1 Categories

Definition 1.1

A Category is a collection of objects, together with a collection of morphisms satisfying:

- *Each morphism f has domain and codomain objects, $f : x \to y$*
- *For any object x, there exists an identity object 1_x, satisfying $1_x x = x$.*
- *For any pair of morphisms, $f : x \to y$, $g : y \to z$, there exists a composite morphism, $gf : x \to z$. This also introduces a law of associativity, $g(fx) = (gf)x$.*

Example 1.1

- Set *forms a category, with its objects being sets, and morphisms being functions between sets.*
- Top *has topological spaces as its objects and continuous functions as its morphisms.*
- Group *has Groups as objects and Group Homomorphisms as morphisms. This example lent the general term "morphisms" to the data of an abstract category. The categories* Ring *of associative and unital Rings and Ring Homomorphisms and* Field *of Fields and field homomorphisms are defined similarly.*

Example 1.2

Keep in mind that categories need not have objects as sets. Though many familiar categories will deal only with sets and functions, our only requirement for a category is that we have objects and morphisms.

- *Take a poset* P*. This forms a category, with objects of P being objects in the category, and if $x \le y$, we have a morphism $x \to y$. Transitivity allows for composition, and identity is guaranteed from reflexivity.*
- *A group defines a category* BG*, with a single object. Morphisms are the elements of G, which can be composed, are associative, and we have an identity in any group G, which becomes the identity morphism on our object.*