Time series modeling

Jesse James PRINCE A.

May 24, 2022

Table of Contents

Basic time Models: AR

- Auto Regressive (AR)

Current predicted output depends on previous outputs

$$Y(k) = \theta_1 Y(k-1) + \xi(k) \qquad AR(1)$$

$$Y(k) = \theta_1 Y(k-1) + \theta_2 Y(k-2) + \xi(k) \qquad AR(2)$$

$$\vdots$$

$$Y(k) = \sum_{i=1}^{n} \theta_i Y(k-i) + \xi(k) \qquad AR(n)$$

Basic time Models: AR

- Auto Regressive (AR)

Current predicted output depends on previous outputs

$$Y(k) = \theta_1 Y(k-1) + \xi(k) \qquad AR(1)$$

$$Y(k) = \theta_1 Y(k-1) + \theta_2 Y(k-2) + \xi(k) \qquad AR(2)$$

$$\vdots$$

$$Y(k) = \sum_{i=1}^{n} \theta_i Y(k-i) + \xi(k) \qquad AR(n)$$

Basic time Models: AR

- Auto Regressive (AR)

Current predicted output depends on previous outputs

$$Y(k) = \theta_1 Y(k-1) + \xi(k) \qquad AR(1)$$

$$Y(k) = \theta_1 Y(k-1) + \theta_2 Y(k-2) + \xi(k) \qquad AR(2)$$

$$\vdots$$

$$Y(k) = \sum_{i=1}^{n} \theta_i Y(k-i) + \xi(k) \qquad AR(n)$$

Basic time Models: MA

- Moving Average (MA)

Current predicted output depends on previous estimation errors

$$Y(k) = \phi_1 \xi(k-1) + \xi(k) \qquad MA(1)$$

$$Y(k) = \phi_1 \xi(k-1) + \phi_2 \xi(k-2) + \xi(k) \qquad MA(2)$$

$$\vdots$$

$$Y(k) = \sum_{j=1}^{p} \phi_j \xi(k-j) + \xi(k) \qquad MA(p)$$

Basic time Models: MA

- Moving Average (MA)

Current predicted output depends on previous estimation errors

$$Y(k) = \phi_1 \xi(k-1) + \xi(k) \qquad MA(1)$$

$$Y(k) = \phi_1 \xi(k-1) + \phi_2 \xi(k-2) + \xi(k) \qquad MA(2)$$

$$\vdots$$

$$Y(k) = \sum_{j=1}^{p} \phi_j \xi(k-j) + \xi(k) \qquad MA(p)$$

Basic time Models: MA

- Moving Average (MA)

Current predicted output depends on previous estimation errors

$$Y(k) = \phi_1 \xi(k-1) + \xi(k) \qquad MA(1)$$

$$Y(k) = \phi_1 \xi(k-1) + \phi_2 \xi(k-2) + \xi(k) \qquad MA(2)$$

$$\vdots$$

$$Y(k) = \sum_{j=1}^{p} \phi_j \xi(k-j) + \xi(k) \qquad MA(p)$$

Mixed time models: ARMA

- Auto Regressive Moving Average (ARMA)

Current predicted output depends on previous outputs and estimation errors

$$Y(k) = \sum_{i=1}^{n} \theta_i Y(k-i) + \sum_{j=1}^{p} \phi_j \xi(k-j) + \xi(k)$$

Mixed time models: others

- Auto Regressive Integrated Moving Average (ARIMA)
  ARMA with differentiation to remove trends.

- Exponential smoothing ARMA (ESARMA)
  Associates to θ_i and ϕ_j a decreasing function to give more importance to new information.

- ARMA with Exogenous Regressors (ARMAX)
  Model includes parallel inputs sequences that are be observed at the same time step as the original time series.

Mixed time models: others

- Auto Regressive Integrated Moving Average (ARIMA)
  ARMA with differentiation to remove trends.

- Exponential smoothing ARMA (ESARMA)
  Associates to θ_i and ϕ_j a decreasing function to give more importance to new information.

- ARMA with Exogenous Regressors (ARMAX)
  Model includes parallel inputs sequences that are be observed at the same time step as the original time series.

Mixed time models: others

- Auto Regressive Integrated Moving Average (ARIMA)

  ARMA with differentiation to remove trends.

- Exponential smoothing ARMA (ESARMA)

  Associates to θ_i and ϕ_j a decreasing function to give more importance to new information.

- ARMA with Exogenous Regressors (ARMAX)

  Model includes parallel inputs sequences that are be observed at the same time step as the original time series.

Mixed time models: others

- Seasonal ARMA (SARMA)
  ARMA with trends and/or seasonal components.

- Markov Switching ARMA (MSARMA)
  ARMA with non linearity due to different functioning regimes

Mixed time models: others

- Seasonal ARMA (SARMA)
  ARMA with trends and/or seasonal components.

- Markov Switching ARMA (MSARMA)
  ARMA with non linearity due to different functioning regimes

Markov Auto Regressive M

A Markov Switching Auto Regressive MSAR model is given by

$$Y_{(k)} = \theta_0^{r(k)} + \sum_{i=1}^{n_{r(k)}} \theta_i^{r(k)} \left(Y_{(k-i)} - \theta_0^{r(k-i)} \right) + \xi_{(k)}^{r(k)}$$

where:

Markov Auto Regressive M

A Markov Switching Auto Regressive MSAR model is given by

$$Y_{(k)} = \theta_0^{r(k)} + \sum_{i=1}^{n_{r(k)}} \theta_i^{r(k)} \left(Y_{(k-i)} - \theta_0^{r(k-i)} \right) + \xi_{(k)}^{r(k)}$$

where:

- The regime sequence $\{r(k)\}$ follows a first order markov chain on the finite space $\{1, \ldots, R\}$;
- The probabilities governing the MC transitions are defined in the transition matrix

$$\mathbf{P} = \begin{bmatrix} p_{1,1} & \cdots & \cdots & p_{1,R} \\ p_{2,1} & p_{2,2} & & \\ \vdots & & \ddots & \\ p_{R,1} & & & p_{R,R} \end{bmatrix}.$$

Markov Auto Regressive M

A Markov Switching Auto Regressive MSAR model is given by

$$Y_{(k)} = \theta_0^{r(k)} + \sum_{i=1}^{n_{r(k)}} \theta_i^{r(k)} \left(Y_{(k-i)} - \theta_0^{r(k-i)} \right) + \xi_{(k)}^{r(k)}$$

where:

- The regime sequence $\{r(k)\}$ follows a first order markov chain on the finite space $\{1, \ldots, R\}$;
- The probabilities governing the MC transitions are defined in the transition matrix

$$\mathbf{P} = \begin{bmatrix} p_{1,1} & \cdots & \cdots & p_{1,R} \\ p_{2,1} & p_{2,2} & & \\ \vdots & & \ddots & \\ p_{R,1} & & & p_{R,R} \end{bmatrix}.$$

Markov Auto Regressive Model

A Markov Switching Auto Regressive MSAR model is given by

$$Y_{(k)} = \theta_0^{r(k)} + \sum_{i=1}^{n_{r(k)}} \theta_i^{r(k)} \left(Y_{(k-i)} - \theta_0^{r(k-i)} \right) + \xi_{(k)}^{r(k)}$$

where:

- The regime sequence $\{r(k)\}$ follows a first order markov chain on the finite space $\{1, \ldots, R\}$;
- The probabilities governing the MC transitions are defined in the transition matrix

$$\mathbf{P} = \begin{bmatrix} 0.8271 & 0.1282 & 0.0440 \\ 0.2091 & 0.4778 & 0.3130 \\ 0.0095 & 0.0093 & 0.9812 \end{bmatrix}.$$