Thomas "Buckets" Practice Final

1. Prove whether each set is a subspace of $\mathbb{M}_{22}$.
   (a) $\mathbb{D}$, the set of all diagonal 2×2 matrices.
   (b) $SL_2(\mathbb{R}) = \{A \in \mathbb{M}_{22} : \det A = 1\}$
   (c) $\left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \right\}$
2. Expand the set $\{1 + x^2\}$to an orthogonal basis for $\mathbb{P}_2$. Show that it is in fact a basis, and that it is orthogonal.
3. Given a linear transformation $T : \mathbb{M}_{22} \to \mathbb{P}_3$, where $\ker T = \{\vec{0}\}$, solve the following. Justify your answers.
   (a) Is T an isomorphism? Prove your answer.
   (b) Explicitly write a linear transformation T satisfying the above, and give its matrix $M_{DB}(T)$, using the bases:

$$B = \left\{ \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}, \quad D = \{1, x, x^2, x^3\}.$$

   (c) Is your matrix invertible?