This assignment requests questions 3-12 from Papa Rudin be done

3. Prove that if f is a real function on a measurable space X such that $\{x : f(x) \geq r\}$ is measurable for every rational r, then f is measurable.

4. Let $\{a_n\}$ and $\{b_n\}$ be sequences in $[-\infty, \infty]$, and prove the following assertions:

   (a)
   $$\limsup_{n\to\infty}(-a_n) = -\liminf_{n\to\infty} a_n.$$

   (b)
   $$\limsup_{n\to\infty}(a_n + b_n) \leq \limsup_{n\to\infty} a_n + \limsup_{n\to\infty} b_n$$

   provided none of the sums is of the form $\infty - \infty$.

   (c) If $a_n \leq b_n$ for all n, then
   $$\liminf_{n\to\infty} a_n \leq \liminf_{n\to\infty} b_n.$$

   Show by an example that strict inequality can hold in (b).