1. Find three examples of adjoint functors not mentioned above. Do the same for initial and terminal objects.

   (a) Let $\mathbb{E}$ be a field extension of another field $\mathbb{F}$. Let G be the Galois group $G = \mathrm{Gal}(\mathbb{E} : \mathbb{F})$ and take the category $\mathcal{E}$, with objects being intermediate field extensions between $\mathbb{E}$ and $\mathbb{F}$, and morphisms as inclusions. Take $\mathcal{G}$, with objects being subgroups, and morphisms $\mathrm{Hom}(H, H') = \{\leq | H \leq H'\}$.

   Then, for $\Bbbk \in \mathcal{E}$, we define the map:

   $$\phi : \mathcal{E} \to \mathcal{G}^{op}, \quad \phi(\Bbbk) = \mathrm{Gal}(\mathbb{E} : \Bbbk).$$

   And for $H \in \mathcal{G}$, define:

   $$\phi : \mathcal{G}^{op} \to \mathcal{E}, \quad \psi(H) = \{x \in \mathbb{E} : \alpha(x) = x \forall \alpha \in H\}.$$

   (Recalling that elements of H are automorphisms of $\mathbb{E}$). Recalling from Galois theory that these maps produce a bijective correspondence of field extensions and subgroups, we see that they are functors which are both adjoint to each other.

   (b)

   (c)

2. 2.1.14 (may be hard)

3. Let G be a group.

   (a) What interesting functors are there (in either direction) between Set and the category $[G, \mathrm{Set}]$ of left G-sets? Which of those functors are adjoint to which?

   (b) Similarly, what interesting functors are there between Vect_k and the category $[G, \mathrm{Vect}_k]$ of k-linear representations of G, and what adjunctions are there between those functors?

4. Let $f : A \to B$ and $g : B \to A$ be order-preserving operations between ordered sets. Prove TFAE:

   (a) For any $a \in A, b \in B$,
   $$f(a) \leq b \iff a \leq f(b).$$

   (b) $a \leq g(f(a))$ for any $a \in A$ and $f(g(b)) \leq b$ for any $b \in B$

5. Show that for any adjunction, the right adjoint is full and faithful if and only if the counit is an isomorphism