SCMA469 Actuarial Statistics

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Contents

Chapter 1

Prerequisites

This is a *sample* book written in **Markdown**. You can use anything that Pandoc's Markdown supports, e.g., a math equation $a^2 + b^2 = c^2$.

The **bookdown** package can be installed from CRAN or Github:

```
install.packages("bookdown")
# or the development version
# devtools::install_github("rstudio/bookdown")
```

Remember each Rmd file contains one and only one chapter, and a chapter is defined by the first-level heading `#`.

To compile this example to PDF, you need XeLaTeX. You are recommended to install TinyTeX (which includes XeLaTeX): https://yihui.name/tinytex/.

Chapter 2

Introduction to Stochastic Processes

You can label chapter and section titles using `{#label}` after them, e.g., we can reference Chapter 2. If you do not manually label them, there will be automatic labels anyway, e.g., Chapter 5.

Figures and tables with captions will be placed in `figure` and `table` environments, respectively.

```
par(mar = c(4, 4, .1, .1))
plot(pressure, type = 'b', pch = 19)
```

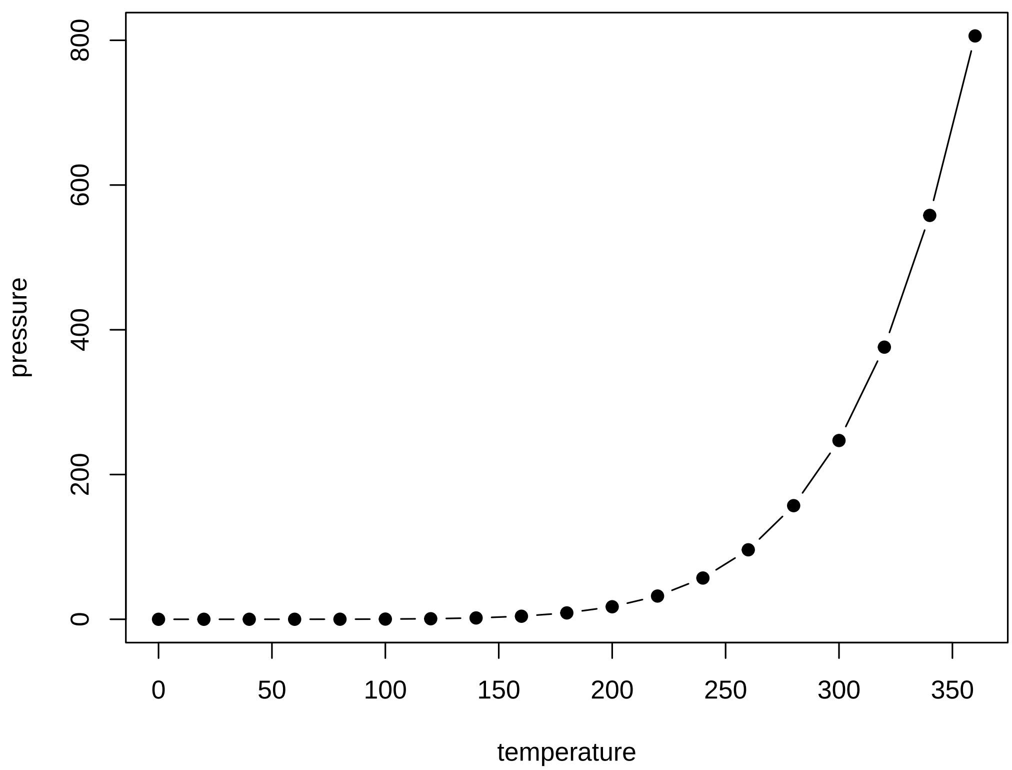

Figure 2.1: Here is a nice figure!

Reference a figure by its code chunk label with the `fig:` prefix, e.g., see Figure 2.1. Similarly, you can reference tables generated from `knitr::kable()`, e.g., see Table 2.1.

Table 2.1: Here is a nice table!

Sepal.Length	Sepal.Width	Petal.Length	Petal.Width	Species
5.1	3.5	1.4	0.2	setosa
4.9	3.0	1.4	0.2	setosa
4.7	3.2	1.3	0.2	setosa
4.6	3.1	1.5	0.2	setosa
5.0	3.6	1.4	0.2	setosa
5.4	3.9	1.7	0.4	setosa
4.6	3.4	1.4	0.3	setosa
5.0	3.4	1.5	0.2	setosa
4.4	2.9	1.4	0.2	setosa
4.9	3.1	1.5	0.1	setosa
5.4	3.7	1.5	0.2	setosa
4.8	3.4	1.6	0.2	setosa
4.8	3.0	1.4	0.1	setosa
4.3	3.0	1.1	0.1	setosa
5.8	4.0	1.2	0.2	setosa
5.7	4.4	1.5	0.4	setosa
5.4	3.9	1.3	0.4	setosa
5.1	3.5	1.4	0.3	setosa
5.7	3.8	1.7	0.3	setosa
5.1	3.8	1.5	0.3	setosa

```r
knitr::kable(
  head(iris, 20), caption = 'Here is a nice table!',
  booktabs = TRUE
)
```

You can write citations, too. For example, we are using the **bookdown** package (Xie, 2021) in this sample book, which was built on top of R Markdown and **knitr** (Xie, 2015).

The course will cover the probabilistic framework for stochastic models of real-world applications with emphasis on actuarial work. We will illustrate some practical actuarial problems for which we will develop mathematical models, tools and techniques for analysing and quantifying the uncertainty of the problems.

Here are some of the examples which will be covered later in the course.

Chapter 3

Examples of real world processes

Example 3.1. Example 1. *(**No claims discount systems (NCD)**) A well-known model widely used by auto insurance companies is the **no claims discount system**, in which an insured receives a discount for a claim free year, while the insured is penalised by an additional premium when one or more accidents occur.*

An example of the NCD system in UK may be structured as follows:

Level	*7*	*6*	*5*	*4*	*3*	*2*	*1*
Premium	*100%*	*75%*	*65%*	*55%*	*45%*	*40%*	*33%*

The rules for moving between these levels are as follows:

- *For a claim-free year, a policyholder moves down 1 level.*
- *Levels 4$-$7:*
  - *For every one claim, the policyholder moves up 1 level or remains at level 7.*
  - *For every two or more claims, move to, or remains at, level 7.*
- *Levels 2$-$3:*
  - *For every one claim, move up 2 levels.*
  - *For every two claims, move up 4 levels.*
  - *For every three or more claims, move to level 7.*
- *Level 1:*
  - *For every one claim, move to level 4.*
  - *For every two claims, move to level 6.*
  - *For every three or more claims, move to level 7.*

The no claims discount system is a form of experience rating consisting of a finite number of levels (or classes), each with its own premium. The 7 levels are experience-rated as described above.

For the NCD model, questions of interest may include:

1. *For 10,000 policyholders at level 7, estimate the* expected numbers* at each discount level at a given time, or once stability has been achieved.*
2. *What is the* probability* that a policyholder who is at a specific discount level (i.e. one of the levels 1-6) has no discount after 2 years?*

3. *What is the* distribution* of being in one of the levels at time 5 years?*

4. *Suppose a large number of people having the same claim probabilities take out policies at the same time. What is the proportion would you expect to be in each discount level in the long run?*

*What would be a suitable model to study the NCD system? As opposed to a **deterministic model** for which its outcomes are fixed, the outcomes of the NCD model are uncertain. It turns out that the NCD system can be studied within the framework of Markov chains, which are examples of stochastic processes. The use of matrix algebra provides a powerful tool to understand and analyse the processes.*

*The evolution of the states or levels can be described the random variables $X_0, X_1, X_2, ...$ and probability distributions, where X_n is the level of the policyholder at time n. In this example, the set of all states called the state space is discrete, which consists of seven levels, and the time variable is also discrete. This is an example of a **discrete time, discrete state space stochastic process**.*

Example 3.2. Example 2. *(**Poisson processes**) Consider the number of claims that occur up to time t (denoted by N_t) from a portfolio of health insurance policies (or other types of insurance products). Suppose that the average rate of occurrence of claims per time unit (e.g. day or week) is given by λ.*

Here are some questions of interest:

1. *On average, 20 claims arrive every day, what is the probability that more than 100 claims arrive within a week?*

2. *What is the expected time until the next claim?*

*In this example, the state space consists of all whole numbers $\{0, 1, 2, ...\}$, while the time variable is continuous. The process is a **continuous-time stochastic process with discrete state space**. The model used to model the insurance claims is an example of **Poisson processes**. The Poisson process is one of the most widely-used counting processes. Even thought we know that claims occur at a certain rate, but completely at random. Moreover, the timing between claims seem to be completely random.*

Later, we will see that there are several ways to describe this process. One can focus on the number of claims that occur up to time t or the times between those claims when they occur. Many important properties of Poisson processes will be discussed.

Example 3.3. Example 3. *(**Markov processes**) Suppose that we observe a total of n independent lives all aged between x and $x + 1$. For life i, we define the following terms:*

- $x + a_i$ *is the age at which observation begins,* $\quad 0 \leq a_i < 1$.
- $x + b_i$ *is the age at which observation ends, if life does not die,* $\quad 0 \leq b_i < 1$.
- $x + t_i$ *is the age at which observation stops, by death or censoring.*
- $d_i = 1$, *if life i dies, otherwise* $d_i = 0$, *if life i censored.*

For example, consider the following mortality data on eight lives all aged between 70 *and* 71.

Life	a_i	b_i	d_i	t_i
1	0	1	1	0.25
2	0	1	1	0.75
3	0	1	0	1
4	0.1	0.6	1	0.5
5	0.2	0.7	1	0.6
6	0.2	0.4	0	0.4
7	0.5	1	1	0.75
8	0.5	0.75	0	0.75

How would one use this dataset to estimate the probability that a life aged 70 dies before age $70+t$ *or survives* *to at least age* $70 + t$*, for* $t \in [0, 1)$*?*

In this example, we can represent the process by $\{X_t\}_{t \geq 0}$ *with two possible states (alive or dead). This model is also an example of a* ***continuous-time stochastic process with discrete state space****.*

Here, we illustrate three actuarial applications which can be modelled by some ***stochastic processes****. We should also emphasis that the outcome of one of the above processes is not fixed or uncertain. The course will provide important tools and techniques to analyse the problems with the goal of quantifying the uncertainty in the system.*

Chapter 4

Discrete-time Markov chains

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Chapter 5

Methods

We describe our methods in this chapter.

Math can be added in body using usual syntax like this

5.1 math example

p is unknown but expected to be around 1/3. Standard error will be approximated

$$SE = \sqrt(\frac{p(1-p)}{n}) \approx \sqrt{\frac{1/3(1-1/3)}{300}} = 0.027$$

You can also use math in footnotes like this*.

We will approximate standard error to 0.027†

*where we mention $p = \frac{a}{b}$

†p is unknown but expected to be around 1/3. Standard error will be approximated

$$SE = \sqrt(\frac{p(1-p)}{n}) \approx \sqrt{\frac{1/3(1-1/3)}{300}} = 0.027$$

Chapter 6

Applications

Some *significant* applications are demonstrated in this chapter.

6.1 DataCamp Light

By default, `tutorial` will convert all R chunks.

eyJsYW5ndWFnZSI6InIiLCJzYW1wbGUiOiJhIDwtIDJcbmIgPC0gM1xuXG5hICsgYiJ9

Chapter 7

Final Words

We have finished a nice book.

Bibliography

Xie, Y. (2015). *Dynamic Documents with R and knitr*. Chapman and Hall/CRC, Boca Raton, Florida, 2nd edition. ISBN 978-1498716963.

Xie, Y. (2021). *bookdown: Authoring Books and Technical Documents with R Markdown*. R package version 0.22.