EIP

Model formulation:

Writing down the formulation to model the mosquito population. The equation describing the susceptible mosquito is as follows :

$$\frac{dS(t)}{dt} = b - \beta S(t)I_H(t) - \mu_S S(t),$$

where is the b is the birth rate and should be equal to the sum of the mortality terms and senescent development terms to ensure a constant population; β is the infection rate where we assume a law of mass action between the susceptible vectors and the infected vertebrate host, I_H; and finally, the background mortality term, μ_S.

The infected mosquitoes then flow into the exposed class E which we describe as the stage where the vectors are not yet infectious to vertebrate hosts:

$$\frac{dE(t)}{dt} = \beta S(t)I_H(t) - \int_{t-\tau}^{t} E(t-\tau)e^{-\mu_I \tau} f(\tau)d\tau - \mu_E E(t),$$

The integrodifferential equation describes the mosquitos that enter the exposed class τ days ago, survive (expressed by the exponential decay), and multiplied by the probability distribution function $f(t)$ of the waiting time. Due to the difficulty of simulating this, we converted the integrodifferential equation into a series of ordinary differential equation by using the 'linear chain trick'. By assuming that the pdf is a gamma-distribution, the above equation can be written as:

$$\frac{dE_1(t)}{dt} = \beta S(t)I_H(t) - n\alpha_E E_1(t) - \mu_E E_1(t) \quad k = 1$$

$$\frac{dE_k}{dt} = n\alpha_E E_{k-1}(t) - n\alpha_E E_k(t) - \mu_E E_k(t) \quad k > 1$$

Here, n is the number of subcompartments within the model and describes the variability in which individuals transition. Specifically, the coefficient of variation (CV) is $\frac{1}{\sqrt{(n)}}$. Exposed individuals than transition into the infectious compartment:

$$\frac{dI}{dt} = n\alpha_E E_n(t) - \alpha_I I(t)$$

The full model formulation with double feeding

We need to modify the full model formulation to create two different compartments. We declare $D(t)$ for exposed mosquitos that have doubled fed and DI(t) for the double infected. We must first modify the exposed class and a new outflow fo the exposed class is $f(V_H)E_0$ which describe the rate at which the exposed class double feed:

$$\frac{dE_1(t)}{dt} = \beta S(t)I_H(t) - f(V_H)E_0(t) - n\alpha_E E_0(t) - \mu_E E_0(t) \quad k = 1$$

$$\frac{dE_k(t)}{dt} = n\alpha_E E_{k-1}(t) - n\alpha_E E_k(t) - f(V_H)E_k(t) - \mu_E E_k(t) \quad k > 1$$

The double-fed states that are still not infectious are then described below:

$$\frac{dD_1}{dt} = f(V_H)E_1(t) - n_D\alpha_D D_1(t) - \mu_D D_1(t) \quad k = 1$$
$$\frac{dD_k}{dt} = f(V_H)D_k(t) + n_D\alpha_D D_{k-1}(t) - n_D\alpha_D D_n(t) - \mu_E D_k(t) \quad k > 1$$

Then we have the double-fed individuals go into the DI compartment:

$$\frac{dDI(t)}{dt} = n_D\alpha_D D_{n_D}(t) - \alpha_{DI} DI(t)$$

Important assumptions in the model we must think about for the second model:

1) Susceptible mosquitos do not double feed (should we change this). I can imagine it changes the model because with double feeding, maybe you can greatly increase the change of being infected by the vertebrate host AND increase your lifespan.
2) When you're an infected mosquito, we assume that the mortality is exponentially distributed.
3) Assume a law of mass action when it comes to susceptible host and vertebrate hosts