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Let b be the last nonzero digit of your UCID number.
My $b = 8$.

1. Suppose that $\sum_{n=0}^{\infty} a_n$ is a series for which the Ratio Test is inconclusive.
   Find the radius of the power series $\sum_{n=1}^{\infty} (-1)^n \frac{a_n}{n^2+1}(4x-8)^{2n}$
   We know that since the Ratio Test is inconclusive, $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1$.
   To find the radius of our series, we can run the ratio test on it.
   $$\begin{aligned} L &= \lim_{n\to\infty} \left| \frac{a_{n+1}(4x-8)^{2n+2}}{(n+1)^2+1} \frac{n^2+1}{a_n(4x-8)^{2n}} \right| \\ &= \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| \cdot \lim_{n\to\infty} \left| \frac{(n^2+1)(4x-8)^{2n+2}}{(n^2+2n+2)(4x-8)^{2n}} \right| \\ &= 1 \cdot \left|(4x-8)^2\right| \cdot \lim_{n\to\infty} \left| \frac{n^2+1}{n^2+2n+2} \right| \\ &= |(4x-8)^2| \end{aligned}$$
   And since $L < 1$ where the series converges,
   $$\begin{aligned} |4x-8|^2 &< 1 \\ |4x-8| &< 1 \\ \left| x - \frac{1}{2} \right| &= \frac{1}{4} \end{aligned}$$
   So, the centre is 1/2 and the radius is 1/4.
2. Consider the series $\sum_{n=0}^{\infty} \frac{(-1)^n 9^n}{(2n)!}$.
   (a) Compute the limit L required to perform the Ratio Test (Write a number or DIV).
   $$\begin{aligned} L &= \lim_{n\to\infty} \left| \frac{9^{n+1}}{(2n+1)!} \frac{(2n)!}{9^n} \right| \\ &= \lim_{n\to\infty} \left| \frac{9}{2n+1} \right| \\ &= 0 \end{aligned}$$
   So $L = 0$.
   (b) Therefore, by the Ratio Test the series converges absolutely.
3. Suppose that a_n is a sequence where we only know the first few terms.
   In particular, suppose that $\{a_n\}_{n=1}^{\infty} = \{2, 3, 8, -8, 1, 5, -3, \dots\}$.
   (a) If the series $\sum_{n=1}^{\infty} a_n$ diverges, then $\lim_{n\to\infty} a_n$ is unknown.
   (b) If the series $\sum_{n=1}^{\infty} a_n$ converges to 267, then $\sum_{n=7}^{\infty} a_n = 256$
4. Fill in the blanks to make the following sentence true.
   The series $\sum_{n=8}^{\infty} \frac{(-1)^{n+1}}{n^{10}}$ converges by the Alternating Series Test.