Problem Formulation

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1 MILP

$$\min_{P_{EV,a,t}} \sum_{T=1}^{T_{end}} \left| \sum_{t=K(T-1)}^{K*T-1} \Big(P_{plan}(T) - P_{BRP}(t) \Big) \Delta t \right| \tag{1a}$$

$$P_{BRP}(t) = \sum_a P_a(t) \tag{1b}$$

$$P_{EV,a}(t) = P_{EV,chrg,a}(t) - P_{EV,dischrg,a}(t) \tag{1c}$$

$$= P_{EV,plan,a}(t) + \Delta P_{EV,a}(t) \tag{1d}$$

$$P_a(t) = P_{grid,a}(t) + P_{PV,a}(t) - P_{load,a}(t) - P_{EV,a}(t) \tag{1e}$$

$$P_a(t) = \sum_b P_{ab}(t) \tag{1f}$$

$$P_{a,min} \leq P_a(t) \leq P_{a,max} \tag{1g}$$

$$SoC_a(t) = SoC_a(t-1) + \frac{P_{EV,chrg,a}(t)\Delta_t}{E_{bat}} \eta_{chrg} - \frac{P_{EV,dischrg,a}(t)\Delta_t}{E_{bat}\, \eta_{dischrg}} \tag{1h}$$

$$SoC_{a,min} \leq SoC_a(t) \leq SoC_{a,max} \tag{1i}$$

$$\rho(t) P_{EV,a,min} \leq P_{EV,chrg,a}(t) \leq \rho(t) P_{EV,a,max} \tag{1j}$$

$$\big(1-\rho(t)\big) P_{EV,a,min} \leq P_{EV,dischrg,a}(t) \leq \big(1-\rho(t)\big) P_{EV,a,max} \tag{1k}$$

2 MPC

Lets consider Δ_t, Δ_T, and t_{sim} respectively as the problem's temporal resolution (in minutes), imbalance length and total horizon with

$$\Delta_T = f(\Delta_t) \tag{2a}$$
$$= \underbrace{15}_{mn} // \Delta_t. \tag{2b}$$

Each instant k_{init} of the problem's horizon is indexed as

$$k_{init} \in \{0, \dots, t_{sim}\}, \tag{3}$$

while each imbalance period is indexed as

$$T'' \in \Big\{0, \dots, \underbrace{T_{end}}_{t_{sim}//\Delta_T}\Big\}. \tag{4}$$

At each instant k_{init}, we consider the MPC horizon that spans from k_{init} to $k_{init} + hor$. Each instant of the MPC's horizon can be defined either as

$$k \in \{k_{init}, \dots, k_{init} + hor\}, \tag{5}$$

or

$$t \in \{0 \dots, hor\}, \text{ with } t = k - k_{init} \tag{6}$$

and the imbalance periods covered by the MPC's horizon are given by

$$T \in \Big\{ \underbrace{T^{hor}_{init}}_{k_{init}//\Delta_T} \dots, \underbrace{T^{hor}_{end}}_{(k_{init}+hor)//\Delta_T} \Big\} \tag{7}$$

where T^{hor}_{init} and T^{hor}_{end} represent the first (initial) and last imbalance period. Figure 1 resumes all the above mention variables.

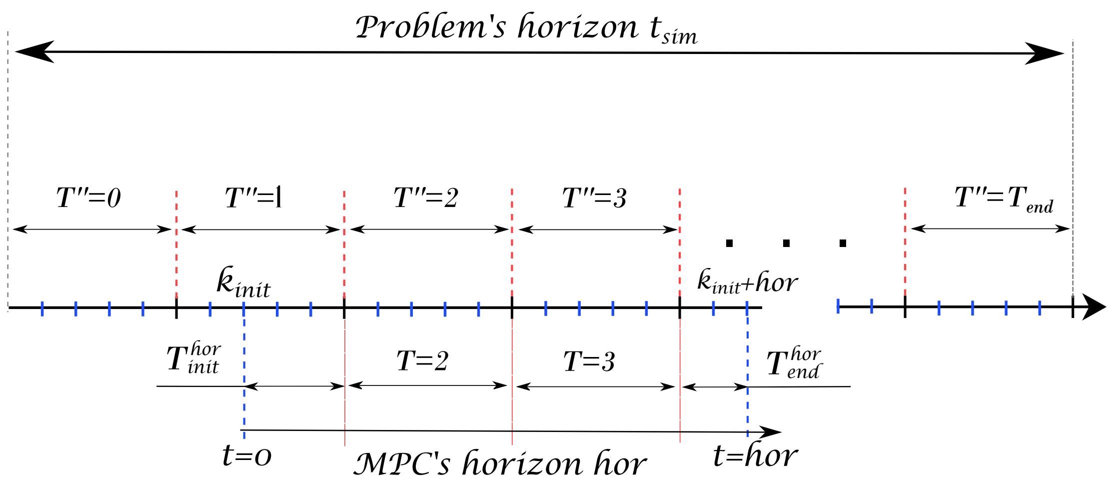

Figure 1: MPC problem variables

2.1 V1G problem formulation

$$\min_{P_{EV,a}(t), SoC_a^{relax}} \left((1-\alpha) f\Big(P_{EV,a}(t)\Big) + \alpha g\Big(Soc_a^{relax}\Big) \right) \tag{8a}$$

$\forall\, t \in \{0, hor\}$

$$P_{BRP}(t) = \sum_a P_a(t) \tag{8b}$$

$$P_{EV,a}(t) = P_{EV,chrg,a}(t) \tag{8c}$$

$$= P_{EV,plan,a}(t) + \Delta P_{EV,a}(t) \tag{8d}$$

$$P_a(t) = P_{grid,a}(t) + P_{PV,a}(t) - P_{load,a}(t) - P_{EV,a}(t) \tag{8e}$$

$$P_a(t) = \sum_b P_{ab}(t) \tag{8f}$$

$$P_{a,min} \leq P_a(t) \leq P_{a,max} \tag{8g}$$

$$SoC_a(t) = SoC_a(t-1) + \frac{P_{EV,chrg,a}(t)\Delta_t}{E_{bat}} \eta_{chrg} \tag{8h}$$

$$SoC_{a,min} \leq SoC_a(t) \leq SoC_{a,max} \tag{8i}$$

$$P_{EV,a,min} \leq P_{EV,chrg,a}(t) \leq P_{EV,a,max} \tag{8j}$$

$$SoC_a(t = t_a^{dep}) \geq SoC_{min}^{dep} - SoC_a^{relax} \tag{8k}$$

$$SoC_a^{relax} \geq 0 \tag{8l}$$

where

$$f(P_{EV,a,t}) = \frac{1}{E_{Mis}^{norm}} \Bigg(\Bigg| \sum_{t=t_{init}(T_{init}^{hor},k)}^{t_{end}(T_{init}^{hor},k)} \Big(P_{plan}(T_{init}^{hor}) - P_{BRP}(t) \Big) + \mathcal{C}(t) \Bigg| \Delta_t + \sum_{T=T_{init}^{Hor}+1}^{T_{end}^{Hor}} \Bigg| \sum_{t=t_{init}(T,k)}^{t_{end}(T,k)} \Big(P_{plan}(T) - P_{BRP}(t) \Big) \Bigg| \Delta_t \Bigg), \quad (9)$$

and

$$\mathrm{g}(Soc_a^{relax}) = \max_{Soc_a^{relax}} \Big\{ \frac{Soc_a^{relax}}{Soc_{min}^{dep} - Soc_{min}} \Big\}. \quad (10)$$

In $f(P_{EV,a,k})$

- T_{init}^{hor} and T_{end}^{hor} represent the first (initial) and last imbalance period over the MPC's Horizon;
- $t_{init}(T', k) = \max(0, \Delta T.T' - k)$ and $t_{end}(T', k) = \min(\Delta T.(T' + 1) - k, hor)$ the first and last instant over the imbalance period T';
- $\mathcal{C}(t)$ is the already computed energy mismatch of previous instant(s) of the first imbalance period defined as

$$\mathcal{C}(t) = \sum_{k'=0}^{t-1} f(P_{EV,a,k'}). \quad (11)$$

  Note that these previous instants are no longer covered by the MPC's horizon however, they are to be considered to compute the total energy mismatch of the first imbalance period;
- the first absolute value is used to compute the first imbalance period energy mismatch;
- the second absolute value is used to compute the remaining imbalance period energy mismatch;