1. Let $M = (Q, \Sigma, T, \delta, q_0, q_{\text{accept}}$ be a Turing machine, where:

   $Q = \{q_0, q_1, q_2, q_3, q_{\text{accept}}\}$,

   $\Sigma = \{0, 1\}$ is the input alphabet

   $T = \{0, 1, \perp\}$ is the tape alphabet (with $\perp$ denoting the blank symbol).

   The transition function δ is defined by the following table:

δ	0	1	$\perp$
q_1	$(q_1, 0, R)$	$(q_1, 0, R)$	$(q_3, \perp, R)$
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	$(q_3, \perp, R)$
q_1	$(q_2, 0, R)$	$(q_0, 1, R)$	$(q_3, \perp, R)$
q_1	$-$	$-$	$-$

   (a) Simulate the behavior of the Turing machine M on the following inputs. For each case, provide the final tape content and the halting state:

      i. 1011

      ii. 111

      iii. 010

   (b) Describe the general behavior of M when the input is of the form 1^k for some $k \in \mathbb{N}$.

   (c) Construct a Turing machine $M' = (Q', \Sigma, T, \delta', q'_0, q'_{\text{accept}})$, where $T = \{0, 1, \perp\}$, that satisfies each of the following:

      i. Replaces the first occurrence of the substring 01 in the input with 10, and leaves the rest unchanged.

      ii. Accepts if and only if the input contains the substring 010.

   Specify only the state transitions relevant to this task (you may assume the rest lead to a rejecting state or halt).

2. Let $\Sigma = \{0, 1\}$. Define the language:

$$L' = \{0^n 1^n 0^n 1^n | n \in \mathbb{N}_0\}.$$

   (a) Design a Turing machine that accepts the language L'.

   (b) Prove that if the Turing machine accepts a string x, then $x \in L'$.

   (c) Modify the Turing machine so that it replaces the input $x \in L'$ with the string xx (i.e., it duplicates the input).

   (d) Prove that the modified machine correctly duplicates the input only if $x \in L'$.