Fall Semester 2021-2022

Continuous Assessment Test - I (October 2022)

Slot: B2+TB2 **Class No.** VL2021220106763

Course Code: BMAT101L **Course Title:** Calculus

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Max. Time: 50 minutes **Max. Marks: 30**

Answer all the questions

1(a) **Easy** 10 marks **CO1** **BL1**

Verify Lagrange's mean-value theorem for $f(x) = x^4 - 16$ on $[1, 2]$, and find its absolute maxima and absolute minima in $[1, 2]$. Also, find the equation of the chord joining the points $(1, -15)$ and $(2, 0)$.

1(b) **Easy** 10 marks **CO1** **BL1**

Consider $f(t) = t^5 - 81t$ for all real t. Verify Rolle's theorem to f on $[-3, 3]$ and use it to find the appropriate critical points of f. Find the intervals of monotonicity and the points of inflection of f.

1(c) **Easy** 10 marks **CO1** **BL1**

Using the first and the second derivative tests, find the relative extrema and the points of inflection of $f(x) = x^4 - 8x^2 + 10$.

1(d) **Easy** 10 marks **CO1** **BL1**

Using the first and the second derivative tests, find the relative extrema and the points of inflection of $f(x) = x^4$.

2(a) **Tough** 10 marks **CO1** **BL3**

Let $f(x) = x\sqrt{4 - x^2}$. Find its points of intersection of the graph of f with the x-axis. Use this information to compute the area of the region $\mathscr{R}$ enclosed by the graph of $f(x)$ and the x-axis. Further, if the region $\mathscr{R}$ is revolved about the x-axis, what is the volume of the solid so generated?

2(b) **Tough** 10 marks **CO1** **BL3**

Using an appropriate substitution, find the area of the region $\mathscr{R}$, enclosed by the curve $y = x\sqrt{2 - x}$ between the ordinates $x = 0$ and $x = 2$. If $\mathscr{R}$ is revolved about the x-axis, compute the volume of the solid so generated.

2(c) **Tough** 10 marks **CO1** **BL3**

Find the area of the region $\mathscr{R}$, enclosed by the curve $y = x^{2/3}\sqrt{1 - x^{2/3}}$ between the ordinates $x = 0$ and $x = 1$. Also, compute the volume of the solid of revolution of the region $\mathscr{R}$ about the x-axis.

2(d) **Tough** 10 marks **CO1** **BL3**

Find the points of intersection of the curves $y = x^2$ and $y = x^3$. Use this information to find the area of the region $\mathscr{R}$, enclosed by these two curves. Also, compute the volume of the solid of revolution of the region $\mathscr{R}$ about the x-axis.

3(a) **Medium** 10 marks **CO2** **BL3**

Find the rate of change of $f(x, y, z) = xy^2 - yz^3 + 2zx^2$, as (x, y, z) follows the curve $x = \sin t$, $y = \cos t$, $z = t/2$. What is its value at the point $(1, 0, \pi/4)$?

3(b) **Medium** 10 marks **CO2** **BL3**

Consider $p = x - 2y + 3z$, $q = 2x + y - z$, $r = x - \frac{y}{3} + \frac{2z}{3}$. Compute the Jacobian $J\left(\frac{p,q,r}{x,y,z}\right)$ at any point (x, y, z). Also examine the possibility of functional dependence of p, q and r. If so, find the relation among them.

3(c) **Medium** 10 marks **CO2** **BL3**

Examine the continuity of

$$f(x) = \begin{cases} \frac{(1-x)^2 \log x}{(x-1)^2+y^2}, & (x, y) \neq (1, 0) \\ 0, & (x, y) = (1, 0). \end{cases}$$

3(d) **Medium** 10 marks **CO2** **BL3**

Examine the continuity of

$$f(x) = \begin{cases} \frac{x^3+y^3}{x+y}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0). \end{cases}$$

Remark. The Bloom's Taxonomy levels should be mentioned as per the question