

Abstract: Advanced linguistic theories have an adverse complexity profile in which the number of calculations required to verify them against nontrivial datasets increases exponentially as a function of data complexity. The task implies millions of calculation steps even for regular sentences and soon becomes infeasible for all traditional paper-and-pencil methods. In this article we propose a computational solution to this problem. We formalize a variant of the bottom-up minimalist grammar (with lexicon, bare phrase structure, labeling algorithm, Merge, head Merge, head movement, A-chains, \bar{A} -chain, adjuncts/adjunction, minimal search, linearization, subcategorization, interface legibility conditions) in a machine-readable notation, embed the result inside a computational infrastructure which mechanizes the core parts of linguistic reasoning, and then consider the use of this construct in the development, justification and testing of linguistic theories and hypotheses in the manner originally proposed by Chomsky (1957).

1 Introduction

Most grammars are inherently combinatorial: they create linguistic expressions by combining primitive parts (“words”) into larger units (“phrases”). The grammar is said to be observationally adequate if it generates an expression iff that expression is attested in some (or several, all) languages. The combinatorial nature of grammatical theories presents a problem, however: the deductive chains from initial lexical selections to the actual expressions soon become so protracted

¹ This is a draft (17. 2. 2024) accompanying the source code
<https://github.com/pajubrat/Template3>.

and long as to be virtually impossible to construct and verify by any paper-and-pencil methodology.² Here we regard this issue as a computational problem and propose a computational solution to it.³ Section 2 lays down the foundations, Sections 3–5 discuss certain special topics such as the nature of the lexicon, head movement, phrasal movement, adjunction and other topics. Section 6 is reserved for broader discussion. All solutions proposed and discussed in this article are available in the source code repository together with the datasets used in the demonstrations.⁴

2 Derivational search function

Before we can look at justification specifically we need some empirical claims to be justified. The choice is to some extent arbitrary as long as the grammar is presented in a sufficiently rigorous way to make full formalization a feasible approach. Perhaps the simplest grammatical theory available today that satisfies this requirement is the minimalist grammar originally designed by Chomsky (1995) and then developed by the author (Chomsky, 2000, 2001, 2008, 2013) and many others. In this theory linguistic phrase structures are binary sets $\{X\ Y\}$, thus a sentence such as *the dog bites the man* would be represented as $\{\{the\ dog\}\ \{bites\ \{the\ man\}\}\}$. Although this is very economical, the downside is that linearization as well as subcategorization/selection (and thus labeling/head

² The fairly standard or conservative generative model described later in this article requires $f(2) = 2$; $f(3) = 18$; $f(4) = 228$; $f(5) = 4580$; $f(6) = 137430$; $f(7) = 5772102$ calculation steps as a function of the size of the initial numeration. To translate these numbers into a more concrete linguistic context, calculating that the properties of an expression such as *the man believes that the dog barks* follow from a fairly standard grammatical theory requires approximately 15 million calculation steps. It is not sufficient that these calculations are performed once; they must be executed each time the theory is changed, and in the context of a realistic research program they must be executed against several expressions that bear upon the hypothesis under consideration.

³ The solution presented in this article is based on earlier ideas first sketched by Chomsky (1957). Chomsky was concerned with a “formalized general theory of linguistic structure” that was applied to linguistic data by comparing the predictions of the theory with observed reality. For example, he observed that by “pushing a precise but inadequate formation to an unacceptable conclusion, we can often expose the exact source of this inadequacy and, consequently, gain a deeper understanding of the linguistic data” (p. 5).

⁴ <https://github.com/pajubrat/Template3>. To install, test and replicate what is documented in this manuscript, first clone this project into a directory in the local machine and write “python Template3.py” into the command prompt when inside the said directory. To run the script the user must have Python (3x) installed on the local machine.

algorithm) become nontrivial problems that would take us too far from the main topic. Let us assume the minimalist framework but with a less controversial asymmetric binary-branching bare phrase structure $[X\ Y]$ as the starting point. Having settled on an initial phrase structure formalism we can begin to look at the complexity problem.

We need a computational function that explores all logical implications of the grammar. Let us assume that there is a syntactic working memory (sWM) which contains all syntactic phrase structure objects (henceforth, *syntactic objects*) currently under active consideration. The derivation begins by populating the sWM with zero-level syntactic objects together said form the *numeration*. First we make the simplification that the initial numeration corresponds to the “words” from which the expression are built by using the rules of the grammar.⁵ For example, an expression such as *the dog bites the man* can be derived from an initial numeration $\{the, dog, bites, the, man\}$ by merging the elements together in some well-defined order. To model this operation in a precise way we posit a recursive *derivational search function* that takes the contents of the syntactic working memory as an input and applies all operations in the grammar (currently only Merge) to all syntactic objects in sWM in a well-ordered sequence, updates the contents of the sWM, and calls the same function recursively with the updated sWM. The derivation ends if only one object is left, after which the result is evaluated and, if it passes as a well-formed output, linearized into a sentence accepted and

⁵ We do not reject the possibility of feeding the numeration with complex syntactic objects, although this option will not be considered in this article. Moreover, the notion of primitive syntactic object is neither trivial nor easy to characterize, see Section 3. Another possibility is to populate the numeration with linguistic features which are then composed into lexical items by further combinatorial rules. This would make no difference to what we say below.

enumerated by the grammar.⁶ The derivational search function is provided below in pseudocode; the Python implementation is in the source code.⁷

(1) *Derivational search function*

- a. Assume a set sWM of syntactic objects as input;
- b. if there is only one syntactic object, evaluate and print it out;
- c. otherwise:
 - c.1 for each operation O in the grammar:
 - c.2 for each pair X, Y of items in sWM:
 - c.2.1 apply $O(X, Y) = \alpha$;
 - c.2.2 create updated sWM* which contains α but not X, Y;
 - c.2.3 call (1) with sWM*.

When applied to four empty words *a*, *b*, *c* and *d* algorithm (1) generates 144 phrase structure representations when it surveys all possible ways of merging them into asymmetric bare phrase structure representations. The first three steps of the derivation are shown below:

⁶ If we intent to compare the output of the grammar with concrete linguistic expressions, the latter which are typically and here as well conceived as linear strings of word-like objects (say, phonological words), then the theory must have a linearization algorithm. Furthermore, depending on how the internal structure of words is represented we might also need a component for producing linearized morpheme sequences and morpheme boundaries of various types.

⁷ <https://github.com/pajubrat/Template3/blob/main/Template3.py>, function *derivational_search_function*.

1.

[a], [b], [c], [d]

Merge(a, b)
= [a b]

[a b], [d], [c]

2.

[a b], [d], [c]

Merge([a b], d)
= [[a b] d]

[[a b] d], [c]

3.

[[a b] d], [c]

Merge([[a b] d], c)
= [[[a b] d] c]

[[[a b] d] c]

|== [[[a b] d] c] <= ACCEPTED: a b d c

We can then assume (in the lack of justification for anything more complex) the standard left-to-right depth-first linearization algorithm which turns the 144 phrase structures into linearized sentences, in the above example the final output sentence was *a b d c*.

Some terminological issues must be clarified before discussing more realistic examples. We want to draw a clear distinction between *computational*, *algorithmic* and *implementation* level descriptions, as originally proposed by Marr (1982). A computational level description is concerned with abstract mappings and ignores both the algorithm and the physical implementation. If we ignore the algorithm (1), what is left is a mapping between a lexical selection and a set of pairs of linearized surface sentences and phrase structure analyses.⁸ For example, the above system mapped

⁸ It is not a coincidence that this mapping corresponds to the standard Y-architecture of the minimalist theory, in which lexical choices branch into concrete sentences and syntactic phrase structure representations, in the minimalist parlance to the PF-interface representations (leading into sensorimotoric systems) and LF-interface representations (leading into conceptual-intentional systems).

the numeration $\{a, b, c, d\}$ into 144 linearized sentence + phrase structure pairs, of which the first 12 are shown below:

- (1) a b d c [[[a b] d] c]
- (2) c a b d [c[[a b] d]]
- (3) d a b c [d[[a b] c]]
- (4) a b c d [[[a b] c] d]
- (5) d a b c [[d[a b]] c]
- (6) c d a b [c[d[a b]]]
- (7) a b d c [[a b][d c]]
- (8) d c a b [[d c][a b]]
- (9) d c a b [d[c[a b]]]
- (10) c a b d [[c[a b]] d]
- (11) c d a b [[c d][a b]]
- (12) a b c d [[a b][c d]]

The same mapping, however, could be generated by an unbounded number of different algorithms, for example, we could reverse-engineer (1) and generate the same mapping by beginning from the surface sentences. We could also consider a variation of (1) that mimics real language processing and/or comprehension. Both approaches could implement the same computational level mapping.⁹ The main point, however, is to avoid confusing algorithms with the computational level descriptions they implement.

This approach does not depend on whether the grammar is representational or derivational. A *representational* grammar captures grammaticality by positing well-formedness conditions that apply to complete grammatical representations. The derivational search function then constructs the representations satisfying the well-formedness constraints and tests them against data. A *derivational* grammar differs from the representation grammar in that the construction process itself, and not just the output, is subject to grammatical laws. From the point of view of the derivational search function the difference is relatively inconsequential: both theories generate grammatical objects by rules and in both cases the output is compared with observation. If the

⁹ The implementation level description concerns the ways in which the algorithm is turned into a physical system that performs the actual task. We use the Python programming language for implementation in this article; any general-purpose programming language could do. In the case of the real natural languages the underlying implementation platform is the human brain.

grammar is derivational, then at least some properties of the derivation itself are part of the empirical content of the theory.

We must also distinguish enumerative grammars from recognition grammars. An *enumerative grammar*, such as the one just examined, is a procedure which generates a set of expressions; everything else logically derivable from the same lexical items but not derived by the grammar is judged ungrammatical. Most grammatical theories available today are formulated in this way. The derivational search function described above performs the enumeration. A *recognition grammar*, in contrast, is a procedure which decides for any given input expression whether it is grammatical or ungrammatical. Instead of enumerating the set of grammatical expressions it provides a characteristic function for the set. Enumerative and recognition grammars are equivalent in the sense that it is possible to construct one from the other (as long as they compute in finite time).¹⁰

Testing the derivational search function with empty words is useful in making sure that the algorithm explores the whole derivational space, but unrealistic as a linguistic model. Real words have features, such as subcategorization, which restrict their distribution and in turn relies on a labeling. We begin with the following head/labelling algorithm:

(2) *Labeling/head for any syntactic object α*

- a. If α is primitive, it will be the head;
- b. suppose $\alpha = [X Y]$, then if X is primitive, it will be the head; otherwise
- c. if Y is primitive, it will be the head; otherwise
- d. apply (2) recursively to Y.

The output will change into the following:

¹⁰ When applied to linguistic theorizing, however, they do have significant differences: recognition grammars are often interpreted implicitly or explicitly as models of “comprehension,” while enumerative grammars are conceived as “production” models. This characterization is imprecise and informal, perhaps misleading, but not unimportant when working with empirical linguistic theories.

- (1) c d a b [_aP [_cP c d][_aP a b]]
- (2) a b c d [_cP [_aP a b][_cP c d]]
- (3) d c a b [_dP d[_cP c[_aP a b]]]
- (4) c a b d [_dP [_cP c[_aP a b]] d]
- (5) d c a b [_aP [_dP d c][_aP a b]]
- (6) a b d c [_dP [_aP a b][_dP d c]]
- (7) d a b c [_cP [_dP d[_aP a b]] c]
- (8) c d a b [_cP c[_dP d[_aP a b]]]
- (9) a b c d [_dP [_cP [_aP a b] c] d]
- (10) d a b c [_dP d[_cP [_aP a b] c]]
- (11) a b d c [_cP [_dP [_aP a b] d] c]
- (12) c a b d [_cP c[_dP [_aP a b] d]]

where the calculated label/head L of complex syntactic object is denoted as $[_LP X Y]$. Is (2) plausible? Because we are working with a computational algorithm with the explicit purpose of testing the theory later against data, at this point we can regard (2) as pure speculation. The important point, instead, is that once we know the head, we can add subcategorization. Let us stipulate two lexical features $[!COMP:F]$ and $[-COMP:F]$ which mandate and ban, respectively, feature F to/from the head of the complement of the lexical item having either feature. Feature F will usually be a major lexical category, but other features are also possible. This grammar will be able to regulate head-complement structures, but cannot rule out ungrammatical specifier-head constructions, so we introduce $[!SPEC:F]$ and $[-SPEC:F]$ for specifier selection with the notion of specifier denoting all left phrases inside the projection from head X (we discuss adjuncts later). Having introduced subcategorization, we must decide where it applies. The possibilities are: (i) at the stage where grammatical operations apply; (ii) at some intermediate construction (“phase”, in the sense of (Chomsky, 2000, 2001)); (iii) at the final output. The first buys us considerable and ultimately necessary savings in computational complexity, so we assume it on such grounds alone; (iii) is also an obvious choice and will later become “interface legibility conditions.” Assumption (ii) could be examined if the derivations were broken down into separate sub-derivations, but since this option was not implemented (see however Section 6), this alternative is irrelevant. Assumption (i) requires that we block derivations violating subcategorization inside the derivational search

function. We add the test to (1), step c.2.1, such that Merge(X, Y) is performed if and only if Y is compatible with the selection features of X.

These assumptions, though still extremely rudimentary, allow us to create and test a simple VP-grammar which enumerates sentences like *the dog bites the man* and *the man bites the dog* with the internal structure [_{VP} [_{DP} *the man*][_{VP} *bites* [_{DP} *the dog*]]] and rules out ungrammatical word permutations. Provided with reasonable subcategorization features the grammar executes 128 derivational steps and generates the following 12 output sentences:

```
(1) the dog bite the man  [_VP [_DP the dog][_VP bite[_DP the man]]]
(2) the man bite the dog  [_VP [_DP the man][_VP bite[_DP the dog]]]
(3) the dog bite the man  [_VP [_DP the dog][_VP bite[_DP the man]]]
(4) the man bite the dog  [_VP [_DP the man][_VP bite[_DP the dog]]]
(5) the dog bite the man  [_VP [_DP the dog][_VP bite[_DP the man]]]
(6) the man bite the dog  [_VP [_DP the man][_VP bite[_DP the dog]]]
(7) the dog bite the man  [_VP [_DP the dog][_VP bite[_DP the man]]]
(8) the dog bite the man  [_VP [_DP the dog][_VP bite[_DP the man]]]
(9) the man bite the dog  [_VP [_DP the man][_VP bite[_DP the dog]]]
(10) the man bite the dog  [_VP [_DP the man][_VP bite[_DP the dog]]]
(11) the man bite the dog  [_VP [_DP the man][_VP bite[_DP the dog]]]
(12) the dog bite the man  [_VP [_DP the dog][_VP bite[_DP the man]]]
```

The output contains several identical sentences and even identical phrase structures. This is because solutions (1–12) represents different derivations, not different sentences (the mappings from derivations into output phrase structures into linearized sentences are all many-to-one). The following screenshot shows what happened during the first three steps of the derivation:

1.

```
[the], [man], [bite], [the], [dog]

Merge(the, man)
= [_DP the man]

[the], [dog], [bite], [_DP the man]
```

2.

```
[the], [dog], [bite], [_DP the man]

Merge(the, dog)
= [_DP the dog]

[_DP the man], [_DP the dog], [bite]
```

3.

```
[_DP the man], [_DP the dog], [bite]

Merge(bite, [_DP the man])
= [_VP bite[_DP the man]]

[_DP the dog], [_VP bite[_DP the man]]
```

Since the derivational search function (if written correctly) performs an exhaustive search, sentences (1–12) are the *only* sentences this grammar enumerates from the given numeration: all other permutations and structures are implicitly judged ungrammatical. Thus, the grammar judges **the man the dog bite* ungrammatical, although this fact is not explicitly stated anywhere. However, we haven't constructed any real proof that the grammar is correct – we eyeballed the output and compared it with the intended output we “had in mind.” We can fix this issue by providing the derivational search function with an explicit set of target sentences. Moreover, both the numeration and the target sentences can be written into an external input file so that they can be controlled outside of the source code itself. For example, we can write the following three lines into the external input file:

```
Numeration=the,man,bite,the,dog
the man bite the dog
the dog bite the man
```

The first line declares the numeration, the two that follow are interpreted as the target sentences against which the grammar is evaluated. Running the model now shows that the it is observationally

adequate. External files like this will become the *datasets* that we use to justify grammatical hypotheses. For example, we can now return to the question of whether the head algorithm (2) is correct. We could assume a variation in which the head algorithm recurses into the left and not to the right and run the model with the same (still trivial) dataset. The new algorithm is still observationally adequate, so we need additional data to distinguish the two variations of (2). Indeed, in the context of a more realistic research project the dataset will be much more complex than suggested by the simple examples analyzed so far and involve several numerations paired with the target sentences so that the hypotheses can be tested against large batches of data.

While this procedure is able to justify grammars against datasets, we could have accomplished the same result by a table-lookup system which pairs input sentences directly with the intended grammaticality judgments. This would be a linguistically uninteresting solution. This is not a problem, however, since the output was generated by minimalist grammatical representations and operations that (hopefully, we surmise) had independent justification. But how do we know what the grammar did when it derived the sentences and whether it made any sense? Some of the screenshots above containing explicit examples of Merge were taken from the *derivational log file* that the model creates at runtime when it derives sentences on the basis of the input in the dataset files. Specifically, every linguistically meaningful step executed by the derivational search function is recorded into the derivational log file. See the actual source code for how to do this in Python.

3 Lexicon, complex words and head chains

The simple VP-grammar tested above relied on an intuitive notion of “word” that formed the basis of both the derivations and the output sentences. Technically these words were provided in their own data structure (call it the *lexicon*) as items defined by a set of lexical features, including the subcategorization features and major lexical categories, and were transformed into primitive

syntactic objects that formed the initial numeration (i.e. there was a $\text{Lex} \rightarrow \text{SO}$ mapping).¹¹ This meant that complex words such as *bites* were represented as such in the lexicon; the third person singular suffix and the present tense were represented as lexical features, though these features had no functional rule in the grammar with such one operation Merge. This assumption is not empirically implausible and characterizes a whole category of lexicalist approaches to word formation (Chomsky, 1970; Jackendoff, 1975; Aronoff, 1976; Lapointe, 1980; Anderson, 1982, 1992; Jensen & Stong-Jensen, 1984; Grimshaw & Mester, 1985; Di Sciullo & Williams, 1987; Borer, 1991; Zwicky, 1992; Sells, 1995; Bresnan & Mchombo, 1995; Scalise & Guevara, 2005; Kiparsky, 2017). In such theories the lexical items that form the starting point of syntactic derivations are generated by separate word formation rules. We can either ignore word formation and write the words directly into the lexicon, as we did in Section 2, or add word formation into the derivational search function.

On the other hand, in many generative theories there is a separate syntactic operation which creates complex words by combining primitive syntactic objects. Here we do not make any more specific empirical assumptions about what the domain of this operation is and instead assume for the sake of the example that such an operation exists (as it does exist in many minimalist theories) and that it can create at least some complex words, in particular we will assume that it creates tensed verbs by combining a syntactically less prominent verbal head with a tense head that selects the verb phrase as its complement (Koopman, 1984; Travis, 1984; Baker, 1985, 1988; Pollock, 1989; Borer, 1991; Hale & Keyser, 1993; Roberts, 2001; Julien, 2002: §2; Matushansky, 2006; Dékány, 2018). In schematic terms the operation produces $[_{YP} ({}_Y X Y)^0 [\dots \cancel{X} \dots]]$ where X is a silent copy of the head that has been copied and adjoined to the higher head Y . It therefore represents a variation of Merge, but with two extra properties: it creates complex zero-level categories instead of

¹¹ Here it does not matter if the lexical items are sets of lexical features or are represented by more complex structures. If the latter, then the lexical entries would be defined by a custom-made data structure that defines what these more complex entries are.

regular phrases and “copies” something from an existing structure. Let us add this operation next into the theory.

First we posit a feature ‘zero-level’ to distinguish complex zero-level categories $(_Y X Y)^0$ from complex phrasal categories $[_{\alpha P} X Y]$ and modify the head algorithm such that it responds to this property and not only to the lack of daughter constituents. At this point we can regard the zero-level property as a stipulation. Then we post an operation *Head Merge* which merges two constituents but creates a zero-level category instead of a regular phrase, the latter which is the output of regular Merge.¹² That is, regular Merge creates $[_{\alpha P} X Y]$, Head Merge $(X Y)^0$.¹³ Next we determine how the properties of the newly created complex zero-level categories are calculated on the basis of their constituents. One possibility (the preferred one, in our view) is to change the head algorithm (2) such that it calculates heads also for complex zero-level categories and then let the features of $(X Y)^0$ depend on the features of its head, in the sense of Williams (1981), Selkirk (1982) and Di Sciullo & Williams (1987); another is to posit separate feature inheritance inside the complex head. We used the latter since the former requires a notion of adjunction that we will introduce only later. Feature inheritance was implemented inside Head Merge and copies the features of Y to $(X Y)^0$. Let us assume that the morpheme boundaries corresponding to Head Merge are represented by # in the output sentences.

If we just added Head Merge to the list of syntactic operations accessed by the derivational search function, a rather large volume of new derivations would emerge in which the grammar produces new complex words and inserts them into the derivations. Although this is one possible

¹² This assumption is not required if we can control the distribution of the ‘zero-level’ features elsewhere, but having a separate function makes everything explicit and allow us to encapsulate the code that handles the creation of complex words.

¹³ Instead of positing HM, we could have relied on regular Merge and posited additional rules and code for classifying some outputs as zero-level objects. At this point, however, it is generally better to posit a specialized function that “encapsulates” the properties of HM into one place, so that we can better manipulate and control its properties.

starting point and not empirically impossible if the operation is controlled in some way,¹⁴ this does not match with the original specification $\alpha = [_{YP} (Y X Y)^0 [_{XP} \dots X \dots]]$ which copies X from within the merge partner XP. So far we have assumed that the derivational search function operates with a set of syntactic objects that it combines into new objects; now we have the additional step where something is “copied” from XP.

One possibility is that Head Move is a grammatical operation like Merge and applies freely: α will become the composite operation of Merge + Head Move which occasionally happen to apply in a sequence but do not need to do so. This presupposes that the objects selected as targets for syntactic operations inside the derivational search function include not only the syntactic objects in the syntactic working memory but also the daughters of those elements.¹⁵ This approach maintains full generality and consequently opens up derivations where all syntactic operations can target the internal constituents of the syntactic objects in the syntactic working memory. The hypothesis leads to two issues that ultimately caused us to reject it for the purposes of this study. First, it introduces an enormous amount of derivational paths that we never see in reality, for example, it becomes possible to perform sideward head movement. This forces us to posit constraints to counterbalance all the illicit derivations, so much so that the restrictions will effectively nullify the whole idea that Head Move is a completely general operation. Another consequence is that the operation is countercyclic: it assumes that X can be inserted inside [Y XP] which requires that we open up previously established phrase structure geometry and then repair it, requiring nontrivial code.¹⁶

A second solution is to posit a special category of “internal” syntactic rules into the derivational search function which targets one syntactic object X and then tampers with its internal properties.

¹⁴ Several minimalist models assume an operation of this type, see López (2015) and Embick (2004) and the literature cited therein.

¹⁵ Either all daughters or daughters selected by some additional criteria. Regardless of the choice, enormous amounts of new derivations will open up.

¹⁶ Specifically, a countercyclic derivation must (i) detach Y from its mother β , (ii) create a new constituent $\alpha = (X Y)^0$ by Head Merge and (iii) insert α as the new daughter for β . Operations (i, iii) must be added to the formalism, while (ii) is Head Merge.

Head Move targets some X, locates its head (or all heads) and applies the operation either freely or as a reflex of some triggering condition. This hypothesis still requires countercyclic operations and the special internal grammatical rules, but it generates much less superfluous derivations than the first solution.

The two solutions could be implemented and experimented with, in fact we gave both serious consideration and implemented part of the second approach. However, there is a third alternative: Head Move is part of Merge, in the sense that right before [Y XP] is created Head Move is applied to the head X of XP (thus, in agreement with the Head Movement Constraint (Koopman, 1984; Travis, 1984)) if and only if Y is a zero-level object with a feature making it a bound morpheme. The operation then copies X and merges it with Y by Head Merge introduced earlier, silences the original X phonologically and creates $[_{VP} (X Y)^0 [_{XP} \dots \text{X} \dots]]$. The operation is cyclic, does not require modifications to phrase structure geometry established before and applies under restricted contexts. For example, all types of sideward head copying are automatically excluded and the operation satisfied HMC. We can perhaps think of Head Move as some kind of “repair” operation executed before Merge.

To test this model against data let us posit a bound tense head T^* into the grammar and assume that it selects for a VP. To verify this grammar against data we assume the numeration $\{T^*, \text{the}, \text{dog}, \text{bark}\}$ where *bark* is an intransitive verb and target *the man bark#T** (dataset #3).¹⁷ Running the grammar with these assumptions produces the following:

```
(1) bark#T* the dog  [_TP (bark T*)[_VP [_DP the dog] __ ]]
Derivational steps: 10
Errors 2
Should not generate: {'bark#T* the dog'}
Should generate: {'the dog bark#T*'}
```

¹⁷ It is assumed that *bark#T** would be replaced with the vocabulary item *barks*, but this process, which is trivial to implement by brute force, is not useful here because it would “mask” the derivational history of the word.

From (1) we can verify that the grammar created a complex tensed verb $(V, T^*)^0$ by copying the verb from within the VP. But as it did so, it generated an VS word order that was not among the target sentences. The reason this grammar did not put the argument to the preverbal SpecTP position is because we assumed in the lexicon that the intransitive verb must have a DP specifier. If we remove this assumption, the grammar generates the correct word order *the man barks* but with a questionable analysis $[_{TP} [_{DP} \text{the man}] [_{TP} (\text{bark}_1 T^*) \text{---}_1]]$:

```
(1) the dog bark#T*  [_TP [_DP the dog][_TP (bark T*) -- ]]
(2) the dog bark#T*  [_TP [_DP the dog][_TP (bark T*) -- ]]
(3) bark#T* the dog  [_TP (bark T*)[_VP [_DP the dog] -- ]]
Derivational steps: 10
Errors 1
Should not generate: {'bark#T* the dog'}
Should generate: set()
```

Although the surface sentence is now correct, the phrase structure analysis is most likely judged as implausible. It is usually assumed that all thematic arguments must be merged inside the VP to receive a thematic role (Fukui & Speas, 1986; Sportiche, 1988; Koopman & Sportiche, 1991), and that the subcategorization features of verbal elements are checked inside the VP. We return to the problem of English V-initial sentences below.

Transitive verbs are usually analyzed as being created by merging verbal stems with a transitivizer *v* which introduces the external (agent, causer) argument to its specifier, so that all thematic arguments are initially generated inside the vP where they receive thematic roles and satisfy subcategorization. Consequently we introduce a bound morpheme *v* into the lexicon such that it requires a DP-specifier and selects for a VP. Adding *v* into the numeration generates 16 sentences after 4586 derivational steps:

```
(1) bite#v#T* the dog the man  [_TP ((bite v) T*)[_vP [_DP the dog][_vP -- [_VP -- [_DP the man]]]]]
(2) bite#v#T* the dog the man  [_TP ((bite v) T*)[_vP [_DP the dog][_vP -- [_VP -- [_DP the man]]]]]
(3) bite#v#T* the man the dog  [_TP ((bite v) T*)[_vP [_DP the man][_vP -- [_VP -- [_DP the dog]]]]]
(4) bite#v#T* the dog the man  [_TP ((bite v) T*)[_vP [_DP the dog][_vP -- [_VP -- [_DP the man]]]]]
```

Notice the familiar “snowball” profile of the three zero-level categories making up the final tensed transitive verb, a consequence of the way head movement was defined above. The subcategorization features of *v* and *V* require that the two DPs are merged inside the VP, which

again leaves the verb at the first position. Although this configuration is grammatical in some languages (Alexiadou & Anagnostopoulou, 1998), it is ungrammatical in English, a problem we address in the next section. Notice that running the model with an elementary transitive numeration consumed already 4586 derivational steps: this is trivial for a computer but unfeasible for paper-and-pencil methodology.

Because head movement was defined as a local operation, this grammar cannot derive sentences in which heads skip over potential targets or where they combine with lower heads. For example, sentences such as (i) **bite#T the man v the dog* are correctly judged ungrammatical. Also (ii) **v#T the man bite the dog* is underivable: we assumed that head movement is always executed when Merge is executed, hence the merger of *v* will create $(V, v)^0$ as a grammatical reflex. It follows that $[(X Y)^0 [\dots \text{X} \dots]]$ occurs if and only if *Y* selects for *XP* and is a bound morpheme.

The above experiment shows that, with the exception of the V-initial configurations, the model works as intended. To show that the grammar is adequate in a linguistically interesting sense requires that we test it against many more numerations. For example, if we simulate this grammar with a numeration that lacks *v* but contains both DP-arguments, it will pass SVO and VSO sentences with analyses $[_{TP} T [_{VP} DP [_{VP} V DP]]]$ and $[_{TP} DP [T [_{VP} DP V]]]$. This happens because we have assumed, tacitly as it were, that *v* is an “optional” element that can be added freely to the sentence to regulate transitive clauses; if we leave it out, the grammar will either insert both arguments inside the VP or it merges one inside TP. Whether we want this to happen or not this shows that serious linguistic hypotheses must be checked against several numerations. This is not an unimportant issue because the entire force of the justification will depend on the nature of the dataset(s), but as this thought experiment shows the selection of the relevant numerations remains a free parameter. For example, if our grammar has problems in handling missing elements, as it had above, we could ignore this test experiment (even worse, implicitly) from the dataset to make it look as though the model is successful. We could also boost the success rate by adding repetitive

variations that we know the model can solve. Thus, the dataset configuration process must be standardized in some manner.

4 Phrasal movement

The final experiment (dataset #4) performed in Section 3 produced verb-initial sentences that are ungrammatical in English. The traditional solution to this problem going back to (Chomsky, 1981, 1982) is to assume that functional heads can have a special nonthematic specifier selection feature, call it EPP, which forces them to have a specifier that is not merged directly (or “externally”) to the specific position but is copied (“internally”) from within the complement to create $\alpha = [_{YP} ZP [_{YP} Y [_{XP} \cancel{ZP} [X WP]]]]$ in an operation we call *phrasal movement*. Implementing an operation of this type without countercyclic derivations requires that we instantiate the operation as a response to the EPP feature right after Merge has created $[Y_{EPP} XP]$. However, and despite the fact that we will follow this template in this article, the two other solutions introduced in the previous section are possible as well and would seem to merit full examination. We could add phrasal movement to the catalog of grammatical operations applied at every derivational step and then allow all operations to target both the syntactic objects in the working memory and their internal constituents, or we could create a special category of rules which affect the internal structure of single syntactic objects. Perhaps the claim that the operations apply “freely” (Chomsky, 2008) refers to an architecture of this type. However, adding this rule directly to Merge suffices to remove VS(O) sentences from the output. For example, numeration $\{T, the, dog, bark\}$ will produce the following output in which the DP-argument is dislocated to SpecTP as a consequence of Merge due to the EPP-feature at T (dataset #6):

```
(1) the dog bark#T  [_TP [_DP the dog]:1 [_TP (bark T)][_VP __:1  __ ]]]
Derivational steps: 10
Errors 0
```

The part of the derivation leading into the accepted output is as follows.

6.

[T], [the], [dog], [bark]

Merge(the, dog)
= [_DP the dog]

[_DP the dog], [T], [bark]

7.

[_DP the dog], [T], [bark]

Merge([_DP the dog], bark)
= [_VP [_DP the dog] bark]

[_VP [_DP the dog] bark], [T]

8.

[_VP [_DP the dog] bark], [T]

Head Chain (T, bark)
Chain ((bark T), __:1)
Merge(T, [_VP [_DP the dog] bark])
= [_TP [_DP the dog]:1 [_TP (bark T)][_VP __:1 __]]]

[_TP [_DP the dog]:1 [_TP (bark T)][_VP __:1 __]]]

|== [_TP [_DP the dog]:1 [_TP (bark T)][_VP __:1 __]]] <= ACCEPTED: the dog bark#T

Both head movement and A-chains are generated during the last step when T is merged above the VP. First, T is combined with the verb to create a finite tensed verb; the resulting complex verb triggers A-chain and copies the grammatical subject to SpecTP. This operation is caused by the EPP feature at T. The verb-initial solution is removed from the output.¹⁸ We can test the model also against transitive clauses, which provides correct outputs, but notice that these mechanisms do not yet correlate A-movement with any “anchoring” properties such as topicness, agreement or case assignment, and we did not restrict the type of phrases that can be targeted: the fact that the grammatical subject was moved was an accidental consequence of the fact that it was merged to

¹⁸ EPP-induced A-movement has generated substantial debate in the literature (Fernández-Soriano, 1999; Chomsky, 2000, 2001, 2008; Holmberg, 2000; Miyagawa, 2001, 2010; Bošković, 2002, 2007; Rezac, 2004; Epstein & Seely, 2005; Rackowski & Richards, 2005; Landau, 2007) that we cannot review here, and indeed it is not our purpose to provide an empirically motivated solution to this issue but rather to show how an operation of this type could be added to the derivational search function.

SpecvP and thereby formed the required local [T vP] configuration for A-movement. This is an obvious problem, but not an implausible starting point since there are languages, such as Finnish (Holmberg & Nikanne, 2002), in which the EPP feature can be checked by nonsubject topics.

In addition, not all phrasal movement is A-movement. Operator movement observed in English interrogatives such as *which man did the dog bite* __, where *which man* is a special interrogative DP that needs to be dislocated to the left periphery of the interrogative clause, have different properties. Moreover, English interrogativization is associated with Aux-inversion, which is also visible in yes/no questions (*did the dog* __ *bite the man?*). Let us first fix the issue with the auxiliary. Suppose *did* represents T and that the force of the sentence (e.g., interrogative, declarative, imperative) is represented by C subcategorizing for TP. We can test the results with dataset (#9)

```
Numeration=C,the,man,did,v,bite,the,dog
C the man did bite#v#T the dog
C the dog did bite#v#T the man
```

which derives

```
(1) C the man did bite#v the dog  [_CP C[_TP [_DP the man]:1 [_TP did[_VP __:1 [_VP (bite v)[_VP __ [_DP the dog]]]]]]]
(2) C the dog did bite#v the man  [_CP C[_TP [_DP the dog]:1 [_TP did[_VP __:1 [_VP (bite v)[_VP __ [_DP the man]]]]]]]
(3) C the dog did bite#v the man  [_CP C[_TP [_DP the dog]:1 [_TP did[_VP __:1 [_VP (bite v)[_VP __ [_DP the man]]]]]]]
(4) C the dog did bite#v the man  [_CP C[_TP [_DP the dog]:1 [_TP did[_VP __:1 [_VP (bite v)[_VP __ [_DP the man]]]]]]]
```

and 12 other sentences, all generating one of the target sentences. Since tense is expressed by *did*, the verb remains at a lower position. All we have to do to model English Aux-inversion is to assume that an interrogative C is a bound morpheme, which generates without any further assumptions the Aux-inversion pattern (#10)¹⁹:

```
(1) did#C(wh) the man bite#v the dog  [_CP (did C(wh))[_TP [_DP the man]:1 [_TP __ [_VP __:1 [_VP (bite v)[_VP __ [_DP the dog]]]]]]]
(2) did#C(wh) the dog bite#v the man  [_CP (did C(wh))[_TP [_DP the dog]:1 [_TP __ [_VP __:2 [_VP (bite v)[_VP __ [_DP the man]]]]]]]
(3) did#C(wh) the dog bite#v the man  [_CP (did C(wh))[_TP [_DP the dog]:1 [_TP __ [_VP __:1 [_VP (bite v)[_VP __ [_DP the man]]]]]]]
(4) did#C(wh) the dog bite#v the man  [_CP (did C(wh))[_TP [_DP the dog]:1 [_TP __ [_VP __:1 [_VP (bite v)[_VP __ [_DP the man]]]]]]]
```

Having these mechanisms in the model, we can model interrogativization. The numeration will be {*C(wh)*, *the*, *man*, *did*, *v*, *bite*, *which*, *dog*} and the target sentences as follows:

¹⁹ Notice that if we remove the auxiliary, this model produces verb-inversion (*bites#v#T#C the dog the man?*) which is ungrammatical in English but grammatical in many other languages, such as Italian and Finnish, so the derivation is not completely implausible, though something must block it in English.

which dog did#C(wh) the man bite#v
 which man did#C(wh) the dog bite#v
 which dog did#C(wh) bite#v the man
 which man did#C(wh) bite#v the dog

Let us assume that C(wh) has an interrogative feature WH which triggers the search for the corresponding interrogative operator that we assume to be D with feature WH (=which). Let us assume, furthermore, that after Merge has created [X YP], X is checked for the interrogative feature WH and, if the feature is present, copy and merge is attempted. The search algorithm, call it *minimal search* after Chomsky (2008), moves downstream following the projectional spine of each head H and then enters the complement of H if any, returning first constituent with a WH-feature at its head. This grammar enumerates the following outputs (among others)(dataset #12)

```
(1) which dog did#C(wh) bite#v the man  [_CP [_DP which dog]:3 [_CP (did C(wh))[_TP __:3 [_TP __ [_VP __:3 [_VP (bite v)[_VP __ [_DP the man]]]]]]]]
(2) which dog did#C(wh) the man bite#v  [_CP [_DP which dog]:4 [_CP (did C(wh))[_TP [_DP the man]:3 [_TP __ [_VP __:3 [_VP (bite v)[_VP __ __:4 ]]]]]]]
(3) which dog did#C(wh) bite#v the man  [_CP [_DP which dog]:3 [_CP (did C(wh))[_TP __:3 [_TP __ [_VP __:3 [_VP (bite v)[_VP __ [_DP the man]]]]]]]]
(4) which dog did#C(wh) bite#v the man  [_CP [_DP which dog]:1 [_CP (did C(wh))[_TP __:1 [_TP __ [_VP __:1 [_VP (bite v)[_VP __ [_DP the man]]]]]]]]
```

which is the result we were looking for. Both the external argument and the internal argument can be moved to SpecCP, depending on where the interrogative operator *which* is. Because minimal search follows labeling and head-complement configurations, we can already suspect that the CED-effects (Huang, 1982b, 1982a) follow in the sense that left phrases can never be searched. However, the numeration posited above does not quite show this, since all operators fell to the minimal search paths. To show that the CED effects are captured we must craft a numeration from a sentence such as *the dog from which city barks* and show that **which city the dog from __ barks* is not in the output. First, adding the preposition *from* into the grammar with reasonable subcategorization features and testing it with an intransitive numeration together with C and T (dataset #13) produced 276 accepted derivations (several overlaps), among them the irrelevant solutions (3)a–b and the relevant analyses (3)c–d.

- (3) a. C the dog barks from the city.
- b. From the city C the dog barks (PP base-generated to SpecCP)

- c. C the city from the dog barks.
- d. C the dog from the city barks.

Once we know that the relevant analysis (3)d is in the output, we can replace the second definite article with *which* and the declarative C with an interrogative C(wh) and run the model to verify that **which city the dog from __ barks* is not among the output (#14). This proves that the grammar does not allow extraction from subjects (extraction from adjuncts cannot be tested since we do not yet have adjuncts in the grammar). However, the experiment again generated a lot of ungrammatical sentences such as those in (4) which show that the grammar far from being even observationally adequate.

- (4) a. [PP from the dog [CP which city₁ C(wh) __₁ barks]] (“relative construction”)
- b. From which city C(wh) barks the dog? (base-generated PP to SpecCP)
- c. [CP From the dog [CP which city C __ barks]]? (Double-filled SpecCP)
- d. Barks₁ the dog __₁ from which city? (cf. ‘Does the man bark from which city?’)

The reader can go through the 84 solutions and consider various options for filtering the unwanted outputs. It is also worth noting that these assumptions generate successive-cyclic A-movement provided that there is a sequence of local heads with the EPP feature. For example, the numeration {*the, dog, T, seem, to, bark*} generates the following output if the raising verb and the infinitival *to* have the EPP feature (dataset #15).

```
(1) the dog seem#T to bark [_TP [_DP the dog]:1 [_TP (seem T)][_VP __:1 [_VP __ [_T/infP __:1 [_T/infP to[_VP __:1 bark]]]]]]]
Derivational steps: 481
Errors 0
```

This property of the grammar allows us to model English personal passives. If we assume that there exists a special “passive” *v** with an unthematic specifier position (EPP) instead of projecting an external agent argument, we get sentences in which the direct object is moved successive-cyclically to the SpecTP position:

```
(1) the man was bite#v*  [_TP [_DP the man]:2 [_TP was[_VP __:2 [_VP (bite v*)[_VP __ __:2 ]]]]]
Derivational steps: 58
Errors 0
```

The numeration contains *was* = T and the special passive v^* , $(\text{bite } v^*)^0$ would be spelled out as *bitten*. The direct object is raised successive-cyclically from the VP into Specv*P and then to SpecTP. Notice that we must assume that A-chains can target not only specifiers but also complements. The key portion of the derivation is the following:

57.

```
[was], [v*], [_VP bite[_DP the man]]

Head Chain (v*, bite)
Chain ((bite v*), __:2 )
Merge(v*, [_VP bite[_DP the man]])
= [_VP [_DP the man]:2 [_VP (bite v*)[_VP __ __:2 ]]]

[was], [_VP [_DP the man]:2 [_VP (bite v*)[_VP __ __:2 ]]]
```

58.

```
[was], [_VP [_DP the man]:2 [_VP (bite v*)[_VP __ __:2 ]]]

Chain (was, __:2 )
Merge(was, [_VP [_DP the man]:2 [_VP (bite v*)[_VP __ __:2 ]]])
= [_TP [_DP the man]:2 [_TP was[_VP __:2 [_VP (bite v*)[_VP __ __:2 ]]]]]

[_TP [_DP the man]:2 [_TP was[_VP __:2 [_VP (bite v*)[_VP __ __:2 ]]]]]

|== [_TP [_DP the man]:2 [_TP was[_VP __:2 [_VP (bite v*)[_VP __ __:2 ]]]]] <= ACCEPTED: the man was bite#v*
```

First the passive v^* is merged, which triggers both head movement and phrasal A-movement and derives the extended verb phrase $[_{v^*P} \text{the man}_1 [_{v^*P} (\text{bite } v^*) [_{VP} \text{bite } __1]]]$ (step 57) and which is then followed by A-movement to SpecTP (step 58).

5 Adjunction

In this section we consider the status of adjuncts and adjunction and argue that they are “parallel” objects residing in the syntactic working memory and are attached to their host constituents via one-way mother-of dependency. We can depict them as “virtual constituents” of their hosts or alternatively think that they increase the “dimensionality” of their host structure.

Consider a sentence such as *the dog bites the man frequently*. This construction behaves as a regular transitive clause but with an additional adverbial modifier *frequently* that is (i) invisible for all head calculations within the projectional spine of the transitive clause, (ii) optional and (iii)

occurs in a curious rightward position that cannot be the complement position of the verb, being filled by the patient. Semantically the adverb (iv) modifies the whole event, not the man, the dog or even biting as such; what happens frequently is that the dog bites the man. Another property is that (v) we can add more adverbials to the clause, seemingly without restrictions. If we assume that bare events are represented by VPs, then (vi) the adverb should be connected in some way to the verb phrase, but there is nothing in our grammar that connects anything directly with a phrasal level representation. In fact, there is nothing that reproduces any of the properties (i–vi). In the minimalist theory as well as in many other grammars elements like the adverbial *frequently* are treated as *adjuncts*, elements that are merged inside grammatical projections in a more loose manner such that they are neither specifiers nor complements.²⁰

So far we have assumed that the derivational search function takes the contents of syntactic working memory as input, applies an operation OP to a pair of syntactic objects X and Y and replaces them with the outcome OP(X, Y). The syntactic working memory therefore contains a list of syntactic objects that we can think of being under active consideration. We can use this feature and assume that some operations create *parallel objects* inside the syntactic working memory instead of replacing them with a new object and then model adjuncts and adjunction on the basis of parallelism.

Let us assume that there is an operation Adjoin(X, Y) which connects X to Y via one-way mother-of dependency but leaves the two elements otherwise intact in the sWM. Thus, the mother constituent of *frequently* could be the verb phrase, while all other properties of the verb phrase, including its daughter constituents, remain the same. This would make the adverb a “virtual” constituent of the verb phrase that exists as a parallel structure. Let us assume that when X is connected in this way to Y by adjunction, the result is notated as Y|X which means “X is adjoined

²⁰ Some grammars collapse specifiers and adjuncts into the same category, but we provide a separate adjunct category in order to develop a fully general model. In a grammar without separate category of adjuncts the option can simply be removed.

to Y and linked to it via a mother-of dependency”. For example, the adverb *frequently* would be notated as VP|*frequently* when it is linked with the verb phrase as its host structure (mother). We leave X into sWM as a parallel structure.

Not all syntactic objects can be adjoined, and those which can be adjoined can have a limited number of positions. For example, we want to create a theory in which VP-adverbs such as *frequently* must be adjoined inside the VP. To both restrict the class of adjoinable objects and their distributions let us assume that adjoinability is defined by a feature $\alpha:F$ where feature F must occur inside the head of the projection to which the adjunct is linked with. A VP-adverb would therefore be defined by feature $\alpha:V$. Only syntactic objects headed by this feature can be adjoined. The vice versa is not assumed: adjoinable objects can also be merged, but must then satisfy subcategorization.²¹ Putting some of the additional technical details aside, let us consider a simple intransitive sentence *the dog barks frequently* with the numeration {T, *the*, *dog*, *bark*, *frequently*} and where we assume that the adverbial has feature $\alpha:V$. The output (dataset #17) is as follows:

```
(1) the dog bark#T frequently  [_TP [_DP the dog ]:2  [_TP (bark T ) [_VP __:2  [_VP __ frequently ] ] ] ]
(2) frequently the dog bark#T  [_AdvP frequently [_TP [_DP the dog ]:1  [_TP (bark T ) [_VP __:1  __ ] ] ] ]
(3) the dog bark#T frequently  [_TP [_DP the dog ]:1  [_TP (bark T ) [_VP __:1  __ ]* ] ] + { VP|frequently° }
(4) the dog bark#T frequently  [_TP [_DP the dog ]:1  [_TP (bark* T ) [_VP __:1  __ ] ] ] + { V|frequently° }
(5) the dog bark#T frequently  [_TP [_DP the dog ]:2  [_TP (bark T ) [_VP __:2  [_VP __ frequently ] ] ] ]
(6) the dog bark#T frequently  [_TP [_DP the dog ]:1  [_TP (bark* T ) [_VP __:1  __ ] ] ] + { V|frequently° }
Derivational steps: 106
Errors 0
```

Solutions (1, 2–6) are generated by either merging the adverbial to the complement position of the verb (1, 5) or by adjoining it either to the VP (3) or V (4, 6). Solution (2) is generated by merging the adverb above TP, but although this sentence is grammatical the analysis is not plausible since it regards the whole phrase as an adverb phrase.²² Notice how the adverb is written outside of the host structure as a parallel object, with its link with the host structure now expressed by the X|Y notation (the host marked by an asterisk). That is, *frequently* is not an ordinary constituent of the VP but

²¹ This property can easily be removed from the grammar.

²² This could be avoided if we assumed that frequently is derived by $[_{AdvP} (frequent-ly)^0 [_{AP} frequent]]$, but we do not go into the details here.

connected to it virtually only by a mother-of dependency. All label/head calculations inside the host structure remain the same: the adjunct is invisible. The newly introduced operation Adjoin is visible in the log file:

53.

```
sWM: frequently°, [_VP [_DP the dog ] bark ], T°
Adjoin(frequently, [_VP [_DP the dog ] bark ])
sWM': [_VP [_DP the dog ] bark ]*, T° + { VP|frequently° }
```

54.

```
sWM: [_VP [_DP the dog ] bark ]*, T° + { VP|frequently° }
Merge(T, [_VP [_DP the dog ] bark ]*)   Head Chain (T, bark)
Chain ((bark T ), __:1 )

sWM': [_TP [_DP the dog ]:1  [_TP (bark T ) [_VP __:1  __  ]* ] ] + { VP|frequently° }

[_TP [_DP the dog ]:1  [_TP (bark T ) [_VP __:1  __  ]* ] ] + { VP|frequently° }
^ ACCEPTED: the dog bark#T frequently
```

The VP-adjunction is performed in the step 53, after which the addition of T (step 54) results in a grammatical sentence. Running the model with a transitive numeration shows, however, that the adverb is always positioned between the verb and the object (*the dog bite frequently the man*), while it should be positioned after the object. This brings up the question of linearization. In the above simulations we assumed (without comment) that the adjunct is always linearized to the left of its host, before any other constituent. It appeared at the post-verbal position in the intransitive clause because we nowhere assumed that verb copying should update the virtual adjunct link; thus, the adjunct is still linked to the silenced verb at the base position and is linearized in relation to that element as shown in (5).

(5) The dog (bark₁, T) [_{VP} frequently ____].

The result is the same if the adjunct is linked with the verb phrase. Changing the linearization to right will handle both intransitive and transitive clauses, but will create problems later because it makes all adjectives (which we assume to be adjuncts) postnominal. More generally, we want to have both left and right adjuncts. Since the linearization of adjuncts is subject to language-specific

variation, we can perhaps assume that it is controlled by a lexical linearization feature. If we assume that adverbs like *frequently* are linearized to the right, the grammar will generate the variants in (6) after 80k derivational steps (dataset #18).

- (6) a. The dog bite the man frequently. (Adverb adjoined to VP)
 b. The dog bite frequently the man. (Adverb adjoined to V)
 c. Frequently the dog bit the man. (Adverb merged to TP)

If adjectives are linearized to left, they will occupy a position between D and N. A numeration $\{the, angry, dog, bark, T\}$ will generate the following output (datasets #18, 19) if we restrict adjective distribution to NPs:

(1) the angry dog bark#T [_TP [_DP the dog*]:1 [_TP (bark T) [_VP __:1 __]]] + { N|angry° }
 Derivational steps: 55
 Errors 0

The derivation for *the angry dog* is as follows:

4.

```
sWM: the°, angry°, dog°
Adjoin(angry, dog)
sWM': dog*°, the° + { N|angry° }
```

5.

```
sWM: dog*°, the° + { N|angry° }
Merge(the, dog*)
sWM': [_DP the dog* ] + { N|angry° }

[_DP the dog* ] + { N|angry° }
^ ACCEPTED: the angry dog
```

First *angry* is adjoined to *dog* (step 4), then the definition article is merged with N to generate $[_{DP} D N]$ with the adjective existing as a parallel object linked with N (step 5). Linearization to left creates the final output sentence. We could create postnominal adjectives by assuming that they linearize to the right of their host.

Implementing the above adjunct analysis involves several technical questions that concern the implementation. For example, linearization is much simpler to implement and easier to understand

if host structures have pointers to their adjuncts, so they were added as an additional property to the syntactic objects. These pointers are not part of the linguistic theory or the (intended) algorithm, rather they belong to the implementation layer. It is assumed that in an empirically more realistic theory adjuncts would be “collapsed” to ordinary constituents before linearization. Currently the system is implemented by allowing linearization algorithm to access the adjunct pointers so this extra step does not have to be implemented.

6 Discussion

The source code together with the dataset containing all experiments (#1–20) are available in the source code repository. It is written in the form of one relatively short Python script containing approximately 500 lines of commented code that can be modified and expanded as required by the theory and data. Here we consider some of the most important extensions.

Let us begin by considering the complexity profiles of the grammars examined in this article. The amount of derivational steps required to calculate the logical implications of sentences that contain unrestricted empty words, which define the worst case scenario, is enormous. Thus, while calculating the derivations for four words consumes 228 derivational steps, seven words require almost six million. The number increases exponentially and soon becomes intractable even for a computer. However, subcategorization and other restrictions can be used to nudge the complexity wall forward. The range of calculations required for simple T-VP-grammars in a realistic grammar formalism was around several thousands, but transitive clauses raised the number of tens of thousands, with the experiments involving an interrogative clause with a PP argument calculated through more than one hundred thousand derivations. Although these numbers can most likely be reduced by adding derivational constraints, and are easily managed by modern computers, the growth function is exponential and will, eventually, hit a wall. For example, testing the model with

the man believes that the dog barks calculated through 15 million steps.²³ What this shows, however, is that even with a tightly constrained and (in our view, no doubt) realistic grammar the number of calculations required to show that the theory matches with the data is too high to be accomplished with any paper-and-pencil methodology, leaving us with no choice but to use mechanical methods such as the one proposed here or some other conceivable alternative. Notice that in a realistic linguistic project it is rarely sufficient to show that the grammar matches with a few target sentences: the testing must be repeated after any change in the theory and executed for several numerations and target sentences.²⁴ This raises the requirements to the level of hundreds of millions, if not more. The alternative – that grammatical theories are not justified rigorously by observation – is simply not acceptable.

Short Python programs can be provided in the form of simple scripts that are contained in one text file. Since the template used in the present article was designed to serve as a starting point for larger projects, it too is contained in just one file. The file contains definitions for data structures (e.g., lexicon, phrase structure) and functions (e.g., the derivational search function). The script reads the dataset file (specified inside the script) and executes all the experiments #1–20 reported above. Larger projects, however, are typically not organized into one script file but are instead dissolved into several files called *modules* that contain code designed for specific tasks. For example, the derivational search function maps initial numerations into phrase structure

²³ A possible solution is to break the initial numeration into sub-numerations (“phases,” Chomsky 2000) which are derived independently, and then connect the outputs together in a separate step. Thus, instead of providing the derivational search function with an initial numeration it is provided with a set of numerations which puts restrictions on which items can be merged with which items. This has many consequences for movement operations, but the exact predictions will depend on what the phases are. This method differs from the one used here in that we restrict the combinatorial possibilities before the derivation begins; it is easy to imagine several schemes which accomplish the same output.

²⁴ While batch testing is easy to implement technically by writing the tests into the one external input file, as was done here, but involves additional concerns since the selection of the input data is still free. One solution is to develop standardized datasets. For example, we could imagine a range of datapoints that any theory of say English passivization has to deduce correctly and then develop of a standardized batch dataset that expresses these properties.

representations, the latter which are processed by one additional function generating the sentence/phrase structure -pairs for accepted solutions. These two functions correspond to the three branches of the Y-architecture: the first corresponds to the horizontal branch while the second corresponds to the two vertical branches. In a linguistically realistic model the syntactic objects constituting the end point of the derivation would be sent to two separate modules, the first which handles mapping into PF and beyond and contains whatever postsyntactic operations are required to calculate the data (Marantz, 1984; Embick & Noyer, 2001; Hale & Keyser, 2002; Matushansky, 2006; Harizanov, 2018) and a second module which maps the objects to the LF-interface that feeds a separate module that handles semantic interpretation. The SO-PF mapping will generate formal-morphological surface modifications that are not visible for semantic interpretation, while the SO-LF mapping generates covert syntax. Thus, what in the template used in this study is expressed by two functions will in a more realistic system be implemented by three separate modules. These modules can be brought under the same overall architecture by creating one higher-level “speaker model” container class which maintains the separate modules, so that instantiating objects from the higher-level container class will automatically create instances of the module classes. Items from the numeration can be feed to the container class which manages all data processing in and out of the modules and thus defines the whole architecture.

In addition to Merge and Move, the standard minimalist theory posits a third operation Agree that is responsible for feature covariance dependencies, such as standard subject-verb agreement. This operation was not implemented; instead, the code already contains the kernel of an operation of this type. Recall that we assumed that C(wh) searches for a corresponding operator element inside its complement before phrasal movement is executed. This was implemented by a minimal search function which takes a wh-feature as input and locates a head that has the same feature. The operation resembles long-distance Agree (or probe-goal dependency). A similar assumption was made tacitly in connection with A-movement, which located the phrase from the complement but

did not restrict the nature of the selected element; consequently, if we want to search for elements of specific type, such as grammatical subjects, a feature-based search function is an obvious candidate. It is possible that a theory of Agree could be developed from these mechanisms without positing a separate operation. This approach would, however, rule out upward-directed agreement systems (Chomsky, 1993; Koopman, 2006; Chandra, 2007; Baker, 2008; Merchant, 2011; Zeijlstra, 2012; Carstens, 2016; Bjorkman & Zeijlstra, 2019; Baker & Camargo Souza, 2020; Keine & Dash, 2022) and is therefore neither empirically inconsequential nor something that could be assumed without giving it careful consideration.

We pointed out in connection with several examples that derivations which are possible in many other languages are not possible in English. For example, the current grammar can form yes/no questions by fronting finite verbs, which is not possible in English. To model language-specific crosslinguistic variation we need a theory of the said variation and some technical implementation for “different grammars.” The easiest way to accomplish a system of this type is to create several language-specific speaker models and then use them on the basis of the language in the numeration and/or input sentence. Each speaker model would contain a different lexicon (depending on the language) plus different grammar rules and/or parameters, depending on how the variation is modeled. If it is modelled by positing different rules (implausible, but theoretically possible), then the list of grammatical rules for each speaker model should be populated from a larger pool of possible rules when the speaker model instances are generated; if the rules differ only in terms of finite number of parameters, then each speaker model would contain a data-structure holding the values of these parameters in addition to the rules. If languages differ only in terms of their lexicon, then the solution is to pair each speaker model with a different lexicon depending on the language.

The derivational search function in and itself is not a model of an actual speaker/hearer, but part of the executive layer “running the grammar.” It can be expanded easily into a more realistic

model, however. Suppose we want to create a realistic model for language production and assume that the endpoint of the derivation as depicted in this article constitutes a “motoric plan” which guides speech production. If we assume that the step-by-step derivation is a realistic description of the processes which build up plans for externalized speech, then we can add psycholinguistic plausibility functions to the derivational search function determining the order at which the various elements are merged together and associate each step in the derivation with a cognitive cost. The theory will now make precise predictions of cognitive costs associated with the production of sentences.²⁵ We can also model and thus predict the order in which real speakers create sentences from lists of words. However, it is far from trivial that a bottom-up model of this type has a measurable connection to language production. It seems to describe some type of abstract “logic” or characteristics that linguistic representations must have.

7 Conclusions

Grammatical theories and linguistic hypotheses are logically deep in the sense that the chain of reasoning connecting them with data are long, especially when the data is nontrivial. This renders traditional paper-and-pencil techniques almost completely useless. A rigorous computational methodology was suggested that calculates through all logical implications of the theory and compares the results automatically with data.

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²⁵ A theory of this type requires, however, an additional component which specifies the meaning of the intended sentence so that the derivation can converge towards the target.

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