1 Integration

Definition 1.0.1

Here is the first principle definition for integration, if it proves helpful.

$$\int_a^b f(x)\mathrm{d}x = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(x_{i,n}^*\right) \cdot \frac{b-a}{n}$$

1.1 Integration by parts

The formula for integration by parts can be found by taking the indefinite integral of the formula for the power rule.

Definition 1.1.1

The formula for integration by parts is as follows with du being $u(x)$ and dv being $v(x)$.

$$\int udv = uv - \int vdu$$

2 Convergence Tests

Definition 2.0.1

Divergence test: Take

$$\sum_{n=a}^{\infty} a_n$$

$$\lim_{n\to a_n} \neq 0$$

Implies that the series diverges.

Definition 2.0.2

Limit comparison test. Suppose:

$$\lim_{n\to\infty} \frac{a_n}{b_n} = c$$

For some positive finite number c. Then:

$$\sum_{n=0}^{\infty} a_n, \sum_{n=0}^{\infty} b_n$$

Either both converge or both diverge.

Definition 2.0.3

Ratio/Root test. For some infinite series with terms a_n, if:

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L = \lim_{n\to\infty} \sqrt[n]{|a_n|}$$

Where L is some positive and finite number, $\sum_{n=0}^{\infty} a_n$ behaves like a geometric series with ratio L and converges according to the rules of a geometric series. (if $L < 1$, the series converges absolutely) Note that if the test yields 1, it is inconclusive.

Definition 2.0.4

Alternating Series Test (AST)
For some series with terms a_n containing some product where a term is $(-1)^n$ or $(-1)^{n+1}$, If:

- $\lim_{n\to\infty} = 0$
- a_n *is decreasing*

Then $\sum_{n=0}^{\infty} a_n$ converges.