

LECTURE NOTES
CPSC 351 — Winter 2025
**Theoretical Foundations of Computer
Science II**

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Chapter 1: Discrete Probability Theory
Chapter 2: Random Variables

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