This assignment requests questions 3-12 from Papa Rudin be done

3. Prove that if f is a real function on a measurable space X such that $\{x : f(x) \geq r\}$ is measurable for every rational r, then f is measurable.

4. Let $\{a_n\}$ and $\{b_n\}$ be sequences in $[-\infty, \infty]$, and prove the following assertions:

   (a)
$$\limsup_{n\to\infty}(-a_n) = -\liminf_{n\to\infty} a_n.$$

   Proof: We first begin by showing that $\sup -a_n = -\inf a_n$. The result follows from properties of limits, namely $\lim_{n\to\infty} -a_n = -\lim_{n\to\infty} a_n$ provided the limit exists.
   Let $\beta = \inf_{m>n} a_n$. Then for any $m > n$, we have $\beta < a_m$, and therefore $-\beta > -a_m$. So $-\beta$ is an upper bound for $-a_m$.
   Since is the greatest lower bound for $A = \{a_m : m > n\}$, we have that if $\gamma > \beta$ is a lower bound for A, then necessarily $\gamma = \beta$.
   Suppose that α is a lesser upper bound for ■

   (b)
$$\limsup_{n\to\infty}(a_n + b_n) \leq \limsup_{n\to\infty} a_n + \limsup_{n\to\infty} b_n$$

   provided none of the sums is of the form $\infty - \infty$.

   Proof: Suppose that a_n, b_n are as above.

$$\limsup_{n\to\infty}(a_n + b_n) = \lim_{n\to\infty} \sup_{m>n}(a_n + b_n)$$

   .

   ■

   One needs only to choose constant sequences (or one constant and one non-constant), in order for strict equality to hold.

   (c) If $a_n \leq b_n$ for all n, then
$$\liminf_{n\to\infty} a_n \leq \liminf_{n\to\infty} b_n.$$