Final Assessment Test
Model Question Paper

Course: BMAT101L Calculus **Slot:** ________
Max Marks: 100 **Time:** 3 hours

Answer Any Five Questions Each question carries 20 marks

1. (a) Examine the continuity and the differentiability of

$$f(x) = \begin{cases} x\sin(1/x), & x \neq 0, \\ 0, & x = 0. \end{cases}$$

at the origin. (10 marks)

(b) Find the area of the region under the graph of $f(x) = x\sqrt{4-x^2}$ between the ordinates $x=-2$ and $x=2$. Further, use washer's method to obtain the volume of the solid generated by revolving the curve $y = f(x)$ between the limits $x=-2$ and $x=2$. (10 marks)

2. (a) If $f(t) = \sin^2 2t$, find the Laplace transforms of $tf(t)$ and $f(t)/t$. (10 marks)

(b) Find the inverse Laplace transform of $\dfrac{1}{(s+1)(s+2)(s^2+2s+10)}$. (10 marks)

3. (a) A delivery company accepts only rectangular boxes the sum of whose length and the perimeter of a cross-section does not exceed 108 inches. Find the dimensions of an acceptable box of largest volume. (10 marks)

(b) Let $f(x, y) = \sin 2x \cos 3y$. Then find all the partial derivatives of upto third order at the origin, and then obtain a cubic approximation of f near the origin. (10 marks)

4. (a) Using the Gamma function, evaluate $\iint xy^2 dxdy$ over the region R enclosed by the coordinate axes and the astroid $x^{2/3} + y^{2/3} = 1$ in the first quadrant. (10 marks)

(b) If $u = x^2 + 4y^2 + 9z^2$, $v = 4xy + 12yz + 6xz$ and $w = x + 2y + 3z$, compute the Jacobian of u, v and w with respect to x, y and z by the direct formula. Then obtain a functional relation among u, v and w. Also, find the Jacobian of u, v and w with respect to x, y and z by using the functional relation and the chain rule of partial differentiation.(10 marks)

5. (a) Sketch the region of integration of $I=\int_0^a\int_0^{b\sqrt{1-x^2/a^2}} x^3 y\,dy\,dx$, and evaluate it directly and also by using the transformation $x=ar\cos\theta,\ y=br\sin\theta$.

(10 marks)

(b) If $f(x,y,z)$ is the electrical charge-density of a solid bounding a volume V, it is known that $\iiint\limits_V f(x,y,z)\,dV$ gives the charge inside the solid. If a uniform charge-density of $f(x,y,z)=3$ Coulombs/cc prevails inside a solid, which is bounded above by the unit sphere $x^2+y^2+z^2=1$ and bounded below by the cone $z=\sqrt{x^2+y^2}$, sketch the region, specify the appropriate limits and the volume element of triple integral, and then find charge stored in it. (10 marks)

6. (a) Using appropriate vector-identities, show that $\mathbf{f}=\dfrac{x\mathbf{i}+y\mathbf{j}+z\mathbf{k}}{\left(x^2+y^2+z^2\right)^{3/2}}$ is both solenoidal and irrotational. (10 marks)

(b) Find the tangent plane and the normal line L to the level surface $x^2+y^2+z-9=0$ at the point $(1,2,4)$. What will be the directional derivative of x^2+y^2+z-9 at that point along the normal L, and in the direction perpendicular to the normal L? (10 marks)

7. (a) Verify Green's theorem in the plane for $\mathbf{F}=x^2(\mathbf{i}+\mathbf{j})$ around the rectangle with vertices $(0,0)$, $(2,0)$, $(0,1)$ and $(2,1)$. (10 marks)

(b) Use Gauss' divergence theorem, compute $\iint \mathbf{f}\cdot\mathbf{N}\,dS$ over the convex portion of the cylinder $x^2+y^2=9$, $0\le z\le 2$, where $\mathbf{F}=y\mathbf{i}+x\mathbf{j}+z^2\mathbf{k}$.

(10 marks)