Name: Thomas Boyko; UCID: 30191728

1. Let $M = (Q, \Sigma, T, \delta, q_0, q_{accept})$ be a Turing machine, where:

$$Q = \{q_0, q_1, q_2, q_3, q_{\text{accept}}\},\$$

 $\Sigma = \{0, 1\}$ is the input alphabet

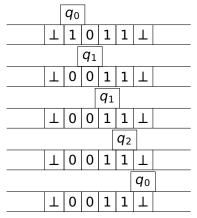
 $T = \{0, 1, \bot\}$ is the tape alphabet (with \bot denoting the blank symbol).

The transition function δ is defined by the following table:

δ	0	1	
$\overline{q_0}$	$(q_1, 0, R)$	$(q_1, 0, R)$ $(q_2, 1, R)$ $(q_0, 1, R)$	(q_3, \perp, R)
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	(q_3, \perp, R)
q_2	$(q_2, 0, R)$	$(q_0, 1, R)$	(q_3, \perp, R)
q_3	_	_	_

- (a) Simulate the behavior of the Turing machine M on the following inputs. For each case, provide the final tape content and the halting state:
 - i. 1011

Solution: Begin with the input string and the state q_0 .



And at this point we transition to q_3 which is a halting state. We are left with the final tape content:

ii. 111

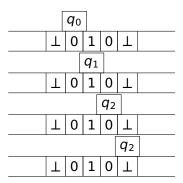
Solution:

And again, we will switch states to q_3 and halt, leaving us with the final tape:

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iii. 010

Solution:



And again, we will switch states to q_3 and halt, leaving us with the final tape: $\boxed{\perp 0 \ 1 \ 0 \ \perp}$

(b) Describe the general behavior of M when the input is of the form 1^k for some $k \in \mathbb{N}$.

Solution:

Now, we can see that we are in the same starting state, only now we are working with the string 1^{k-3} . So we repeat the above process on this substring, and continue until we find the character \bot , at which point we will be stuck in q_3 , and halt. So we can say that the machine takes a string 1^k and converts every third 1 to a zero, starting with the first 1.

- (c) Construct a Turing machine $M' = (Q', \Sigma, T, \delta', q'_0, q'_{accept})$, where $T = \{0, 1, \bot\}$, that satisfies each of the following:
 - i. Replaces the first occurrence of the substring 01 in the input with 10, and leaves the rest unchanged.
 - ii. Accepts if and only if the input contains the substring 010.

Specify only the state transitions relevant to this task (you may assume the rest lead to a rejecting state or halt).

2. Let $\Sigma = \{0, 1\}$. Define the language:

$$L' = \{0^n 1^n 0^n 1^n | n \in \mathbb{N}_0\}.$$

(a) Design a Turing machine that accepts the language L'.

Solution: Let $M = (Q, \Sigma, T, \delta, q_0, q_{accept})$ be a Turing machine, where:

 $Q = \{q_0, q_1, q_2, q_3, q_{\text{accept}}\},\$

 $\Sigma = \{0, 1\}$ is the input alphabet

 $T = \{0, 1, \bot\}$ is the tape alphabet (with \bot denoting the blank symbol).

The transition function δ is defined by the following table:

Provided for readability is a summary of each state and its purpose:

- i. Initial state. As well, this state scans right until it finds \bot , then moves to the left and switches to state 1.
- ii. When this state is reached, the position on the tape should be on the furthest left 0 or 1. If it's a 1, we switch to state 2. If it's a 0 or \bot we halt.
- iii. This state scans left until it finds \bot , then moves to the right and switches to state 3.
- (b) Prove that if the Turing machine accepts a string x, then $x \in L'$.
- (c) Modify the Turing machine so that it replaces the input $x \in L'$ with the string xx (i.e., it duplicates the input).
- (d) Prove that the modified machine correctly duplicates the input only if $x \in L'$.