Written Assignment 2 - Thomas Boyko - 30191728

Consider the power series:

$$\sum_{n=0}^{\infty} \frac{3^n}{n^2+1}(2x-1)^n$$

(a) Explain how you know that the centre of the series is $\frac{1}{2}$.

We can set $2x - 1$ equal to zero, yielding $x = \frac{1}{2}$, and at at that value, each term of the series will equal zero.

(b) Show that the radius of convergence is $\frac{1}{6}$.

To show this, we can use the ratio test and see where $L < 1$.

$$\lim_{n\to\infty} \left| \frac{3^{n+1}(2x-1)^{n+1}}{(n+1)^2+1} \cdot \frac{n^2+1}{3^n(2x-1)^n} \right| = L$$
$$\lim_{n\to\infty} \left| \frac{3(2x-1)(n^2+1)}{n^2+2n+2} \right| = L$$
$$|6x| = L$$
$$|6x| < 1$$
$$|x| < \frac{1}{6}$$

So the radius of convergence is $\frac{1}{6}$.

(c) Show that $x = \frac{1}{3}$ is included in the interval of covergence of the power series.

Our series will now become:

$$\sum_{n=0}^{\infty} \frac{3^n}{n^2+1}(\frac{2}{3}-1)^n$$
$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+1}$$

Now we can use the alternating series test.

$$\lim_{n\to\infty} \frac{(-1)^n}{n^2+1} = 0$$

(since the denominator tends to infinity.)

Now we must show $a_n \leq a_{n+1}$.

$$a_n = \frac{1}{n^2+1}$$
$$a_{n+1} = \frac{1}{n^2+2n+2}$$

Since $n^2 + 2n + 2 > n^2 + 1$ for all positive n, $a_{n+1} \geq a_n$, we can use the Alternating Series test to say that this series converges.

(d) Show that $x = \frac{2}{3}$ is included in the interval of convergence of the power series.

Our series becomes:

$$\sum_{n=0}^{\infty} \frac{3^n}{n^2+1}(\frac{4}{3}-1)^n$$
$$\sum_{n=0}^{\infty} \frac{1}{n^2+1}$$

Now we can use the Comparison Test with $\frac{1}{n^2}$.

$$\lim_{n\to\infty} \frac{n^2}{n^2+1} = 1$$

Since the limit of the ratio of the two test equals a positive, finite number, both series must converge (since $\frac{1}{n^2}$ is a p-series with $p = 2$).