounding

5. bi-weekly compounding
6. weekly compounding
7. daily compounding

### Model formulation

1. **Set up input variables:** Referring to the corresponding excel file, create the labels for the input variables and enter the values in cells B4:B7.
2. **Calculate the output values:** The effective annual interest rate  $i\%$  which is equivalent to the given nominal rate  $i^{(p)}$  convertible  $p$  times a year can be calculated by

$$i = \left(1 + \frac{i^{(p)}}{p}\right)^p - 1.$$

Hence, the value of the investment at time  $n$  years is

$$FV = P(1 + i)^n.$$

The output values are calculated and placed in cells B10 and B11, respectively.

### 3. Set up data table (a one-input data table) :

1. Create the labels and values for different compounding frequencies.
2. In cells C16 and D16, create the references to cells B10 and B11, which are the effective annual interest rate and future value of investment, respectively.

The positions of the cells C16 and D16, the formulas used to calculate the required values, are **one row above and to the right** of the column of values. The data table is column-oriented because the variable values are in a column, B17:B23 in this example.

3. Select the range of cells that contains the formulas and values that you want to substitute, i.e. the range B16:D23.
4. Choose Data What-If Analysis Data table. Then type the cell reference for the input cell, cell B5 in this case, in the **Column input cell** because the data table is column-oriented. Click OK and Excel will complete the data table.

### Notes

1. A new formula can be added to the existing one-variable data table by typing the new formula in a blank cell to the right of the existing formula(s) in the top row of the data table. Then repeat Step 3.4 as given above.
2. The contents of the data table are generated with a multicell array formula (details will be discussed in a subsequent section):

`{=TABLE(,B5)}.`

**Example 4.4.** Suppose you borrow  $L$  from a bank to be repaid by the end of  $n$  years at an interest rate of  $i\%$  per annum effective. If you agree to repay the loan and the interest in level instalments throughout term of loan.

1. Create a model to calculate the amount of annual instalments  $X$  for various values of number of instalments  $n$  and interest rate of  $i\%$  per annum effective.
2. Set up a two-input data table to show annual instalments  $X$  for different values of  $n$  and  $i$ .
3. Comment on the results obtained.

## 5 Analysing data using Goal Seeking and Solver

### 5.1 Analysing data using Goal Seeking

Goal Seek feature allows a user to find the value that is an input for a formula (a single input cell) to produce a required result. For example, given a series of cashflows  $C_1, C_2, \dots, C_n$  (positive and negative) payable at times  $t_1, t_2, \dots, t_n$ , you can set up your worksheet to calculate the present value at time 0 defined by

$$PV_i(0) = \sum_{i=1}^n C_i \left( \frac{1}{1+i} \right)^{t_i}.$$

The solution  $i$  of the equation

$$PV_i(0) = 0,$$

which is the yield of the cashflows, can be directly obtained by using Goal Seek. Here the yield  $i$  is the value for a worksheet input that makes the value of the formula  $PV_i(0)$  (or the present value function expressed in terms of  $i$ ) match the goal, which is zero in this case.

**Note** Goal Seek works only with one variable input value. You can use the Solver add-in when more than one input value is needed; for example, both the loan amount and the monthly payment amount for a loan.

We first illustrate how to use Goal Seek to find the interest rate from a loan in the following example.

**Example 5.1.** Suppose that you borrow 5,000 for a term of 3 years. The loan is to be repaid by 3 level annual repayments of 2,010.57 at the end of each year. Calculate the interest rate for this loan.

#### Model formulation for Example 5.1

1. **Set up input variables** Create the labels for the input variables in cells A4:A6 for loan amount, term of the loan, and payment, respectively. Then enter the values 5,000, 3 and 2,010.57 in cells B4-B6, respectively. Create the label for the output variable (i.e. the interest rate  $i$ ). You may enter the guessing value for the interest rate, say for e.g.  $i = 1\%$  in the corresponding cell (i.e. the cell B9)
2. **Construct year-by-year cashflows** Use the guessing value of  $i$  to construct year-by-year cashflow to calculate the present value of the cashflows (in cell E12 in our example)
3. **Use Goal Seek to calculate the interest rate.** Choose

**Data → Data Tools → What-If Analysis → Goal Seek**

1. Then in the **Set cell** box, enter the reference for the cell that contains the formula (or the goal we want to match), i.e. cell E12.
2. In the **To value** box, enter the result of the the formula, i.e. the value of the present value of the cashflows which is 0.
3. In the **By changing cell** box, enter the reference for the cell that you want to adjust, i.e. cell B9.
4. **Goal Seek status box** then displays the target value and the value that excel calculated.

#### Note

1. In case that Goal Seek cannot find a solution but there is a solution to the problem.
  1. You should change the value used in the by changing cell box to a value that should be close to the solution.
  2. Increase the Maximum Iterations setting which allows Excel to try more possible solutions. To set the Maximum Iterations, go to

**File → Excel → Options → Formula tab**

2. In order to increase the accuracy of the (approximated) solution, set the option value for Maximum Change (located in **Excel Options Formula tab**) to a small value. The default value is 0.001 which gives only 3 decimals accuracy.

**Example 5.2.** Consider a transaction from an investment that offers

- to pay an investor of amounts  $B_1, B_2, \dots, B_n$  at time  $t_1, t_2, \dots, t_n$
- in return for outlays (amounts to be paid by the investor) of amounts  $A_1, A_2, \dots, A_n$  at these times, respectively.

Only one of  $A_i$  and  $B_i$  will be non-zero in general. Make all the important variables of the problem input variables. Build the model to calculate the yield of the investment for the two examples in the lecture note.

## 5.2 Analysing data using Solver

Goal Seek has a limitation that can solve for only one adjustable cell. Solver extends this Goal Seek function and can do the following:

- work with multiple adjustable cells.
- specify constraints for the values of adjustable cells.
- find an optimum value for a formula.
- generate multiple solutions to a problem.

## 5.3 A simple Solver example

We will describe how to use Solver to solve same the problem as Goal Seek.

1. **Set up input variables** This is similar to Goal Seek section.
2. **Set up a formula for the output variable** This is similar to Goal Seek section.
3. **Use Goal Seek to calculate the interest rate.** Choose

**Data → Analysis → Solver**

1. Then in the **Set Objective** box, enter the reference for the cell that contains the formula, i.e. cell B6.
2. In the **To** box, choose **Value of** box and then enter the result of the the formula.
3. In the **By Changing Variable cells** box, enter the reference for the cell that you want to adjust, i.e. cell B5.

## 6 Analysing data with pivot tables

Excel provides a powerful tool namely **pivot table** to summarise, analyse, explore and present data.

- Users can simply create useful summary of data and rearrange the information in any form with a few mouse clicks.
- Pivot tables can create frequency distributions and cross-tabulations of several different data dimensions.
- The users can also create **pivot charts** based on pivot tables, which provide meaningful, graphical data representation.

The following examples illustrate how to create pivot tables in Excel. The dataset used in the examples contains data on sales from a superstore. The information on this data includes

- Order and ship dates,
- Ship modes,
- Customer details,
- Product details,
- Sale and profit amounts.

The file can be downloaded from <https://community.tableau.com/docs/DOC-1236>.

**Example 1** Suppose that the manager is interested in profits, broken down by region and sub-category. Instead of sorting the data and creating formulas to answer the question, you can summarise the data by creating a pivot table, which can be done in the following steps:

1. Select any cell or table containing the data.
2. From the **Insert** tab and click **PivotTable**.
3. Specify the data and location for the pivot table in the **PivotTable dialog box**.
4. A blank **PivotTable** and **PivotTable Fields List** will appear.
5. Drag the field names (or column headers) at the top to one of the four boxes at the bottom of the **PivotTable Fields List**.

For this example,

- **Drag the Profit field into the Values area.** At this point, the pivot table displays the total of all the values in the Profit column.
  - **Drag the Sub-Category field into the Row Labels area.** The pivot table shows the total profit for each of the sub categories.
  - **Drag the Region field into the Column Labels area.** The pivot table shows the total profit for each of the sub categories, cross-tabulated by region.
6. The pivot table will calculate and summarise the selected fields. The result is shown in the Table 1.
  1. The pivot table automatically updates the table with every change you make in the PivotTable Field List.
  2. You can sort data in a PivotTable. Just right-click any cell in the column to sort and choose Sort from the short-cut menu.
  3. You can apply number formatting, for example changing the Number Format to Currency.
  4. Pivot table data is usually summarised using a sum. You can also summarise your data using different summary techniques, such as Sum, Count, Average, etc.

Table 2: Table 1: The pivot table from Example 1.

Sum of Profit	Region				
Sub-Category	Central	East	South	West	Grand Total
Accessories	7251.63	11195.86	7004.54	16484.6	41936.64
Appliances	-2638.62	8391.41	4123.94	8261.27	18138.01
Art	1195.16	1899.94	1058.59	2374.1	6527.79
Binders	-1043.64	11267.93	3900.66	16096.8	30221.76
Bookcases	-1997.9	-1167.63	1339.49	-1646.51	-3472.56

Sum of Profit	Region				
Chairs	6592.72	9357.77	6612.09	4027.58	26590.17
Copiers	15608.84	17022.84	3658.91	19327.24	55617.82
Envelopes	1777.53	1812.41	1465.48	1908.76	6964.18
Fasteners	236.62	263.99	173.72	275.19	949.52
Furnishings	-3906.22	5881.41	3442.68	7641.27	13059.14
Labels	1073.08	1129.28	1040.77	2303.12	5546.25
Machines	-1486.07	6928.64	-1438.89	-618.93	3384.76
Paper	6971.9	9015.37	5947.06	12119.24	34053.57
Phones	12323.03	12314.69	10767.28	9110.74	44515.73
Storage	1969.84	8389.37	2274.3	8645.32	21278.83
Supplies	-661.89	-1155.14	1.88	626.05	-1189.1
Tables	-3559.65	-11025.38	-4623.06	1482.61	-17725.48
Grand Total	39706.36	91522.78	46749.43	108418.45	286397.02

## 6.1 Filters

Filters can be used to display part of your data that you need. For example, you can determine how various discount rates affect the profits.

1. Drag a field to the **Filters** section. In this example, we will use Discount field.
2. The **filter** will appear above the pivot table. Then use a drop-down list to filter the displayed data by one or more items.

The pivot table filtered by discount field is shown in Table 2.

Table 3: Table 2: The pivot table is filtered by discount.

Discount	0.5				
Sum of Profit	Region				
Sub-Category	Central	East	South	West	Grand Total
Bookcases		-4255.81			-4255.81
Machines			-7635.23		-7635.23
Tables	-4309.74			-4305.64	-8615.39
Grand Total	-4309.74	-4255.81	-7635.23	-4305.64	-20506.43

## 6.2 Modifying the pivot table

You can add further summary information by using the **PivotTable Fields List**. For example, you can add a second field (category) to the Row Label area. Here you also need to change the order in which the fields are listed, i.e. Category should be on the top level followed by sub-category. Table 3 shows the pivot table when two fields are added in row labels.

Table 4: Table 3: Two fields are used for row labels.

Sum of Profit		Region				
Category	Sub-Category	Central	East	South	West	Grand Total
Furniture	Bookcases	-1997.9	-1167.63	1339.49	-1646.51	-3472.56
	Chairs	6592.72	9357.77	6612.09	4027.58	26590.17
	Furnishings	-3906.22	5881.41	3442.68	7641.27	13059.14
	Tables	-3559.65	-11025.38	-4623.06	1482.61	-17725.48

Sum of Profit		Region				
Furniture Total		-2871.05	3046.17	6771.21	11504.95	18451.27
Office Supplies	Appliances	-2638.62	8391.41	4123.94	8261.27	18138.01
	Art	1195.16	1899.94	1058.59	2374.1	6527.79
	Binders	-1043.64	11267.93	3900.66	16096.8	30221.76
	Envelopes	1777.53	1812.41	1465.48	1908.76	6964.18
	Fasteners	236.62	263.99	173.72	275.19	949.52
	Labels	1073.08	1129.28	1040.77	2303.12	5546.25
	Paper	6971.9	9015.37	5947.06	12119.24	34053.57
	Storage	1969.84	8389.37	2274.3	8645.32	21278.83
	Supplies	-661.89	-1155.14	1.88	626.05	-1189.1
Office Supplies Total		8879.98	41014.58	19986.39	52609.85	122490.8
Technology	Accessories	7251.63	11195.86	7004.54	16484.6	41936.64
	Copiers	15608.84	17022.84	3658.91	19327.24	55617.82
	Machines	-1486.07	6928.64	-1438.89	-618.93	3384.76
	Phones	12323.03	12314.69	10767.28	9110.74	44515.73
Technology Total		33697.43	47462.04	19991.83	44303.65	145454.95
Grand Total		39706.36	91522.78	46749.43	108418.45	286397.02

**Example 2** Create a pivot table to answer the following questions

1. What are the total sales obtained from the top 5 customer who have spent the most, broken down by sub-category? How many orders did each customer have for each product?
2. How does the sales in the Central region compared with the other regions combined?
3. In which region does the superstore obtain the highest profit for each year?
4. Create a frequency distribution of the profit, filtered by year.

### 6.3 Working with Nonnumeric Data

Pivot tables can be used for nonnumeric data. In this case, a useful pivot table can be created that counts the items rather than sums of them.

The following Pivot table cross-tabulates the segment for each combination of segment and region. Here, we specify each of the Pivot table fields as follows:

- The Segment field is used for the Columns.
- The Region field is used for the Rows.
- The Segment field is also used for the Values and is summarised by Count.
- A second instance of the Segment field is added to the Values section. To display percentages, in Field Setting, choose Show Values As Percent of Column Total.

width=1

Table 5: Table 4: The Pivot Table for Nonnum

		Segment			
		Consumer		Corporate	
		Count of Segment	Count of Segment by Percentage	Count of Segment	Count of Segment by Percentage
Central		1212	23.35%	673	22.28%
East		1469	28.30%	877	29.04%



	Segment			
South	838	16.14%	510	16.89%
West	1672	32.21%	960	31.79%
Grand Total	5191	100.00%	3020	100.00%

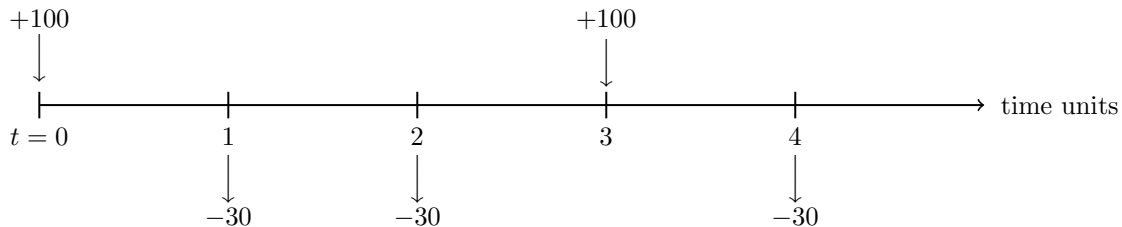
## 7 Tutorial 1

- Calculate the following accumulation:
  - Accumulate \$5,000 for 4 years at 7.5% per annum effective.
  - Accumulate \$800 for 2.7 years at 3% per quarter-year effective.
  - Accumulate \$10,000 for 27 months at 4.25% per half-year effective.
- Calculate the present values on 1 January 2015 of the following payments at the given rates of interest:
  - \$1,000 on 1 January 2016, at 7.5% per annum effective.
  - \$100 on 1 October 2016, at 3% per quarter-year effective.
  - \$10,000 on 1 April 2016, at 4.25% per half-year effective.
- If the effective rate of interest is 4% per annum, calculate the effective rate of interest per month?
  - If the effective rate of interest is 6.5% per half-year, calculate the effective rate of interest per quarter-year?
- The effective rate of interest per annum was 4% during 2015, 5% during 2016 and 6% thereafter.
  - Calculate the accumulation of \$500 from 1 January 2015 to 1 January 2018.
  - Calculate the accumulation of \$2000 from 1 April 2015 to 1 October 2017.
  - Calculate the accumulation factor from 1 January 2015 to 1 January 2018.
- You deposit \$ 3000 to an account that earn 2.5% compounded annually. How much will you have in three years?
- A person borrows a sum of \$5,000 and agrees to pay this back at the end of 1 year with interest calculated at an effective rate of 10% per annum. Calculate the amount to be repaid for the loan.
- You want to have \$1000 in 2 years and \$2000 in 4 years. How much should you deposit now into an account earning the effective rate of 5.75% semiannually?
- Katy deposits 100 into a saving account which pays interest at  $i$  **per quarter** effective.  
 At the same time, Taylor deposits 500 into a different saving account which pays a simple interest at an annual rate of  $i$ .  
 During the last 3 months of the 4th year, they both earn the same amount of interest. Calculate  $i$ .
- An ordinary annuity is a series of equal payments made at the end of consecutive periods over a fixed length of time. Draw a timeline for the following annuity having cashflow of 1 unit at the end of each of the next  $n$  time units.
- Draw a timeline to illustrate this insurance benefit: Whole Life Insurance - payable immediately on death - has following conditions:
  - death benefit (sum insured) of 1

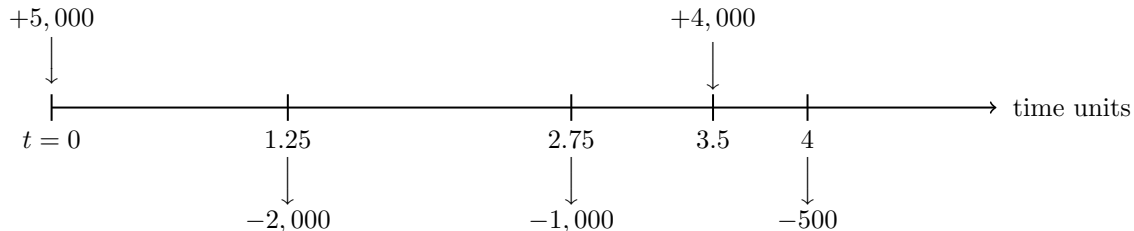
- payable immediately on the death
  - of an individual currently aged  $x$
  - for death occurring any time in the future.
11. (Excel) It is a good exercise to check whether the Excel worksheet you have developed so far for calculating the present value and future value can be applied to the questions in this Tutorial. What would you do to improve the Excel worksheet that can be applied to a more general scenario?

## 8 Tutorial 2

- Starting at 1 January 2015, the effective rate of interest per annum was 3% per quarter-year for 9 months, 4% per half-year for 15 months and 2% per month thereafter.
  - Calculate the accumulation factor from 1 January 2015 to 1 January 2018.
  - Calculate the accumulation of \$5,000 from 1 July 2015 to 1 October 2017.
  - Calculate the accumulation of \$100 from 1 March 2016 to 1 August 2018.
  - Calculate the present value at 1 January 2015 of \$ 25,000 receivable on 1 July 2016.
  - Calculate the present value at 1 April 2015 of \$ 8,000 receivable on 1 October 2017.
  - Calculate the discount factor from 1 July 2015 to 1 October 2016.
- The effective rate of interest is 7.25% per time unit. Cashflows are shown in the following time line.
  - Calculate the accumulation at time time  $t = 4$  units of these cashflows.
  - Calculate the accumulation at time time  $t = 8$  units of these cashflows.
  - Calculate the present value at time time  $t = 0$  units of these cashflows.

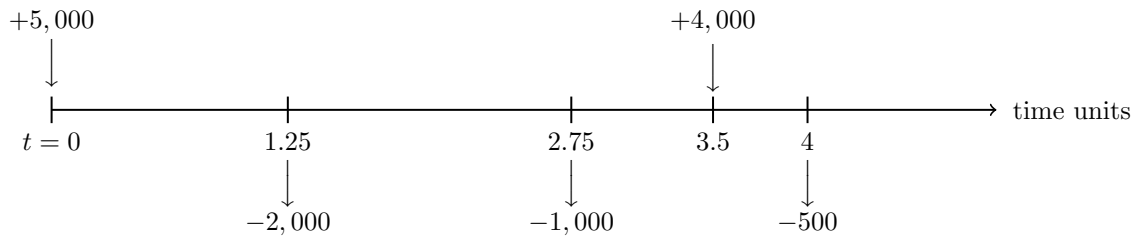


- The effective rate of interest is 6% per time unit. Cashflows are shown in the following time line.
  - Calculate the accumulation at time time  $t = 5$  units of these cashflows.
  - Calculate the value at time time  $t = 2$  units of these cashflows.
  - Calculate the present value at time time  $t = 0$  units of these cashflows.



- The effective rate of interest per annum was 4% during 2015, 3% per half-year until 1 October 2017 and 1.5% per month thereafter. Cashflows are shown in the following time line.

1. Calculate the accumulation on 1/1/2019 of these cashflows.
2. Calculate the present value on 1/1/2015 of these cashflows.
3. Calculate the value at time time 1/7/2017 of these cashflows.



5. (Excel) It is a good exercise to check whether the Excel worksheet you have developed so far for calculating the present value and future value can be applied to the questions in this Tutorial. What would you do to improve the Excel worksheet for a more general scenario?

## 9 Tutorial 3

1. Calculate the present value now of an annuity payable monthly in advance. The annual amount of the annuity will be \$ 2,400 for the first 10 years and \$ 3,600 for the next 15 years, after which payment will cease. Assume that the effective rate of interest is 2% per annum.
2. Assume that the effective rate of interest will be 3% for 5 years from now, 4% for the next 5 years and 5% thereafter. Calculate the following values:
  1. The present value of an annuity of \$ 1,000 per annum, payable in arrear for 15 years.
  2. The present value of an annuity due of \$ 500 per annum, payable at the beginning of the year for 20 years.
  3. The accumulation value of an increasing annuity payable yearly in arrear for 30 years. The first annual payment is \$ 100, and payments will be increase by \$ 100 each year.
  4. The accumulation value of an increasing annuity payable yearly in advance for 18 years. The first annual payment is \$ 1,000, and payments will be increase by 2% each year (compound).
  5. The present value of an annuity of \$ 200, payable in arrear for 10 years and deferred for 3 years.
3. You borrow \$ 240,000 from a bank to be repaid by the end of 5 years. Assume that the interest rate is 4% per annum. Consider the following four possible options for the loan to be repaid.
  1. Calculate the amount of the repayments to repay if you choose to repay the loan as late as possible.
  2. You may choose to repay interest only during the 5 years term of loan and repay the capital at the end of the term. Calculate interest to be repaid and draw the timeline to illustrate the cashflows for the repayment of the loan.
  3. Calculate the amount X of level instalments to repay the loan which will be paid at the end of each year for 5 years and draw the timeline to illustrate the cashflows for the repayment of the loan.
  4. Calculate the amount Y of level instalments to repay the loan which will be paid at the end of each month for 5 years and draw the timeline to illustrate the cashflows for the repayment of the loan. **Instalment** is a sum of money due as one of several equal payments for something, spread over an agreed period of time.

4. A person now age 30 has received a pension from a company. When he retires at age 60, he will be paid on each birthday from the 60 to the 85th inclusive. The first annual payment will be half of his salary when he retires, and payments will then increase by 2% compounding each year. Currently, he receives a salary of \$ 20,000 and will increase by 3% each year compounding in line with inflation. Assume that the effective rate of interest will be 4% for the next 20 years and 5% thereafter. Calculate the present value now of this pension.
5. (Excel) Use Excel worksheet you have developed so far to calculate the results from the questions in this Tutorial.

## 10 Tutorial 4

1. Show that the following series of cashflows are equivalent given that an interest rate is 4% per annum effective.
  1. One single payment of amount 14,802.44 at year 10.
  2. a level annuity of 400 payable yearly in arrear for the next 10 years plus a lump sum of 10,000.
  3. a level annuity of 1,232.91 payable yearly in arrear for the next 10 years.
2. You invest in a project which requires you to pay 2,000 and receive back 300 at the end of each of the next 8 years. Calculate the yield of this investment. ANS = 4.2394551%
3. You pay a price of 5,000 for an investment that will repay you 600 per annum payable half-yearly in arrear for the next 12 years. Calculate the yield of this investment. ANS = 3.1491266%
4. An investor pays 100,000 in order to receive 20,000 back at the end of the first 3 years and 25,000 back at the end in the next 4 years. Calculate the yield of this investment. ANS = 12.6209232%
5. You invest in a project which requires you to pay 500,000 at the start of each of the calendar years 2018, 2019 and 2020. The project is expected to return profits of 400,000 for 6 years at the end of each calendar year 2024 to 2029 inclusive. Calculate the yield of this investment. ANS = 5.7285486%
6. (Modified from CT1 2014 IFoA Exam)

An investor is considering two projects, Project A and Project B. Project A involves the investment of 2,000,000 in a retail outlet. Rent is received quarterly in arrear for 25 years, at an initial rate of 100,000 per annum. It is assumed that the rent will increase at a rate of 5% per annum compound, but with increases taking place every five years. Maintenance and other expenses are incurred quarterly in arrear, at a rate of 12,000 per annum. The retail outlet reverts to its original owner after 25 years for no payment.

Project B involves the purchase of an office building for 1,000,000. The rent is to be received quarterly in advance at an initial rate of 85,000 per annum. It is assumed that the rent will increase to 90,000 per annum after 20 years. There are no maintenance or other expenses. After 25 years the property reverts to its original owner for no payment.

Calculate the annual effective internal rate of return for both Projects A and B. Which project is preferable?

7. (Excel) Use Excel worksheet you have developed to calculate the results from the questions in this Tutorial.

## 11 Tutorial 5

1. You borrow 30,000 for a term of 6 months to be repaid in arrear by level monthly instalments. The rate of interest will be 4% pa effective.
  1. Calculate the monthly repayment.
  2. Construct the complete loan schedule
2. A loan of 800,000 is repayable by equal monthly repayments for 10 years, with interest rate payable at 6.5% pa effective.
  1. Calculate the amount of each monthly payment.
  2. Calculate the interest and capital contents of the 96th repayment.
3.
  1. An investor takes out a loan of 100,000 from a bank to be repaid by level annual instalments in arrear over 12 years where the bank charges an effective annual rate of interest of 7%. Immediately after the 6th repayment has been made, the investor may
    1. extend the term of the loan by extra 2 year, or
    2. miss the next two repayments.Calculate the revised repayment amount in each case.
  2. Suppose the bank allows the investor to miss the next two repayments but the capital outstanding will be charged interest at 10% pa effective while the investor is not making repayments. Calculate the revised repayment.
  3. Suppose in Question 3.2 that the investor will miss the next two repayment and extend the term of the loan by extra 4 years. Calculate the revised repayment.
4. An investor borrows 50,000 for a term of 12 years. The rate of interest will be 4% pa effective for the first 6 years and 5% pa effective thereafter. The loan will be repaid level annual repayments for the first 6 years, and then increasing to twice the origin level for the last 6 years. Calculate the annual repayment.
5. An investor borrows 40,000 for a term of 10 years. The rate of interest will be 6.5% pa effective The loan will be repaid level annual repayments, increasing at 2% per annum.
  1. Calculate the first annual repayment.
  2. Calculate the capital outstanding after the 7th repayment is made.
  3. Calculate the interest content of the 8th repayment.
6. (Excel) Suppose you borrow  $L$  from a bank to be repaid by the end of  $n$  years at an interest rate of  $i\%$  per annum effective. If you agree to repay the loan and the interest in equal annual instalments throughout term of loan and the first payment is made at the end of the first year.

Create a model to produce a loan amortisation (or loan schedule) table. Make the interest rate, loan life, initial loan, and other necessary variables input variables. The loan amortisation table should include the following columns:

- The year-beginning balance
- The annual repayment
- Interest Component
- Capital content
- Capital outstanding (the year-end balance)

## 12 Solutions to Tutorial 1

1. The solutions to each question are as follows:

1.  $5000(1.075)^4 = 6677.345703$
2. Let  $i\%$  be the annual rate effective equivalent to 3% per quarter-effective,  $i = (1.03)^4 - 1$ . Hence, the accumulation is

$$800(1+i)^{2.7} = 800(1.03)^{4 \times 2.7} = 1100.859802.$$

3. Let  $j\%$  be the monthly rate effective equivalent to 4.25% per half-year effective,  $j = (1.0425)^{2/12} - 1$ . Hence, the accumulation is

$$10000(1.0425)^{(2/12) \times 27} = 12059.86056.$$

2. The solutions to each question are as follows:

1.  $\frac{1000}{1.075} = 930.232558$ .
2.  $\frac{100}{(1.03)^7} = 81.309151$ .
3. Let  $j\%$  be the quarterly rate effective equivalent to 4.25% per half-year effective,  $j = (1.0425)^{2/4} - 1$ . Hence, the present value is

$$10000 \times (1+j)^{-5} = 10000(1.0425)^{-(5/2)} = 9011.764643.$$

3. The solutions to each question are as follows:

1. 0.3274%
2. 3.1988%

4. The solutions to each question are as follows:

1.  $500(1.04)(1.05)(1.06) = 578.76$
2.  $2000(1.04)^{3/4}(1.05)(1.06)^{3/4} = 2259.299$
3.  $(1.04)(1.05)(1.06) = 1.15752$

5. The account balance in 3 years is  $3000(1.025)^3 = 3230.67$ .

6. The amount to be repaid for the loan is  $5000(1.1) = 5500$ .

7. Let  $X$  be the amount to be deposited now.

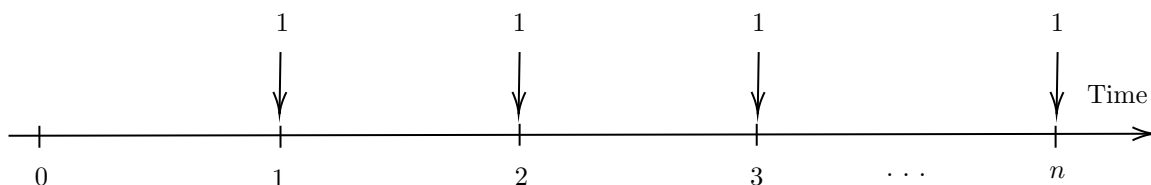
$$X = \frac{1000}{(1.0575)^4} + \frac{2000}{(1.0575)^8} = 2078.36.$$

8. At time 3.75 years, Katy has a balance of  $100(1+i)^{15}$ . The interest on this balance over the next 3 months is  $100(1+i)^{15} \cdot i$ . Taylor earns simple interest on the original amount which is equal to  $500i \cdot \frac{3}{12}$ . Therefore, we solve for  $i$  from the following equation:

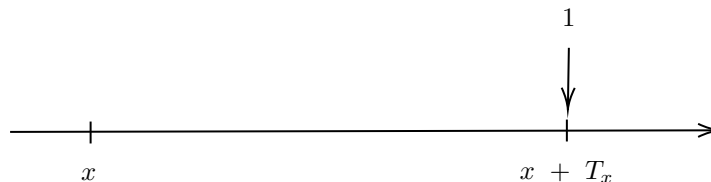
$$100(1+i)^{15} \cdot i = 500i \cdot \frac{3}{12},$$

which gives  $i = 0.014987$ .

9. The timeline for the following annuity having cashflow of 1 unit at the end of each of the next  $n$  time units is given in the figure below:



10. (More details in the course “Life Contingencies I”). We need to define a random variable  $T_x$  = the remaining future life time of a life aged  $x$ .



The quantity of interest is the present value of the death benefit assuming the interest rate of  $i\%$  p.a. effective. It is also a random variable,

$$PV = \frac{1}{(1+i)^{T_x}}.$$

It turns out that the premium rate of this whole life insurance is  $E[PV]$ , the expected value of the present value,  $PV$ .

## 13 Solutions to Tutorial 2

1. The solutions to each question are as follows:

1.  $(1.03)^3(1.04)^{2.5}(1.02)^{12} = 1.528611$
2.  $5000A(0.5, 2.75) = 5000(1.03)(1.04)^{2.5}(1.02)^9 = 6788.786068$
3. We first find the rate  $j\%$  per month effective that is equivalent to the rate of 4% per half-year effective.

$$j = (1.04)^{1/6} - 1 = 0.00656.$$

The accumulated value is  $100A(1 + 2/12, 3 + 7/12) = 100(1 + j)^{10}(1.02)^{19} = 155.522118$ .

4.  $25000V(0, 1.5) = \frac{25000}{(1.03)^3(1.04)^{1.5}} = 21571.39968$ .
5.  $8000V(0.25, 2.75) = \frac{8000}{(1.03)^2(1.04)^{2.5}(1.02)^9} = 5720.455921$ .
6.  $V(0.5, 1.75) = \frac{1}{(1.03)(1.04)^2} = 0.897627$ .

2. The solutions to each question are as follows:

1. With  $i = 7.25\%$  per time period,  $V(4) = 100(1+i)^4 - 30(1+i)^3 - 30(1+i)^2 + 100(1+i) - 30 = 138.041762$ .
2.  $V(8) = V(4) \cdot (1+i)^4 = 182.641597$ .
3.  $PV(0) = V(4) \cdot (1+i)^{-4} = 104.332903$ .

3. The solutions to each question are as follows:

1. With  $i = 6\%$  per time period,  $V(5) = 5000(1+i)^5 - 2000(1+i)^{3.75} - 1000(1+i)^{2.25} + 4000(1+i)^{1.5} - 500(1+i) = 6897.948585$ .
2.  $V(2) = V(5)(1+i)^{-3} = 5791.650645$ .
3.  $PV(0) = V(5)(1+i)^{-5} = 5154.548456$ .

4. The solutions to each question are as follows:

1.  $V(1/1/2019) = 100(1.04)(1.03)^{3.5}(1.015)^{15} - 30(1.03)^{3.5}(1.015)^{15} - 30(1.03)^{1.5}(1.015)^{15} + 100(1.015)^{12} - 30 = 152.955693$ .
2.  $PV(0) = V(1/1/2015) = \frac{152.955693}{(1.04)(1.03)^{3.5}(1.015)^{15}} = 106.074596$ .
3.  $V(1/7/2017) = PV(0)(1.04)(1.03)^3 = 120.546998$ .

## 14 Solutions to Tutorial 3

1. Let  $j$  be the effective rate per month equivalent to  $i = 2\%$ . We have

$$j = (1.02)^{1/12} - 1 = 0.001652.$$

Hence,

$$PV(0) = 200\ddot{a}_{120}^j + 300\ddot{a}_{180}^j \left( \frac{1}{1.02} \right)^{10} = 60148.03.$$

2. The solutions to each question are as follows:

1. The cashflows have been splitted into three periods: (a) from time point 0-5, (b) 5-10 and (c) time point 10 onward.

$$PV(0) = 1000(a_5^{3\%} + 1.03^{-5}a_5^{4\%} + 1.03^{-5}1.04^{-5}a_5^{5\%}) = 11489.49$$

2. We have

$$PV(0) = 500(\ddot{a}_5^{3\%} + 1.03^{-5}\ddot{a}_5^{4\%} + 1.03^{-5}1.04^{-5}\ddot{a}_{10}^{5\%}) = 7229.67$$

3. The accumulated value is

$$100(Is)_5^{3\%}(1.04)^5(1.05)^{20} + [100(Is)_5^{4\%} + 500s_5^{4\%}](1.05)^{20} + [100(Is)_{20}^{5\%} + 1000s_{20}^{5\%}] = 78929.01$$

4. Let  $i_1 = 3\%$ ,  $i_2 = 4\%$  and  $i_3 = 5\%$ . The accumulated value is given by

$$\begin{aligned} V(18) &= [1000(1+i_1)^5 + 1000(1.02)(1+i_1)^4 + 1000(1.02)^2(1+i_1)^3 + \dots + 1000(1.02)^4(1+i_1)](1.04)^5(1.05)^8 \\ &\quad + [1000(1.02)^5(1+i_2)^5 + 1000(1.02)^6(1+i_2)^4 + 1000(1.02)^7(1+i_2)^3 + \dots + 1000(1.02)^9(1+i_2)](1.05)^8 \\ &\quad + [1000(1.02)^{10}(1+i_3)^8 + 1000(1.02)^{11}(1+i_3)^7 + 1000(1.02)^{12}(1+i_3)^6 + \dots + 1000(1.02)^{17}(1+i_3)] \\ &= 1000(1.02)^5 \left[ \left( \frac{1+i_1}{1.02} \right)^5 + \left( \frac{1+i_1}{1.02} \right)^4 + \dots + \left( \frac{1+i_1}{1.02} \right) \right] (1.04)^5(1.05)^8 \\ &\quad + 1000(1.02)^{10} \left[ \left( \frac{1+i_2}{1.02} \right)^5 + \left( \frac{1+i_2}{1.02} \right)^4 + \dots + \left( \frac{1+i_2}{1.02} \right) \right] (1.05)^8 \\ &\quad + 1000(1.02)^{18} \left[ \left( \frac{1+i_3}{1.02} \right)^8 + \left( \frac{1+i_3}{1.02} \right)^7 + \dots + \left( \frac{1+i_3}{1.02} \right) \right] \end{aligned}$$

Let  $1+j_1 = \frac{1+i_1}{1.02}$ . Then,  $j_1 = 0.009804$  and

$$\left[ \left( \frac{1+i_1}{1.02} \right)^5 + \left( \frac{1+i_1}{1.02} \right)^4 + \dots + \left( \frac{1+i_1}{1.02} \right) \right] = \frac{(1+j_1)^5 - 1}{j_1/(1+j_1)} = 5.148995.$$

Let  $1+j_2 = \frac{1+i_2}{1.02}$ . Then,  $j_2 = 0.019608$  and

$$\left[ \left( \frac{1+i_2}{1.02} \right)^5 + \left( \frac{1+i_2}{1.02} \right)^4 + \dots + \left( \frac{1+i_2}{1.02} \right) \right] = 5.301921.$$

Let  $1+j_3 = \frac{1+i_3}{1.02}$ . Then,  $j_3 = 0.029412$  and

$$\left[ \left( \frac{1+i_3}{1.02} \right)^8 + \left( \frac{1+i_3}{1.02} \right)^7 + \dots + \left( \frac{1+i_3}{1.02} \right) \right] = 9.134790.$$

Therefore,  $V(18) = 32814.45$ .



5. The present value is

$$PV(0) = \left( \frac{200}{(1.03)^4} + \frac{200}{(1.03)^5} \right) + 200a_5^{0.04}(1.03)^{-5} + 200a_3^{0.05}(1.03)^{-5}(1.04)^{-5} \\ = 350.2192 + 768.0362 + 386.1574 = 1504.413$$

3. The solutions to each question are as follows:

1.  $240000(1.04)^5 = 291996.7$
2. The interest amounts are  $0.04 \times 240000 = 9600$ .
3. By the Principle of Equivalence, we have

$$240000 = Xa_5^{0.04}.$$

This gives  $X = 53910.51$ .

4. Level installments are payable monthly, which follows

$$240000 = Ya_{60}^j,$$

where  $j = (1.04)^{1/12} - 1$ . This gives  $Y = 4412.23$ .

4. The person retires in 30 years, when his salary is expected to be  $20000 \times (1.03)^{30} = 48545.25$ . The first payment will be half of this which is equal to 24272.62. The present value at age 60 of his pension is

$$24272.62 \times \ddot{a}_{26}^{0.029412} = 449717.9$$

(the precise value is 449719.051954). Here we use  $\frac{1.05}{1.02} = 1.029412$  and the annuity is paid from the 60th to the 85th birthday inclusive so there are 26 payments made in advance. Therefore, the present value of this at age 30 is

$$449717.9 \times (1.05)^{-10} \times (1.04)^{-20} = 126002.9.$$

(the precise value is 126003.181173)

## 15 Solutions to Tutorial 4

1. To examine whether the cashflows are equivalent, we compare their present values.

a. The present value of single payment of amount 14,802.44 at year 10 is

$$PV(0) = \frac{14,802.44}{1.04^{10}} = 10000.$$

b. The present value of the level annuity of 400 payable yearly in arrears for the next 10 years plus a lump sum of 10,000 is

$$PV(0) = 400a_{10}^{0.04} + \frac{10000}{1.04^{10}} = 10000.$$

c. The present value of the level annuity of 1,232.91 payable yearly in arrears for the next 10 years.

$$PV(0) = 1,232.91a_{10}^{0.04} = 10000.$$

It follows that the values of these cashflows are the same, i.e. equivalent.

2. The annual yield of this investment  $i$  is the solution of the equation of value:

$$f(i) = -2000(1+i)^8 + 300s_8^i = 0.$$

If we solve using software, we get  $i = 4.2394551$ . Instead of using software, you can also use linear interpolation to approximate the solution.

3. Working in time unit of half year, the equation of value is

$$f(i) = -5000(1+i)^{12 \times 2} + 300s_{24}^i = 0.$$

The yield  $i$  per half year is  $i = 3.1491266\%$  and hence the annual yield is  $6.397423\%$ .

4. The equation of value is

$$f(i) = -100(1+i)^7 + 20s_7^i + 25s_4^i = 0.$$

The annual yield is  $12.6209232\%$ .

5. You are suggested to draw the time line for these cashflows. The equation of value is

$$f(i) = -5\ddot{s}_3^i(1+i)^9 + 4s_6^i = 0.$$

The annual yield is  $5.7285486\%$ .

## 16 Solutions to Tutorial 5

1. The monthly repayment can be calculated from this equation

$$X = \frac{30000}{a_6^j} = \frac{30000}{5.931847} = 5057.45,$$

where  $j = (1.04)^{1/12} - 1 = 0.003274$ .

The complete loan schedule is illustrated below:

Time	Repayment	Interest Content	Capital Content	Capital Outstanding
0	-	-	-	30000
1	$X$	98.21	4959.23	25040.77
2	$X$	81.98	4975.47	20065.30
3	$X$	65.69	4991.76	15073.54
4	$X$	49.35	5008.10	10065.44
5	$X$	32.95	5024.50	5040.94
6	$X$	16.50	5040.94	0

2. The monthly repayment can be calculated from this equation

$$X = \frac{800000}{a_{120}^j} = \frac{800000}{88.806749} = 9008.32,$$

where  $j = (1.065)^{1/12} - 1 = 0.005262$ .

The capital outstanding after 95th repayment (25 payments left) is

$$L_{95} = 9008.32a_{25}^j = 210506.84.$$

Hence, the interest content of the 96th repayment is

$$j \times L_{95} = 0.005262 \times 210506.84 = 1107.62.$$

The capital content of the 96th repayment is

$$X - 1107.62 = 7900.70.$$

3. 1.

a. **Extending the term of the loan by extra 2 year:** The original repayment is

$$X = \frac{100000}{a_{12}^{0.07}} = \frac{100000}{7.942686} = 12590.20,$$

The capital outstanding after 6th repayment (6 payments left) is

$$L_6 = Xa_6^{0.07} = 60011.68.$$

By extending the term of the loan by extra 2 year, the revised repayment  $X'$  can be obtained (for 8 payments) from

$$X' = \frac{L_6}{a_8^{0.07}} = 10050.02$$

b. **Missing the next two repayments:** From the previous result, the capital outstanding after 6th repayment  $L_6 = 60011.68$ . Then in 2 years, with interest at 7% per annum, this accumulates to

$$L_6 \times (1.07)^2 = 68707.37.$$

This must now be repaid by only 4 annual repayments, so the new repayment  $X''$  can be obtained from

$$X'' = \frac{68707.37}{a_4^{0.07}} = 20284.35.$$

2. The capital outstanding will accumulate (at 10%) to

$$L_6 \times (1.1)^2 = 72614.14.$$

The new repayment amount  $X'''$  is

$$X''' = \frac{72614.14}{a_4^{0.07}} = 21437.73.$$

3. The new repayment amount will be

$$\frac{72614.14}{a_8^{0.07}} = 12160.53.$$

4. You are suggested to draw the time line for these cashflows. Let  $X$  be the level of repayment of the first 6 years ( $2X$  will be repaid after this period for the last 6 years). It can be obtained from

$$50000 = X(a_6^{0.04} + 2a_6^{0.05}(1.04)^{-6}) = 3769.34.$$

5. 1. Let  $X$  be the first annual repayment. Then,

$$40000 = X(v + v^2(1.02) + v^3(1.02)^2 + \dots + v^{10}(1.02)^9),$$

where  $v = 1/(1.065)$ . By rewriting the above equation, we have

$$40000 = \frac{X}{1.02} \left( \frac{1.02}{1.065} + \left( \frac{1.02}{1.065} \right)^2 + \left( \frac{1.02}{1.065} \right)^3 + \dots + \left( \frac{1.02}{1.065} \right)^{10} \right),$$

Let  $i' = \left( \frac{1.065}{1.02} - 1 \right) = 0.044118$ . Hence,

$$40000 = \frac{X}{1.02} \cdot a_{10}^{i'},$$

and  $X = 5133.91$ .

2. We will calculate the capital outstanding after 7th repayment,  $L_7$  (3 payments left). We first find the amount  $X_8$  of the 8th repayment,

$$X_8 = X(1.02)^7 = 5897.25.$$

So the capital outstanding after the 7th repayment is equal to the present value of the remaining 3 repayments (see the table below).

Time	7	8	9	10
Payment	$L_7 = ?$	$X_8$	$X_8(1.02)$	$X_8(1.02)^2$

It follows that

$$\begin{aligned}
 L_7 &= X_8(v + v^2 + v^3) \\
 &= \frac{X_8}{(1.02)} a_3^{i'} \\
 &= 15919.94,
 \end{aligned}$$

where  $v$  and  $i'$  are the same as above.

3. The interest content of the 8th repayment is

$$L_7 * i = 15919.94 \times 0.065 = 1034.80.$$