In the surface M_g of genus g, let C be a circle that separates G_g into two compact subsurfaces M_h' and $M_{k'}$ obtained from the closed surfaces M_h and M_k by deleting an open disk from each. Show that M_h' does not retract onto its boundary circle C and hence M_g does not retract onto C. [Hint: abelianize π_1.] But show that M_g does retract onto the nonseparating circle C' in the figure (Hatcher).

Begin with a quick computation of the fundamental group of the punctured surface of genus g, M_g. Recall the construction of M_g consisted of a single 0-cell, $2g$ 1-cells, and a single 2-cell. Puncturing the 2-cell with a hole or by removing a single point allows us to retract the hole onto the boundary of the cell. After gluing the boundary to the 1-cells, we will just be left with the 1-cells, and we will have a homotopy equivalent space,

$$\pi_1(S^1) =$$

Now suppose for the sake of contradiction that $M_{h'}$ did retract onto the boundary circle $CS^{\{1\}}$. Then the inclusion $\iota_* : \pi_1(S_1) \to \pi_1(M_{h'})$ would be injective.

In addition, we have a quotient map $q : \pi_1(M_{h'}) \to (\pi_1(M_{h'}))^{\{ab\}}$

$$\begin{tikzcd}
\pi_1(S_1) \arrow[r, two heads, "\iota_*"] \arrow[dr, "\iota_* q"'] & \pi_1(M_{h'}) \arrow[d, "q"] \\
& (\pi_1(M_{h'}))^{ab}
\end{tikzcd}$$