1 Statements

A **Statement or Proposition** is a sentence that is true or false but not both. Here's some examples:

Example 1.0.1

- *The sky is blue.*
- *5=2*
- *All prime numbers are divisible by two.*

A **Conditional Statement** is one of the form "If p, then q." We denote this correlation with a $\Rightarrow$ symbol. If we have a statement $p \Rightarrow q \equiv r$, then r is always true unless p is false and q is true. In a conditional statement, we consider p to be the hypothesis, and q to be the conclusion.

Note that the hypothesis of a conditional statement can be false, as long as the conclusion is as well. This means that $0 = 1 \rightarrow 1 = 2$ is true.

Statements can take other forms, such as using and ($\wedge$), or ($\vee$), not ($\neg$), as well as some other forms seen especially when writing proofs.

Two mathematical forms often seen are: "There exists / For some" ($\exists$), and "For all / For every" ($\forall$).

The **negation** of a Statement is the opposite of the statement. For all true values of P, $\neg P$ will be false. and vice versa.

Definition 1.0.2

Negations of some basic statements:

$$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$$

$$\neg(P \vee Q) \equiv \neg P \wedge \neg Q$$

$$\neg(\exists x, P(x)) \equiv \forall x, \neg P(x)$$

$$\neg(\forall x, Q(x)) \equiv \exists x, \neg Q(x)$$

$$\neg(P \rightarrow Q) \equiv P \wedge \neg Q$$

Definition 1.0.3

- *The converse of $P \rightarrow Q$ is $Q \rightarrow P$.*
- *The contrapositive of $P \rightarrow Q$ is $\neg P \rightarrow \neg Q$.*

Note that the contrapositive of a statement is logically equivalent, but the converse may not be.

2 Integers

Integers posess the following properties:

Remember that theorems can be used in assignment proofs, but lemmas must be used more carefully with explanation.

Theorem 2.0.1

$\forall n \in \mathbb{Z}$, *n is either even or odd but not both.*

Theorem 2.0.2

$\forall a, b \in \mathbb{Z},\ a|b \implies b \geq a$

Theorem 2.0.3

Products and sums of integers are integers.

Lemma 2.0.4

Suppose m, n are integers. If m and n are both odd, mn is odd.

Proof 2.0.5

Assume $m, n \in \mathbb{Z}$. Suppose m and n are odd.
Then, $m = 2k + 1, n = 2l + 1$ for some $l, k \in \mathbb{Z}$.
$mn = (2k + 1)(2l + 1)$
$mn = 2(4kl + k + l) + 1$
So, mn can be expressed in the form $2q + 1$, where $q = 4kl + k + l$, and q is an integer.

Definition 2.0.6

Let $n \in \mathbb{Z}$.
n is prime $\iff \exists r, s$ so that $rs = n$, and $r = 1, s = n$ or $r = n, s = 1$.
n is composite $\iff \exists r, s$ so that $rs = n$ and nither r or s is equal to 1.

Definition 2.0.7

Let $a, b \in \mathbb{Z}$. a divides b ($a|b$) if $a \neq 0$ and $ak = b$ for some $k \in \mathbb{Z}$.

3 Rationals

Definition 3.0.1

Let $x \in \mathbb{R}$. x is rational ($x \in \mathbb{Q}$) $\iff$ $x = \frac{a}{b}$ for some $a, b \in \mathbb{Z}, b \neq 0$

Lemma 3.0.2

If $x \in \mathbb{Z}$, $x \in \mathbb{Q}$.

Proof: *Any $x \in \mathbb{Z}$ can be written as $x = \frac{x}{1}$. Since both 1 and x are integers, x is rational.* ∎

Lemma 3.0.3

Like with integers, the sum and product of any rationals is also rational.