Assignment 2 - Thomas Boyko - 30191728

1. Consider the following probability distribution table for a discrete random variable X.

x	-10	0	5
P(X=x)	0.3	0.5	0.2

   (a) Find $E[X]$.

$$\begin{aligned} E[X] &= \sum xp(x) \\ &= -10(0.3) + 0(0.5) + 0.2(5) \\ &= -3 + 1 \\ &= -2 \end{aligned}$$

   (b) Find $VAR[X]$.

$$\begin{aligned} E[X^2] &= \sum x^2 p(x) \\ &= 100(0.3) + 0(0.5) + 25(0.2) \\ &= 30 + 0 + 5 \\ &= 35 \\ VAR[X] &= E[X^2] - E[X]^2 \\ &= 35 - (-2)^2 \\ &= 31 \end{aligned}$$

   (c) Find $E[3X]$.

$$\begin{aligned} E[3X] &= 3E[X] \\ &= 3(-2) \\ &= -6 \end{aligned}$$

2. In an experiment two weighted (unfair) coins are being flipped simultaneously. The first coin will land "heads" with a probability of 0.7 and the second coin will land "heads" with a probability of 0.8. Assume that the results of the flips are independent. Let X represent the total number of "heads" that result from this experiment.

   (a) Find $E[X]$.
   We can find the expected value from the formula $\sum xp(x)$.

$$E[X] = 0(0.3 \cdot 0.2) + 1(0.3 \cdot 0.8 + 0.7 \cdot 0.2) + 2(0.7 \cdot 0.8) = 1.5$$

   So the expected number of heads is 1.5.

   (b) Find $SD[X]$.

$$\begin{aligned} SD[X] &= \sqrt{E[X^2] - E[X]^2} \\ &= \sqrt{0(0.3 \cdot 0.2) + 1(0.3 \cdot 0.8 + 0.7 \cdot 0.2) + 4(0.7 \cdot 0.8) - 1.5^2} \\ &= \sqrt{2.62 - 2.25} \\ &= 0.6082762 \end{aligned}$$

   So $SD[X] = 0.60828$.

3. Four fair coins are flipped. Let X represent the number of "heads" observed.

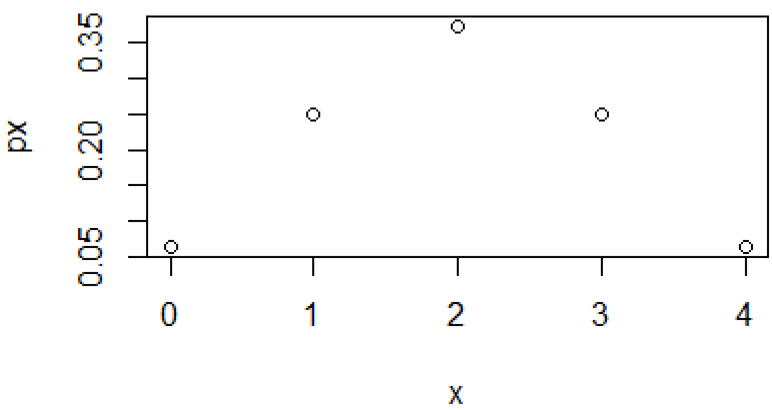
   (a) Construct a probability distribution table for X.

```
x=0:4
px=dbinom(x,4,.5)
```

x	0	1	2	3	4
P(X=x)	0.0625	0.2500	0.3750	0.2500	0.0625

   (b) Create a probability distribution graph for X (a simple sketch is fine!). Include the graph/sketch in your assignment submission. How would you describe the shape of the graph?

```
##Continuing from a)
plot(x,px)
```



   The shape of the graph is symmetric around the mean of 2. The graph is roughly bell-shaped

   (c) What is the probability that at least two "heads" are observed when these four fair coins are flipped?

```
##Continuing from a),b)
sum(dbinom(2:4,4,0.5))
#0.6875
```

   So the probability of at least two heads is 0.6875.

4. A factory produces a type of screw that is sold in small boxes of 20 screws each. Suppose the probability of any given screw being defective is 0.01.

   (a) What is the probability that a box contains no defective screws?

```
dbinom(0,20,0.01)
#0.8179069
```

   So the probability that the box contains no defective screws is 0.81791.

   (b) What is the probability that a box contains exactly one defective screw?

```
dbinom(1,20,0.01)
#0.1652337
```

   So the probability of there being exactly one screw that is defective is 0.0.16523.

(c) The company that sells the boxes offers a full refund on the purchase of a box if the box contains more than one defective screw. What is the probability that the company would have to offer a refund for a randomly selected box?

```
sum(dbinom(2:20,20,0.01))
#0.01685934
```

So the probability that the company will have to refund a randomly selected box is 0.01686.

5. Suppose $Y \sim binomial(120, 0.3)$.

(a) Find $E[4Y-2]$.
First we will find $E[Y]$.

$$E[Y] = np = 120(0.3) = 36$$

From this,

$$E[4Y-2] = 4E[Y] - 2 = 4(36) - 2 = 142$$

So $E[4Y-2] = 142$.

(b) Find $VAR[4Y-2]$.
First we find $VAR[Y]$.

$$VAR[Y] = npq = 120(0.7)(0.3) = 25.2$$

And we know that:

$$VAR[4Y-2] = 4^2 VAR[Y] = 16(25.2) = 403.2$$

So $VAR[4Y-2] = 403.2$.

6. An urn contains two red balls, four blue balls, and three green balls. You randomly select a ball from the urn, record its color, and return it to the urn. You do this process twice (for a total of two recorded colors). Let X represent the number of red balls that you observed.

(a) Create a probability distribution table for X.

```
dbinom(0:2,2,2/9)
```

x	0	1	2
P(X=x)	0.60493827	0.34567901	0.04938272

(b) How many red balls should you expect to select in this process?

$$E[X] = np = 2\frac{2}{9} = \frac{4}{9}$$

7. Bob is playing a game in which a fair die is rolled. If the die lands on a 6, Bob wins the round. If it lands on any other number, he loses the round.

(a) What is the probability that Bob's first win will occur on the fourth round?

```
dbinom(1-1,4-1,1/6)/6
#0.09645062
```

So the probability that his first win occurs on the fourth round is 0.09645.

(b) What is the probability that Bob's second win will occur on the third round?

```
dbinom(2-1,3-1,1/6)/6
#0.0462963
```

So the probability that Bob's second win occurs on the third round is 0.04630.

8. According to the M&M's website, 24% of peanut M&M's are yellow. Consider a package containing 23 peanut M&M's.

   (a) What is the probability that more than 10 of the M&M's in the pack are yellow?

   ```
   sum(dbinom(11:23,23,0.24))
   #0.01087513
   ```

   The probability of more than 10 yellow M&M's in the pack is 0.01088.

   (b) What is the probability that between five and 10 (inclusive) of the M&M's in the pack are yellow?

   ```
   sum(dbinom(5:10,23,0.24))
   #0.6674023
   ```

   The probability of between five and 10 (inculsive) M&M's are in the pack is 0.66740.

   (c) If the number of yellow M&M's in the pack is between five and 10 (inclusive), what is the probability that there are exactly seven yellow M&M's in the pack?

   $$P(x = 7|5 \leq x \leq 10) = \frac{P(x = 7 \cap 5 \leq x \leq 10)}{P(5 \leq x \leq 10)} = \frac{P(x = 7)}{P(5 \leq x \leq 10)}$$

   ```
   dbinom(7,23,0.24)/sum(dbinom(5:10,23,0.24))
   #0.20871
   ```

   So, given that there are between 5 and 10 inclusive yellow M&M's in the pack, there is a 0.20871 chance that there are 7 yellow M&M's.