
Röntgenstrahlung

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TEIL III

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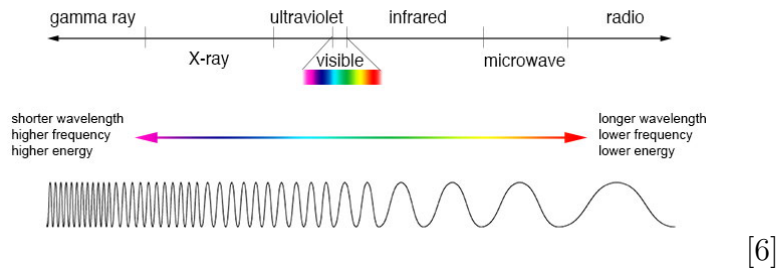
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1 Abstract

This experiment deals with X-rays, which were discovered by Conrad Röntgen in 1895 and named after him. In the electromagnetic spectrum, it is located at a wavelength of less than 10nm. In this experiment the K_α and K_β characteristics are considered. Their energy and characteristic wavelength are calculated using a molybdenum anode, the absorption of a zirconium foil is determined and the lattice constant of a LiF crystal is determined.



2 Theory

2.1 Generation of X-rays

X-rays are electromagnetic waves generated by the deceleration of fast electrons. There is the following relationship between the energy $[E]$ of the electrons and the wavelength $[\lambda]$ of the generated radiation:

$$E = h \cdot \nu = h \cdot \frac{c}{\lambda} \quad (1)$$

with h being Planck's constant, ν the frequency and c the speed of light.

X-rays are often generated with an X-ray tube. In this process, fast electrons are decelerated in solid matter. The electrons emitted by a hot cathode are accelerated with a high voltage U . This causes them to hit the anode with a high velocity. When the electrons penetrate the anode material, X-rays are generated. The heating voltage U_H can be used to regulate the current of the accelerated electrons and the intensity of the X-rays.

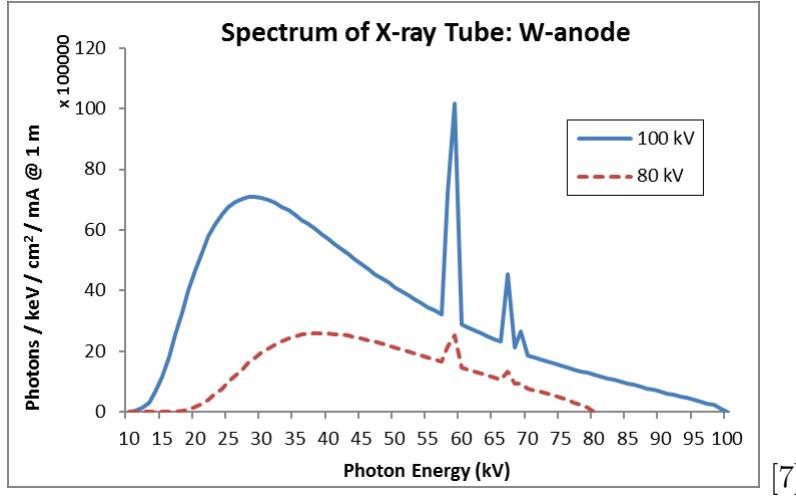


Figure 1: Example of X-ray spectrum with Bremsstrahlung and characteristic lines clearly visible.

The spectrum of the X-ray radiation consists of the continuous spectrum of the bremsstrahlung, on which the line spectrum of the characteristic X-ray radiation is superimposed.

2.2 Bremsstrahlung

When the electrons hit the anode they have the energy:

$$E = e \cdot U \quad (2)$$

This can happen in one step. Then the emitted radiation has the wavelength:

$$\lambda_0 = \frac{h \cdot c}{e \cdot U} \quad (3)$$

Radiation with shorter wavelength cannot be produced. The counting rate of the bremsstrahlung starts at the cut-off wavelength λ_0 , then passes through a maximum and asymptotically approaches zero with increasing wavelength.

2.3 Characteristic radiation

Characteristic radiation occurs when the electrons ionize atoms of the anode material during deceleration. This occurs when an electron is transported from an inner nuclear shell into the continuum beyond the ionization energy. The resulting unoccupied electron states are closed by electrons from higher shells. When an electron transitions to a lower energy level, the corresponding discrete energy differences are released in the form of X-rays, causing the line spectrum of X-ray radiation characteristic of the particular anode material. In the ionization processes, the atomic nucleus must absorb some of the momentum. The probability for the generation of unoccupied electron states is therefore highest in the inner shells (K and L), because the coupling to the nucleus is stronger than in the outer shells.

2.4 Absorption of X-Rays

The following relationship taken from [1] describes the decrease in intensity of an electromagnetic wave when passing through an absorbing material of thickness d :

$$I(d) = I_0 \cdot e^{-\mu d} \quad (4)$$

I_0 is the intensity before and $I(d)$ is the intensity after the absorber. μ is the material-dependent absorption coefficient. Three microscopic processes are responsible for the absorption of X-rays:

- the photoelectric effect
- the Compton effect
- the elastic scattering (basically Rayleigh-scattering)

For absorbers of higher atomic number, the photoelectric effect provides the largest contribution. It is also true that the absorption coefficient μ depends very strongly on both the wavelength of the radiation and the atomic number Z of the atoms of the absorber material. The absorption increases roughly with Z^5 and λ^3 . In addition to this monotonic increase with wavelength, abrupt changes in μ occur at certain wavelengths. These are called absorption edges. With decreasing λ , i.e. increasing energy, the binding energies of the electrons from deeper shells are exceeded. This makes more electrons available for photon absorption, which explains the discontinuous increase of μ . The bond energies of the absorption electrons can be precisely determined via the absorption edges. The final states of photoabsorption lie in the continuum. Therefore, the absorption edge of a shell is always at higher energy than the characteristic X-ray lines belonging to it. Consequently, all materials are well transparent to their own radiation. In the experiment, the absorption behavior of a material is determined by measuring the transmission. The intensity $I(\lambda)$ of the X-rays after passing through the absorber is determined as a function of the wavelength. If I_0 is the intensity without absorption, then the wavelength dependence of the ratio is called transmission curve:

$$T(\lambda) = \frac{I(\lambda)}{I_0} \quad (5)$$

The absorption then becomes

$$A(\lambda) = 1 - T(\lambda) = \frac{I_0 - I(\lambda)}{I_0} \quad (6)$$

2.5 Measure of the Röntgen radiation with the Geiger-Müller tube

In addition to blackening photographic plates or fluorescence phenomena, X-rays can be detected by the Geiger-Müller counter tube. It consists of a metal tube filled with gas and a wire clamped on one side in the tube axis. The entrance opening of the tube is closed by a thin mica window and the tube and the wire are

electrically insulated from each other. An electrical voltage of several hundred Volts is present between them. The X-rays entering through the window ionize gas molecules. Electrons are released and accelerated in the electric field in the direction of the positively charged wire. They generate further electrons by ionization. The resulting avalanche of electrons, which finally reaches the wire, generates a charge pulse there that charges a capacitor and slowly flows away to ground via an electrical resistor. The voltage pulse generated at the capacitor is amplified in the downstream electronics and both digitally displayed and made audible. It should be noted that after the detection of an X-ray quantum, a detector requires a certain amount of time until it is again ready to register further events. This time is called dead time τ . The X-ray quanta arriving in this time period cannot be detected. It follows that the measured count rate R_z is always smaller than the actual rate R of the arriving quanta, even if the detection probability of a single quantum would be of a single quantum would be one.

Two types of dead time can be distinguished. These depend on the design of the detector and the connected measurement electronics:

- non-extendable dead time: X-ray quanta hitting the detector during the dead time are not registered, but otherwise have no influence on the dead time.
- extendable dead time: X-ray quanta hitting the detector during the dead time are not counted, but start a new dead time. The Geiger-Müller counting tube used in the experiment is an example of a detector with extendable dead time.

For the extendable dead time, the counting rate R_z is a function of the dead time τ and the rate R of the actual arriving quanta as shown in the equation below taken from [1]:

$$R_z = R \cdot e^{-R \cdot \tau} \quad (7)$$

2.6 X-rays on a metal grid

If one wants to achieve a wavelength-resolved measurement, one can use diffraction at the lattice. To provide the small lattice parameter required for X-ray diffraction, crystals are used as lattice. The X-rays strike the lattice plane array at the angle θ with the lattice plane spacing d of a crystal. The wave is diffracted by the atoms in the different lattice planes. The scattered waves interfere with each other. Diffraction maximums can be observed when the net difference between the adjacent scattered waves is a path difference of integer multiples $[n]$ of the wavelength. In this case the Bragg condition is valid:

$$n \cdot \lambda = 2d \cdot \sin(\theta_n), n = 1, 2, 3, \dots \quad (8)$$

For these diffraction maximums to be detected by a detector, it must be moved through twice the angle, $2\theta_n$ with respect to the direction of the X-ray beam incident on the crystal. For this purpose a protractor is used in the X-ray spectrometer, which ensures this θ - 2θ -coupling.

3 Experimental procedures

First, a NaCl crystal is placed in the holder provided. As a first measurement, at 35kV, 0,8mA an angular step size of $\Delta\theta = 0,1^\circ$ and a measurement time of $\Delta t = 1\text{s}$ per angular step, the first measurement is made on an angular interval from 2° to 25° . A computer records the measured values for the count rate at each angle and plots the resulting emission spectrum as a line graph. The measured values must be saved manually after each measurement. Then the measurement is repeated on the interval from 18° to 23° at 1,0mA, on $\delta t = 8\text{s}$ extended measurement time and otherwise the same settings. Next, a 0,05mm thick zirconium foil is placed in front of the detector and measured in the interval from 4° to 10° at a current of 0.8mA and a measurement time of $\Delta t = 1\text{s}$ and otherwise unchanged settings. The foil is then removed again. Now the NaCl crystal is rotated by 180° and the first measurement is repeated in the interval from 2° to 10° and otherwise identical ice positions. Then the NaCl crystal is exchanged for a LiF crystal with the yellow mark on the crystal pointing towards the X-ray source. With settings otherwise unchanged from the previous measurement, the spectrum is now recorded in the range from 2° to 25° . Then, in the interval between 0.1mA and 1.0mA, the current is varied in 0,1mA steps, measuring each time from 8° to 11° at 35kV, $\delta t = 4\text{s}$ and otherwise unchanged settings. Finally, six measurements are made with the following settings:

Table 1: Settings for the experimental finding of the Planck constant.

U(kV)	I(mA)	$\Delta t(\text{s})$	$\theta [^\circ]$
35,0	0,8	8	4,0-6,5
32,5	0,8	8	4,5-7,0
30,0	0,8	8	5,0-7,5
28,0	1,0	16	5,5-8,0
26,0	1,0	16	6,0-8,5
24,0	1,0	16	6,5-8,5

4 Results and Discussion

4.1 Determination of the uncertainty in the angle measurements

The detector has a given angular uncertainty of $0,05^\circ$, by confronting the two measurements of the K_α and K_β lines of NaCl crystal(The second one rotated by 180°) it is possible to determine an additional angular uncertainty.

Figure 2: Superimposed spectrum of the two measurements of the NaCl crystal.

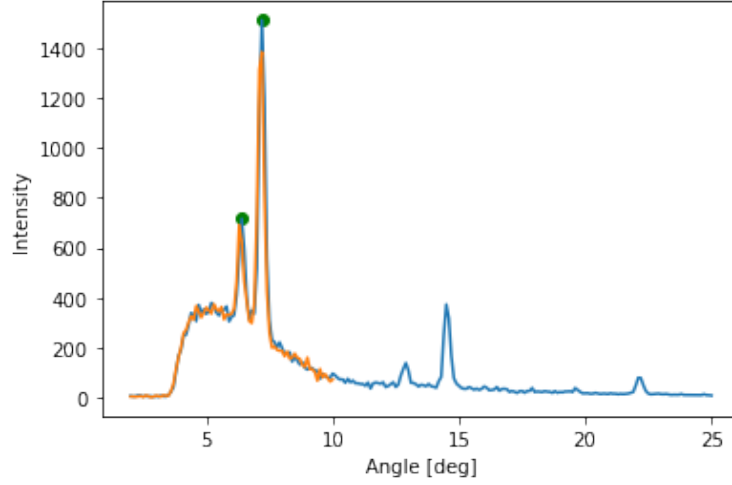


Table 2: Superposition of the two measurements.

$\theta[^\circ]$	$K_\beta[^\circ]$	$K_\alpha[^\circ]$
0	$6,4 \pm 0,05$	$7,2 \pm 0,05$
180	$6,3 \pm 0,05$	$7,2 \pm 0,05$

The total error thus amounts to 0.08° , the weighted average of the difference between the two measurements.

4.2 Emission spectrum of the molybdenum X-ray tube

Table 3: Angles at which the different orders of the K_α and K_β lines are located.

Order	$K_\beta[^\circ]$	$K_\alpha[^\circ]$
1	$6,4 \pm 0,08$	$7,2 \pm 0,08$
2	$12,9 \pm 0,08$	$14,5 \pm 0,08$
3		$22,1 \pm 0,08$

The value calculated in 4.1 was used as angular uncertainty in this part. No other uncertainty was added, (like the halfwidth of the peak) since they were orders of magnitude smaller than 0,08. There is no K_β line for the third order since even after multiple measurements it wasn't possible to get a clear line that stood out from the background as can be seen in graph 2. By using equation 8 it is possible to calculate the corresponding wavelengths

Table 4: Wavelengths of the K_α and K_β lines.

Order	K_β [pm]	K_α [pm]
1	$62,87 \pm 0,78$	$70,69 \pm 0,78$
2	$62,96 \pm 0,38$	$70,61 \pm 0,38$
3		$70,73 \pm 0,01$

By using equation 1 it's then possible to obtain the corresponding energy levels of the lines shown in table [5].

Table 5: Energies of the K_α and K_β lines.

Order	K_β [keV]	K_α [keV]
1	$19,74 \pm 0,25$	$17,55 \pm 0,19$
2	$19,07 \pm 0,12$	$17,57 \pm 0,09$
3		$17,54 \pm 0,06$

The weighted averages shown in table [6].

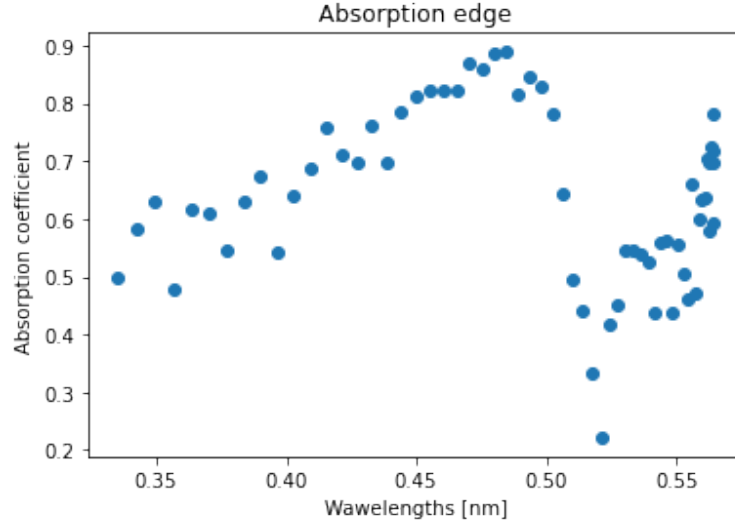
Table 6: Weighted averages of the energies of the K_α and K_β lines.

	K_β	K_α
Wavelength [pm]	$(62,94 \pm 0,46)$	$(70,70 \pm 0,31)$
Energy[keV]	$(19,71 \pm 0,14)$	$(17,56 \pm 0,08)$

4.3 Transmission of zirconium foil

The absorption edge is at $(2 \pm 0,2)\text{nm}$. The uncertainty was determined the fact that the absorption edge does not go straight down but has a certain horizontal width between the its highest and lowest points. The energy is then $(617 \pm 61)\text{eV}$.

Figure 3: Wavelength vs Absorption coefficient of zirconium foil.



4.4 Grid constant of the LiF crystal

By solving equation 8 for d we obtain:

$$d = \frac{n \cdot \lambda}{2 \cdot \sin(\theta_n)} \quad (9)$$

Since the wavelengths are known from 4 and the angles of the K_α and L_β remain to be found. This is done in graph [4].

Figure 4: X-ray spectrum of LiF crystal.

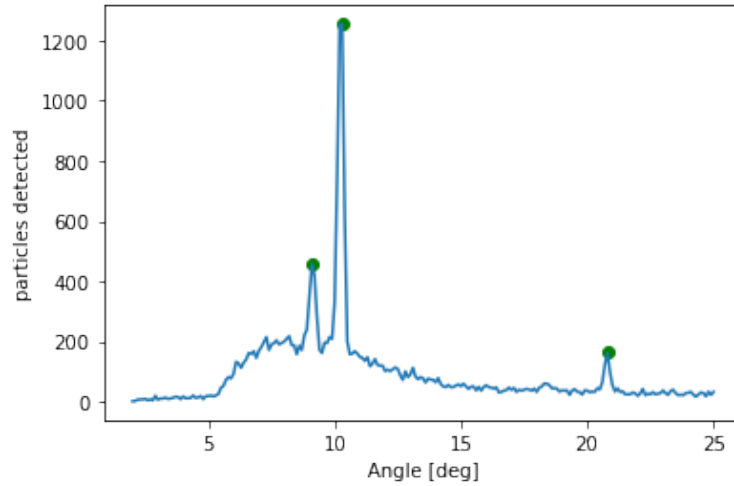


Table 7: Angles at which the different orders of the K_α and K_β lines are located for LiF.

Order	$K_\beta[^\circ]$	$K_\alpha[^\circ]$
1	$9,1 \pm 0,08$	$10,3 \pm 0,08$
2		$20,8 \pm 0,08$

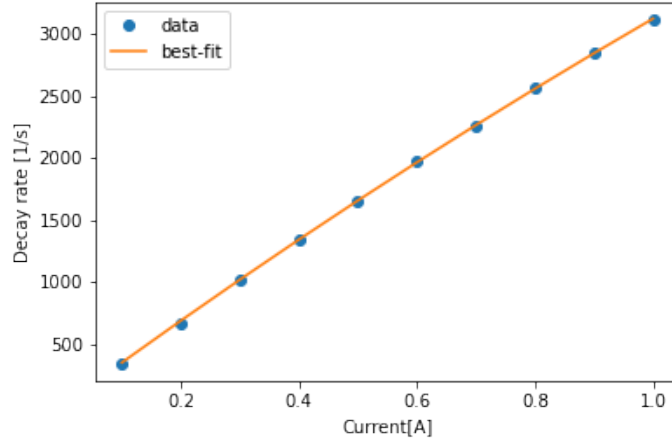
The uncertainty once again comes from 4.1 and only 3 lines are seen in this measurement. By calculating a weighted average for $a = 2 \cdot d$ the lattice parameter is equal to $(397,27 \pm 3,49)\text{pm}$. This aligns with the literature value of 403pm [2].

4.5 Calculating the dead time of the detector

The fitting function for the decay rate is:

$$R_Z = r \cdot I \cdot e^{-r \cdot I \cdot \tau} \quad (10)$$

R_Z being the decay rate, so the sum of all decays in our measurements series divided by $\Delta t = 4\text{s}$, with I being the corresponding current. τ and r are the fitting parameters with τ being the dead time of the detector.



The calculated fit parameters are shown in table [8]

r	τ
$(3514.23 \pm 23.25) A^{-1} s^{-1}$	$(33,25 \pm 2.13) \mu s$

Table 8: Fit parameters.

The value of τ given in the specifications of the detector is $\leq 90\mu s$ [5] which is perfectly in accordance with the result calculated above.

4.6 Determination of the Planck constant

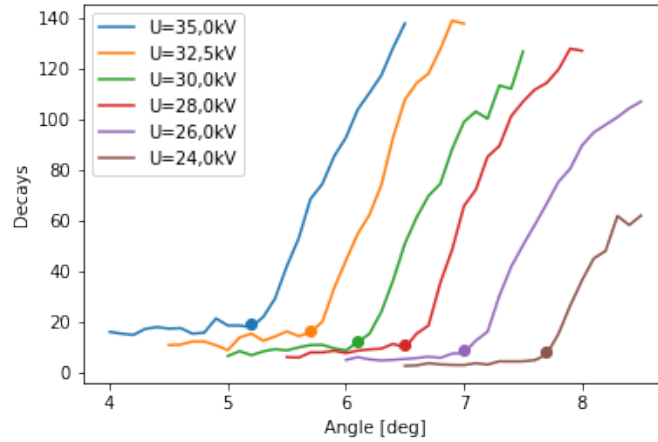
To calculate the Planck constant from our data two equations are needed: first equation 8 to find the corresponding frequency to the critical angle:

$$\lambda_0 = \frac{2d \cdot \sin(\theta_0)}{n} = \frac{a \cdot \sin(\theta_0)}{n} \quad (11)$$

with a the lattice constant from 4.4, and then equation 3 solved for h , which gives:

$$h = \frac{e \cdot U \cdot \lambda_0}{c} \quad (12)$$

The critical angles, the angles where the Bremsstrahlung stops, can be seen in graph 4.6. By using equations 11 and 12 and averaging the results the value for h computes to $(6,76 \pm 0,34) \cdot 10^{-34} \text{m}^2\text{kg/s}$ which is in accordance with the theoretical value of $6,62 \cdot 10^{-34} \text{m}^2\text{kg/s}$ [8]. The uncertainty of the calculated Planck constant comes from assigning an error of $0,2^\circ$ to the critical angle since it was determined by looking at the graph and seeing where the sudden increase started, so the choice is not always consistent or correct since no formal criteria for locating the point were set in advance.



5 Questions

1) To which order could the Bragg angle of the K_α line of molybdenum still be detected with the LiF and the NaCl crystal? At which angle do these orders occur?

Since the detector can maximally move 90° using 8 n_{max} calculates to $n = \frac{2 \cdot d}{\lambda}$. For LiF this is equal to the 5th order and is located at $63,3^\circ$, while for NaCl it's the 7th order, located at $61,34^\circ$. The numbers used to computer these results where taken from the findings of the previous exercises.

2) How can, using the results obtained in this experiment, the L_α line be obtained?

The energy of the L_α line is equal to the difference between the energies of the K_β and K_α lines. So it's equal to $(2,15 \pm 0,16)\text{keV}$.

6 Appendix

6.1 Error propagation

For the error propagation the python library uncertainties [4] was used. The weighted average was computed according to formulas in [3].

References

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