

NEW ADVANCES IN STATISTICAL NEUROIMAGE
PROCESSING: TACKLING THE SMALL SAMPLE SIZE
PROBLEM

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New Advances in Statistical Neuroimage Processing: Tackling the Small Sample Size Problem

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TITLEBACK

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ABSTRACT

The rise of neuroimaging in the last years has provided physicians and radiologist with the ability to study the brain with unprecedented ease. This led to a new biological perspective in the study of neurodegenerative diseases, allowing the characterization of different anatomical and functional patterns associated with them. Computer Aided Diagnostic (CAD) systems use statistical techniques for preparing, processing and extracting information from neuroimaging data pursuing a major goal: optimize the process of analysis and diagnosis of neurodegenerative diseases and mental conditions.

With this thesis we focus on three different stages of the CAD pipeline: pre-processing, feature extraction and validation. For preprocessing, we have developed a method that target a relatively recent concern: the confounding effect of false positives due to differences in the acquisition at multiple sites. Our method can effectively merge datasets while reducing the acquisition site effects. Regarding feature extraction, we have studied decomposition algorithms (independent component analysis, factor analysis), texture features and a complete framework called Spherical Brain Mapping, that reduces the 3-dimensional brain images to two-dimensional statistical maps. This allowed us to improve the performance of automatic systems for detecting Alzheimer's and Parkinson's diseases. Finally, we developed a brain simulation technique that can be used to validate new functional datasets as well as for educational purposes.

Guide:

<https://plg.uwaterloo.ca/~migod/research/beck00PSLA.html>

RESUMEN

Resumen de la tesis en español.

DECLARACIÓN

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*We have seen that computer programming is an art,
because it applies accumulated knowledge to the world,
because it requires skill and ingenuity,
and especially because it produces objects of beauty.*

— Donald Ervin Knuth

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ACRONYMS

PCA	Principal Component Analysis
ICA	Independent Component Analysis
FA	Factor Analysis
SPECT	Single Photon Emission Computed Tomography
CT	Computed Tomography
PET	Positron Emission Tomography
AD	Alzheimer's Disease
PD	Parkinson's Disease
PKS	Parkinsonism
ASD	Autism Spectrum Disorder
MRI	Magnetic Resonance Imaging
fMRI	functional MRI
SWPCA	Significance Weighted Principal Component Analysis
SBM	Spherical Brain Mapping
VBM	Voxel Based Morphometry
SPM	Statistical Parametric Mapping
SPM8	Statistical Parametric Mapping Software, version 8
CTL	Control Subject
VAF	Voxels As Features
CAD	Computer Aided Diagnosis
ADNI	Alzheimer's Disease Neuroimaging Initiative
PPMI	Parkinson's Progression Markers Initiative

VDLN	Virgen de las Nieves Hospital
VDLV	Virgen de la Victoria Hospital
MRC-AIMS	Medical Research Council Autism Imaging Multicentre Study
MNI	Montreal Neurological Institute
synT ₁	simulated T ₁ - weighted Inversion Recovery
qT ₁	quantitative T ₁ - weighted
qT ₂	quantitative T ₂ - weighted
GM	grey matter
GLM	General Linear Model
WM	white matter
CSF	cerebro-spinal fluid
ANOVA	Analysis Of Variance
SVD	Singular Value Decomposition
SVM	Support Vector Machine
SVC	Support Vector Classifier
CBM	Component Based Morphometry
KDE	Kernel Density Estimation
MCI	Mild Cognitive Impairment
EM	Expectation-Maximization
PDF	Probability Density Function
FWE	Family Wise Error rate
RF	radiofrequency
SNR	Signal-To-Noise Ratio
rCBF	regional Cerebral Blood Flow
DAT	Dopamine Transporters

FBP	Filtered Back Projection
FDR	False Discovery Rate
ROI	Region of Interest
HMM	Hidden Markov Model
AC	Anterior Commissure
LBP	Local Binary Patterns

Part I
INTRODUCTION

1

INTRODUCTION

1.1 MOTIVATION

In recent years, there has been a rise in the use of neuroimaging in the clinical practice. It has improved and speeded the procedure of diagnostic, providing unprecedented insight into the brain. Neuroimaging is very extended in research as well. Different fields such as psychiatry, neurology, psychology, behavioural science or biology make extensive use of brain imaging in their studies.

The basis of these studies are common: a selection procedure by which a representative set of subjects is recruited, the fulfilment of an experiment on (or by) each subject and a statistical analysis of the acquired data. Particularly, when studying a certain disease, it is common to recruit subjects affected by the disease and non-affected, healthy subjects, usually known as Control Subjects (**CTLs**). Then, in this typical example, both affected and **CTLs** are scanned, and brain anatomy or function is analysed using statistical tools. The result of this analysis is a list of significant differences between structure or function that could be linked to the disease.

Computer Aided Diagnosis (**CAD**) systems provide a set of tools to help setting up and performing these studies. It is currently a thriving area of research involving multidisciplinary teams, combining computer science, mathematics, medicine, artificial intelligence, statistics, machine learning, and many others [**Martinez-Murcia2016**]. The main aim is to assist clinicians in the procedure of diagnosis and study of the diseases by providing software that can effectively recognize disease patterns, characterize differences and make predictions.

One fundamental issue often found in this studies is the sample size. The number of subjects frequently ranges from tens to hundreds, whereas the number of features (namely voxels) to be analysed can add up to millions. This causes the so-called *Small Sample Size Problem* [**Duin2000**] which negatively affects the statistical power of any experiment performed using these datasets [**Button2013**].



Figure 1.1: Illustration of the separation between points in one-dimensional and two-dimensional spaces.

1.2 THE SMALL SAMPLE SIZE PROBLEM

The *Small Sample Size Problem* refers to a problem that arises when the proportion between number of subjects and number of features is large. Think of, e.g., 15 points in a one-dimensional line, as in Figure 1.1. If we think of a subject as a vector, we would have 15 subjects in a one-dimensional space. Now, imagine that we add a second dimension. It is easy to see that our subjects would be farther than in the two-dimensional world. And the same would happen if we move to four, ten or thousands of dimensions. The farther our points are, the more difficult is for a statistical tool to extract information. That is what we call *almost empty spaces* [Duin2000, Stoeckel04], in contrast to *dense spaces*, where points are closer.

Neuroimaging provides hundreds of thousands, or even millions of voxels, in what could mean millions of features. That implies that any calculation performed in those almost empty spaces will eventually lack information. This implies a loss of statistical power of the methods used, usually producing false negatives (the system is unable to detect real signal) and false positives (the system detects signal where there is not). These are known in statistics as Type I and Type II errors respectively.

In differential diagnosis studies, the small sample size problem leads to wrong conclusions about where real differences are located. This, in addition to untracked confounding variables are one of the fundamental sources of non-reproducibility in current neuroimaging studies [Button2013].

The solution might seem straightforward: increase sample size. But this is not always possible, since neuroimaging studies do their best at recruiting as many

people as they can with a limited budget. Many efforts have been put into establishing multi-centre collaborations that allow the recruitment of a larger population, but despite offering a higher statistical power, these studies still suffer from a number of confounding variables such as population bias or scanner differences [haar2014anatomical]. In Chapters 7 and 8 we explore different approaches to this solution.

Another option involves reducing the number of features, via feature selection or feature extraction. This has been widely used in computed-aided methodology for neuroimaging [DeMartino2007, xu2009source, Gorriz2010, Illan2011, Martinez-Murcia2016] with great success, and solutions using this approach will be treated in Chapters 4, 5 and 6.

The Small Sample Size problem is directly related to the *Curse of Dimensionality* [Krishnaiah1982], which proves that, in contrast to what could be expected, once a certain classifier performance has been achieved, it holds or even decreases when feeding more features to the classifier. The problem also affects statistical hypothesis testing, a tool widely used for inference in neuroimaging, in what is known as the *Multiple Comparisons* problem [Benjamini2010], a particular field that is still being studied.

1.3 AIMS AND OBJECTIVES

This thesis aims to contribute new approaches to overcome the small sample size problem in neuroimaging. This can provide more accurate CAD systems by reducing the number of false positives, increasing the reliability of their results.

We will take two different approaches, as commented in previous sections: increasing the sample size and reducing the feature space. Therefore, we can define the following objectives:

- Develop and evaluate algorithms that reduce the feature space, in which is usually known in the field as feature extraction and feature selection strategies.
- Develop and evaluate new strategies to increase the sample size in neuroimaging studies.

Most of the studies in the literature focus on the first objective. Feature extraction algorithms that use Principal Component Analysis (PCA) [Khedher2015, Towey2011] or Partial Least Squares [Segovia2013], among others, are widely studied. We have developed three different approaches to those:

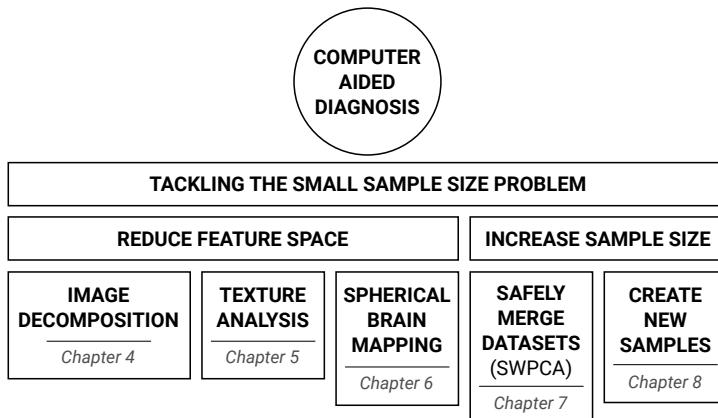


Figure 1.2: Structure and connexions between the different strategies proposed in this thesis, organized by chapters and parts.

- A combination of image decomposition algorithms and feature selection. In this approach we have used three criteria to select the most significant voxels from the images, and then applied Factor Analysis ([FA](#)) and Independent Component Analysis ([ICA](#)) to decompose the data and significantly reduce their feature space.
- A feature extraction based on texture analysis.
- A novel strategy called Spherical Brain Mapping ([SBM](#)). This feature extraction technique uses a spherical coordinate system to map statistical measures to a bidimensional plane. It builds paths used as feature selection vectors where several measures are computed.

On the other hand, we have evaluated newer ways to increase sample size in neuroimaging studies. We have developed:

- A system to reduce undesired variance in structural multicentre studies, called Significance Weighted Principal Component Analysis ([SWPCA](#)). This system is intended to reduce the amount of false positives in large collaborations, providing more homogeneous images and improving their statistical analysis.
- An algorithm to simulate functional brain images using existing data, and therefore, increase sample size.

1.4 ORGANIZATION OF THIS THESIS

This thesis work is organized in four parts plus appendices, each of which is subdivided in several chapters. In the first part, we introduce the motivations and main aims of this work (Chapter 1), examine the state of the art in medicine, neuroimaging and CAD systems (Chapter 2) and present a general methodology that will be followed throughout this thesis, including preprocessing and evaluation (Chapter 3).

Parts ii and iii refers to each of the solutions outlined in the previous section, and disaggregated in Figure 1.2. In part ii we focus on the feature reduction techniques, including decomposition methods (Chapter 4), texture analysis (Chapter 5), and the novel algorithm Spherical Brain Mapping (Chapter 6). On the other hand, Part iii is focused on two different strategies used to increase the sample size: the Significance Weighted Principal Component Analysis algorithm (Chapter 7), used to safely merge structural images acquired at different centres, and a neuroimage simulation algorithm (Chapter 8) that can be used to extend existing functional datasets.

Finally, in Part iv we provide a general discussion of the results presented in this thesis, conclusions about the methods and prospective work that could be performed with this basis.

1.5 CONTRIBUTIONS

Some ideas and figures have appeared previously in the following publications, that we divide here in articles and conference presentations.

1.5.1 Articles

1.5.2 Conferences

1.5.3 Books

2 | STATE OF THE ART

We have already stated the motivation and objectives of this thesis. Now, we will describe in detail some of the more relevant issues for the state of the art. First, in Section 2.1, we will make an introduction to the different neuroimaging modalities used in our experiments. Afterwards, we will provide some insights into the neurological and psychiatric disorders treated here in Section 2.2 or the most extended voxel-wise analyses used in the neuroimaging community at Section 2.3. Finally, at Section 2.4 we will explore recent contributions to the field that use Machine Learning.

2.1 INTRODUCTION TO NEUROIMAGING

Medical imaging refers to all types of 2D, 3D and 4D images used in clinical practice. These involve many different modalities, among them X-rays, ultrasound, endoscopy, microscopy, etc. In neuroimaging, the most extended is by far Magnetic Resonance Imaging ([MRI](#)), which provides intensity maps that represent the internal structure of the brain. Other modalities are aimed at studying the function of the brain, by injecting radioactive ligands that, linked to a receptor, can measure its distribution. This is the case of Positron Emission Tomography ([PET](#)) and Single Photon Emission Computed Tomography ([SPECT](#)).

2.1.1 Magnetic Resonance Imaging

Magnetic Resonance Imaging ([MRI](#)) is perhaps the most widespread in neuroimaging, given its ability to visualize both structural and functional (in functional [MRI](#)) properties of the brain, and, in contrast to other imaging modalities, is considered non-invasive. [MRI](#) uses strong magnetic fields to excite certain atomic nuclei, that can absorb and emit this energy.

[MRI](#) combines the magnetic field with a radiofrequency ([RF](#)) emission to excite the atomic nuclei present in corporal structure, resulting in a image of the distribution of certain atoms in the body. Most [MRI](#) use hydrogen atoms, since they are present in water (which adds up to around 70% of body mass) and

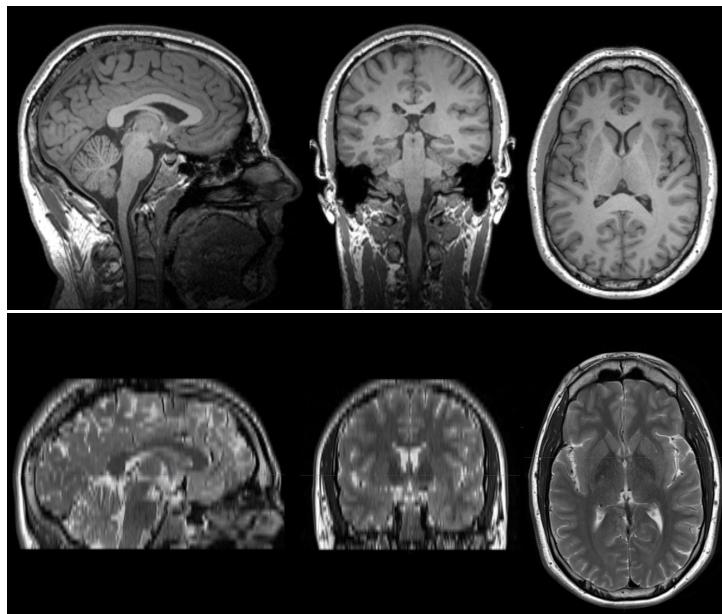


Figure 2.1: Example of T₁ and T₂-weighted MRI images of the same subject (me).

the signal derived is stronger than other atoms, increasing the Signal-To-Noise Ratio ([SNR](#)), and therefore, the image quality.

The procedure uses a strong magnetic field B_0 to align the magnetic moment of the hydrogen nuclei in parallel or anti-parallel (depending of their initial spin). This way, the magnetic moment of all nuclei will increase up to a stable state, in contrast to their null value in absence of B_0 . Within this magnetic field, the hydrogen atoms precess around an axis along the direction of the field.

A given nuclei has a resonance frequency which is proportional to the intensity of B_0 , which, by using strong fields, allow us to resonate hydrogen far below potentially damaging frequencies. The precession frequency is determined by the Larmor equation (2.1):

$$f_0 = \frac{\gamma}{2\pi} B_0 \quad (2.1)$$

where γ depends on the nuclei, which in the case of hydrogen, $\gamma = 42.6 \text{ MHz/T}$. When a subject is introduced in the [MRI](#) scanner, it is submitted to the magnetic field B_0 , so that the hydrogen nuclei are aligned to the field, with a precession frequency f_0 . Then, a [RF](#) pulse of the same frequency is generated, which is then absorbed by the nuclei, forcing them to place perpendicular to the field. Once the [RF](#) emission is interrupted, the nuclei return to its equilibrium state by means of a procedure called relaxation. In this procedure, they emit part of

the absorbed energy, which is then captured by a RF receptor. Usually, position information is encoded in the RF signal by varying B_0 using gradient coils.

The RF signal is measured during the relaxation time, and two different relaxation times are set: the T₁ (spin-lattice) relaxation time and the T₂ (spin-spin) relaxation time. The T₁ time is the time during which nuclei emit energy to the adjacent tissue and realign to the longitudinal plane (z axis), whereas the T₂ time refers to the time when nuclei realign to the transversal plane (y axis). These times are used to create T₁-weighted and T₂-weighted images (see Figure 2.1). T₁-weighted images allow to distinguish between GM and WM in the cerebral cortex, to identify fatty tissue, and generally, obtain structural information. Conversely, T₂-weighted images are used to assess cerebro-spinal fluid (CSF) or to visualize and identify WM lesions.

2.1.2 Single Photon Emission Computed Tomography

The Single Photon Emission Computed Tomography (SPECT) is based on the principles of Computed Tomography (CT), by which a series of signal acquisition at different angles can be reconstructed back into a bidimensional distribution of the signal. In SPECT, a gamma photon emitting radioisotope is linked to a pharmaceutical that binds to a given biomarker, generating a radiopharmaceutical or agent. This agent is injected into the patient, and after a certain time in which the radiopharmaceutical is distributed, the patient is introduced into the SPECT-CT scan.

Afterwards, the scanner performs a series of acquisitions at different planes and angles from the body, from which the gamma signal is measured. For each plane, all acquisitions at each angle are pooled and a single two-dimensional image is reconstructed using a Filtered Back Projection (FBP) algorithm, or Radon inversion formula [Herman2009], which derives from the Fourier's Theorem. A total of 180 projections per plane, using an angular resolution of 2 degrees, are usually taken.

There exist a number of radiopharmaceutical used in clinical practice, and therefore, we will focus on the two varieties used in this thesis. First, we use an agent called ^{99m}Tc-HMPAO, which consists of two stereoisomers of hexametazime (HMPAO) linked to the radioisotope technetium 99-metastable. This agent is usually used to assess regional Cerebral Blood Flow (rCBF), which can be used to diagnose neurological diseases or cancer.

Additionally, we use images generated using the agent Ioflupane (¹²³I), a cocaine analog with high binding affinity for Dopamine Transporters (DAT). It is used fundamentally in the assessment of Parkinson's Disease (PD), given that

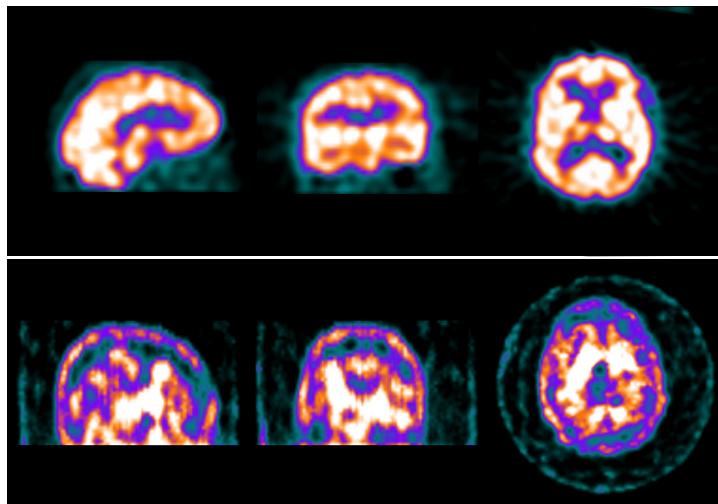


Figure 2.2: Example of SPECT images, a SPECT-HMPAO and a SPECT-DaTSCAN.

the disease is associated with a loss of dopaminergic neurons in the striatal region.

2.1.3 Positron Emission Tomography

The Positron Emission Tomography ([PET](#)) is a technique similar to [SPECT](#), but in this case, the agent used and the equipment is designed to deal with a pair of gamma photons resulting of the annihilation of a positron with its corresponding antiparticle, the electron. The pair of photons are generated in opposite directions, and the detection depends on them being simultaneously or coincidently detected at the receptor. The receptor comprises a scintillator which emits light when the gamma photon incides, and a detector, usually a photomultiplier tube or silicon avalanche photodiodes.

It uses the same [FBP](#) algorithm as [SPECT](#) in the reconstruction of the images, and a similar strategy for acquiring the signal at different angles. However, the amount of data is smaller than in [SPECT](#), and therefore, the reconstruction procedure is harder. As a result, [PET](#) scanner operation is considered more costly than [SPECT](#) [[Carlson2016](#)].

The agent used in the images that we have processed is PET-FDG, also known as Fludeoxyglucose (^{18}F). It is a glucose analogue that allows us to measure the glucose metabolism in the brain. It is widely used in neurology [[Newberg2002](#)] and cancer detection [[Kelloff2005](#)], since it can be correlated with cellular activity.

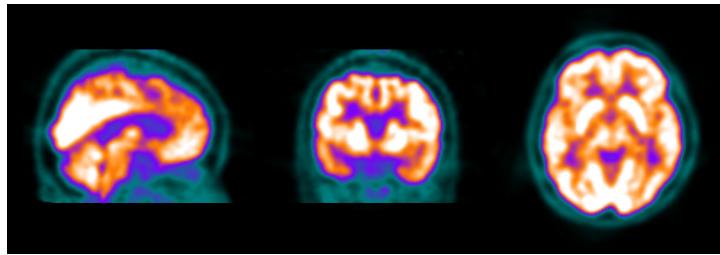


Figure 2.3: Example of a PET-FDG image.

2.2 MEDICAL BACKGROUND

2.2.1 Alzheimer's Disease

Alzheimer's Disease ([AD](#)) is the most common cause of dementia in the world, with more than 46 million people affected, and it is likely to increase up to 135.5 million by 2050 [[Association2016](#)]. Its causes are still not clear, but it is characterized by deposits of high amounts of structures such as Amyloid- β ($A\beta$) plaques or neurofibrillary tangles accompanied by synaptic dysfunction and neurodegeneration that eventually lead to cell death [[Ballard2011](#), [Sevigny2016](#)].

Diagnosis of [AD](#) is often based on the clinical history of the patient, using cognitive tests along with medical imaging and blood tests. A definite diagnosis can only be addressed post-mortem, via a direct examination of the brain tissue [[Ballard2011](#)]. Cognitive tests such as *Mini Mental State Exam* or *Clinical Dementia Ratio* are widely used in clinical practice.

Initial symptoms of [AD](#) are often mistaken for normal ageing, leading to a state known as Mild Cognitive Impairment ([MCI](#)). Not all [MCI](#)-affected subjects develop [AD](#). In fact, the prediction of [MCI](#) conversion is the most urging challenge in [AD](#) research, since an early diagnosis can lead to an improvement in life expectancy and quality of life of the patients.

[MRI](#) brain images have been extensively used in the diagnosis of [AD](#) by assessing neurodegeneration on [GM](#) and [WM](#) tissues. Research has shown in [[Baron2001](#), [Misra2009](#), [Pievani2013](#), [Dubois2007](#)] that neurodegeneration in Alzheimer's Disease mainly occurs in the [GM](#) tissue. Particularly grey matter loss has been described in the Hippocampus and Parahippocampal lobes, according to the NINCDS-ADRDA criteria for AD diagnosis [[Dubois2007](#)], with further atrophy described in the medial temporal structures, the Posterior Cingulate gyrus and adjacent Precuneus [[Baron2001](#)]. Moreover, significantly lower volumes of certain regions in [GM](#) and [WM](#) have been considered a promising biomarker and predictor of the progression of [AD](#) in a longitudinal study involv-

ing MCI patients [Misra2009], and some structures in the striatum (putamen and caudate nucleus) have shown important volume abnormalities [Pievani2013]. All these data suggest that many of the symptoms of AD can be observed in anatomical MRI images even in early stages of the disease, which could be of great help in its successful diagnosis and treatment.

Nuclear imaging, such as PET or SPECT, have also been used in clinical practice, especially to discard other diseases. In typical PET-FDG or SPECT-HMPAO, AD is characterized by reduced brain activity in bilateral regions, such as the posterior cingulate gyri and precunei, as well as the temporo-parietal region [Claus1994]. It also affects the frontal cortex and the whole brain in severe cases [Leon1983, Kogure2000].

Recently, new more specific radiopharmaceuticals have been developed, among them the Pittsburgh compound B (PiB). This drug binds to fibrillar A β allowing *in vivo* visualization of A β plaques in the brain [Ikonomovic2008]. However, due to their technical requirements -a relatively small half-life of the radioactive element-, they are unusual in clinical practice.

2.2.2 Parkinsonism

Parkinsonism (PKS) or Parkinsonian Syndrome is the second most common neurodegenerative disease in the world, with a prevalence of 1-3% in the elder population (over 65 years)[Eckert2007]. It is characterized by hypokinesia, rigidity, tremor and postural instability [Eckert2007]. It is not a single disease itself, but a wide range of conditions that share similar symptoms. The most common cause of PKS is Parkinson's Disease (PD), but other possible causes include toxins, a few metabolic diseases, and other extrapyramidal syndromes such as Multiple System Atrophy, Progressive Supranuclear Palsy or Cortico-Basal Degeneration [Christine2004, tatsch2008extrapyramidal].

2.2.2.1 Parkinson's Disease

The etiology of Parkinson's Disease (PD) involves the progressive loss of Dopamine Transporters (DAT) of the nigrostriatal pathway. This causes a decrease in the dopamine content of the striatum, since the pathway connects the substantia nigra to the striatum.

In PD, structural imaging such as MRI has limited value, since structural abnormalities can be seen only in the latter stages. Nevertheless, in molecular imaging, a number of radiopharmaceuticals have been proposed to assess the levels of pre and post-synaptic DAT at the striatum. Among them, the ^{123}I -ioflupane (better known by its tradename DaTSCAN) is perhaps the

most popular. DaTSCAN binds to the **DAT** at the striatum [Winogrodzka2003, PunalRioboo2007, Eckert2007], allowing the estimation of **DAT** density by means of a **SPECT** scanner. In DaTSCAN images, a reduced uptake at the striatum is a clear indication of **DAT** loss, and therefore, of **PD** [PunalRioboo2007].

2.2.2.2 Extrapiramidal Syndroms

Among the extrapiramidal syndroms of **PKS**, the most relevant diagnoses are Multiple System Atrophy, Progressive Supranuclear Palsy and Cortico-Basal Degeneration [tatsch2008extrapiramidal]. An accurate diagnosis can positively impact in the health of these patients, avoiding wrong treatment decisions.

Structural imaging, as in the previous case, has little value here. To establish a differential diagnosis with **PD**, different drugs have been proposed. DaTSCAN is widely used to assess pre-synaptic dopaminergic loss, but in a post-synaptic level, one of the most extended is ¹⁸F-DMFP-PET [Segovia2016a]. However, in this thesis we have focused only on the pre-synaptic level, and therefore, in the diagnosis of **PD**.

2.2.3 Autism Spectrum Disorder

Autism Spectrum Disorder (**ASD**) is a neurodevelopmental syndrome characterized by social and communication impairment as well as restricted, repetitive patterns of behaviour, interests or activities. Its origins are still unknown, although research suggest [Szatmari1999] that there exist genetic risk factors.

The evidence of either functionally or structurally affected areas in the brain is a major concern [Ecker2014, haar2014anatomical]. In the latter years, many strategies have been explored to recruit large samples in order to detect significant differences between affected and non-affected subjects. This has been addressed by initiatives such as the Medical Research Council Autism Imaging Multicentre Study (**MRC-AIMS**) [Ecker2012, Ecker2013] or the Autism Brain Imaging Data Exchange (ABIDE) [DiMartino2014].

2.3 VOXELWISE ANALYSES

Traditional analysis of neuroimaging involves visual analysis by experts clinicians, or semi-quantitative analysis of Regions of Interest (**ROIs**). With the rise of neuroimaging in the mid-nineties, some computer-aided solutions appeared, starting with the widely known Statistical Parametric Mapping (**SPM**) [Friston1994], its extension to structural imaging Voxel Based Morphometry

([VBM](#)) [[Ashburner2000](#)] and later, the first application of classifiers to medical imaging, called Voxels As Features ([VAF](#)) [[Stoeckelo4](#)].

2.3.1 Statistical Parametric Mapping

Statistical Parametric Mapping ([SPM](#)) is a new methodology to automatically examine differences in brain activity in functional imaging studies involving functional [MRI](#) ([fMRI](#)) or [PET](#), firstly proposed by Friston in [[Friston1994](#)]. The technique can be applied either to static images (e.g., [PET](#)) or timeseries ([fMRI](#)), using inference techniques based on hypothesis testing, in order to construct the General Linear Model ([GLM](#)) that better describes the variability in the data.

Statistical hypothesis testing involves constructing a pair of hypotheses: H_0 , or the null hypothesis, that states no relationship between variables; and H_1 , the alternative hypothesis. In neuroimaging, H_0 usually means that there are no relevant differences between classes (for example, between patients affected by Alzheimer's Disease ([AD](#)) and [CTL](#)), and H_1 implies that there is a significant difference. Many different tests such as massive univariate t-Test or Analysis Of Variance ([ANOVA](#)) (see Chapter 4 for more information on these techniques) can be used in the [SPM](#) software [[spm_book](#)], by using a design matrix that describes a t or F based contrast (for t-Test and [ANOVA](#) respectively). These terms are generally referred to as Z-values, namely the signed number of standard deviations an observation is above the mean.

The test are computed voxel-wise, from which a p-value can be obtained, nominally the probability of obtaining equally or more extreme Z values than the one actually found. p-values are very extended in neuroimaging, representing the probability of a Z value being equal or more extreme than the reference value given. In many studies $p < 0.05$ is used for measuring statistical significance, which means that only a 5% of the times a experiment is repeated we would obtain that result or a more extreme one. The use of the significance threshold $\alpha = 0.05$ implies that any voxel with a p-value smaller than 0.05 is considered sufficient to reject the null hypothesis.

[SPM](#) outputs maps like the one shown in Figure 2.4. There, significant Z-values according to a given threshold ([FWE](#) uncorrected or corrected, see Section 2.3.3) are displayed over an anatomical reference. The resulting maps allow a visual inspection of the active brain areas, which can later be related to a certain disease or task.

Although [SPM](#)'s main feature is the estimation of differences, the term has been extended to cover the whole process performed by the [SPM](#) software. That is, it generally involves registration to a template, intensity normalization, smooth-

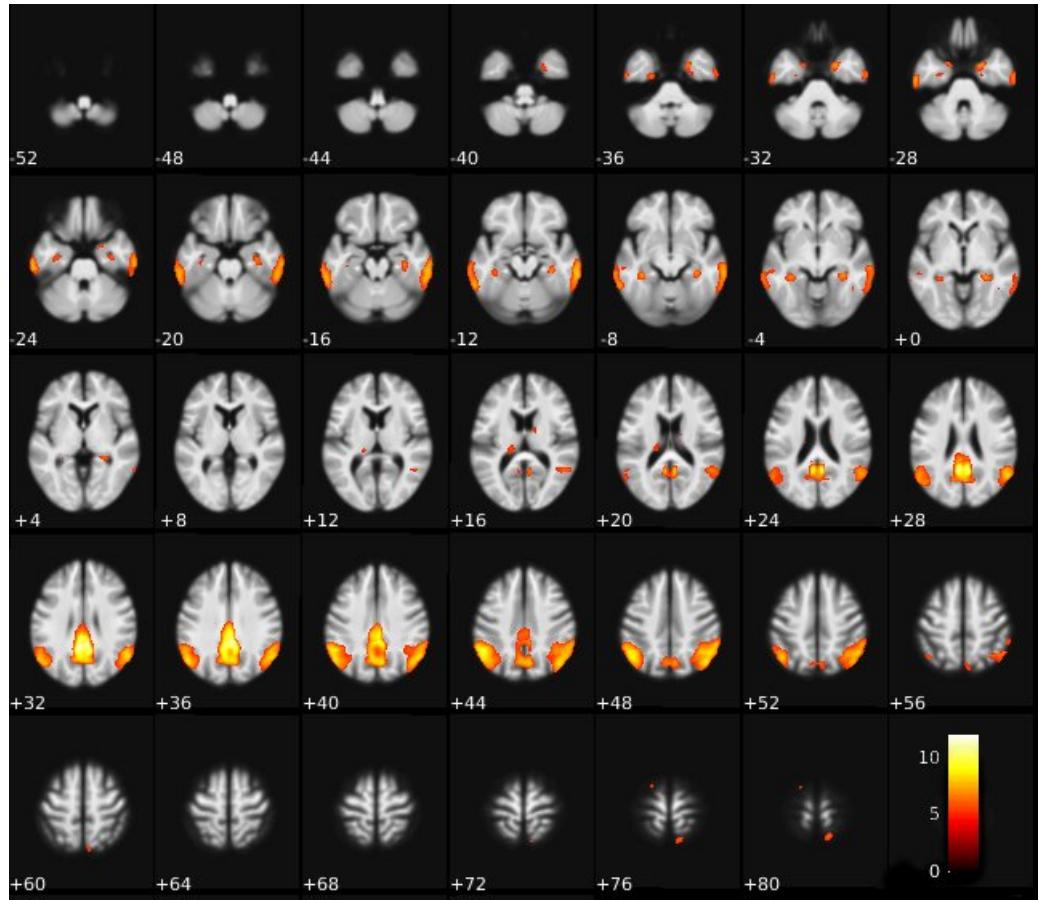


Figure 2.4: Example of a SPM analysis on a PET dataset displaying the differences between AD and CTL, using $p < 0.05$ and FWE correction.

ing, the proper [SPM](#) difference estimation and the display of the results. An overview of these procedures is provided at Chapter [3](#).

2.3.2 Voxel Based Morphometry

Voxel Based Morphometry ([VBM](#)) can be considered an extention of [SPM](#) applied to structural [MRI](#) images [[Ashburner2000](#)]. The procedure involves preprocessing (see Chapter [3](#)), where smoothing is applied to reduce smaller anatomical differences. Afterwards, a [GLM](#) is applied to each voxel in the images, and a Z-score map similar to Figure [2.4](#) is produced.

Smoothing is more important in [VBM](#) than in regular [SPM](#), since [MRI](#) images have higher resolution and are less noisy than functional images. Larger smoothing kernels will miss out smaller regions, while smaller kernel can lead to artifacts in the generated Z-maps, including misalignment of brain structures, differences in folding patterns or misclassification of tissue types [[Martinez-Murcia2016book](#)]. Therefore, the kernel size must be carefully chosen, usually using a-priori knowledge about the regions affected, and always double checking for artifacts and reproducibility.

The idea behind [VBM](#) has been extended in a number of papers, using multivariate approaches that takes into account all voxels at once, and not their individual differences. Some of them include [ICA](#) decomposition of the dataset and a posterior conversion to Z-scores in what was called Source Based Morphometry [[xu2009source](#)], or multidimensional Tensor Based Morphometry [[bossa2010tensor](#)].

2.3.3 The Multiple Comparisons Problem

The Multiple Comparisons problem arises when using hypothesis testing to assess statistical significance. This is widely used in neuroimaging, where statistical tests such as the t-Test or [ANOVA](#) are used to quantify voxel-wise differences, and state their statistical significance, or p-value. The p-value, as described above, is the probability of any value being more extreme than a certain threshold under a given hypothesis. In our problem, given the t-value T_i for the i^{th} voxel ($i = 1, \dots, N$) of the images, and a threshold T_{th} under the hypothesis H_0 , the significance can be assessed by checking:

$$P(T_i > T_{\text{th}} | H_0) < \alpha \quad (2.2)$$

where α is the significance level.

Choosing α is not trivial in neuroimaging. The use of the significance level $\alpha = 0.05$ implies that any voxel with a p-value smaller than 0.05 is considered

sufficient to reject the null hypothesis. This does not directly imply the necessity of accepting the alternative hypothesis H_1 , although it is often thought so. Neither it yields the probability of the null hypothesis [Dixon2003].

If we apply $p < 0.05$ directly to a medical image of, for example, 300,000 voxels, that could mean the possibility of almost 15,000 voxels being false positives. Controlling the apparition of false positives when applying a massive univariate test is not trivial. It implies a balance between the true positive rate (sensitivity) or true negative rate (specificity), given that, for example, controlling the amount of false negatives will result in many false positives and vice-versa.

Usually, two options for controlling the amount of false positives are given: the Family Wise Error rate (**FWE**) and the False Discovery Rate (**FDR**). The **FWE** is the probability of obtaining at least one type I error. Mathematically, the null hypothesis for the i^{th} voxel H_{0i} states that there is no activation in that voxel. Therefore, the family-wise null hypothesis for our problem is:

$$H_0 = \bigcap_i H_{0i} \quad (2.3)$$

If we reject a single null hypothesis ($T_i > T_{\text{th}}$), we reject H_0 . Therefore, we want to control the probability of a single voxel being significant if the family-wise null hypothesis is valid:

$$P \left(\bigcup_i \{T_i > T_{\text{th}}\} | H_0 \right) < \alpha \quad (2.4)$$

In this case, we must obtain the critical value T_{th} , which is the higher t value that matches that expression. Many options have been proposed to this problem, among them the conservative Bonferroni correction, methods that use random field theory or permutation tests.

2.3.3.1 The Bonferroni Correction

The Bonferroni correction [Shaffer1995] rewrites eq. 2.2 setting $\alpha = \frac{\alpha}{N}$ so that:

$$P(T_i > T_{\text{th}} | H_0) < \frac{\alpha}{N} \quad (2.5)$$

That way, using the Boole's inequality:

$$\text{FWE} \leq \sum_i^N \frac{\alpha}{N} = \alpha \quad (2.6)$$

Therefore, we can comply with the imposed restriction for a maximum **FWE**, or in our case, a maximum rate α of false positives. This is considered a rather

conservative approach. In the example cited above, if we want to keep the **FWE** below 0.05, we should divide it by N, therefore obtaining a T_{th} that makes $\alpha = 0.05/N = 1.67 \times 10^{-7}$.

Other less conservative options try to compute a critical value T_{th} that minimizes the **FWE** using spatial information. This is the case of using an approximation of the distribution of the maximum statistic over the image, or the spatial correlation, including elements from random field theory (the approach used in **SPM** [[spm_book](#)]).

2.3.3.2 Random Field Theory

In the random field approach, the maps of the statistic are treated, under the null hypothesis, as a lattice representation of smooth isotropic three dimensional random fields of test statistics. This approximation to the problem allow us to approximate the upper tail of the maximum distribution, the part needed for defining an event that occurs when the map exceeds the critical value T_{th} . Further information about random field theory and how it is applied to neuroimaging can be found at [[spm_book](#)].

The other approach, based on the **FDR**, aims at controlling the proportion of false positives in the total number of voxels declared significant. The most extended procedure for controlling the **FDR** is that proposed by Benjamini and Hochberg [[Benjamini1995](#)]. The Benjamini and Hochberg method start with calculating the p-values of all voxels and ranking them so that:

$$p_1 \leq p_2 \leq \dots \leq p_i \leq \dots \leq p_N \quad \forall i = 1 \dots N \quad (2.7)$$

2.3.3.3 FDR Controlling Procedures

Let q be the a maximum **FDR** value that we can afford, for example 0.05. For each i , we compute:

$$p_i \leq \frac{i}{N} q \quad (2.8)$$

The maximum i value that holds Eq. 2.8 is used as α , the significance level, and its corresponding statistical value (T_i in the case of a t-test) is used as the critical value. This test, under the family-wise null hypothesis H_0 , is equivalent to controlling the **FWE**. However, **FDR** methods are less conservative than other approaches such as the Bonferroni or other **FWE**-based corrections, leading to a gain in statistical power.

2.3.3.4 *Permutation Tests*

An empirical way to obtain p-values without relying on any parametric assumption is permutation testing [Anderson2001, Winkler2014]. Permutation tests evaluate a statistic such as the F-statistic or the t-test using randomly target variables, in our case, the classes. The procedure is applied many times (up to 10,000), and for each permutation, only the maximum value of the computed statistic is considered. These values are used to build the null distribution, from which the family-wise corrected p-values are computed. We assume t

Results obtained in permutation tests are comparable to those obtained using Random Field Theory [Winkler2014], and far less conservative than when applying the Bonferroni correction. Many permutation schemes can be applied. The one that we use here is based upon [Anderson2001], as it was demonstrated to achieve more sensitivity than other alternatives.

2.4 MACHINE LEARNING IN NEUROIMAGING

Machine learning is the subfield of computer science that provides computers with the ability to learn from data instead of being programmed for an explicit task. The application of new machine learning techniques in CAD systems is a current trend. Works on this topic have increased exponentially in the past ten years, and it is expected to grow even more. Machine learning explores the study and construction of algorithms that can learn from and make predictions on data, and therefore, it is very useful in neuroimaging.

Two approaches exist in machine learning. Supervised learning explores the patterns that lead to a certain outcome, e.g. the brain activation patterns that are related to a certain disease. On the other hand, unsupervised learning explores the underlying structure of the data. Machine learning in CAD is mostly based on supervised learning, since it is focused on the prediction and analysis of patterns related to a certain disease.

In its simplest form, a machine learning pipeline for neuroimaging consists of a single classifier, just as the VAF approach that we mentioned. However, classifiers can improve their detection power if higher level features are extracted from the data, e.g., features that represent the distribution of the voxel intensities, the texture of the images, or the sources of variance of the maps. This is known as feature extraction, and the most common technique is image decomposition.

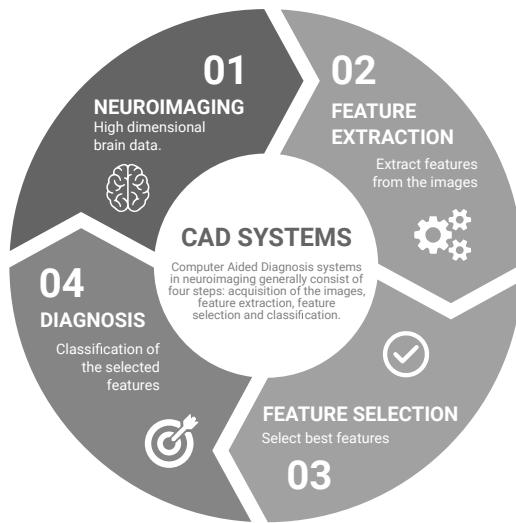


Figure 2.5: Illustration of a typical neuroimaging CAD system.

To overcome this problem, most **CAD** systems in the literature use feature extraction and feature selection [Martinez-Murcia2016], in order to reduce the feature space size (see Figure 2.5).

In recent years, many efforts have been put into discovering new automatic or semi-automatic algorithms for analyzing neurological images, in has been known as Computer Aided Diagnosis (**CAD**)

2.4.1 Voxels as Features

Voxels As Features (**VAF**) [Stoeckelo4] is the most direct application of machine learning to neuroimaging. It was firstly proposed for evaluating and performing automatic diagnosis of **AD** using functional **SPECT** imaging. It is the simpler machine learning approach used in this thesis, using a standard preprocessing (registration, intensity normalization) and a Support Vector Classifier (**SVC**) (See Appendix B) to predict the class of an image using all its intensities as features. Feature extraction, here, considers all voxels, and then there is no feature selection applied.

It has been used in many works as a baseline [Salas-Gonzalez2009], since it is comparable to the performance achieved by expert physicians using visual analysis [Stoeckelo4]. The weight vector of the **SVC** can be inverse transformed to the dimension of the original images, and therefore provide a visual map that reflects the most influential voxels, in a similar way to the Z maps of **SPM** and **VBM**.

2.4.2 Multivariate Analyses

In contrast to traditional visual inspection a semiquantitative analysis of neuroimaging, machine learning is nowadays a trend in the field. Machine learning is the subfield of computer science that provides computers with the ability to learn from data instead of being programmed for an explicit task. Applications range from automatic processing of images to ,

Statistical techniques such as [PCA](#), are used for feature extraction and hypothesis testing for feature selection in systems intended to

This thesis explores how to use signal processing and machine learning to overcome the small sample size problem in neuroimaging.

[CAD](#)

3

GENERAL METHODOLOGY

To perform most automated analyses on neuroimaging, it is fundamental that images are comparable. Preprocessing comprises a series of algorithms that, applied after the acquisition and reconstruction of the images, produce directly comparable images in both structure and magnitude.

In this section we present the preprocessing algorithms used in this thesis. Whether they have been used in one or all experiments, they can be classified in two major categories: spatial and intensity preprocessing. Later, in Section 2.3, we present some voxelwise analyses, commonly used in clinical practice, that we have set as a baseline in our experiments.

3.1 SPATIAL PREPROCESSING

Spatial processing usually accounts for the differences in position, angles and structure that are commonly found between images. A common pipeline in, for example, [MRI](#) preprocessing, is the one found at Figure 3.1, where the images are registered (or spatially normalized) to a template, smoothed and finally segmented. The smoothing is an optional step, generally used in procedures like segmentation or [VBM](#).

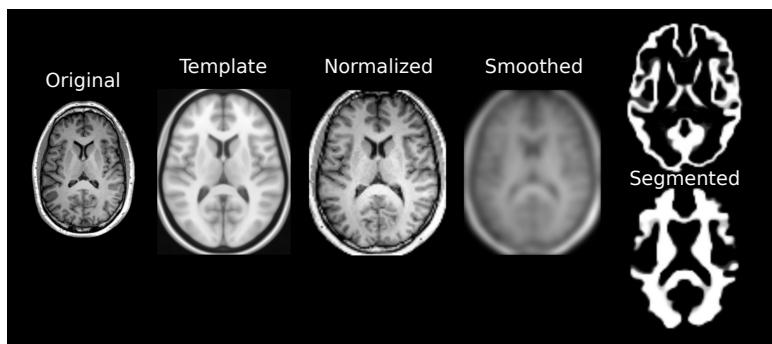


Figure 3.1: Typical pre-processing pipeline in [MRI](#).

In this thesis, all the experiments in all image modalities involve spatial normalization. Smoothing, as well as segmentation, is only applied in some exper-

iments that use [MRI](#) images, such as the segmented images in Chapter 6 or the whole-brain analysis performed in Chapter 7.

3.1.1 Spatial Normalization or Registration

Spatial Normalization, also known as Registration, is the procedure that by which every subject's brain is mapped from their individual space to a standard reference system. Registered images allows our system to overcome the individual differences in position and anatomy by establishing a common reference space in which a given coordinate represent the same anatomical position in all brains in the dataset.

There exist a number of pieces of software widely used for registering images, such as FreeSurfer [[Reuter2010](#)] or FSL (in the FLIRT and FNIRT package) [[Smith2004](#)], most of them perform linear, non-rigid and elastic transformations or a combination of these. In this work we have used the software SPM8 [[spm_book](#)] to perform registration of all the datasets, including [MRI](#), [SPECT](#) and [PET](#) images. So, from this moment, we will focus on the registration as performed in the Statistical Parametric Mapping Software, version 8 ([SPM8](#)).

Linear registration usually refers to the affine transformation, a matrix multiplication that includes 12 parameters for translation, rotation, scale, squeeze, shear and others:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{20} & a_{21} & a_{22} & a_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \quad (3.1)$$

This matrix multiplication is performed globally, as it transforms the whole image, not accounting for local geometric differences. In equations 3.2, 3.3 and 3.4 we give an example of the parameters that are computed for scale, translation and shear in 3D:

$$\text{scale} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.2)$$

$$\text{translation} = \begin{bmatrix} 1 & 0 & 0 & \Delta x \\ 0 & 1 & 0 & \Delta y \\ 0 & 0 & 1 & \Delta z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.3)$$

$$\text{shear} = \begin{bmatrix} 1 & h_{xy} & h_{xz} & 0 \\ h_{yx} & 1 & h_{yz} & 0 \\ h_{zx} & h_{zy} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.4)$$

The combination of all these operations result in the estimation of the twelve parameters that we found in Eq. 3.1, which are the ones used in [SPM8](#). The estimation of these parameters is performed via the optimization of a cost function, that in [SPM8](#) can be the minimum squared difference between the source image and the template [`spm_book`] in the case of within-modality registration, or the mutual information in between-modality registration. These functions are also used in FLIRT [[Jenkinson2001](#)], whereas FreeSurfer uses the Tukey's biweight function (in `mri_robust_template`) [[Reuter2012](#)].

After the affine transform, the software usually performs a fine-tuning step via nonrigid transformations, to account for relevant anatomical differences between subjects. Nonrigid transformations range from the use of radial basis functions, physical continuum models and the large deformation models, or diffeomorphisms, that [SPM8](#) uses. These procedures work by estimating a warp-field and then, apply it to the affine-registered images. An example of the differences of using only affine registration and applying diffeomorphisms can be found at Figure 3.2.

3.1.1.1 Co-registration

Sometimes we have several image modalities of the same subject, for example [MRI](#) and [PET](#) or functional [MRI](#), often acquired at the same time. In this particular case, we can use the higher resolution [MRI](#) image to calculate the affine parameters and warping, and apply those to all modalities of the same subject. To do so, we perform a first co-registration, that is, a registration of the lower-resolution images (e.g. [PET](#)) to its correspondent [MRI](#) image. Being anatomically

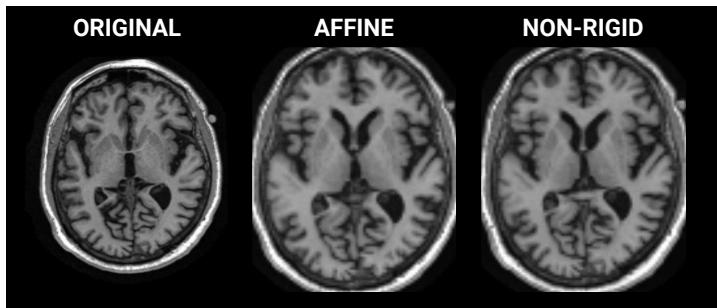
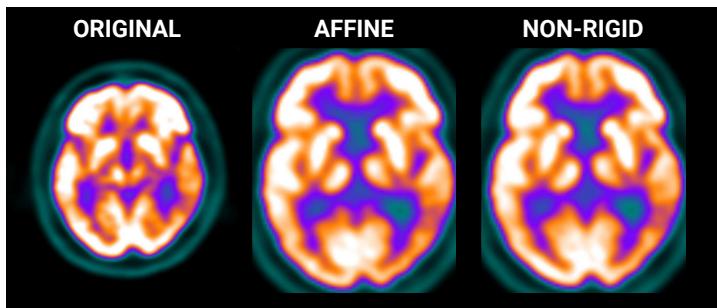
(a) Comparison in [MRI](#).(b) Comparison in [PET](#).

Figure 3.2: Comparison of the affine registration and the application of non-linear transformations to both [MRI](#) and [PET](#) images of the same [ADNI](#) subject.

similar, the co-registration usually comprises a single affine transformation. Afterwards, we can proceed with the registration of that [MRI](#) image to the template, and apply the same transformation to all its co-registered images.

3.1.1.2 *The MNI Space*

In this thesis, all images are coregistered to the Montreal Neurological Institute ([MNI](#)) space [Mazziotta2001]. This is the most widely used coordinate system, recently adopted by the International Consortium for Brain Mapping (ICBM) as its standard template. The three-dimensional coordinate system defined in [MNI](#) was intended to replace the Tailarach space, a system based on a dissected brain, that was used to compose an atlas by Tailarach and Tournoux [Talairach1988c]. The current template is known as ICBM152, and features the average of 152 normal [MRI](#) scans matched to an older [MNI](#) template using a nine parameter affine registration.

3.1.2 Segmentation

When using [MRI](#) images in this thesis, we often refer to grey matter ([GM](#)) and white matter ([WM](#)) maps, which is the result of the segmentation of the original data. Segmentation aims at producing maps of the distribution of different tissues, and it generally addresses [GM](#), [WM](#) and [CSF](#) classes, although lately some software can output data for bone, soft tissue or very detailed functional regions and subregions [[Fischl2002](#)].

In this thesis we have used the [VBM](#) toolbox of the [SPM8](#) software, which yields [GM](#), [WM](#) and [CSF](#) maps. It features an Expectation-Maximization ([EM](#)) algorithm to model the distribution of the tissue classes as a mixture of gaussians and, by combining this distribution-based information with tissue probability maps using a bayesian rule, the software produces joint posterior probability maps for each tissue. To clean up the segmentation maps, a series of iterative dilations and erosions are used. Finally, since brain regions are expanded or contracted at the spatial normalization step, we can scale the segmented maps using modulation, producing final maps where the total amount of grey matter is preserved.

3.2 INTENSITY NORMALIZATION

Generally, structural modalities such as T₁ and T₂-weighted images are considered unitless, in contrast to functional imaging, in which each voxel's intensity represent the distribution of some biomarker, such as glucose metabolism, dopamine transporters, etc. These amounts are affected by many sources of variability that can affect the final values: contrast uptake, radiotracer decay time, metabolism, etc. Therefore, along with the previous spatial normalization, there is a need to normalize the intensities of the images, so that the amount they represent are comparable.

In the case of intensity normalization, the method acts as a linear transformation of the image, preserving fundamental information such as contrast between regions. This approximation estimates the new intensity values I' as:

$$I' = I/I_p \quad (3.5)$$

where I_p is a constant parameter that is unique for each image. After this division, the new intensities would be directly comparable. The technique used to compute the normalization parameter varies, ranging from the simplest normalization to the maximum [[Salas-Gonzalez2009](#), [Martinez-Murcia20129676](#)] to complex methodologies that use assumptions about the image's Probability Density Function ([PDF](#)).

The *normalization to the maximum* strategy computes I_p as the average value of the 95th bin of the histogram of the image. In other words, this mean averaging the 5% higher intensity values and use this mean as I_p . Another useful approach is the so-called *integral normalization*, which computes I_p as the sum of all values in the image.

Other approaches involves some a-priori knowledge about the intensity distribution of normal subjects in a certain modality. This is the case of setting I_p to the Binding Potential (BP), a ratio between the intensities at specific and non-specific areas [Scherfler2005].

Finally, more advanced approaches use a general linear transformation of the image:

$$I' = aI + b \quad (3.6)$$

The parameters a and b are so that the [PDF](#) of a given matches a reference [PDF](#). There exist methods that use the histogram [Arndt1996], the gaussian distribution or the alpha-stable distribution [Salas-Gonzalez2013]. In this latter case, the parameters a y b are computed as linear transformations of some distribution's parameters:

$$a = \frac{\gamma^*}{\gamma}, \quad b = \mu^* + \frac{\gamma^*}{\gamma}\mu \quad (3.7)$$

where γ^* and γ are the dispersion parameters of the alpha-stable intensity distribution of the non-normalized and the reference image respectively, and μ^* and μ are the location parameters of the same images.

Despite traditionally structural modalities such as [MRI](#) did not use intensity normalization, there exist a new tendency towards the use of quantitative T₁ - weighted ([qT₁](#)) and quantitative T₂ - weighted ([qT₂](#)) images [Weiskopf2013] that provide biomarkers for absolute measures such as myelination, water and iron levels. This strategy is especially designed to overcome different sources of variability that affect multicentre studies, e.g. magnetic field inhomogeneity, noise, evolution of the scanners, etc. The role of those in multi-centre studies is addressed at Chapter 7.

See Figure 3.3 for a comparison between different strategies of intensity normalization on the same images.

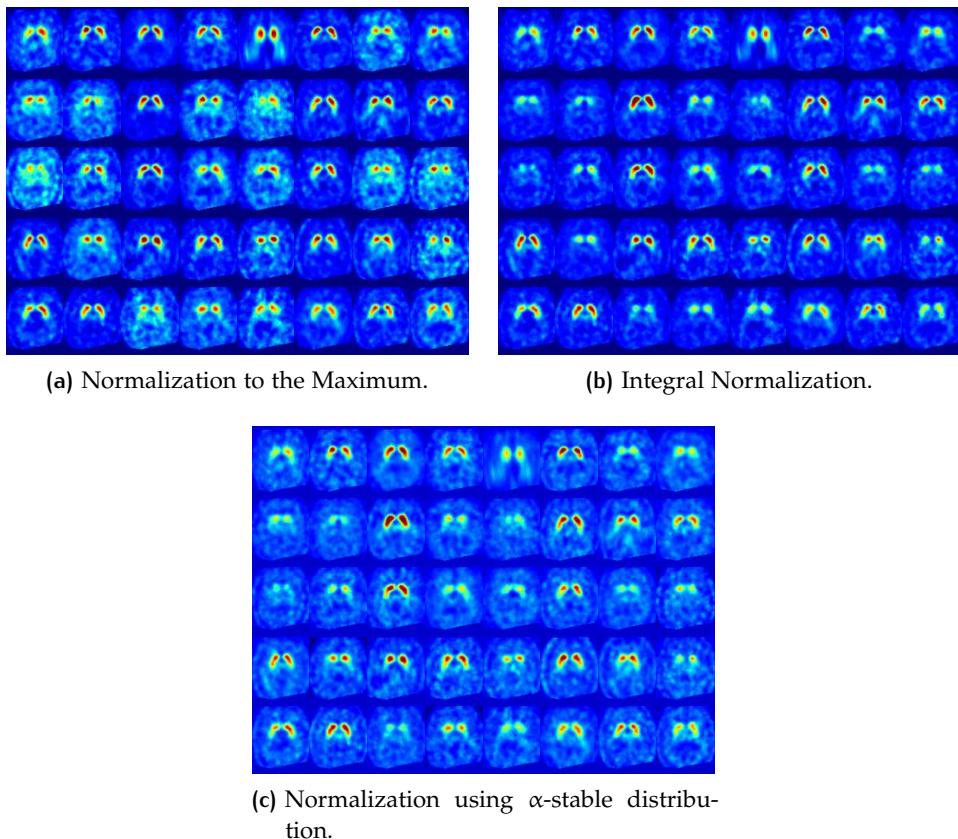


Figure 3.3: Comparison between different types of intensity normalization, applied to the VDLN-DAT dataset (see Appendix A).

3.3 EVALUATION PARAMETERS AND METHODOLOGY

3.3.1 Cross-validation

The classification was validated using stratified 10-fold cross-validation (Kohavi, 1995). In brief, 9 subsets of the dataset were used for extraction of the PCs and training of the classifier with the remaining subset used for testing. This procedure was repeated for each subset, repeated 10 times to avoid possible bias and random effects of the partitions. The average and standard deviation of the accuracy (acc), sensitivity (sens) and specificity (spec) values for each repetition were recorded.

3.3.2 Classification Performance

The proposed methodology has been tested on the three previously described databases, using a cross-validation method called leave-one-out to extract several performance parameters: accuracy, sensitivity, specificity, positive likelihood (PL) and negative likelihood (NL). This method achieves an almost unbiased error estimate, however it might be affected by the database topology, reason why we used different databases to test our method.

Along with the widely used sensitivity, specificity and accuracy (which are prevalence dependent), we use the Positive and Negative Likelihood ratios (PL and NL) to provide some prevalence independent parameters that are also widely used in clinical medicine, where values of PL greater than 5 or NL values less than 0.2 can be applied to the pre-test probability of a patient having the disease tested for to estimate a post-test probability of the disease state existing [McGEE2002]. A positive result for a test with an PL of 8 adds approximately 40% to the pre-test probability that a patient has a specific diagnosis.

To provide a better understanding of the accuracy distribution (mean, range, skewness), we have made use of the box plot (Fig. 5.2). In these plots, the median for each group of data is indicated by the red center line, and the first and third quartiles are the edges of the blue box, which is known as the inter-quartile range (IQR). The extreme values (within 1.5 times the inter-quartile range from the upper or lower quartile) are the ends of the lines extending from the IQR. Points at a greater distance from the median than 1.5 times the IQR are plotted individually, representing potential outliers.

Part II

REDUCING THE FEATURE SPACE

4

IMAGE DECOMPOSITION

In this, we will focus on

[Martinez201141, Martinez-Murcia20129676, Martinez-Murcia2013255, Martinez-Murcia2013255]

Signal decomposition algorithms are the first feature extraction algorithms that we will deal with. They are aimed at modelling a set of samples as a linear combination of latent variables. These latent variables or components can be thought of as the basis of a n-dimensional space where each sample is projected and represented as a n-dimensional vector. In a general case applied to our neuroimaging data, the source images can be modelled as:

$$\mathbf{X} = s_0 \mathbf{w}_0 + s_1 \mathbf{w}_1 + \cdots + s_N \mathbf{w}_N + \epsilon = \mathbf{sW} + \epsilon \quad (4.1)$$

Where s_i is the coordinate (or component score) of the current image in the i -th dimension of the new space defined by all the base vectors \mathbf{w}_i (component loadings), and ϵ is the error of the estimation. Figure 4.1 shows an illustration of the process.

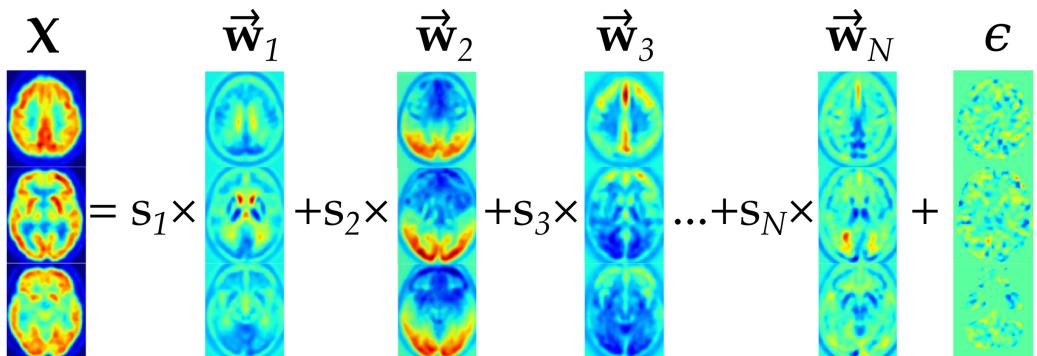


Figure 4.1: Illustration of how decomposition algorithms such as FA and ICA work on a PET-FDG brain image.

Signal decomposition techniques are widely used in many applications, ranging from one-dimensional signals such as audio or Electroencephalography (EEG) to multidimensional arrays, and are frequently applied as feature reduction to overcome the small sample size problem, that is, the loss of statistical power due to a larger number of features compared to the number of samples.

The pipeline that will be used throughout this chapter is displayed at Figure 4.2, involving a feature selection (for reducing the dimensionality), decomposition of the feature vectors and classification.

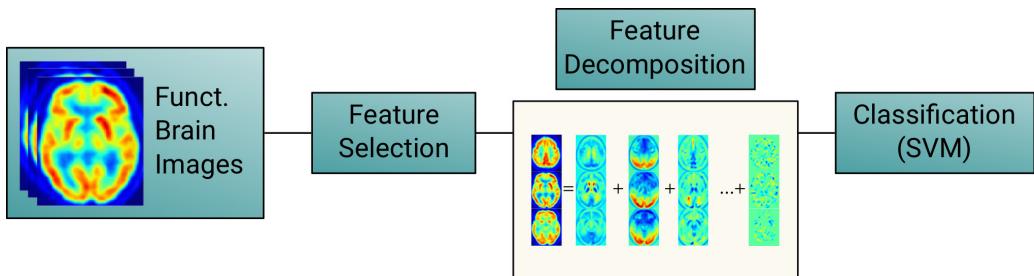


Figure 4.2: Illustration of the system used in Chapter 4.

4.1 FEATURE SELECTION

Feature selection is the first strategy used for feature reduction [CITAR WILEY MIO], and it is often used along with feature extraction in order to build more complex pattern recognition systems. It refers to any strategy intended to find a subset of the original features containing the more suitable ones according to a certain criterion. Therefore, irrelevant features are discarded, and resultant models are faster and more cost-effective [Guyon03]. However, it usually requires an additional optimization to find the parameters for the optimal feature subset, and furthermore, it is impossible to guarantee that the optimal features for the subset are the same of the full feature set [DaelemansHosteMeulderEtAl2003].

In this work, we will use filtering methods to perform feature selection. Filtering methods are based on the computation of a feature relevance score directly on the data, before any interaction with the classification model. The relevance score is used to sort the different features, discarding those with a lower score, and it is usually computed independently for each feature, in what is called a univariate approach [SaeysInzaLarranaga2007].

Explicar por qué usamos feature selection -> bajar tiempo de computación.
[Martinez201141, Martinez-Murcia20129676]

4.1.1 t-Test

Student's t-Test (t-Test) is a widely used statistical test which quantifies the differences between two different classes. It uses a common estimation of variance for both classes. The value of statistical t can be computed [Fay10] as:

$$t = \frac{\bar{\Omega}_1 - \bar{\Omega}_2}{\sigma_{\Omega_1 \Omega_2}^2 \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \quad (4.2)$$

where

$$\sigma_{\Omega_1 \Omega_2}^2 = \sqrt{\frac{(n_1 - 1)\sigma_{\Omega_1}^2 + (n_2 - 1)\sigma_{\Omega_2}^2}{n_1 + n_2 - 2}} \quad (4.3)$$

$\sigma_{\Omega_1 \Omega_2}^2$ is an estimator of common standard deviation of both samples, $\bar{\Omega}_1$ and $\bar{\Omega}_2$ are the mean of each class, n_1 is the number of samples in class ω_1 and n_2 is the number of samples in class ω_2 .

4.1.2 Mann-Whitney-Wilcoxon

Mann-Whitney-Wilcoxon U-test(MWW) uses the absolute value of the statistical U to rank voxels. Calculation of U value is done by the following expression [Fay10]:

$$U_i = R_i - \frac{n_i(n_i + 1)}{2} \quad (4.4)$$

where n_i is the sample size for sample i , and R_i is the sum of the ranks in sample i (where $i = 1, 2$). Smaller U_i value is taken as the final U value.

This statistical test measures the dissimilarity between two groups of values, and, although is similar to Student's t-Test, is less likely than it to spuriously indicate significance because of the presence of outliers. As we make use of real data, and we have no knowledge about the images' statistical distribution, MWW test could be a very good choice [Fay10].

4.1.3 Relative Entropy

Relative Entropy (or Kullback-Leibler divergence) is a non-symmetric measure of the difference between two probability distributions Ω_1 and Ω_2 . Because of its non-symmetric property, we can make use of this to evaluate the difference between CTRL and AD images for each voxel. Relative Entropy can be calculated with equation 4.5 [Theodoridis1999].

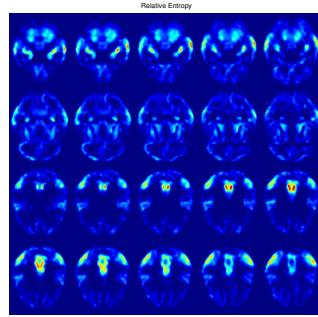


Figure 4.3: Several selected slices of the significance map obtained for each brain coordinate by applying the Relative Entropy criterium to the ADNI database.

Let Ω_1 and Ω_2 be two discrete random variables. The Kullback-Leibler Divergence, or Relative Entropy is defined as:

$$KL_{\omega_1 \omega_2} = \left(\frac{\sigma_2^2}{\sigma_1^2} + \frac{\sigma_1^2}{\sigma_2^2} - 2 \right) + \frac{1}{2} (\mu_2 - \mu_1)^2 \left(\frac{1}{v_2} + \frac{1}{v_1} \right) \quad (4.5)$$

where μ_c is the mean of class c , and σ_c^2 its variance. Figure 4.3 depicts values of Relative Entropy for each voxel.

4.2 DECOMPOSITION ALGORITHMS

CITE WE WILL USE COMPONENT AND FACTOR INDEPENDENTLY.

4.2.1 Factor Analysis

[Martinez201141, Martinez-Murcia20129676]

To this purpose, a number of key features (K) are extracted using Factor Analysis technique [Harman73]. It is assumed that each image in the database is a different observation of the experiment. Factor analysis models each of the n observations as the expression of fewer unobserved variables, which are called factors. Each observation has N variables, which are modelled as lineal combinations of the K factors, plus errors, as described in Eq. 4.6.

$$\mathbf{x} = \boldsymbol{\mu} + \boldsymbol{\lambda}\mathbf{F} + \boldsymbol{\epsilon} \quad (4.6)$$

where \mathbf{x} is the vector containing the observed variable (dimension n), $\boldsymbol{\mu}$ is the mean of the variable, $\boldsymbol{\lambda}$ is a vector of K factor loadings for this observation, \mathbf{F} is

a matrix of dimension $N \times K$ which contains the common factors and ϵ is the error of reconstruction. For all observations, Eq. 4.6 can be rewritten as:

$$\mathbf{X} = \boldsymbol{\mu} + \boldsymbol{\Lambda}\mathbf{F} + \boldsymbol{\epsilon} \quad (4.7)$$

where \mathbf{X} is a matrix of observed variables of dimension $n \times N$, $\boldsymbol{\mu}$ is a vector of means of length n , $\boldsymbol{\Lambda}$ is a matrix of dimension $n \times K$ which contains the maximum likelihood estimate of the factor loadings for each observation, \mathbf{F} is a matrix of dimension $N \times K$ which contains the common factors and $\boldsymbol{\epsilon}$ is a vector of length n containing reconstruction errors.

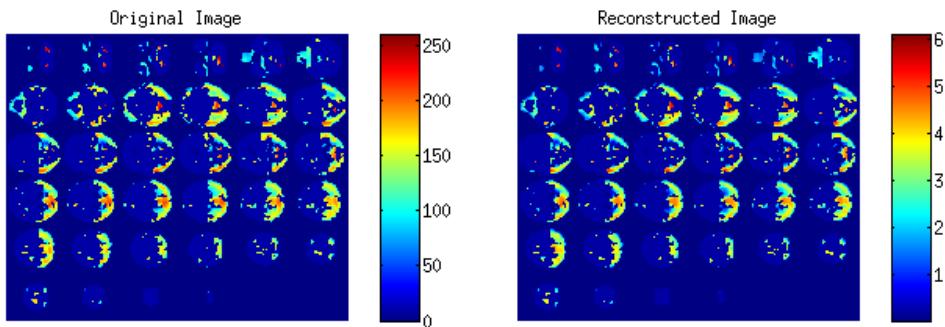


Figure 4.4: a) Original PET image composed by $N = 7000$ selected voxels and b) reconstruction using Factor Analysis with $K = 13$ factors extracted.

Original input image can be reconstructed with computed factors and factor loadings, as we have seen in Eq. 4.6. In Fig. 4.4 original image and Factor Analysis reconstruction are shown. The selected regions pinpoint the disease affected areas which are class-discriminative. Specifically, it highlights the posterior cingulate gyri and precunei, as well as the temporo-parietal region, both considered as typically affected by glucose hypometabolism in the AD [Claus1994]. A closer look shows that it also selects small thalamus regions, which has never been described as a relevant region for diagnosis. To compute representativeness of different factors, we can rewrite Eq. 4.7 as:

$$\text{Cov}(\mathbf{X} - \boldsymbol{\mu}) = \text{Cov}(\boldsymbol{\Lambda}\mathbf{F} - \boldsymbol{\epsilon}) \quad (4.8)$$

If Σ stands for $\text{Cov}(\mathbf{X} - \boldsymbol{\mu})$, Eq. 4.8 can be rewritten as:

$$\Sigma = \boldsymbol{\Lambda}\text{Cov}(\mathbf{F})\boldsymbol{\Lambda}^T - \text{Cov}(\boldsymbol{\epsilon}) \quad (4.9)$$

$$\Sigma = \Lambda \Lambda^T - \Psi \quad (4.10)$$

and extract Ψ , the diagonal matrix containing the specific variances of the reconstruction error.

The choice of K requires a deeper analysis. If K is large, the image can be modelled very well, and therefore, the reconstruction error should be small. However, a large number of features can be counter-productive for the performance of the classifier due to the small sample size problem. Thus, we should find a trade-off between the length of the feature vectors (K) and the ability of reconstruction. According to Fig. 4.5, the variance reconstruction error tends to stabilize as K increases, and the improvements are no longer significant. In the experimental Section ?? a detailed discussion about K parameter selection is shown in addition with other experimental findings.

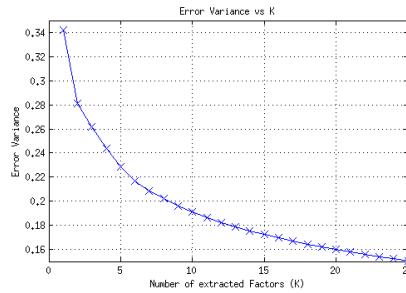


Figure 4.5: Specific variance of reconstruction error Ψ using Factor Analysis, in function of number of factors extracted (K) for ADNI database (the behaviour is similar in other datasets).

4.2.2 Independent Component Analysis

[Martinez-Murcia2013255, Martinez-Murcia201458]

Independent Component Analysis (ICA) [Hyvarinen2000], is a statistical technique that represents a multidimensional random vector as a linear combination of non-gaussian random variables (the so-called "independent components") to be as independent as possible, and has been used widely on segmentation and clustering of medical images [DeMartino2007, Alvarez2009]. It can be considered as a non-gaussian version of Factor Analysis. Assume that we observe n linear mixtures x_1, x_2, \dots, x_n of length N that can be modelled as an expression of K independent components (IC). These independent components are defined as $S = (s_1, s_2, \dots, s_K)$, where each s_K vector has a length of N . So, each random

vector \mathbf{x}_n can be described as a linear combination of K independent components:

$$\mathbf{x}_n = a_{1n}\mathbf{s}_1 + a_{2n}\mathbf{s}_2 + \dots + a_{Kn}\mathbf{s}_K \quad (4.11)$$

Without loss of generality we can assume that both the observed vectors and the independent components are zero mean. If the previous conditions are not met, the \mathbf{x} variables can be centered by subtracting the sample mean. To use a vector-matrix notation, more convenient in this case, we denote as matrix \mathbf{X} the random vector whose elements are $\mathbf{x}_1, \dots, \mathbf{x}_n$. We also denote as \mathbf{A} the matrix that contains all a_{Kn} elements, the "mixing matrix" that projects each image into the space defined by the IC. Using this notation, the mixing model above remains as follows:

$$\mathbf{X} = \mathbf{AS} \quad (4.12)$$

The starting point of ICA is the assumption that all components \mathbf{s}_K are statistically independent. To measure independence, we assume that all independent components have a non-gaussian statistical distribution. It is assumed that a sum of independent signal trends to gaussianity, so if non-gaussianity is maximized with any independence criteria F , for instance, the kurtosis or negentropy, we obtain signals that are more independent than the previous ones [Hyvarinen1999, Hyvarinen2000]. After estimating the matrix \mathbf{A} , we can compute its inverse, \mathbf{W} and obtain the projection \mathbf{S} of the images in the dataset into the IC space with:

$$\mathbf{S} = \mathbf{WX} \quad (4.13)$$

4.2.2.1 FastICA

Adaptive algorithms based on gradient descend can be problematic when they are used on an environment in which adaptation is not necessary, like this case. The convergence is often slow, and depends on the choice of convergence parameters. As a solution to this problem, block algorithms based on fixed-point iteration [Oja1997, FastICA99] can be used. In [Oja1997] a fixed-point algorithm based on kurtosis is introduced. In [FastICA99], this algorithm, known as FastICA, is generalized to general contrast functions. The single unit FastICA algorithm has the following form:

$$\mathbf{w}(k) = E\{\mathbf{x}g(\mathbf{w}(k-1)^T\mathbf{x})\} - E\{g'(\mathbf{w}(k-1)^T\mathbf{x})\}\mathbf{w}(k-1) \quad (4.14)$$

where the loadings vector \mathbf{w} is normalized to unit norm in each iteration, and the function $g(x)$ is a derivative of the contrast function G defined in [Hyvarinen1999].

The expected values are estimated in practice by using the mean of a significantly high number of samples of the input data. The speed of convergence of the fixed-point algorithms is clearly superior to more neural algorithms. Improvements between 10 and 100 times the speed are observed frequently [Giannakopoulos1998].

4.3 RESULTS

In this work we will analyse the behaviour of the system proposed in the introduction and illustrated at Figure 4.2. The system comprises the selection of the most relevant voxels using filtering methods (we will focus on t-test, relative entropy and wilcoxon) and a feature decomposition of these using either FA or ICA. Finally, the feature vectors are classified using a SVC with linear kernel, and performance values are obtained via cross-validation (see Section 3.3 for more information).

We vary the number of selected voxels and the number of factors or components depending on the algorithm and the dataset used and evaluate the system with those characteristics. That way, we obtain an estimation of the performance of the system in different situations, so that we can draw conclusions on the disease patterns and the ability of the system in the detection of different diseases.

4.3.1 Alzheimer's Disease

We begin by applying the proposed feature selection plus decomposition pipeline to the two functional neuroimaging datasets: ADNI-PET and VDNLN-HMPAO. For this experiment we will use a maximum of 20,000 selected voxels and 25 components.

4.3.1.1 Factor Analysis

First, we use FA as a decomposition technique. In Figure 4.6 we average the accuracy over the number of voxels or the number of components respectively, to look at how these variables affect the performance of the system, and we do this for the three filtering methods used.

We can observe that the results are always better when using the ADNI-PET dataset than with the VDNLN-HMPAO, and this is especially notorious when using the relative entropy selection criterion. The performance tends to slightly increase with the number of voxels selected, but it is not the case with the number of components. By looking at figures 4.6b, 4.6d and 4.6f, it seems that

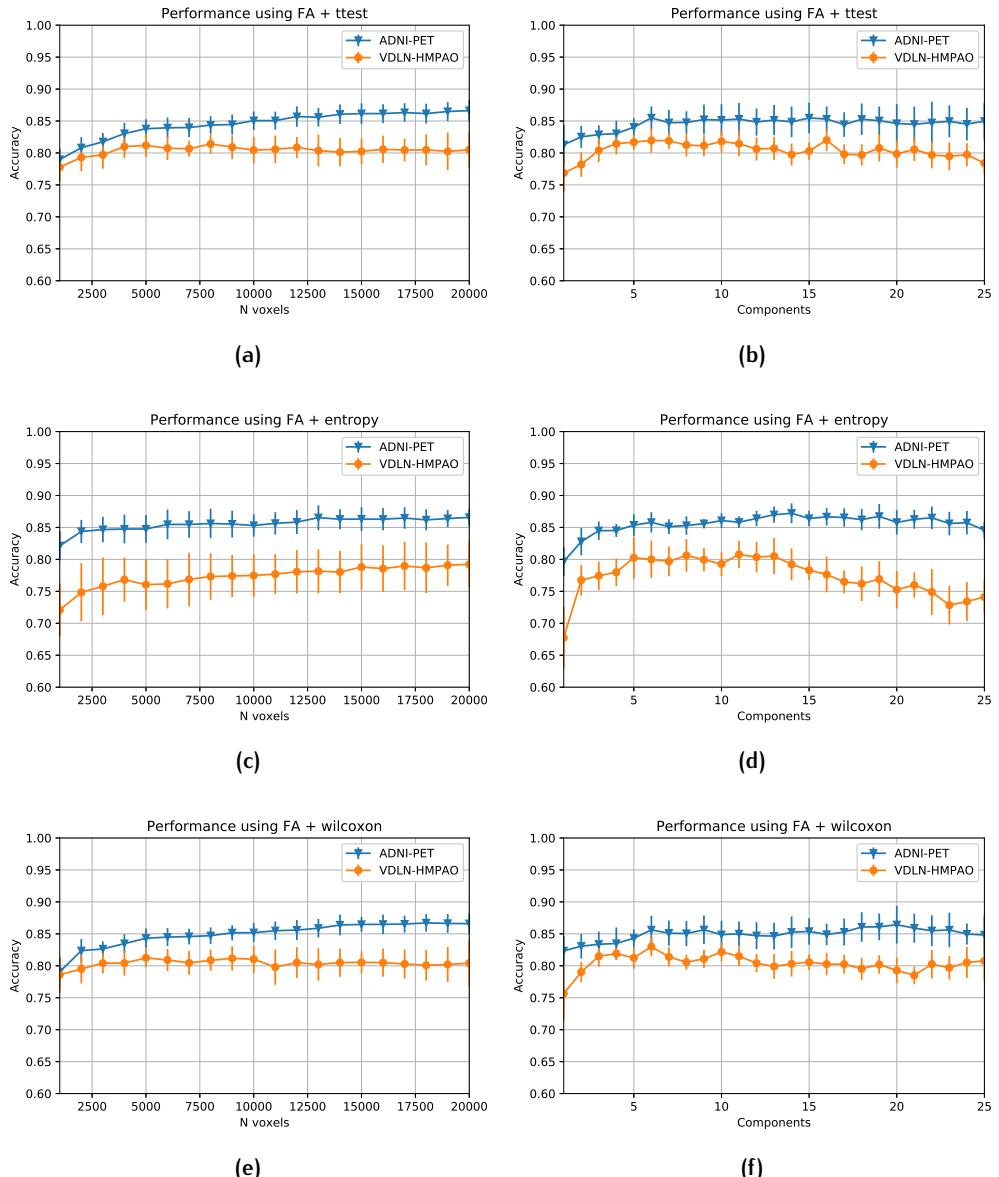


Figure 4.6: Average performance and standard deviation of the proposed system using the two AD datasets, FA and the three feature selection criteria: t-test ((a) and (b)), relative entropy ((c) and (d)) and wilcoxon ((e) and (f)).

a relatively small number of components (approximately 6) is enough to obtain good performance, and afterwards, the performance holds or even decreases.

4.3.1.2 *Independent Component Analysis*

In this section, we compute the results of applying [ICA](#) to the ADNI-PET and VDNL-HMPAO datasets. Figure 4.7 depicts the average accuracy over the number of voxels or the number of components respectively for the different selection criteria.

The case is similar to the one presented in Section 4.3.1.1, where the performance slightly improves when increasing the number of selected voxels. The performance is again better when using the ADNI-PET dataset than with the VDNL-HMPAO, although the behaviour is similar.

The results change when varying the number of components. In this case, although good performance is obtained within the first 5 components in most cases, the performance does not decrease with a higher number, and sometimes increases (for example, in the case of [ICA](#) and the t-test or the wilcoxon selection criteria).

4.3.1.3 *At the Operation Point*

Now we focus on non-averaged values, the values for which our system is optimal: the operation point.

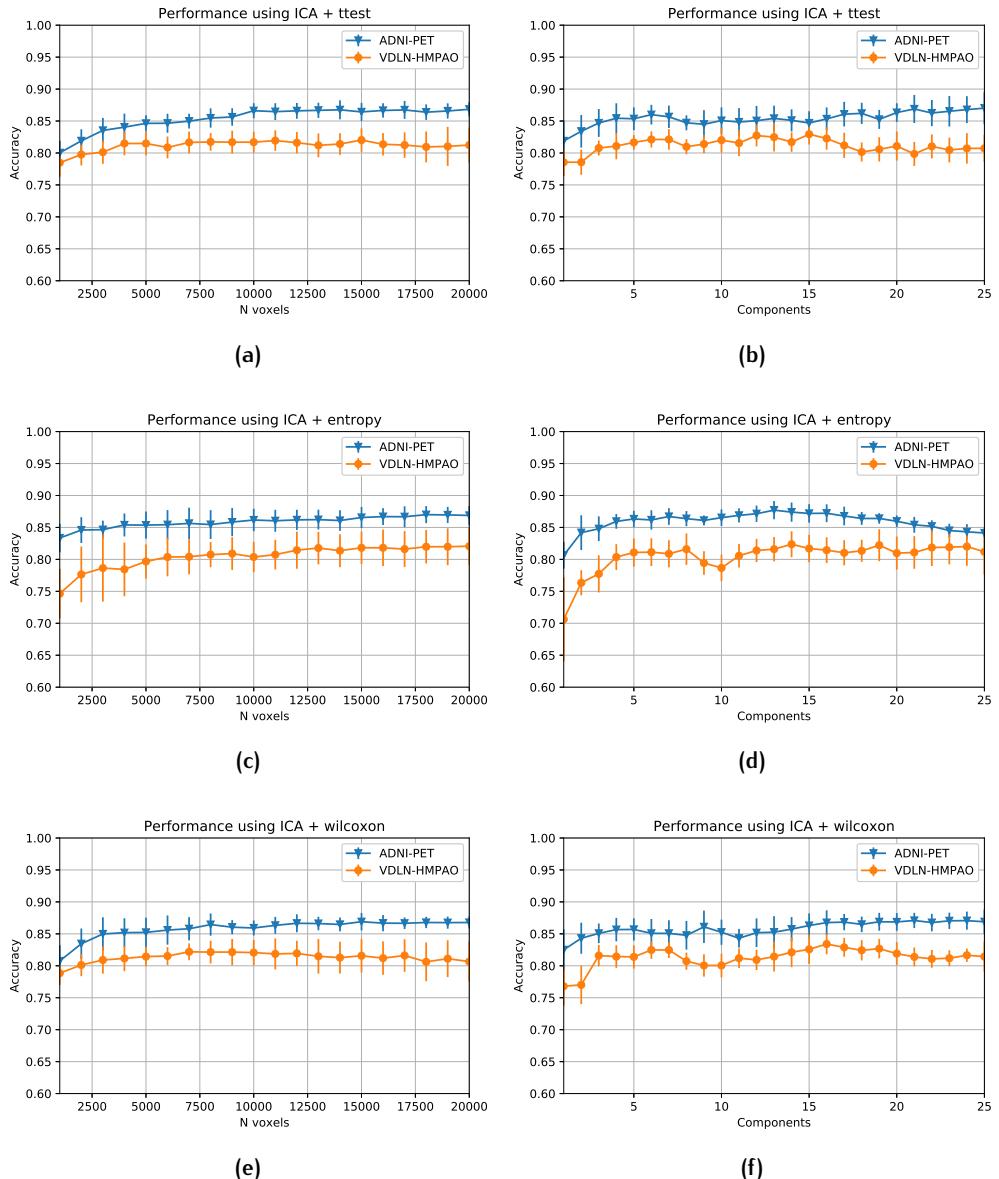


Figure 4.7: Average performance and standard deviation of the proposed system using the three AD datasets, ICA and the three feature selection criteria: t-test ((a) and (b)), relative entropy ((c) and (d)) and wilcoxon ((e) and (f)).

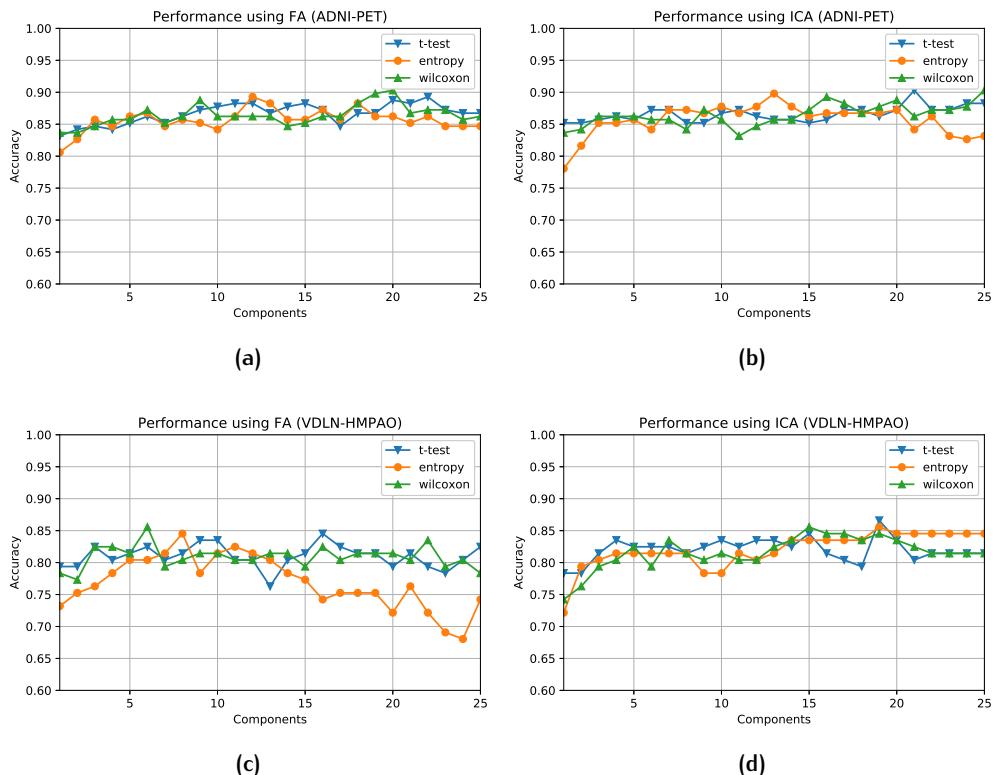


Figure 4.8: Performance of the proposed system using the two [AD](#) datasets: ADNI-PET and VDLN-HMPAO at the operation point, and how they vary over the number of components used in the decomposition.

DB	DEC.	CRITERION	ACCURACY	SENSITIVITY	SPECIFICITY
ADNI-PET	FA	t-test	0.893 ± 0.074	0.886 ± 0.119	0.901 ± 0.101
		entropy	0.893 ± 0.074	0.894 ± 0.092	0.891 ± 0.088
		wilcoxon	0.903 ± 0.066	0.917 ± 0.079	0.891 ± 0.082
	ICA	t-test	0.903 ± 0.071	0.893 ± 0.100	0.910 ± 0.107
		entropy	0.898 ± 0.059	0.917 ± 0.088	0.881 ± 0.084
		wilcoxon	0.903 ± 0.066	0.906 ± 0.097	0.901 ± 0.094
VDLN-HMPAO	FA	t-test	0.845 ± 0.102	0.843 ± 0.158	0.855 ± 0.140
		entropy	0.845 ± 0.052	0.893 ± 0.146	0.785 ± 0.163
		wilcoxon	0.856 ± 0.084	0.857 ± 0.137	0.855 ± 0.151
	ICA	t-test	0.866 ± 0.085	0.873 ± 0.144	0.855 ± 0.154
		entropy	0.856 ± 0.101	0.853 ± 0.138	0.855 ± 0.151
		wilcoxon	0.856 ± 0.086	0.857 ± 0.156	0.855 ± 0.160

Table 4.1: Accuracy, sensitivity, specificity, and their standard deviation at the operation point for each method and its corresponding feature selection criterion, using two AD datasets.

4.3.2 Parkinson's Disease

4.3.2.1 Factor Analysis

4.3.2.2 Independent Component Analysis

4.3.2.3 At the Operation Point

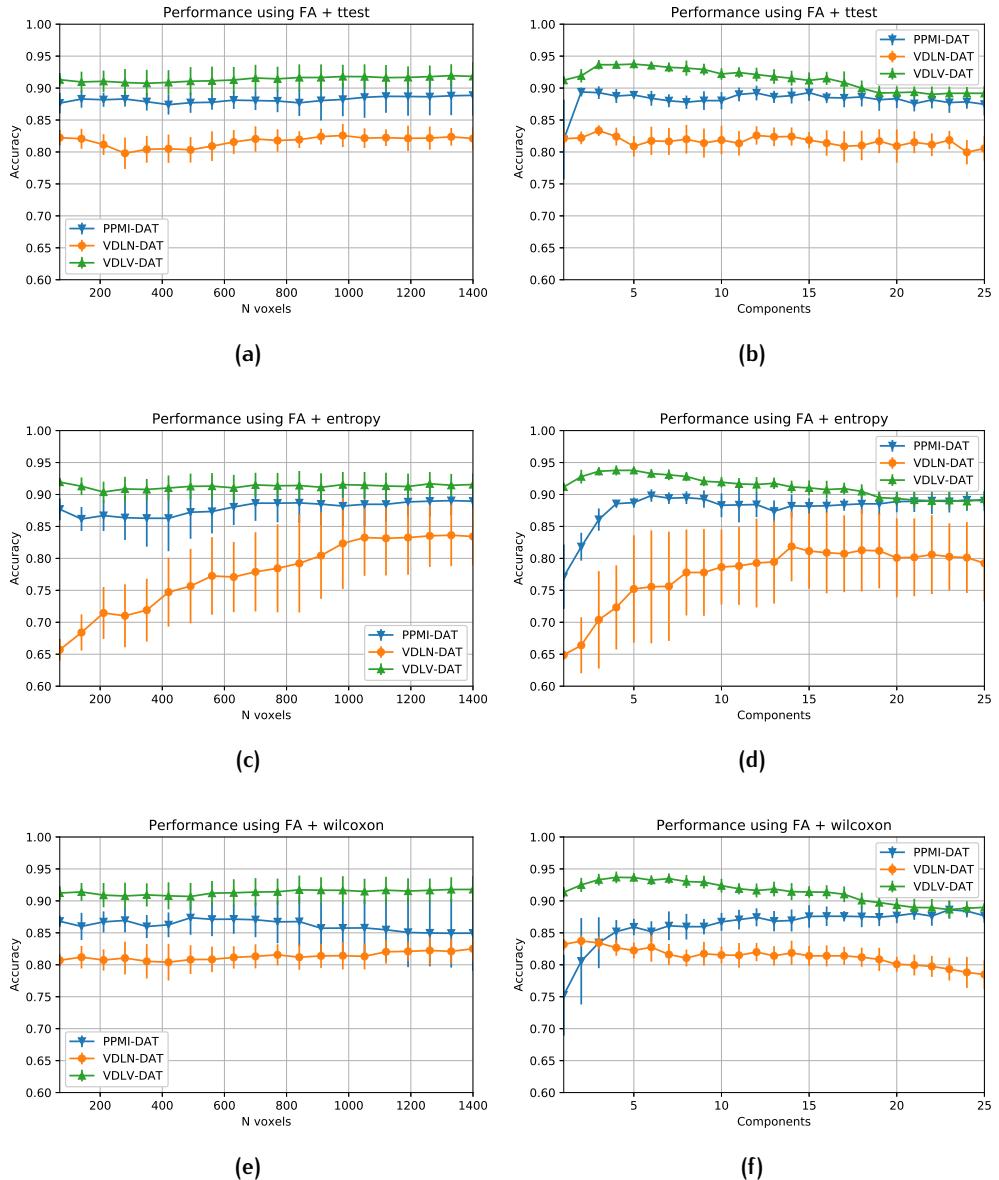


Figure 4.9: Average performance and standard deviation of the proposed system using the three PKS datasets, FA and the three feature selection criteria: t-test ((a) and (b)), relative entropy ((c) and (d)) and wilcoxon ((e) and (f)).

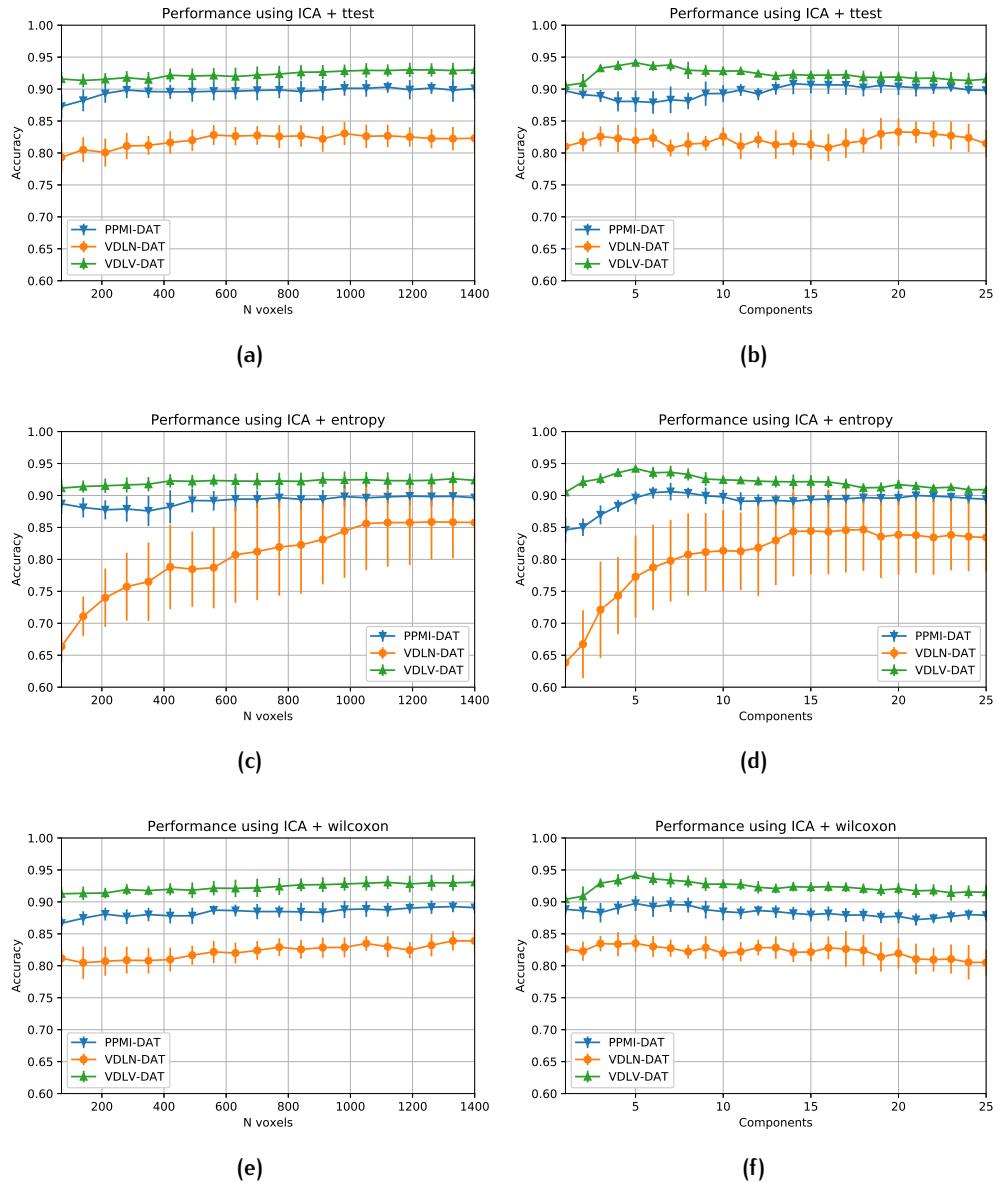


Figure 4.10: Average performance and standard deviation of the proposed system using the three PKS datasets, ICA and the three feature selection criteria: t-test ((a) and (b)), relative entropy ((c) and (d)) and wilcoxon ((e) and (f)).

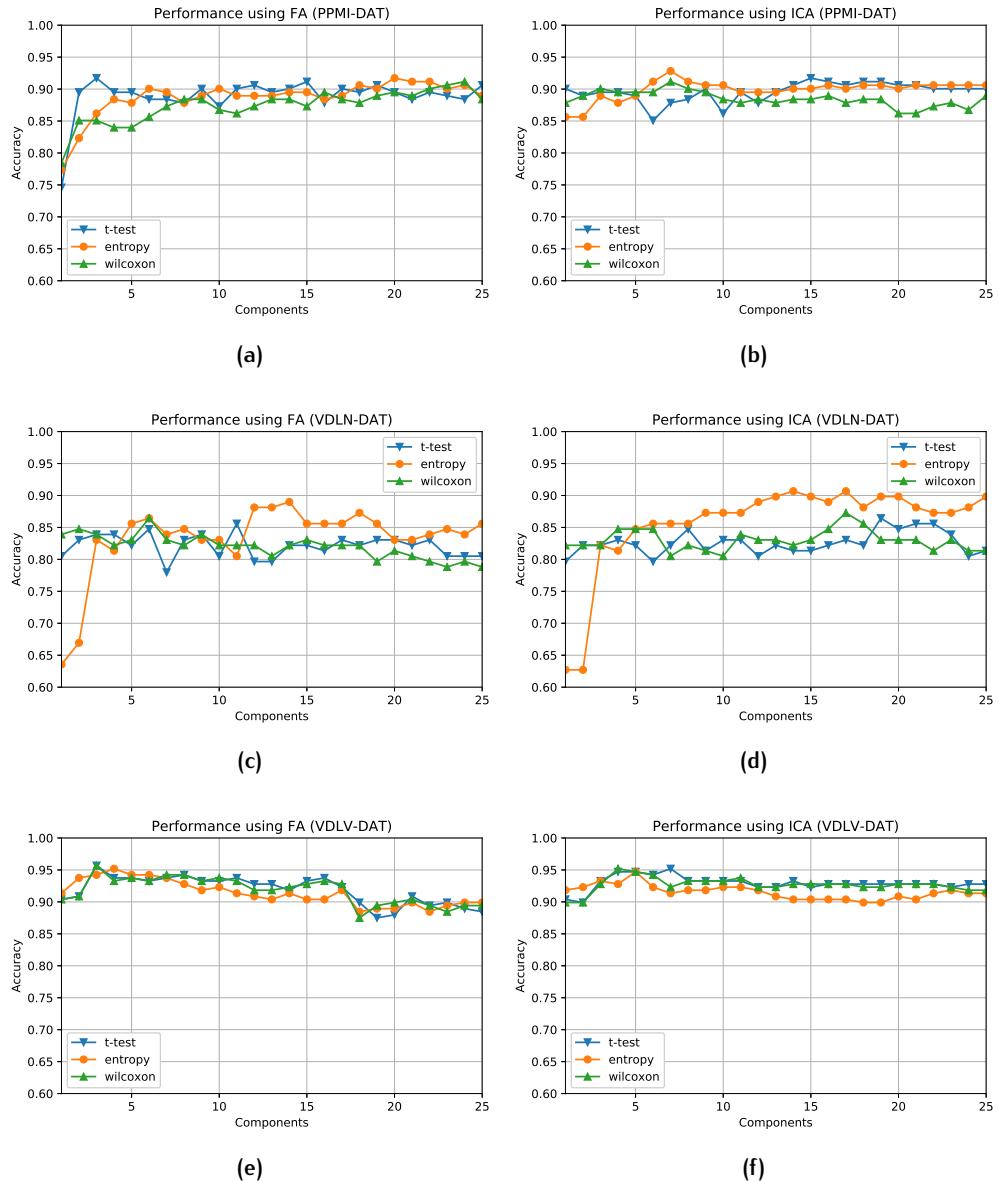


Figure 4.11: Performance of the proposed system using the two PKS datasets: PPMI-DAT, VDLN-DAT and VDLV-DAT at the operation point, and how they vary over the number of components used in the decomposition.

DB	DEC.	CRITERION	ACCURACY	SENSITIVITY	SPECIFICITY
VDLN-DAT	FA	t-test	0.856 ± 0.111	0.887 ± 0.178	0.795 ± 0.164
		entropy	0.890 ± 0.098	0.875 ± 0.118	0.910 ± 0.116
		wilcoxon	0.864 ± 0.070	0.916 ± 0.114	0.780 ± 0.183
	ICA	t-test	0.864 ± 0.101	0.873 ± 0.174	0.840 ± 0.166
		entropy	0.907 ± 0.075	0.889 ± 0.124	0.935 ± 0.131
		wilcoxon	0.873 ± 0.108	0.859 ± 0.181	0.890 ± 0.151
VDLV-DAT	FA	t-test	0.957 ± 0.033	0.940 ± 0.066	0.973 ± 0.065
		entropy	0.952 ± 0.037	0.940 ± 0.066	0.964 ± 0.064
		wilcoxon	0.957 ± 0.033	0.940 ± 0.066	0.973 ± 0.065
	ICA	t-test	0.952 ± 0.037	0.940 ± 0.066	0.964 ± 0.064
		entropy	0.947 ± 0.045	0.940 ± 0.066	0.955 ± 0.076
		wilcoxon	0.952 ± 0.037	0.940 ± 0.066	0.964 ± 0.064
PPMI-DAT	FA	t-test	0.917 ± 0.037	0.918 ± 0.095	0.918 ± 0.091
		entropy	0.917 ± 0.060	0.918 ± 0.076	0.921 ± 0.120
		wilcoxon	0.912 ± 0.056	0.927 ± 0.098	0.889 ± 0.102
	ICA	t-test	0.917 ± 0.056	0.900 ± 0.095	0.948 ± 0.109
		entropy	0.928 ± 0.055	0.909 ± 0.091	0.961 ± 0.090
		wilcoxon	0.912 ± 0.070	0.909 ± 0.100	0.920 ± 0.118

Table 4.2: Accuracy, sensitivity, specificity, and their standard deviation at the operation point for each method and its corresponding feature selection criterion, using three PKS datasets

4.4 DISCUSSION

5

TEXTURE FEATURES

5.1 INTRODUCTION

Texture analysis is any procedure intended to classify and quantify the spatial variation of voxel intensity throughout the image. In neuroimaging, they are more commonly used to classify images or to segment them (which can be also considered a form of classification). Depending on the number of variables studies, we can divide the methodology into first, second and higher order statistical texture analysis. In first-order statistics, only voxel intensity values are considered, and values such as average, variance, kurtosis or frequency (histogram) of intensity values are computed.

Second-order statistical texture analysis is by far the most popular form. It is based on the probability of finding a pair of grey-level at a certain distance and orientation on a certain image. Most algorithms are based in the Grey Level Co-occurrence Matrix (GLCM), in which is known as Haralick Texture Analysis [59]. The GLCM can be thought of as a representation of the frequency of the adjacent pixel variations in a certain spatial direction and distance. For a three dimensional brain image and two different grey levels and , at a given offset the co-occurrence value is defined as:

[Martinez-Murcia2013²⁶⁶, Martinez-Murcia2014]

5.2 HARALICK TEXTURE FEATURES

5.2.0.1 *Gray Level Co-occurrence Matrix*

A co-occurrence matrix is a square matrix that counts the number of repetitions of co-occurring values over an image, given an offset that measures the distance from the central to the neighbor voxels. In this work we use a modification of the widely used co-occurrence matrix defined in [Philips2008], that generalizes this matrix to a three-dimensional space. For two different gray levels i

and j , the co-occurrence matrix \mathbf{C} is defined over a $n \times m \times k$ three-dimensional image \mathbf{I} , given an offset Δ as follows:

$$\mathbf{C}_\Delta(i, j) = \sum_{p=(1,1,1)}^{(n,m,k)} \begin{cases} 1, & \text{if } \mathbf{I}(p) = i \text{ and } \mathbf{I}(p + \Delta) = j \\ 0, & \text{otherwise} \end{cases} \quad (5.1)$$

where i and j are different gray levels, $\mathbf{p} = (x, y, z)$ is the spatial position where $x = 1 \dots n, y = 1 \dots m, z = 1 \dots k$ and $\Delta = (d, d, d)$ is the offset vector, which has been set as dependent only on the distance d . In this work, the 3D-GLC matrix is computed over a selected subimage, computed as defined in Sec. ??, and we have performed a rescaling of the original data into 16 gray levels, leading to a best-performance.

5.2.0.2 Haralick Texture Features

After computing the 3D-GLC matrix, twelve Haralick Texture Features [Haralick73] are derived, using the following expressions:

Energy	$\sum_{i,j} \{\mathbf{C}_\Delta(i,j)\}^2$
Entropy	$-\sum_{i,j} \mathbf{C}_\Delta(i,j) * \log(\mathbf{C}_\Delta(i,j))$
Correlation	$\sum_{i,j} \frac{(i - \mu_i)(j - \mu_j) \mathbf{C}_\Delta(i,j)}{\sigma_i \sigma_j}$
Contrast	$\sum_{i,j} i - j ^2 \mathbf{C}_\Delta(i,j)$
Variance	$\sum_{i,j} (i - \mu_i)^2 \mathbf{C}_\Delta(i,j) + (j - \mu_j)^2 \mathbf{C}_\Delta(i,j)$
Sum Mean	$\frac{1}{2} \sum_{i,j} (i \mathbf{C}_\Delta(i,j) + j \mathbf{C}_\Delta(i,j))$
Inertia	$\sum_{i,j} (i - j)^2 * \mathbf{C}_\Delta(i,j)$
Cluster Shade	$\sum_{i,j} (i + j - \mu_x - \mu_y)^3 * \mathbf{C}_\Delta(i,j)$
Cluster Tendency	$\sum_{i,j} \{i + j - \mu_x - \mu_y\}^4 * \mathbf{C}_\Delta(i,j)$
Homogeneity	$\sum_{i,j} \frac{\mathbf{C}_\Delta(i,j)}{1+ i-j }$
Max Probability	$\max(\mathbf{C}_\Delta)$
Inverse Variance	$\sum_{i,j} \frac{\mathbf{C}_\Delta(i,j)}{(i-j)^2}$

Thirteen spatial directions are considered to compute the GLC matrix (see [Philips2008]). Furthermore, these GLC matrices can be calculated taking into account all the voxels that are at a distance d from the central voxel, so we

have calculated one matrix for each distance $d = 1 \dots 10$. Therefore, one GLC matrix in each of the 13 spatial directions and each value of d makes $13 \times 10 = 130$ matrices for each image. From each one, 12 Haralick texture features are computed, so we obtain a feature vector \mathbf{x}_0 that contains 1560 Haralick features that characterizes each image. A lower number of features is desirable in the clustering task, so in the next section, some measures are described to reduce the dimension of \mathbf{x}_0 by selecting the most discriminant features.

5.3 RESULTS

5.3.1 Experiment 1

In experiment 1, the influence and effect of using each of the twelve Haralick texture features on the results is tested, as in [martinez2013texture]. GLC matrices are computed at a distance d ranging from 1 to 10 voxels from the central one. As discussed later in this Section, the optimum sub-volume will have a smaller size than $40 \times 40 \times 50$, and so, the maximum value of $d = 10$ correspond to at least 20% of the brain sub-volume selected at that point; lower frequency textural changes can be neglected for diagnosis. Furthermore, as the voxel size of all databases is approximately $2 \times 2 \times 2$ mm, the maximum textural changes are computed within a $20 \times 20 \times 20$ mm area. This is approximately half the size of the striatum, which is enough to correctly extract the textural features of the area.

The computed GLC matrices are used to extract the Haralick Texture Features in two different ways: a "single approach", which only considers one type of feature using only the matrices at a distance d from the central voxel -and using all the spatial directions- and a "cumulative approach" which considers one type of feature too, but this time using all matrices in distances ranging from 1 to d .

As commented before, we will use a normalization to the maximum strategy to normalize all the images in our three databases. After this, the 3D-GLC matrices will be computed over the selected subvolumes, given an intensity threshold I_{th} (see Sec. ??). The optimum value of I_{th} should be high enough to avoid introducing background voxels in the subvolume selected, yet adequately low to select the biggest subvolume containing only brain voxels. This should lead to the best performance, since the 3D GLC matrices (and the Haralick Texture Features) would have enough information, and would not include non-brain textural patterns.

To better illustrate the influence of the subvolume selection process, Figure 5.1 depicts the average accuracy values for the two aforementioned approaches:

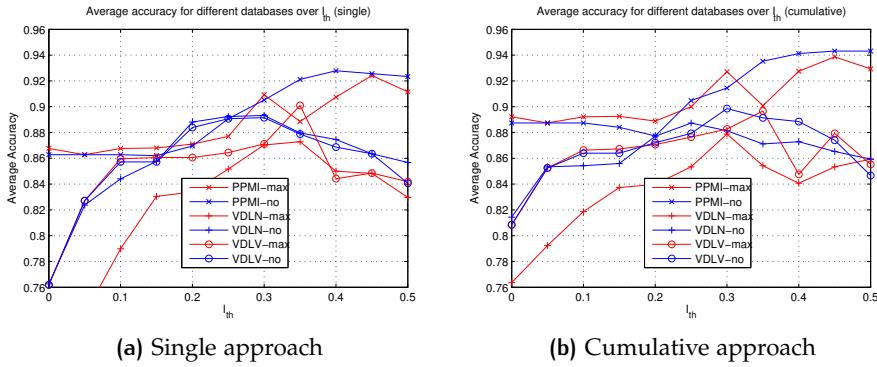


Figure 5.1: Average accuracy values obtained for (a) the single approach and (b) the cumulative approach.

Single (only the textural features computed at a distance d) and Cumulative (all textural features computed in the 1 to d distance range). In both cases, 10 values of d , 13 directions and 12 features have been averaged to obtain the accuracy for each value of the intensity threshold I_{th} .

The effect of removing the background is clearly shown in these pictures, obtaining best results when using a $I_{th} > 0.30 \times I_{max}$ and then increasing the accuracy. Furthermore, the effect of the normalization is also clear in these two images. It is possible to notice that, when using no-normalized databases (those noted with a “-no” suffix) there are wide ranges of I_{th} values in which similar performance values are obtained, while when using the normalized images, there are obvious peaks of accuracy around some values (usually, 0.30 or 0.35). Having applied no normalization to the images, the average image I_{mean} , from which the subvolume coordinates are extracted, is highly affected by the difference of intensities of different anatomical areas. Therefore, the subvolume computed will not be optimum, and for every value of I_{th} there will be an set of samples in which the texture features will be correctly computed, and another set in which those will be poorly extracted.

The behaviour of each of the Haralick texture features can also be analysed using a box plot (see Section 3.3), to show both numerical accuracy values and the properties (robustness, parameter independence) of using each one. Figure 5.2 depicts all 130 accuracy results of the “single approach” for each feature extracted from a subimage that uses $I_{th} = 0.30 \times I_{max}$ at each distance d (ranging from 1 to 10) in each of the 13 spatial directions. In this case, we can observe that best performance is obtained with the Cluster Tendency in all databases. Good values are also achieved using Homogeneity, Contrast and Correlation. This

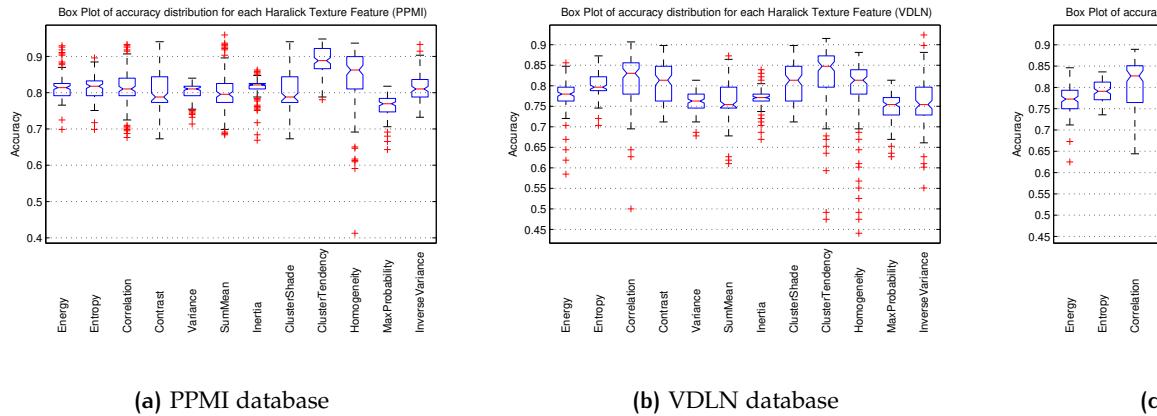


Figure 5.2: Box plot of all 130 accuracy values computed for each feature, using the “single approach”, at 10 distances d (ranging from 1 to 10) and 13 spatial directions, for (a) PPMI database, (c) VDLV database and (b) VDLN database. The red marks represent the outliers.

behaviour is consistent along all three databases, which allow us to propose Cluster Tendency as the best feature to characterize PD patterns.

Finally, to characterize the ability of this single-feature approach, we show its performance at the defined operation point (Using an $I_{th} > 0.30 \times I_{max}$ and a value of $4 < d < 8$) in Table 5.1.

Database - approach	I _{th}	d	Feature	Acc	Sens	Spec	PL	NL
PPMI - cumulates	30	8	Cluster Tendency	0.952	0.973	0.937	15.37	0.029
PPMI - distances	30	6	Cluster Tendency	0.952	0.964	0.943	16.92	0.038
VDLN - cumulates	30	6	Cluster Tendency	0.906	0.911	0.904	9.50	0.098
VDLN - distances	30	6	Cluster Tendency	0.907	0.911	0.904	9.50	0.098
VDLV - cumulates	35	7	Cluster Tendency	0.899	0.879	0.920	10.99	0.130
VDLV - distances	35	6	Cluster Tendency	0.923	0.907	0.940	15.12	0.098

Table 5.1: Accuracy values obtained at the operation point, using Cluster Tendency as a feature. The I_{th} used to compute the GLC matrix is also displayed.

Obviously, the best approach is the cumulative one, given that it contains a bigger amount of information, and thus, describing in a more accurate way the different images. Note that for the cumulative approach, all values of Cluster Tendency computed between $d = 1$ and d are used, while for the single approach, only values of Cluster Tendency at d are considered. However, the

single approach also performs relatively well, proving the value of the Haralick texture features to characterize DaTSCAN images.

Results are particularly good in every case when $I_{th} > 0.30 \times I_{max}$, a phenomenon that was previously shown in PPMI database (see Fig. 5.1), but that also extends here to all other databases.

5.3.2 Experiment 2

For experiment 2, all features computed in the aforementioned experiments (the 13 direction vectors and 10 distances used to compute the 3D cooccurrence matrix, and the 12 Haralick texture features extracted from these matrices) are used as an input to the classifier. But, in order to reduce dimensionality, we use the measures of discrimination ability proposed in Section ?? to rank these features in a descending order of ability in distinguish PD patterns from normal controls, selecting the first N.

Firstly, the impact of our volume selection threshold I_{th} (see Sec. ??) on the quality of the resulting Haralick Features, and thus, the accuracy of the experiment, will be analysed. As commented before, best results should be obtained when taking into account the biggest volume of the brain containing only brain voxels, and thus, eliminating the background. Regarding all databases, we obtain Figure 5.3, in which average values of accuracy for every value of I_{th} are plotted. These average values are computed in a similar way to Fig. 5.1, by averaging all 50 accuracy values that correspond to each value of I_{th} . The accuracy values are obtained using each of the 5 proposed selection methods, and using each value of percentage of features selected (ranging from 1 to 100%, by steps of 10%, of the total amount of 1560 features).

For two out of three databases there is a clear maximum in accuracy for an $I_{th} = 0.30 \times I_{max}$, while the remaining one obtain similar results along a wide range of I_{th} . Furthermore, best average values are obtained using the normalized database, although the PPMI case is slightly different, due to the attenuation correction preprocessing. In Fig. ?? the resulting subimage of applying this threshold was shown, to provide a better understanding of how the textural features are better defined in this. As suggested before, all no-brain voxels are removed from this images, all the textural features correspond only to the internal brain textural changes, and thus, to the textural patterns of the disease, leading to a better performance.

As results suggest, the use of our volume selection strategy with a intensity threshold between 0.25 and 0.45 is profitable in all cases. Also, the use of intensity normalized images, using the normalization to the maximum algorithm

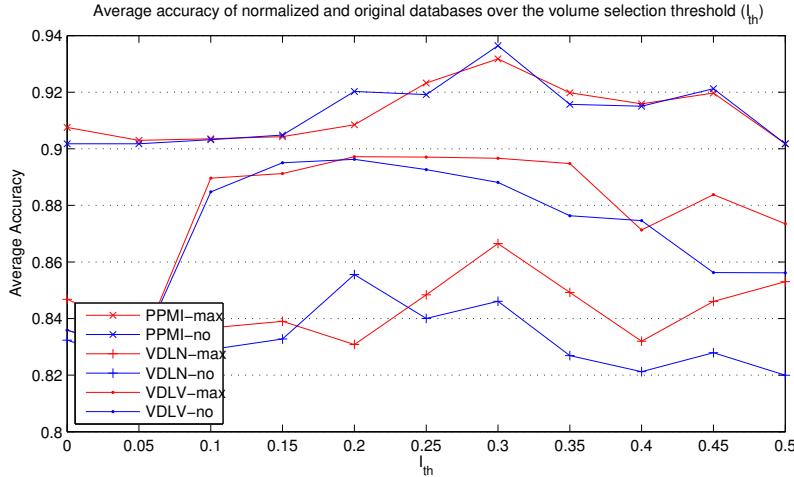


Figure 5.3: Accuracy obtained by averaging all accuracy values using a given volume selection threshold I_{th}

has also a good impact on the performance of the system. In this context, Fig. 5.4 analyses the behavior of our system using each of the discrimination-based ranking methods. On these three figures, the values of the computed average accuracy (using the values for intensity thresholds of 0.10 to 0.45) are plotted over the percentage of selected features (previously ranked from the most to the least discriminant, following different criteria) using the three databases.

Some conclusions about the amount of features that each of our discrimination-ranking methods need can be extracted from these average accuracy graphs. Methods that obtain their maximum accuracy using less than 50% of the features can be considered of great help, as they perform a significant feature reduction. Therefore, methods like Mann-Whitney-Wilcoxon (MWW) can no longer be considered, as it needs more than 50% of features to obtain good results. The opposite behaviour is given by Bhattacharyya Distance (BD) and Relative Entropy (RE), that obtain their maximum average accuracy using the first 10% of features. Fisher's Discriminant Ratio (FDR) and t-Test need a higher amount, but less than 50%.

The aforementioned behaviour correspond to an average behaviour in accuracy. To take a deeper look at the different evaluation parameters and different selection criteria, peak results obtained with each selection criteria are shown on Table 5.2.

This table confirms that the Mann-Whitney-Wilcoxon method can be no longer considered, as it needs more than a 50% of the features to obtain poorer results than all others. Regarding the remaining methods, we observe that those that

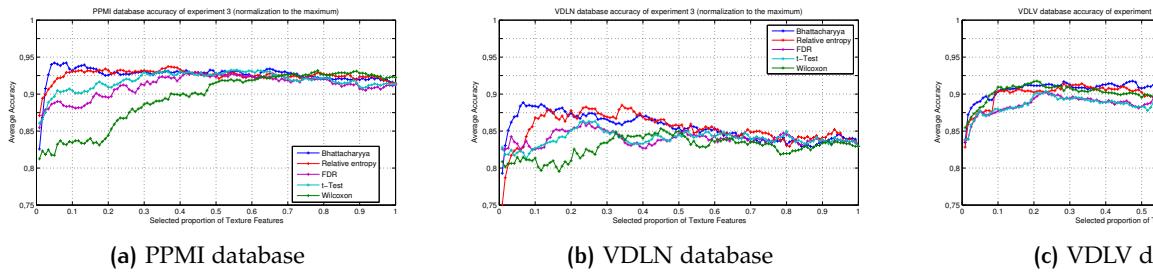


Figure 5.4: Average accuracy computed for each selection criteria, using all accuracy values for intensity thresholds of 0.10 to 0.45. These values are plotted over N , the number of features selected using some of the ranking criteria defined in Sec. ?? (where N ranges from 1% and 100% of the 1560 total Haralick features calculated). These values correspond to the images of the (a) PPMI database, (c) VDLV database and (b) VDLN database (experiment 2).

Database	Criterium	Accuracy	Sensitivity	Specificity	PL	NL	%
PPMI	Bhattacharyya	0.967	0.973	0.962	25.62	0.028	49.1
	Relative Entropy	0.967	0.982	0.956	22.16	0.019	30.8
	FDR	0.974	0.991	0.962	26.10	0.009	34.2
	t-Test	0.974	0.991	0.962	26.10	0.009	35.8
	Wilcoxon	0.959	0.955	0.962	25.15	0.047	85.8
VDLN	Bhattacharyya	0.924	0.956	0.904	9.97	0.049	10.0
	Relative Entropy	0.924	0.933	0.918	11.36	0.073	20.0
	FDR	0.941	0.933	0.945	17.03	0.071	16.7
	t-Test	0.932	0.933	0.932	13.63	0.072	22.5
	Wilcoxon	0.898	0.889	0.904	9.27	0.123	3.3
VDLV	Bhattacharyya	0.938	0.935	0.940	15.59	0.069	40.8
	Relative Entropy	0.933	0.935	0.930	13.36	0.070	45.8
	FDR	0.928	0.963	0.890	8.75	0.042	30.8
	t-Test	0.933	0.935	0.930	13.36	0.070	34.2
	Wilcoxon	0.928	0.926	0.930	13.23	0.080	18.3

Table 5.2: Best results obtained in experiment 2, using three databases, in terms of its accuracy, sensitivity, specificity, Positive Likelihood and Negative Likelihood. The amount of features used to achieve these results is shown as a percentage of the total number of features (1560). Values obtained by leave-one-out.

needed a lower amount of features (BD and RE) obtain here lower values of accuracy than those that needed a higher amount (t-Test and FDR). So, the choice of the best method is, in this case, a matter of trade-off between the computer performance (the number of features to estimate) and the accuracy needed. As in clinical practice, accuracy (and PL) is the parameter that needs to be maximized, we can conclude that FDR and t-Test are the best discrimination-ranking methods to use in this task, although all other methods reveal the ability of our system in the PD detection with an relevant performance (over 90% of accuracy in most cases).

For comparison purposes, we have established a baseline method proposed in Illan et al [Illan2012], where a Voxels-as-Features (VAF) approach with SVM linear, using different normalization strategies were tested. Two additional methods have been compared with our proposed system in order to check the performance versus state-of-the-art algorithms. These methods have been an asymmetrical Single Value Decomposition (SVD) [Segovia2012] that applied SVD on both sides of the brain (since PD often appears only in one hemisphere), and a Empirical Mode Decomposition (EMD) [Rojas2012] using different Independent Mode Functions (IMF), particularly the IMF-3. Table 5.3 compares all the aforementioned methods.

System	Acc	Sens	Spec	PL	NL
Homogeneity	0.959	0.973	0.949	19.22	0.028
Cluster Shade	0.955	0.964	0.949	19.01	0.038
Cluster Tendency	0.955	0.973	0.943	17.10	0.029
Correlation	0.941	0.946	0.937	14.92	0.058
Energy	0.937	0.964	0.918	11.73	0.039
Entropy	0.967	0.982	0.956	22.16	0.019
FDR	0.974	0.991	0.962	26.10	0.009
t-Test	0.974	0.991	0.962	26.10	0.009
VAF	0.840	0.807	0.862	5.88	0.224
VAF-IN	0.913	0.890	0.932	13.08	0.118
SVD	0.940	0.962	0.918	11.73	0.041
EMD-IMF3	0.950	0.951	0.948	18.28	0.051

Table 5.3: Comparison of our proposed system (using different texture features) and some other methods in the bibliography: VAF system using the intensity-normalized images, a combination of intensity normalization strategies and classifiers (VAF-IN) [Illan2012], a SVD-based approach [Segovia2012] and EMD using the third independent mode function (IMF3) [Rojas2012].

In Table 5.3, performance values at the operation point are shown for Experiment 1 (using five texture features: Homogeneity, Cluster shade, Cluster tendency, Correlation and energy) and for Experiment 2 (using Relative Entropy, FDR or Student's t-Test). These values are compared with the VAF, SVD and EMD approaches previously cited. We can observe that the performance values obtained with Experiment 1 are very similar to other state-of-the-art methods, like the proposed in [Segovia2012, Rojas2012], whereas the methodology used in Experiment 2 outperform all previously used methods. Particularly, as we previously mentioned, the use of either FDR or t-Test to select the most discriminant features gives us results over a PL of 26 and sensitivity over 99%, which proves the ability of some Haralick Textures, and the combination of them, in characterizing the different Parkinson's Disease patterns, and the robustness of the proposed methods.

5.4 DISCUSSION

6

SPHERICAL BRAIN MAPPING

6.1 INTRODUCTION

The main aim of this work is to provide a new framework that allows the mapping of a 3D brain image to a two-dimensional space by means of some statistical measures [Martinez-Murcia2015]. The system is based on a conversion from 3D spherical to 2D rectangular coordinates. For each spherical coordinate pair (θ, φ) , a vector containing all voxels in the radius is selected, and a number of values are computed, including statistical values (average, entropy, kurtosis) and morphological values (tissue thickness, distance to the central point, number of non-zero blocks). These values conform a two-dimensional image that can be computationally or even visually analysed. In this paper, we proceed using a statistical mask computed using a two-sample t-Test, and a SVM classifier. We have achieved performance results that match those obtained with a priori information but using a complete automated procedure, which furthermore reduces the dimensionality of the original images from more than two million voxels to hardly tens of thousands pixels. These maps can be successfully used by itself, but also allow further processing by using them combined and applying some univariate or multivariate algorithms for dimensional reduction.

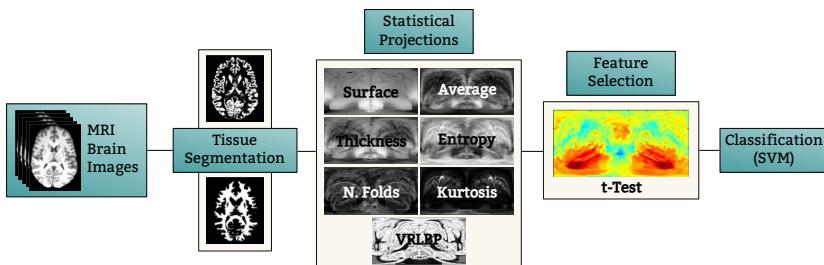


Figure 6.1: Flow diagram of the procedure used in the textural analysis of projected MR brain images.

In [Martinez-Murcia2015] a new framework called Spherical Brain Mapping ([SBM](#)) was proposed. It was intended to perform feature extraction in MR Brain Images by reducing the whole images to bidimensional maps representing a

number a number of statistical and morphological measures. Each pixel in the maps was the result of computing a certain measure on a set of voxels crossed by the mapping vector, a rectilinear vector centred at the Anterior Commissure ([AC](#)) and spanning over all values of azimuth and elevation angles. The bidimensional maps were related to anatomical changes such as brain atrophy or cortical thickness, yielding high performance in differential diagnosis. Furthermore, they provided a significant feature reduction, as well as a visual representation of the underlying information. An extension to the framework using texture and volumetric Local Binary Patterns ([LBP](#)) [[Unay2007](#)] around the mapping vector was proposed in [[Martinez-MurciaVRLBP](#)].

In this work, we propose a new path tracing algorithm, based on Hidden Markov Models ([HMMs](#)), to enhance the mapping procedure in [SBM](#) by replacing the mapping vector with curvilinear paths that adapt to the structural information present in MRI. This allows the computation of the feature maps as well as the direct use of the intensity distribution along the path as a characterization of the structural differences in normal or [AD](#)-affected subjects. Since the Grey Level Co-occurrence Matrix ([GLCM](#)), firstly developed by Haralick [[Haralick73](#)], have been used in the characterization of brain in numerous works [[martinez2014parametrization](#), [sikio2015mr](#)], we propose a possible extension intended to characterize the brain texture along each path and its neighbourhood.

6.2 SPHERICAL BRAIN MAPPING

Original [SBM](#) [[Martinez-Murcia2014225](#), [Martinez-MurciaVRLBP](#), [Martinez-Murcia2015](#), [Martinez-Murcia2016](#)]

The technique proposed to perform the mapping of the 3D brain images to a 2D map using spherical coordinates -from now on Spherical Brain Mapping ([SBM](#))- is based on the use of spherical coordinates in the brain. A base point is set in the central voxel of the MRI image, and a mapping vector $\mathbf{v}_{\theta,\varphi}$ of length N is defined for each inclination (θ) and azimuth (φ) angles in the range $0^\circ < \theta < 180^\circ$ and $0^\circ < \varphi < 360^\circ$ (see Figure 6.2). Therefore, we will define the sampled set $V_{\theta,\varphi}$, a set that contains P voxels crossed by the sampling vector $\mathbf{v}_{\theta,\varphi}$. For each set $\mathbf{v}_{\theta,\varphi}$, a mapping value v is computed from the sampled voxels $V_{\theta,\varphi}$, depending on the measure used. In this section, six basic measures are proposed:

- A basic brain surface approach, that accounts for the distance between the central voxel and the last tissue voxel in $V_{\theta,\varphi}$ that is greater than a thresh-

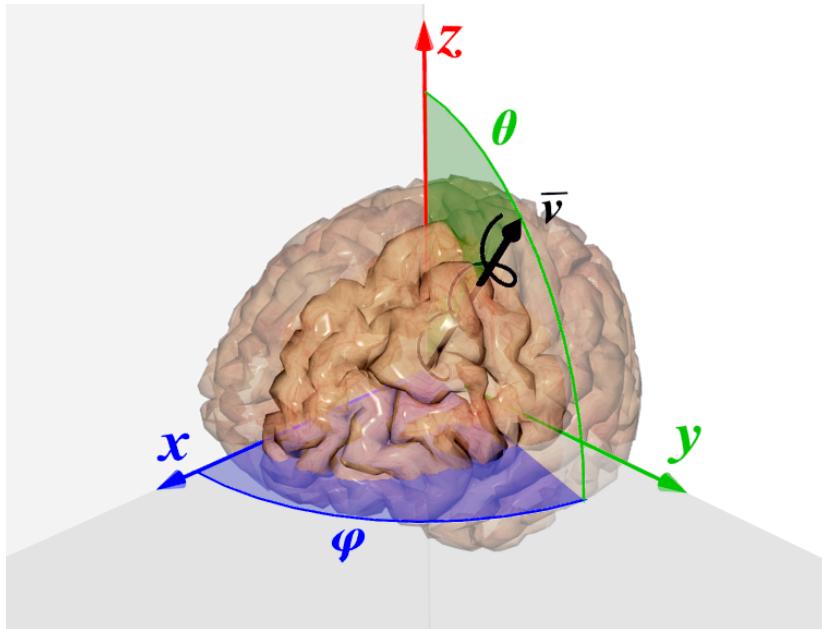


Figure 6.2: Illustration of the computation of the mapping vector $v_{\theta,\varphi}$, the angles θ and φ and the r -neighbourhood of v (see Section 6.3).

old I_{th} . This might allow our system to observe structural degeneration and tissue loss in the surface of the tissue.

$$v_{surf} = \arg \max_i \{V_{\theta,\varphi}(i) > I_{th}\} \quad \forall i = 1, \dots, P \quad (6.1)$$

- Another parameter used is thickness of the tissue. This can be useful when measuring the thickness of segmented Gray Matter or White Matter MR images. It is defined as the distance between the last and first elements in $V_{\theta,\varphi}$ with an intensity greater than a threshold I_{th} (typically 0):

$$v_{thick} = \arg \max_i \{V_{\theta,\varphi}(i) > I_{th}\} - \arg \min_i \{V_{\theta,\varphi}(i) > I_{th}\} \quad \forall i = 1, \dots, P \quad (6.2)$$

- The number of folds represents the number of overlapping segments of tissue in the set $V_{\theta,\varphi}$. It is computed by thresholding $V_{\theta,\varphi}$ using the value I_{th} and counting the number of resulting connected subsets. Let $A_{\theta,\varphi}$ be the set that contains all the indices of the voxels in $V_{\theta,\varphi}$ with an intensity greater than I_{th} :

$$A_{\theta,\varphi} = \{i / V_{\theta,\varphi}(i) > I_{th}\} \quad (6.3)$$

where $A_{\theta,\varphi} \in \mathbb{N}$. Let us divide $A_{\theta,\varphi}$ in J disjoint connected subsets so that:

$$A_{\theta,\varphi} = A_{\theta,\varphi}^1 \cup A_{\theta,\varphi}^2 \cup \dots \cup A_{\theta,\varphi}^J \quad \text{so that} \quad A_{\theta,\varphi}^i \cap A_{\theta,\varphi}^j = \emptyset \quad \forall i,j \quad (6.4)$$

Therefore, our $v_{nf} = J$, the number of disjoint connected subsets in $A_{\theta,\varphi}$.

- An average approach, where the average of all the intensity values in the set $V_{\theta,\varphi}$ is computed as:

$$v_{av} = \frac{1}{N} \sum_i V_{\theta,\varphi}(i) \quad \forall i = 1, \dots, P \quad (6.5)$$

- The entropy assumes that the set $V_{\theta,\varphi}$ is a probability mass vector (probability of belonging to a certain tissue, normalized) and computes v as:

$$v_{ent} = \sum_i V_{\theta,\varphi}(i) * \log(V_{\theta,\varphi}(i)) \quad \forall i \in \arg_i \{V_{\theta,\varphi}(i) > 0\} \quad (6.6)$$

- The uncorrected kurtosis, also known as fourth standardized moment, of the set $V_{\theta,\varphi}$ in which v is calculated using:

$$v_{kurt} = \frac{\frac{1}{N} \sum_i (V_{\theta,\varphi}(i) - \bar{V}_{\theta,\varphi}(i))^4}{\left(\frac{1}{N} \sum_i (V_{\theta,\varphi}(i) - \bar{V}_{\theta,\varphi}(i))^2 \right)^2} \quad \forall i = 1, \dots, P \quad (6.7)$$

where $\bar{V}_{\theta,\varphi}$ is the average of all voxels in $V_{\theta,\varphi}$ (same value as v_{av} , described in Eq. 6.5).

The resulting **GM** and **WM** maps are depicted in Figure 6.3.

The resulting maps will contain the value v mapped in each direction (θ, φ) . As the inclination angle θ ranges from 0° to 180° and the azimuth φ from 0° to 360° , the resulting maps will have a size of 181×361 , where each pixel is the v value for each direction. The whole algorithm can be downloaded at <http://wdb.ugr.es/~fjesusmartinez/portfolio/sbm/>.

This methodology defines the sampling set as the voxels that are crossed by the sampling vector $v_{\theta,\varphi}$. When projecting a structure as complex as the brain, this implies a loss of contextual information of both the neighbourhood and the layers crossed by $v_{\theta,\varphi}$. Two approaches have been suggested to overcome this problem: the first one extends the system by dividing the sampling set $V_{\theta,\varphi}$ in n equal parts in a so-called “Layered approach”, and the second one uses a helical sampling and Local Binary Patterns (LBP) to map the neighbourhood of the sampling vector and characterize the texture of the area.

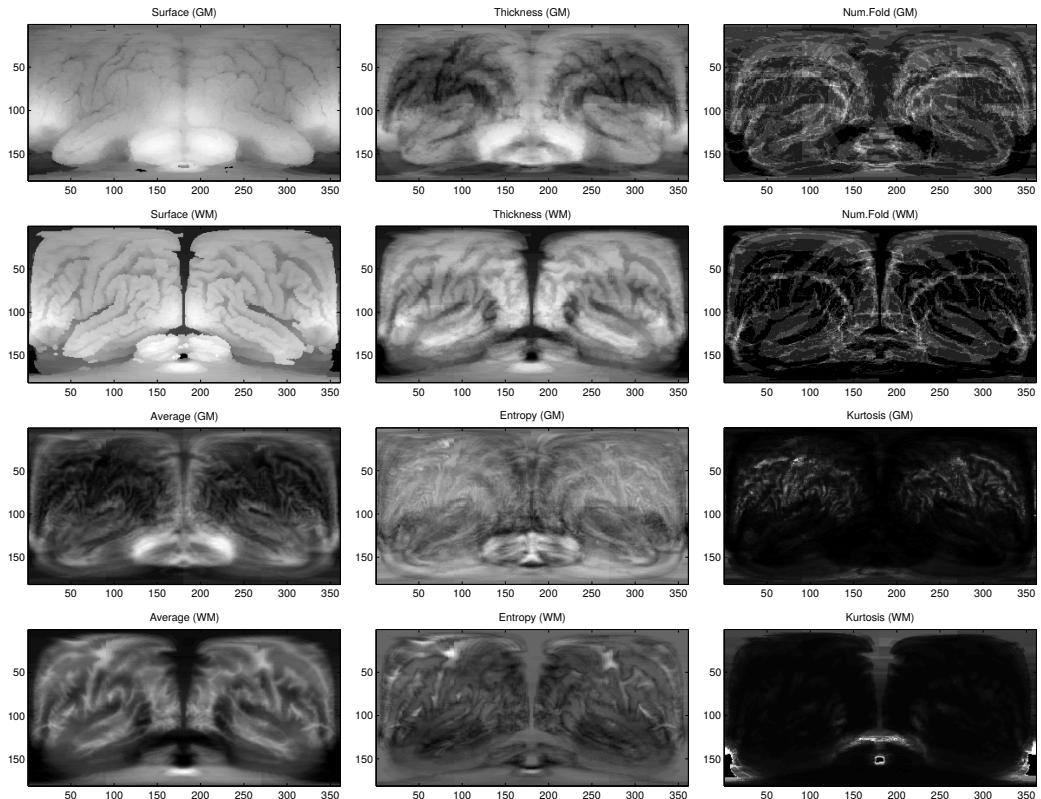


Figure 6.3: Resulting GM and WM maps of the same control subject using the six proposed measures: Surface, Thickness, Number of Folds, Average, Entropy and Kurtosis.

6.2.1 Layered Extension

The first strategy used to improve the descriptive abilities of the mappings is the Layered Extension. In this approach, the sampled voxels set $V_{\theta,\varphi}$ can be divided in n equal subsets, using each one to project one section -or layer- of the brain. If, for example, we use $n = 4$, 4 subsets containing the same number of voxels will be derived from $V_{\theta,\varphi}$, and therefore 4 different maps will be depicted, from the closest to the centre to the farthest. This layered approach reveals the different anatomical structures found in the brain at different depths, potentially revealing a more detailed distribution the features of the mappings . These layered maps and their performance will be discussed later.

6.3 VOLUMETRIC RADIAL LBP

In a second attempt to overcome the loss of per layer information, we have expanded the influence of $\mathbf{v}_{\theta,\varphi}$ to its r -neighbourhood. This has been done by defining a v measure that describes not only the features of the voxels crossed by $\mathbf{v}_{\theta,\varphi}$, but the texture of its neighbourhood using a Volumetric Radial Volumetric Radial Local Binary Pattern (VRLBP) descriptor.

Local Binary Patterns (LBP) were first introduced in [Ojala1996] to describe the texture of an image with application to face recognition. Later, in [Zhao2007], the technique was extended to a Volume LBP (VLBP), defining a 3D texture in a local neighbourhood by using a cylinder oriented in one direction and whose radius define the neighbourhood used to compute the LBP descriptor.

In the VRLBP, the sampling method devised in [Zhao2007] has been updated to follow helical coordinates around the mapping vector $\mathbf{v}_{\theta,\varphi}$ (see helix around $\mathbf{v}_{\theta,\varphi}$ in Figure 6.2). Formally, we note $V_{\theta,\varphi}^{P,r}$ the set of P sampled voxels of the image I in the r -neighbourhood of $\mathbf{v}_{\theta,\varphi}$ taken by helical sampling:

$$V_{\theta,\varphi}^{P,r} = \{I(g_{\theta,\varphi}^{0,r}), I(g_{\theta,\varphi}^{1,r}), I(g_{\theta,\varphi}^{2,r}), \dots, I(g_{\theta,\varphi}^{P-1,r})\} \quad (6.8)$$

where the coordinates $g_{\theta,\varphi}^{p,r}$ of each voxel are computed in the direction of $\mathbf{v}_{\theta,\varphi}$ by:

$$g_{\theta,\varphi}^{p,r} = \begin{cases} x_{\theta,\varphi}^{p,r} = p \sin(\varphi) \cos(\theta) - r \sin(2\pi n \frac{p}{P}) \\ y_{\theta,\varphi}^{p,r} = p \sin(\varphi) \sin(\theta) + r \cos(2\pi n \frac{p}{P}) \\ z_{\theta,\varphi}^{p,r} = p \cos(\varphi) \end{cases} \quad p = \{0, \dots, P-1\}, P \in \mathbb{N} \quad (6.9)$$

being n the number of loops in the helix. Following [Zhao2007], voxels that do not fall exactly at the coordinates computed in the equations 6.9 are estimated by interpolation.

Let us assume, without lost of generality, that P and r are fixed. This way, set of sampled voxels $V_{\theta,\varphi}^{P,r}$ becomes $V_{\theta,\varphi}$, which matches the definition of Section ???. Following this notation, the value v for this VRLBP approach is computed using:

$$v_{VRLBP} = \sum_i s(V_{\theta,\varphi}(i) - V_{\theta,\varphi}(0)) \cdot 2^i \quad \forall i = 1, \dots, P \quad (6.10)$$

where $s(x)$ is the sign function, defined as:

$$s(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases} \quad (6.11)$$

This approach provides textural information about all brain structures in a certain direction, as it can be seen in Figure 6.4.

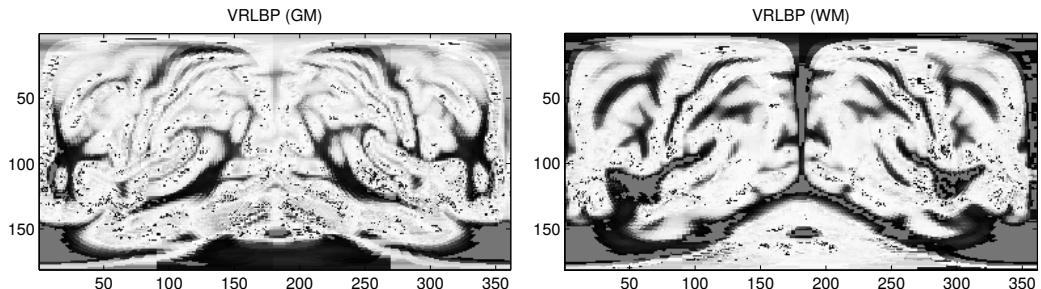


Figure 6.4: An example of the VRLBP projection for GM and WM Tissues.

```

def foo():
    hola amigo
    print('amigo')

eh = foo("amigo")

string title = "This is a Unicode $\u03c0$ in the sky"
/*
Defined as $\pi=\lim_{n\rightarrow\infty}\frac{P_n}{d}$ where $P$ is the
perimeter
of an $n$-sided regular polygon circumscribing a
circle of diameter $d$.
*/
const double pi = 3.1415926535

```

6.4 PATH VIA HIDDEN MARKOV MODELS

We have already talked about the limitations of the rectilinear mapping vector used in [Martinez-Murcia2015] and [Martinez-MurciaVRLBP]. Therefore, it would be desirable to define new mapping paths that use all the available intensity and spatial information in MRI images. This way, the resulting sets of selected voxels (and the resulting maps) would contain information about both the intensities and structure.

To define the paths, we can define 3D Biomedical Images as a tuple containing spatial information in the image range ($\mathbf{x} \in \mathbb{I}$, where $\mathbb{I} \subset \mathbb{R}^3$) as well as intensity information ($I(\mathbf{x}) \in \mathbb{R}$). There exist a number of possibilities in the interpretation of intensity data on the images, from plain intensity values to a sampling of the underlying tissue density (and thus, an estimation of the probability of finding tissue in each position).

Following our [SBM](#) approach, in which some radia are used to extract relevant statistical features from these images, we formulate a 3D path tracing algorithm suitable for extraction of curvilinear structures from 3D biomedical images, and directly linked to each direction (φ, θ) as in the original work. Our objective is to make these paths as representative of the underlying intensity distribution as possible. To do so, we must use both intensity and spatial information to construct maximum intensity-similarity paths oriented in the given direction.

Let us note a 3D path in a certain direction (φ, θ) as a Markov Model [[Chen2008](#)]:

$$\mathbf{X} = \{\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N\} \quad (6.12)$$

Therefore, our optimum path would be the one that maximizes the probability of the path:

$$\mathbf{X}_{\text{opt}} = \arg \max_{\mathbf{X}} \{P(\mathbf{X})\} \quad (6.13)$$

or, similarly, the probability of all the nodes:

$$P(\mathbf{X}) = P(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N) \quad (6.14)$$

where \mathbf{x}_0 is the node located at the AC and \mathbf{x}_N is the theoretical projection of the current direction (φ, θ) in the limits of the image \mathbb{I} . Setting \mathbf{x}_0 at the AC is not a random choice; it is a convention when using the [MNI](#) coordinates [[Evans1993](#)] and furthermore, since it is a point that shares connectivity with both hemispheres, the resulting paths will be optimal, covering most of the brain. We can assume a first-order Hidden Markov Model ([HMM](#)) for the tracing of the path, and the i -th node will be computed as:

$$P(\mathbf{x}_i | \mathbf{x}_{i-1}, \mathbf{x}_{i-2}, \dots, \mathbf{x}_0) \approx P(\mathbf{x}_i | \mathbf{x}_{i-1}) \quad (6.15)$$

and with this assumption, (6.14) becomes:

$$P(\mathbf{X}) = P(x_0, x_1, \dots, x_N) = \prod_{i=1}^N P(x_i | x_{i-1}) \quad (6.16)$$

In our case, we will assume that the hidden state of each node will be its intensity $I(x_i)$. Let $\mathbf{I} = \{I(x_0), I(x_1), \dots, I(x_N)\}$ be the vector of the intensities in each node. With the introduction of these factors, our optimal path defined in (6.13) can be viewed as:

$$\mathbf{x}_{\text{opt}} = \arg \max_{\mathbf{x}} \{P(\mathbf{X}|\mathbf{I})\} \quad (6.17)$$

$$P(\mathbf{X}|\mathbf{I}) = P(x_0, \dots, x_N | I(x_0), \dots, I(x_N)) \quad (6.18)$$

$$= \frac{P(I(x_0), \dots, I(x_N) | x_0, \dots, x_N) \cdot P(x_0, \dots, x_N)}{P(I(x_0), \dots, I(x_N))} \quad (6.19)$$

where:

$$P(I(x_0), \dots, I(x_N) | x_0, \dots, x_N) = \prod_{i=1}^N P(I(x_i) | x_i) \quad (6.20)$$

and $P(I(x_0), \dots, I(x_N))$ is the *a priori* probability of the intensities in the path. We can assume, without lack of generality, that this term is constant along the path, and therefore, it plays no part in the optimization process.

For computational purposes, we will compute all the needed probabilities on a set of candidates $\mathbf{X}_c = \{x_{c,1}, x_{c,2}, \dots, x_{c,M}\}$ defined by x_{i-1} . These candidates are contained inside the L^2 -norm support ball $B_{2,r}(x - x_{i-1})$ of radius r centred in x_{i-1} .

To estimate the individual probabilities needed for (6.20) $P(I(x_i) | x_i)$, we can assume a normal distribution of intensities of the candidates with mean $I(x_{i-1})$ and variance σ_c^2 the variance of the intensities of the candidate set. By assuming this, the probability of a certain intensity in the candidate node x_i increases as the intensity becomes more similar to the intensity of x_{i-1} . Therefore, the assumption supports the tracing of minimal intensity difference paths. The probability of the intensity of a candidate x_i will be:

$$P(I(x_i) | x_i) = \frac{1}{\sqrt{2\pi\sigma_c^2}} \exp\left(-\frac{(I(x_i) - I(x_{i-1}))^2}{2\sigma_c^2}\right) \quad (6.21)$$

The last term $P(x_0, \dots, x_N)$ to be defined in (6.19) is directly related to the SBM framework defined before, as it will depend on the radial direction (φ, θ) that we want to “force” in the path. To define this term, we will define an attractor located in the position x_N (the projection of the current direction (φ, θ) in

the limits of the image \mathbb{I}). We assume that this attractor defines the conditionality, that is, that it affects the transition probability between states by means of an isotropic Gaussian radial basis function (RBF), as defined in (6.22). This definition helps the attractor to lightly condition the direction of the path at first, and more strongly as the path approaches the cortex, leading to a better representation of the underlying structure.

$$P(x_0, \dots, x_N) = P(x_i | x_N) \quad (6.22)$$

$$= \frac{1}{\sqrt{(2\pi)^d |\Sigma|}} \exp \left(-\frac{1}{2} (x_i - x_N) \Sigma^{-1} (x_i - x_N) \right) \quad (6.23)$$

where Σ is the covariance matrix of the given distribution. As we will consider only isotropic gaussian kernel, the matrix can be considered as a 3×3 diagonal matrix whose diagonal elements are a scalar value σ^2 , which we set in each iteration to the euclidean distance between x_i and x_N .

6.4.0.1 Step Size

In this algorithm, instead of using a fixed step size, we evaluate each candidate point $x \in B_{2,r}(x - x_i)$, and thus, the only parameter to regulate is the radius of the L2-norm support ball r . As a trade-off between computational issues and definition of the defined path, we will use a value of $r = 3$ voxels, which gives approximately 200 candidate points in each iteration.

6.4.0.2 Stop Condition

Although the paths could be defined until they reach the last point x_N (remember, the projection of the general direction onto the limits of \mathbb{I}), our interest is to model the paths inside the brain, and therefore we establish a stop condition using an intensity threshold. This threshold is computed using the entropic thresholding, as defined in [Yen1995]. Let $G_m \equiv \{I_0, I_1, \dots, I_m\}$ denote the set of intensity levels of the whole image. We can compute a histogram to obtain the observed frequencies f_{I_i} , and thus, the observed probability of the different Grey levels $p_i = f_{I_i}/N$, where N is the number of voxels in our image \mathbb{I} .

For a given intensity threshold $I_{th} = I_s$, if $\sum_{i=0}^{s-1} p_i$ is larger than zero and smaller than 1, the following distributions can be derived from this distribution after normalization:

$$A \equiv \left\{ \frac{p_0}{P(I_s)}, \frac{p_1}{P(I_s)}, \dots, \frac{p_{s-1}}{P(I_s)} \right\} \quad (6.24)$$

$$B \equiv \left\{ \frac{p_s}{1 - P(I_s)}, \frac{p_{s+1}}{1 - P(I_s)}, \dots, \frac{p_m}{1 - P(I_s)} \right\} \quad (6.25)$$

where $P(I_s) = \sum_i^s p_{I_i}$ is the cumulative density function for the s -th Grey level. Therefore, we choose the threshold so that the total amount of information provided by A and B (foreground and background of the image) is maximized. The total information provided by the s -th Grey level is:

$$TE(s) = E_A(s) + E_B(s) \quad (6.26)$$

$$= - \sum_{i=0}^{s-1} \left(\frac{p_i}{P(I_s)} \right) \log \left(\frac{p_i}{P(I_s)} \right) \quad (6.27)$$

$$- \sum_{i=s}^{m-1} \left(\frac{p_i}{1 - P(I_s)} \right) \log \left(\frac{p_i}{1 - P(I_s)} \right) \quad (6.28)$$

A summary of our HMM-based path tracing method is shown in Algorithm 1. To illustrate the effect of the algorithm on a real example, Fig. 6.5 depicts the resulting set of paths computed over the standard MRI DARTEL template.

Algorithm 1: HMM-based Path Creation

input : MRI Brain Image I of size $U \times V \times W$, x_0

output: List of nodes in the optimum path X_{opt}

Compute the $I_{th} = I_s$ where s maximizes $TE(s)$;

Set x_0 to the AC;

Compute the attractor position x_N in the direction (φ, θ) ;

$x_i \leftarrow x_0$;

while ($i < IterLimit$) $\&$ ($I(x_i) > I_{th}$) $\&$ ($x_i \in \mathbb{I}$) **do**

Get the node candidates $X_c = \{x_{c,1}, x_{c,2}, \dots, x_{c,M}\}$ where

$x_{c,m} \in B_{2,r}(x_{c,m} - x_i)$;

Get the intensities of the candidates $I(x_c) \quad \forall x_c \in X_c$;

foreach $x_c \in X_c$ compute $P(x_c|x_N)$ and $P(I(x_c)|x_i)$;

$x_{i+1} = \arg \max_{x_c} [P(I(x_c)|x_i) \cdot P(x_c|x_N)]$;

$i = i + 1$;

$X_{opt} \leftarrow \{x_0, x_1, \dots, x_N\}$;

6.4.1 Radial Texture Features

The paths extracted with the aforementioned algorithm present a meaningful representation of the structure of the intensity levels (and therefore, the tissue density) of the MRI brain images. At this point, one might consider using the intensity values located in those radii as features, and try to characterize each radius' discrimination ability by means of these intensities.

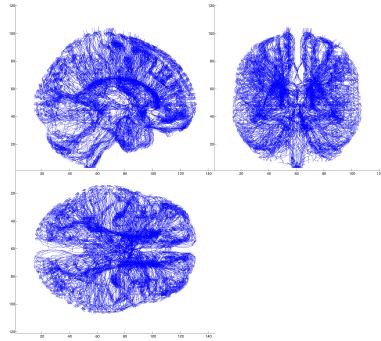


Figure 6.5: Set of [HMM](#) based paths over the MRI DARTEL template.

Conversely, the strategy proposed in [[Martinez-MurciaVRLBP](#)], which uses Local Binary Pattern (LBP) descriptors in the helical neighbourhood of a rectilinear mapping vector, might be a complementary approach. Due to the [HMM](#) paths topology, the helical sampling becomes difficult to compute and not even useful, therefore we propose a modification of the Grey Level Co-occurrence Matrix (GLCM) along the paths.

The GLCM, proposed by Haralick[[Haralick73](#)], is one of the most widely used methods in texture characterization, and it has been successfully applied to medical imaging in the past[[kovalev2001three](#), [martinez2014parametrization](#)]. It works by storing the number of voxel-wise correspondences between K grey levels with a certain position offset Δ on a $K \times K$ matrix (C_Δ) along all the image.

Our modification will compute a node-wise GLC matrix, in which the number of grey-level transitions between adjacent nodes, noted as x_i and x_{i+1} , is stored along the whole path $X = \{x_0, x_1 \dots x_N\}$. Mathematically, the computation of the GLCM in each point in the path will be:

$$C_{\Delta_i}(j, k) = \sum_{i=0}^{N-1} \begin{cases} 1 & I(x_i) = j, I(x_{i+1}) = k \\ 0 & \text{otherwise} \end{cases} \quad (6.29)$$

where the offset is different for each pair of nodes $\Delta_i = x_{i+1} - x_i$.

The definition provided in (6.29) is intended for computing the values in each node. However, we can generalize this construction to include not only the nodes, but the intensity information around each node in the computation, which could potentially lead to more significant texture features. Let us note a

set containing all the voxels in the closed neighbourhood of x_i as X_i . Therefore, (6.29) can be generalized for any voxel $x \in X_i$ as:

$$C(j, k) = \sum_{i=0}^{N-1} \sum_{x \in X_i} \begin{cases} 1 & I(x) = j, I(x + \Delta_i) = k \\ 0 & \text{otherwise} \end{cases} \quad (6.30)$$

From the GLCM, a variety of texture descriptors, or features, can be extracted. In this work we will use ten texture features proposed in the original Haralick's article[Haralick73] as well as in Refs [soh1999texture] and [clausi2002analysis]: Contrast[Haralick73], Correlation[Haralick73], Dissimilarity[soh1999texture], Energy[Haralick73], Entropy[soh1999texture], Homogeneity[Haralick73], Difference Variance[Haralick73] (D. Variance), Difference Entropy[Haralick73] (D. Entropy), Inverse Difference Normalized[clausi2002analysis] (IDN) and Inverse Difference Moment Normalized[clausi2002analysis] (IDMN). The mathematical expressions for these features are presented in Equations 6.31 to 6.40.

$$\text{Contrast} = \sum_j \sum_k \{(j - k)^2 C(j, k)\} \quad (6.31)$$

$$\text{Correlation} = \frac{\sum_j \sum_k (j - \mu_j)(k - \mu_k) C(j, k)}{(\sigma_j \sigma_k)} \quad (6.32)$$

$$\text{Dissimilarity} = \sum_j \sum_k \{|j - k| C(j, k)\} \quad (6.33)$$

$$\text{Energy} = \sum_j \sum_k C(j, k)^2 \quad (6.34)$$

$$\text{Entropy} = -\sum_j \sum_k C(j, k) \log(C(j, k)) \quad (6.35)$$

$$\text{Homogeneity} = \sum_j \sum_k \frac{C(j, k)}{1 + |j - k|} \quad (6.36)$$

$$\text{D. Variance} = \sum_{j=0}^{N_g-1} j^2 p_{x-y}(j) \quad (6.37)$$

$$\text{D. Entropy} = -\sum_{j=0}^{N_g-1} p_{x-y}(j) \log(p_{x-y}(j)) \quad (6.38)$$

$$\text{IDN} = \sum_j \sum_k \frac{C(j, k)}{1 + |j - k| / N} \quad (6.39)$$

$$\text{IDMN} = \sum_j \sum_k \frac{C(j, k)}{1 + (j - k)^2 / N^2} \quad (6.40)$$

6.5 RESULTS

6.5.1 Experimental settings and validation

In this work, have considered a binary classification problem: AD vs. NC, where we have evaluate separately each type of mapping. First, in Section 6.5.2, we will analyse our maps by means of the t statistic, where the areas of higher

statistical significance (AD vs NC) will be highlighted. To better interpret the results, an anatomical reference is provided.

Second, we have performed a classification analysis using feature selection by means of t-Test, then training and testing a linear SVM classifier. The method has been validated using stratified 10-fold cross-validation, as recommended in [Kohavi1995a]. This procedure consists on randomly partitioning the whole datasets into 10 subsets that contain the same proportion of individual of both classes as the whole database. Then, one subset is used for testing and the remaining 9 are used for training. This is repeated for each of the subsets as training sets. Finally, the whole cross-validation strategy will be repeated 10 times to avoid the possible bias and random effects of the partitions, and obtain the average and standard deviation of the performance values.

Values of accuracy (acc), sensitivity (sens) and specificity (spec) along with their standard deviation will be employed to evaluate the performance of the different mappings. Selection of parameter C of the SVM classifier (as implemented in LIBSVM [Chang2001]) will be performed using an inner 5-fold cross-validation on the training subset.

6.5.2 Statistical Significance Analysis

In this section, we will study the statistical significance of the SBM maps by using a two-sample t-Test with pooled variance estimate, as defined in Section ???. The computed t values for each coordinate pair in the maps (θ, φ) will be displayed later in Section 6.5.2.2 for the six original measures, in Section 6.5.2.3 for the layered extension and in Section 6.5.2.4 for the VRLBP. However, to provide a better understanding of these t-maps, an anatomical reference is provided in Section 6.5.2.1.

6.5.2.1 Anatomical Reference

Our SBM technique maps all sampled voxels selected by our mapping vector $\mathbf{v}_{\theta,\varphi}$ to a single point in the projected map. These points cross different anatomical regions, however it is difficult to know at first which regions are crossed, given the coordinate pairs (θ, φ) . To clarify this and provide a better understanding, we have mapped the widely known [AAL] atlas [Tzourio-Mazoyer2002] using SBM, and the regions are displayed in Figures 6.6 and 6.7.

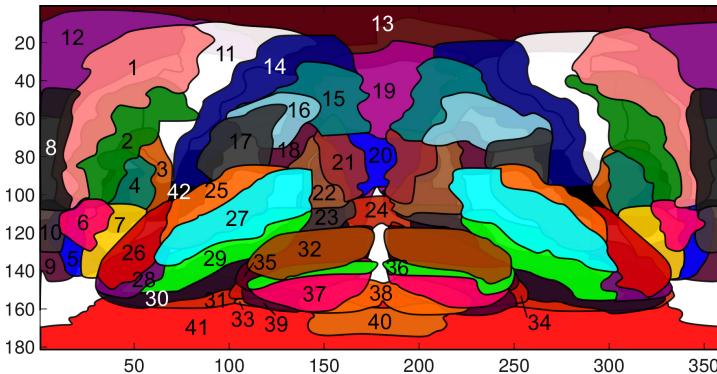


Figure 6.6: Projection of different cortical regions. In the Frontal region, we can find: 1) Frontal Sup., 2) Frontal Mid., 3) Frontal Inf. Oper., 4) Frontal Inf. Tri., 5) Frontal Sup. Orb, 6) Frontal Mid. Orb, 7) Frontal Inf. Orb, 8) Frontal Sup. Medial, 9) Rectus, 10) Frontal Med. Orb., 11) Precentral, 12) Supp. Motor Area. In the Parietal region: 13) Paracentral Lobe, 14) Postcentral, 15) Parietal Sup., 16) Parietal Inf., 17) Supramarginal, 18) Angular. In the Occipital region: 19) Precuneus, 20) Cuneus, 21) Occipital Sup., 22) Occipital Mid., 23) Occipital Inf., 24) Lingual. In the Temporal region: 25) Temporal Sup., 26) Temporal Pole Sup., 27) Temporal Mid., 28) Temporal Pole Mid., 29) Temporal Inf., 30) Fusiform, 31) Parahippocampal. The Cerebellum, divided in: 32) Cerebellum Crus 1, 33) Cerebellum 3, 34) Cerebellum 4-5, 35) Cerebellum 6, 36) Cerebellum 7b, 37) Cerebellum 8, 38) Cerebellum 9, 39) Cerebellum 10. And additionally, the 40) Medulla, 41) Brain Stem and 42) Insula.

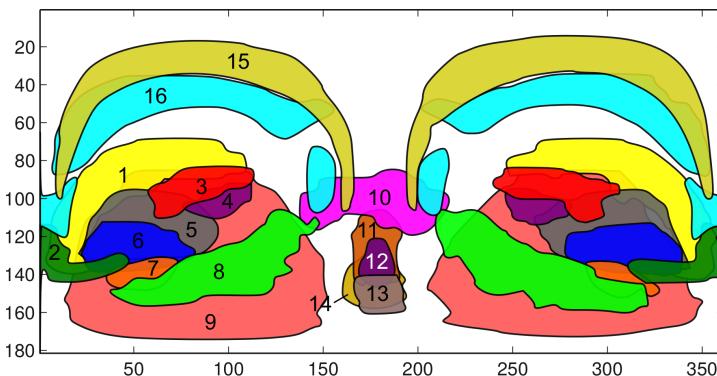


Figure 6.7: Projection of some important subcortical regions and organs. We observe the following subcortical structures: 1) Caudate Nucleus, 2) Olfactory Bulb, 3) Rolandic Operculum, 4) Heschl's gyri, 5) Putamen, 6) Globus Pallidus, 7) Amygdala, 8) Hippocampus, 9) Thalamus, 10) Lingual, 11) Vermis 4-5, 12) Vermis 7, 13) Vermis 9, 14) Vermis 1-2, 15) Cingulate Gyrus, 16) Corpus Callosum

6.5.2.2 Spherical Brain Mapping

In this section, the six measures proposed in Section ?? will be analysed using a t-test. To proceed, a t-value will be computed for each pixel in the maps, yielding a significance map. These significance maps, or t-maps, are presented in Figure 6.8.

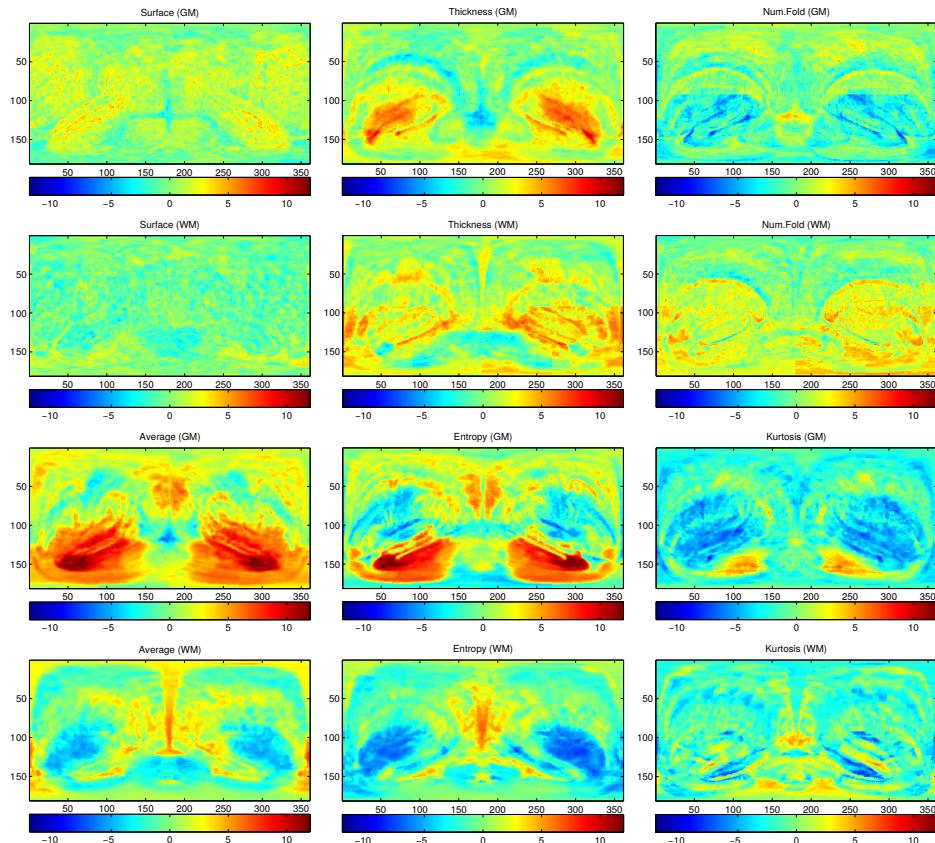


Figure 6.8: t-maps that present the level of statistical relevance in the AD vs. NC paradigm, for each type of mapping and GM and WM.

Absolute t-values higher than 1.96 can be considered to be significant, with a $p < 0.05$. In this case the areas of greater significance are in dark red and dark blue, where red is a positive t-value, meaning a higher value in controls than in AD subjects, and blue is a negative t-value, and conversely, blue means negative values, which are related to a higher value in AD than in controls.

The first thing to note is that the distribution of the t values in the Surface mapping, in both GM and WM, is not relevant at all, with very few significant pixels distributed along the whole image.

In the remaining **GM** mappings, greater t-values are located in the frontal, occipital and parietal lobes, but the most significant areas can be found in the temporal lobe. This is most obvious in the Average and Entropy mappings, but can also be found in Thickness. Number of Folds and Kurtosis present high, but however negative, t-values in these areas as well. This suggests that for **GM**, most of the neurodegeneration is located in the temporal lobe and all the underlying structures that are projected in this area, including the Hippocampus and Parahippocampal gyrus, which are considered a fundamental disease indicator in the NINCDS-ADRDA criteria [Dubois2007]. Additionally, some structures that are located in the same area have been recently related to the progression of the disease, such as the Caudate Nucleus and Putamen [Pievani2013]. These changes are more precisely located when using one of our spherical maps such as the Average or Entropy.

Conversely, in the **WM** mappings the selected regions are different. When using Number of Folds and Thickness, the selected areas are located in the vicinity of those obtained in **GM**. However, our spherical maps, especially Average and Entropy, behave differently. There is still high t-values that correspond to the White Matter of the Parahippocampal gyrus, but large areas of negative t-values that are located in areas corresponding to the Caudate Nucleus, Globus Pallidus and Putamen. The areas corresponding to the Posterior Cingulate gyrus and adjacent Precuneus present also values related to cell loss, as suggested in [Baron2001].

6.5.2.3 *Layered Extension*

The significance levels of the layered mappings has been assessed as well. However, due to space restrictions, we will only analyse the anatomical features of one of the mappings: a four-layered average mapping of the **GM**, that can be checked in Figure 6.9.

It is plain to see that most of the neurological changes in **GM** appear in layers 2 and 3, specifically in the Hippocampus, Parahippocampal lobe and Amygdala (layer 2) and the temporal lobe (layer 3), where the values of the average mapping (equivalent to the density of the tissue) are higher in normal control subjects than in AD affected patients. This reveals atrophy in these organs, as it has been previously reported in the bibliography [Dubois2007, Pievani2013]. In the case of **WM**, however, the changes are negative in the areas where the Rolandic Operculum, Heschl's gyri, Putamen and Globus Pallidus are found, and positive in some sections of the Hippocampus and the White Matter contained in the Parahippocampal lobe and the remaining parts of temporal lobe (layer 2 and 3). Nevertheless, the most significant differences are located in layer

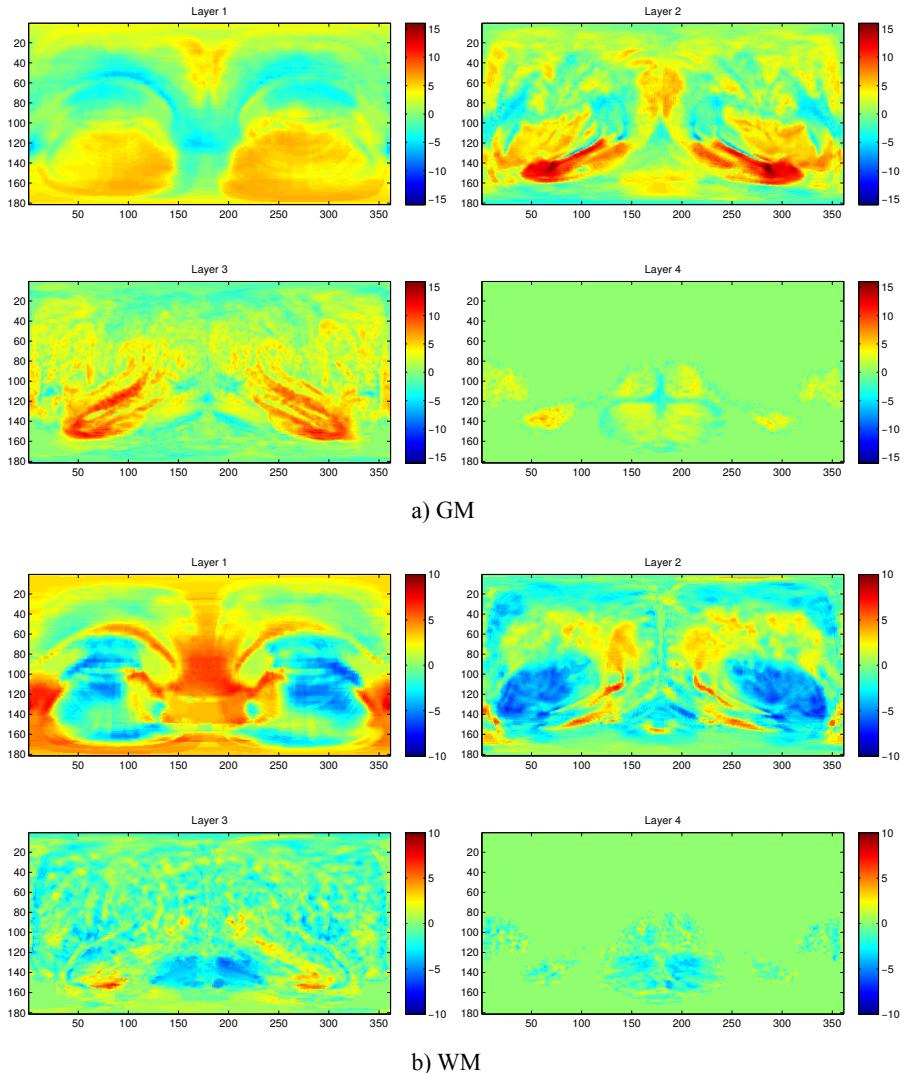


Figure 6.9: t-maps that present the level of statistical relevance in the AD vs. NC paradigm, for a four-layered average mapping over a) GM and b) WM.

1, in the borders between ventricles and Thalamus, and specially in the Cuneus, Precuneus and Posterior Cingulate gyrus, which were reported in [Baron2001].

6.5.2.4 VRLBP

Finally, to end this statistical significance analysis, the t-maps of the more complex VRLBP mapping are presented in Figure 6.10.

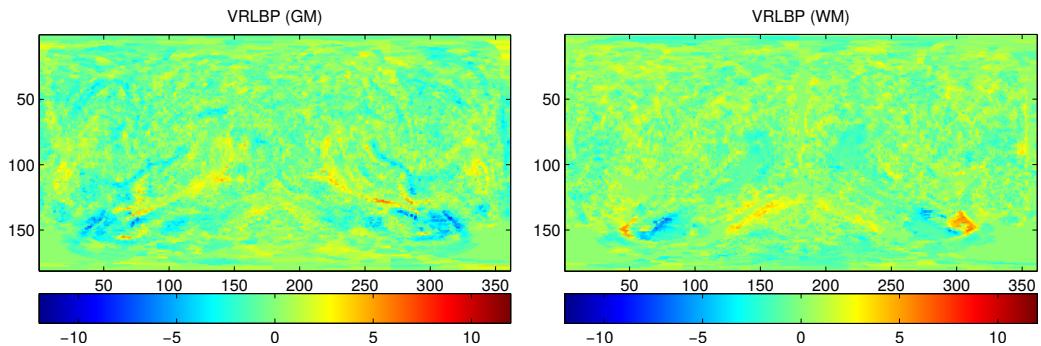


Figure 6.10: t-maps that present the level of statistical relevance in the AD vs. NC paradigm, for the VRLBP projections mapping over a) GM and b) WM.

These maps present low absolute t levels in most of the projection, however some small regions present high significance. These regions correspond to small areas in temporal lobe, Amygdala and Hippocampus in the GM, and even smaller regions in the WM corresponding to the limits between Hippocampus and Amygdala.

6.5.3 Classification Analysis

To obtain comparable performance metrics suitable to analyse the generalization capabilities of SBM, in this section a number of classification results are presented. A baseline is established in Section 6.5.3.1 and then the performance of our maps, included the layered extension and VRLBP, is presented in Section 6.5.3.2.

6.5.3.1 Baseline - VAF

In order to establish a baseline to assess the predictive ability of our maps, we will use the Voxels As Features (VAF) paradigm, described in [Stoeckel04]. This approach uses the whole 3D GM or WM segmented MR images and then uses all voxels of the 3D images as features in the SVM classification, yielding the

performance values shown in Table 6.1. The performance of the SBM maps will be compared to these.

APPROACH	ACCURACY	SENSITIVITY	SPECIFICITY
VAF (GM)	0.768 ± 0.011	0.752 ± 0.016	0.785 ± 0.016
VAF (WM)	0.642 ± 0.009	0.668 ± 0.012	0.617 ± 0.013

Table 6.1: Performance values (Average \pm Standard Deviation) for the Voxels as Features approach in both GM and WM tissues.

6.5.3.2 Spherical Brain Mapping

In this analysis, we have proceed as commented before, by computing the significance of each pixel using a t-test and then selecting a proportion of the most relevant, once they have been ranked according to their t-value. Later, these features are used to train and test a linear SVM classifier.

APPROACH	PERC.	ACCURACY	SENSITIVITY	SPECIFICITY
Surface (GM)	0.100	0.638 ± 0.006	0.660 ± 0.030	0.616 ± 0.024
Surface (WM)	0.100	0.672 ± 0.007	0.692 ± 0.018	0.652 ± 0.018
Thickness (GM)	0.725	0.781 ± 0.007	0.811 ± 0.011	0.751 ± 0.017
Thickness (WM)	0.925	0.758 ± 0.009	0.773 ± 0.017	0.744 ± 0.011
Num.Fold (GM)	0.600	0.749 ± 0.013	0.782 ± 0.019	0.716 ± 0.013
Num.Fold (WM)	0.500	0.757 ± 0.005	0.745 ± 0.006	0.768 ± 0.009
Average (GM)	0.575	0.879 ± 0.005	0.897 ± 0.006	0.861 ± 0.006
Average (WM)	0.150	0.800 ± 0.011	0.802 ± 0.013	0.798 ± 0.009
Entropy (GM)	0.825	0.846 ± 0.008	0.842 ± 0.009	0.849 ± 0.011
Entropy (WM)	0.525	0.796 ± 0.006	0.811 ± 0.009	0.781 ± 0.009
Kurtosis (GM)	1.000	0.753 ± 0.007	0.801 ± 0.011	0.704 ± 0.015
Kurtosis (WM)	0.175	0.697 ± 0.008	0.702 ± 0.018	0.693 ± 0.009
VRLBP (GM)	0.200	0.903 ± 0.010	0.890 ± 0.012	0.916 ± 0.018
VRLBP (WM)	0.150	0.909 ± 0.014	0.899 ± 0.028	0.919 ± 0.018

Table 6.2: Performance values (Average \pm Standard Deviation) for the different SBM approaches.

The results for each type of map are presented in Table 6.2, including the percentage of selected voxels (perc.) at which each value is obtained. Regarding

the Grey Matter, we can observe that the best type of mappings in the diagnosis task, in terms of average accuracy, are the Average (0.879 ± 0.005) and Entropy (0.846 ± 0.008). There results are followed by the measures of Thickness (0.781 ± 0.007), Kurtosis (0.753 ± 0.019) and the worse accuracy estimates are for Number of Folds (0.749 ± 0.013) and Surface (0.638 ± 0.006).

In the case of White Matter, and according to Table 6.2, the performance is again higher in Average (0.800 ± 0.011) and Entropy (0.796 ± 0.006). Thickness and Number of Folds present similar, but lower, performance values, respectively 0.758 ± 0.009 and 0.757 ± 0.005 , and being the Kurtosis (0.697 ± 0.008) and Surface (0.672 ± 0.007) maps the less powerful.

However, VRLBP outperform all these approaches by obtaining an accuracy of 0.903 ± 0.010 for GM and 0.909 ± 0.014 for WM, revealing itself as the best technique.

The evolution of the performance of the maps as the number of selected pixels varies is shown in Figure 6.11. In general, it is possible to see very small differences in the accuracy of the system, which makes its performance almost independent from the number of selected pixels. However, this is not the case of the Surface, and, more remarkable, the VRLBP. In the latter, the performance is the best for both tissues when the proportion of selected pixels is small, but degrades significantly as its number increases.

Regarding the four-layer extension to SBM, the performance values obtained by different mappings at different layers and thresholds (t-values of 2, 4, 8 and 10) is presented in Figure 6.12.

The first thing that we can observe is that for both GM and WM tissues, the better performance is achieved with the second layer. This is specially surprising in the WM case, as the highest t-values were located in layer 1.

6.5.4 Experimental Setup

In order to test our HMM-based path tracing algorithm, we propose the following experiments:

- Firstly, in Sec. 6.5.5 an evaluation of the algorithm over two synthetic 2D and 3D images.
- Secondly, in Sec. 6.5.6, the HMM paths created using the DARTEL template will be evaluated in the differential diagnosis (NC versus AD).
- Finally, in Sec. 6.5.7, the proposed texture feature maps computed along the DARTEL HMM paths will be evaluated in a differential diagnosis as well.

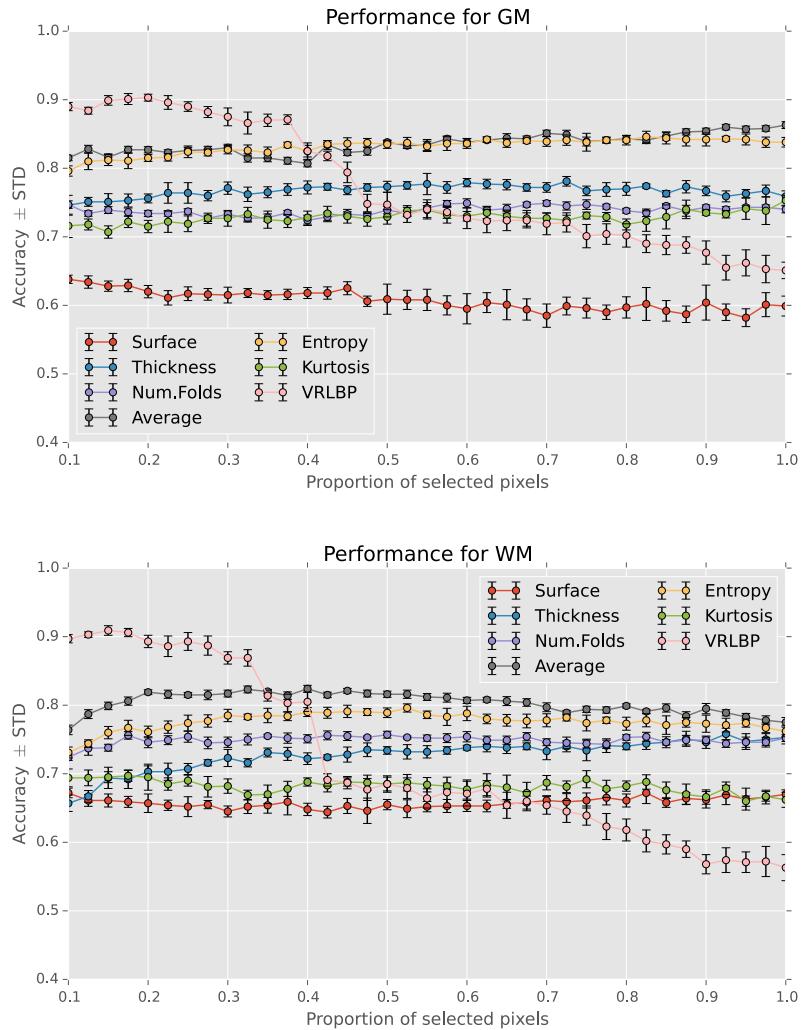


Figure 6.11: Performance for the different SBM approaches over the: a) Grey Matter and b) White Matter.

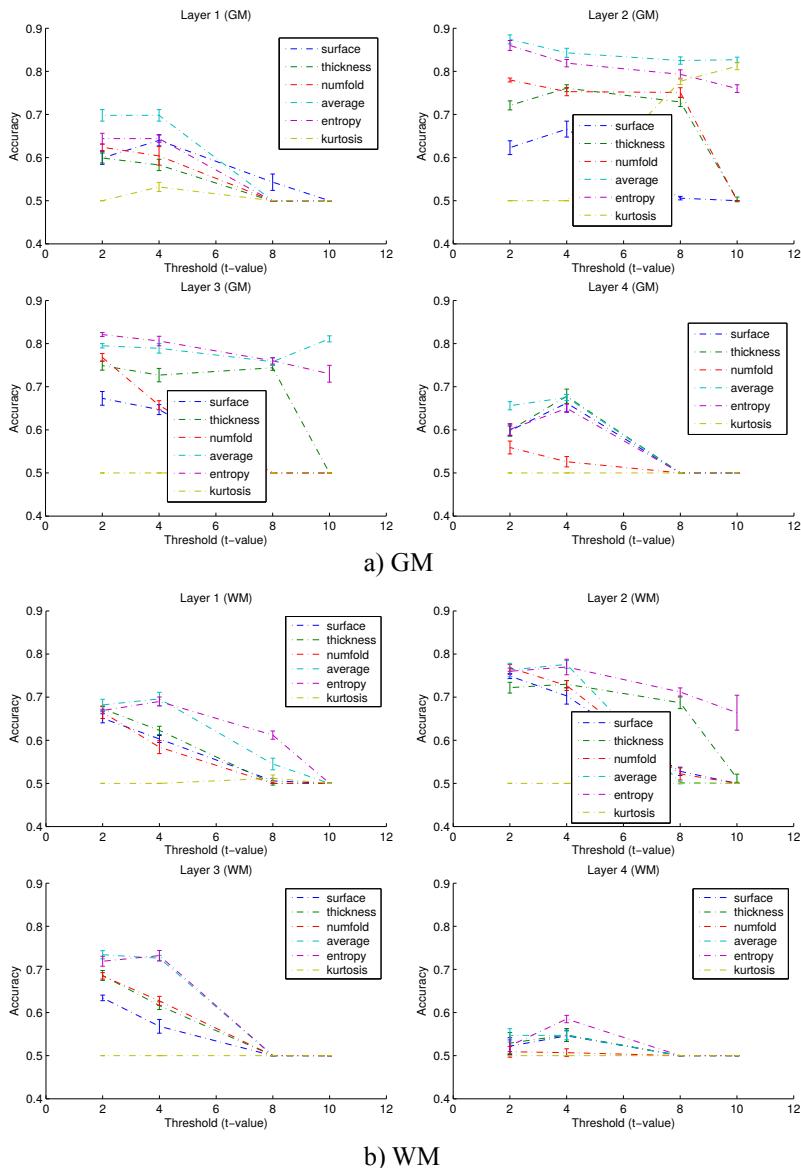


Figure 6.12: Performance for the different four-layered mappings over the: a) Grey Matter and b) White Matter at different levels of statistical significance.

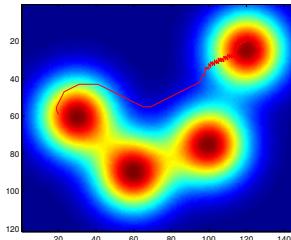


Figure 6.13: Path traced over a gaussian mixture distribution of 4 isotropic gaussian kernels.

For sections 6.5.6 and 6.5.7 a similar strategy is used to obtain performance results. Once a set of features has been extracted (intensity values in each path and statistical measures for Section 6.5.6 and texture features for Section 6.5.7), they are used to train and classify a Support Vector Machine (SVM) classifier with linear kernel, as implemented in LIBSVM[Chang2001], to classify the component scores. The classification was validated using stratified 10-fold cross-validation, as recommended in [Kohavi1995a].

6.5.5 2D and 3D demonstrations

A demonstration of the ability of our HMM path tracing algorithm can be found in Figures 6.13 and 6.14. In Fig. 6.13, the path tracing algorithm has been tested over a synthesized gaussian mixture probability density function using four isotropic gaussian kernels. The initial point was located at $x_0 = (120, 20)$ and the attractor at $x_N = (20, 60)$. The resulting path maximizes both the orientation of the path (towards x_N) and the minimum change in the intensity values, which is specially visible in the last nodes of the path, where it approaches x_N surrounding the nearby kernel. In this case, the chosen L₂-norm of the support ball has been $r = 3$.

The algorithm has been tested on a three-dimensional, helix-shaped point distribution as well (Fig. 6.14). The tracing algorithm needs per-voxel intensity (or probability) values, therefore we have estimated the probability distribution of the points as the number of points within each voxel over the total number of points. Using x_0 as the point with minimum z coordinate in the data distribution and x_N the one with maximum z, the resulting path follows the data distribution consistently until it reaches the attractor.

Finally, we have tested the algorithm on a real world example, using a digital elevation model (DEM) of the Iberian Peninsula, generated by the LANDSAT

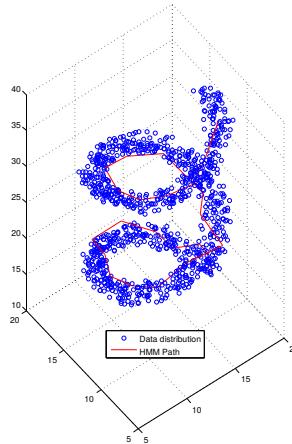


Figure 6.14: HMM path computed inside a density distribution defined by an helix.

SRTM30+ mission (see Fig. 6.15). We have tested a multiple path tracing by establishing sequentially x_0 and x_N in ten cities. The resulting paths optimize both the distance and height variation, as well as resembling -in most cases- the roads that connect these cities in the real world. Given the dimensions of the image, in this case, the L₂-norm of the support ball has been set to $r = 30$.

6.5.6 Intensity paths

In this section, we present the results of the first experiment involving paths in MRI. To do so, we define a set of canonical paths that are computed on the DARTEL template. These DARTEL paths model the anatomy of a normal subject to whom all other images have been registered. This means that we have fixed the location of the nodes to the structural information of the template, and by extension, to the general anatomy of all images in the database. Therefore, we can characterize the structural differences by the intensity distribution –in other words, the tissue density– of the voxels at the path nodes. Comparing the intensity distribution found in controls to the one found in AD affected subjects is thus the first logical step to measure how these paths can distinguish the different classes.

To test the algorithm we use the $180 \times 360 = 64800$ DARTEL paths computed in each spatial direction (φ, θ) , with $\varphi \in [0, 360]$ and $\theta \in [-90, 90]$, to select the intensities in the voxels that are placed at the nodes. The amount of voxels selected ranges from 2 to several dozens. The set of selected intensities are used as features to train and test a SVM classifier. The accuracy reached by each

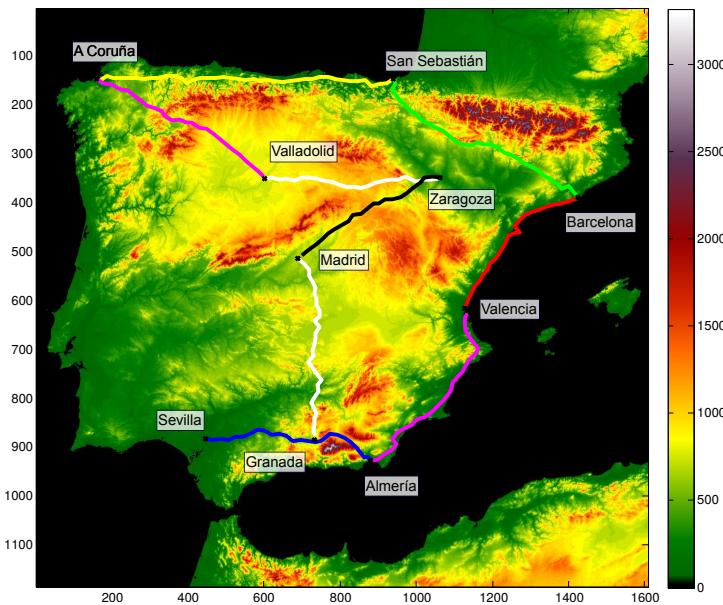


Figure 6.15: Simulation of the HMM-based path tracing over an Iberian Peninsula height map, interconecting different cities.

path (using the aforementioned cross-validation strategy) is presented as colour information in Figure 6.16. The higher accuracy obtained using only one path is 0.8028 ± 0.0873 , and corresponds to the light green paths that cross the temporal lobe.

It is interesting to question if the performance of this differential diagnosis could be improved using the information contained in more than one path at a time. To this end, we first take the higher accuracy (accuracy ≥ 0.7) paths according to the aforementioned performance and select all voxels located in the nodes of these paths. Additionally, we use a t-test over the set of voxels selected by these paths, to further reduce the set to those voxels that have significant ($p < 0.05$) t-values ($|t| > 1.96$). The performance values for the experiment involving all voxels in the paths (first row) and the one that uses only those significant voxels (second row) are presented in Table 6.3.

Finally, we mimic the procedure followed in the SBM article [Martinez-Murcia 2015]. That is, we first compute the average, variance, entropy and kurtosis maps of each brain, but instead of using rectilinear paths, we use the DARTEL paths. Afterwards, all the features contained in these maps are used as an input to the SVM classifier. The performance results are shown in Table 6.4.

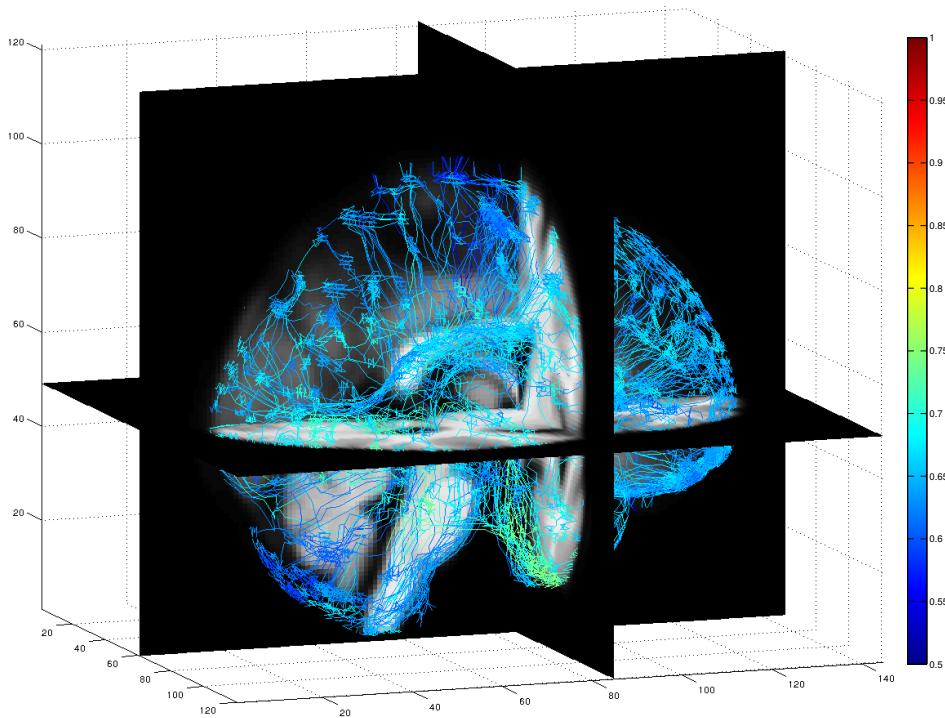


Figure 6.16: DARTEL paths computed in each direction (φ, θ). Each path's colour represent the accuracy in a differential diagnosis. Only one in every five paths are shown for clarity purposes.

SIDE	ACCURACY	SENSITIVITY	SPECIFICITY
Both	0.806 ± 0.069	0.733 ± 0.073	0.878 ± 0.097
Left	0.769 ± 0.035	0.717 ± 0.061	0.822 ± 0.057
Right	0.792 ± 0.080	0.706 ± 0.120	0.878 ± 0.101
Both	0.828 ± 0.054	0.794 ± 0.095	0.861 ± 0.039
Left	0.733 ± 0.037	0.694 ± 0.099	0.772 ± 0.124
Right	0.781 ± 0.085	0.711 ± 0.122	0.850 ± 0.083

Table 6.3: Performance values ($\pm SD$) for the selected paths as features, and using t-test to select the voxels.

FEATURE	ACCURACY	SENSITIVITY	SPECIFICITY
Average	0.594 ± 0.062	0.661 ± 0.121	0.528 ± 0.106
Variance	0.750 ± 0.064	0.633 ± 0.131	0.867 ± 0.102
Entropy	0.603 ± 0.069	0.661 ± 0.071	0.544 ± 0.125
Kurtosis	0.756 ± 0.105	0.733 ± 0.165	0.778 ± 0.150

Table 6.4: Performance values ($\pm SD$) for each of the measures used in the [SBM](#) article.

FEATURE	ACCURACY	SENSITIVITY	SPECIFICITY
Contrast	0.733 ± 0.060	0.689 ± 0.126	0.778 ± 0.105
Correlation	0.672 ± 0.068	0.672 ± 0.112	0.672 ± 0.100
Dissimilarity	0.711 ± 0.085	0.678 ± 0.110	0.744 ± 0.102
Energy	0.689 ± 0.061	0.700 ± 0.115	0.678 ± 0.073
Entropy	0.675 ± 0.101	0.672 ± 0.115	0.678 ± 0.159
Homogeneity	0.697 ± 0.058	0.700 ± 0.115	0.694 ± 0.106
Difference Variance	0.736 ± 0.070	0.683 ± 0.098	0.789 ± 0.090
Difference Entropy	0.725 ± 0.122	0.683 ± 0.176	0.767 ± 0.114
IDN	0.719 ± 0.065	0.683 ± 0.108	0.756 ± 0.105
IDMN	0.717 ± 0.076	0.678 ± 0.125	0.756 ± 0.084

Table 6.5: Performance values ($\pm SD$) for each of the 10 texture features.

6.5.7 Texture features

The second experiment is intended to extract texture features from the DARTEL paths. With this approach, we obtain one single value per texture feature and path in the subjects, values that intrinsically contain information from their location in the path, in contrast to standard [SBM](#) measures. In the end, each subject will be characterized by a 2D, 361×181 array of scalars, one for each texture feature applied to the paths. Performance values for the nine texture features maps from Section 6.4.1 are presented in Table 6.5.

The higher accuracy obtained by the texture maps is 0.736 ± 0.070 , corresponding to Difference Variance. The performance values of the different texture features, all obtaining accuracies higher than 65% (most of them above 70%) reveal the discrimination abilities of these textures, although these values are not as good as those obtained using the voxel intensities or the [SBM](#) features.

6.6 DISCUSSION

6.6.1 Spherical Brain Mapping

The structural changes in MR images during the progression of the Alzheimer's Disease are widely documented in the bibliography [Misra2009, Baron2001, Pievani2013, Stoeckel04, han2006reliability, Fischl2004]. According to our current knowledge, the neurodegeneration and posterior atrophy occurs mainly in the GM tissue, although significant changes are present also in WM.

The mappings defined throughout Sections ??, 6.2.1 and 6.3 account for different properties of the tissues crossed by $v_{\theta,\varphi}$. As it can be seen in Figure 6.17, our mappings show in general a higher performance when using the GM tissue, which is consistent with the literature. There are some exceptions, however, being the clearest the VRLBP, and, to a lesser extent, the number of folds and surface. The different mappings and their utility will be described in the following paragraphs.

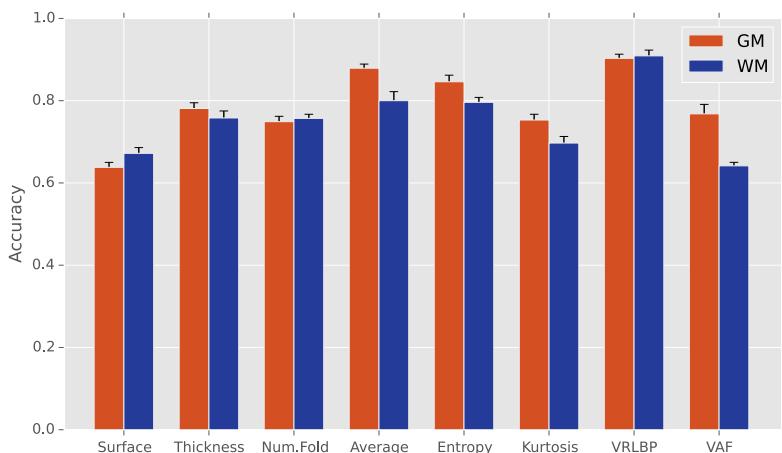


Figure 6.17: Performance at the operation point for the different mappings over the Grey Matter and White Matter, compared with the performance of VAF.

The first three approaches, Surface, Thickness and Number of Folds are easily interpreted, as they intend to represent the surface of the tissue by mapping the distance between the centre of the image and the last voxel, the thickness of the tissue, and a measure of the complexity of the different sulci and gyri.

Surface and Thickness are highly related to other measures provided by widely-used software. However, as they are related to our more general SBM description,

their performance is poor, specially in the case of the Surface mapping. As it can be seen in Fig. 6.3, and later in the t-maps at Fig. 6.8, the detail of the surface map lacks higher detail, specially due to the superposing gyri and sulci. These superposition occur to a lesser extent in WM tissue, and this is probably why this technique obtains higher performance in WM than in GM.

As for the case of Thickness, although similar, it gathers much more information than the surface, without achieving, however, the level of detail of the cortical thickness measures provided by Freesurfer [Fischl2004] or other software. Nevertheless, cortical thickness it is a descriptive, widely accepted as a measure of neurodegeneration in Alzheimer's Disease in the literature [han2006reliability, Fischl2004], and its measures might be relevant for a subsequent analysis.

Number of Folds, however, is intended to model the complexity of the cerebral cortex, and therefore, it is of far more use in the case of GM than in the WM. This can be easily checked when looking at the maps obtained for both GM and WM in Figure 6.3.

The last three measures described in Section ?? are statistical values that describe the variability of the sampling set $V_{\theta,\varphi}$. It would be reasonable to expect the better performance to be linked to the mapping that better models the tissue atrophy.

This is the case of the average of these intensities, which can be interpreted as the total amount of tissue, being therefore a good measure of the level of brain atrophy in each direction (θ, φ) . The average maps show the best performance of all the measures proposed in Section ??, and is higher in GM than in WM. This is consistent with the literature, as atrophy mainly occurs in GM tissues.

Entropy is a more complex statistical concept that comes from information theory, but is usually related to the amount of information, or in other words, the "randomness" of a source. In our particular case it could be interpreted as a measure of texture, that is, the grey-level variability in the direction of $v_{\theta,\varphi}$. These maps perform very similar to the average ones in both GM and WM, suggesting that the entropy accounts for the tissue density as well.

The last mapping defined, Kurtosis, is a fourth-order statistic, often interpreted as the peakedness (width of peak) of a probability distribution. In our context, it is related to the sharpness of the changes in the direction of $v_{\theta,\varphi}$, and thus is related to the number of folds. As in the case of the latter, the Kurtosis performs poorly in both types of tissues, probably because they are measures that are not as directly related to atrophy as other measures such as average, entropy or thickness.

The last of the single measures proposed in this work is the Volumetric Radial LBP defined in Section 6.3. It is a measure of the texture not only in the direction

of $v_{\theta,\varphi}$, but also in the neighbourhood of the mapping vector. Therefore, it is not strange that it obtains the best performance of the whole work, yielding accuracy results above 0.9 for both **GM** and **WM** tissues.

This could seem counter-intuitive, as the t-maps for this technique, presented in Fig. 6.10, show small regions of high significance, when compared to the measures in Sec. ???. Yet, despite its size, it performs fairly well with a relatively small amount of data. It is probably due to the nature of VRLBP, and the areas highlighted in Fig. 6.10 probably correspond to the texture changes associated to the loss of tissue in the Hippocampus.

As for the layered extension, which might seem a powerful method to add detail to the mappings, obtains however similar performance to the methodology above. It seems that the amount of information that can be obtained by each measure does not depend on the number of layers, and accordingly, its benefits are only related to visualization. In this case, best values are obtained in layer 2, which is consistent to the presence of some organs, specially the Hippocampus.

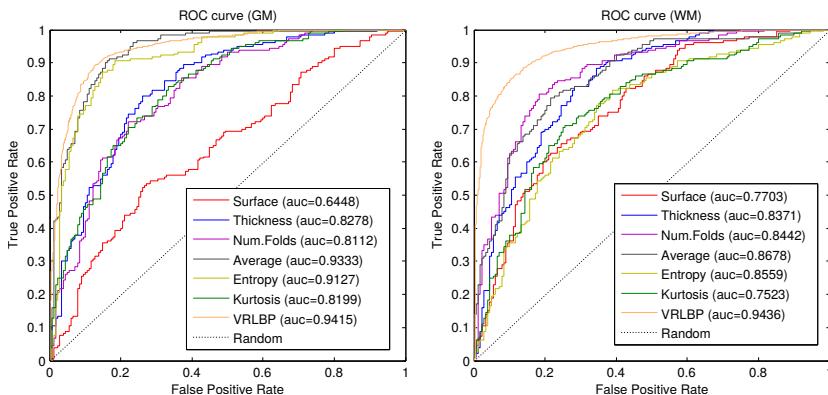


Figure 6.18: ROC curves of the different mappings for the **GM** and **WM** tissues.

Finally, in order to have another look at the performance of our mappings, the ROC curves of each type are presented in Figure 6.18. There we can see how the VRLBP approach outperforms all the other measures, specially in the case of **WM** tissue. In **GM**, Average and Entropy present values really close to VRLBP, as expected. Conversely, the poorest performance is achieved by the Kurtosis and Surface mappings, however the Surface performs better in **WM** than in **GM**. These results confirm the performance values presented in Table 6.2 and Figure 6.11, making our proposed mapping framework a reasonable choice for obtaining both a visual interpretation of otherwise hidden features and a significant dimensionality reduction.

It is important to note that our Spherical Brain Mapping defines a whole framework that can be easily extended with different sampling strategies. This

is the case of the layered extension and the helical sampling in VRLBP, but they are only two examples of what can be done. Since our simplest approach implies a computation of a value from a vector of intensities, measures used to describe time-course data could be added to complete and highlight different properties of the tissues. In this context, high-order statistics [Zhou2008], as well as spectral measures [Locatelli1998] have been successfully applied to analyse electroencephalogram (EEG) signals, and could be therefore applied here to bring different structural properties of the images into focus. Additionally, our mapping method is potentially applicable to other imaging modalities, such as PET and SPECT, where the structural information is sometimes lost [IAIlan2010, Ramirez2009]. Our technique does not need the use of complex co-registering of MRI and functional imaging to locate cerebral structures, as it rely only in their angle and depth. Moreover, in the case of Diffusion Tensor Imaging (DTI), which has proven itself as a good tool for the diagnosis of Alzheimer's Disease [Grana2011, Medina2008], SBM could be modified to replace $v_{\theta,\varphi}$ with each tract, and subsequently project a given feature, resulting in a summary of the tract's behaviour in a single two-dimensional image.

6.6.2 Paths via HMM

In this work we propose a new path tracing algorithm based on Hidden Markov Models used to trace similar intensity paths inside the brain. The paths are meant to be used as a feature extraction tool in the SBM framework either by selecting voxels or computing features. We have performed several experiments to evaluate these approaches in a differential diagnosis of AD using MRI brain images.

Our paths are defined so that they construct a minimum intensity variation path starting at the AC and oriented in a general direction set by the spherical coordinate pair (φ, θ) . As commented before, the AC is the obvious starting point, given its privileged position in the middle of the left and right hemispheres. A different starting point will reveal suboptimal, stopping at disconnected regions such as the ventricles, and yielding incomplete paths.

The paths adapt to the intensity changes in a certain direction in the brain, modelling grey level connectivity in all spherical directions. Since grey level is directly related to tissue density, we can assume that the outcome follows smooth, same-density paths that start in white matter and progressively transition to grey matter in a specific direction. Therefore, they are not functional connectivity maps like Diffusion Tensor Imaging (DTI), which have been used as well in the diagnosis of AD[Grana2011, Medina2008]. While DTI fibers are

the result of a tensor processing over diffusion images that quantify the water molecule motion -in both direction and average magnitude- at the voxel level, our [HMM](#) paths only characterize grey level connectivity in static MRI images, and are meant to be used for feature selection.

Our first experiment uses [HMM](#) paths computed on the DARTEL template to describe how the intensity of the set of voxels corresponding to a certain path can be used as discriminant features in a SVM classifier. The differences in the distribution of intensities between controls and AD affected subjects are used to identify structural changes in AD. Fig. [6.19](#) depicts the paths that achieved best performance (accuracy higher than 0.75) in this differential diagnosis, superimposed to some structures rendered from the Automated Anatomical Labeling (AAL) brain atlas[[Tzourio-Mazoyer2002](#)].

The paths that obtained higher accuracy are those that cross structures such as the Hippocampus, Amygdala, Thalamus, Fusiform and Inferior Temporal Gyrus. Particularly, grey matter loss in the Hippocampus has been described in the NINCDS-ADRDA criteria for AD diagnosis[[Dubois2007](#)] and is widely accepted[[chan2001patterns](#), [Baron2001](#), [Jong2008](#)]. Furthermore, the evidence suggest that atrophy affects the surrounding structures (Amygdala, Parahippocampal and Fusiform Gyrus) as well[[chan2001patterns](#), [Baron2001](#)]. Some studies have found significant atrophy in the Thalamus and Putamen in early AD[[Jong2008](#)] as well. Generally, in advanced AD, most of the neocortex and grey matter suffer from atrophy[[chan2001patterns](#), [Baron2001](#), [Jong2008](#)], which explains why most of the paths that involve the neocortex in Fig. [6.16](#) obtain accuracy rates around 0.7.

A number of feature maps have been computed as well. These are the result of applying some of the [SBM](#) measures to the voxels selected by the [HMM](#) paths. Variance and kurtosis have been proved as the most discriminative maps (with accuracy higher than 0.7). This is coherent with the definition of the paths, where the intensity transitions are minimal. Therefore, average would be the less discriminative in this case, being higher order statistics such as variance or more representative of the tissue density distribution of each class.

Regarding texture analysis, we have again discriminative features (with accuracy that surpass the 70%) yet not very powerful. This situation might be due to the definition of the paths as minimum intensity variation paths, being the textural changes along the path minimal.

However the real utility of these texture features could be in its application to longitudinal studies, since texture can be related to evolution of the disease[[sikio2015mr](#)]. It is very convenient to use a scalar to characterize a measure (in our case, texture features) in each direction. The texture obtained in each session can be used to construct a function of neurodegeneration that al-

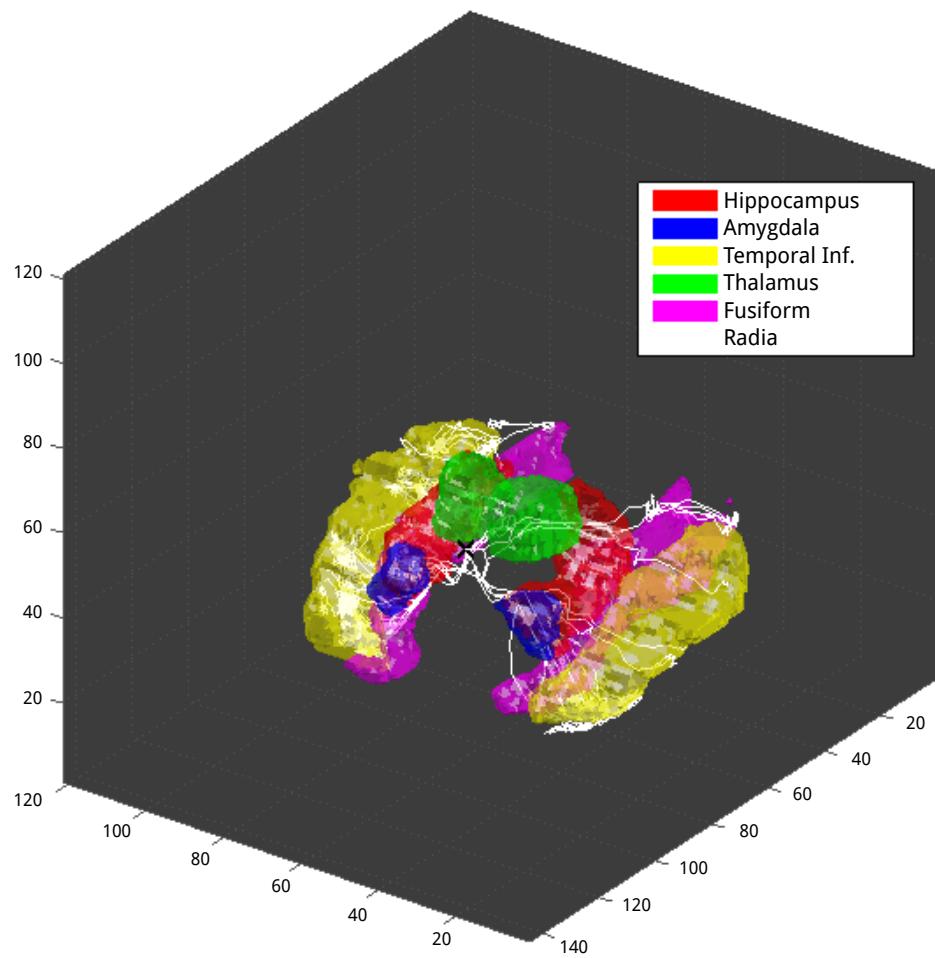


Figure 6.19: Paths that obtain more than 75% accuracy, and a three-dimensional representation of the structures crossed by them.

FEATURE	ACCURACY	SENSITIVITY	SPECIFICITY
Paths	0.806 ± 0.069	0.733 ± 0.073	0.878 ± 0.097
Selected Paths	0.828 ± 0.054	0.794 ± 0.095	0.861 ± 0.039
Variance	0.750 ± 0.064	0.633 ± 0.131	0.867 ± 0.102
Kurtosis	0.756 ± 0.105	0.733 ± 0.165	0.778 ± 0.150
Texture (Difference Variance)	0.736 ± 0.070	0.683 ± 0.098	0.789 ± 0.090
VAF	0.768 ± 0.011	0.752 ± 0.016	0.785 ± 0.016
SBM-average (GM)	0.879 ± 0.005	0.897 ± 0.006	0.861 ± 0.006
SBM-average (WM)	0.800 ± 0.011	0.802 ± 0.013	0.798 ± 0.009
SBM-VRLBP (GM)	0.903 ± 0.010	0.890 ± 0.012	0.916 ± 0.018
SBM-VRLBP (WM)	0.909 ± 0.014	0.899 ± 0.028	0.919 ± 0.018
LVQ-SVM (GM)	0.869 ± 0.101	0.822 ± 0.120	0.890 ± 0.102
SCA (GM)	$0.880 \pm 0.0^*$	$0.926 \pm 0.0^*$	$0.845 \pm 0.0^*$
SCA (WM)	$0.808 \pm 0.0^*$	$0.817 \pm 0.0^*$	$0.800 \pm 0.0^*$

* SCA used leave-one-out cross-validation. SD is 0.

Table 6.6: Comparison between our algorithm performance values (best values for selected voxels in all paths and texture features) (\pm SD) and other methods in the bibliography

lows the exploration of the different stages of the disease as the changes in the brain texture along the time within a single patient.

Table 6.6 presents some of the best results of our methodology involving HMM paths in this order: the performance of a single path and using the selected paths as features (Section 6.5.6), the performance of using projected maps (in this case, variance and kurtosis) like in the SBM paper (Section 6.5.6) and the results of computing texture maps using radial GLCM and Haralick Texture Features (Section 6.5.7). It is compared with the methods using in the SBM paper [Martinez-Murcia2015], the SBM-VRLBP [Martinez-MurciaVRLBP], the Voxels As Features (VAF) [Stoeckel04] algorithm and different approaches used in the ADNI database and involving SVM classifiers such as the LVQ-SVM [Ortiz2013] or Spatial Component Analysis (SCA) [Illan2014].

VAF is often used as a baseline when comparing different methodology, as it has been described as a good estimator of the accuracy obtained by means of visual analysis [Stoeckel04]. As we commented before, the raw voxel intensities selected by our DARTEL paths achieve higher accuracy than statistical or texture features, and it is the only strategy that outperforms VAF. Texture and statistical features obtain poorer, although still good, performance (around 75% accuracy). When compared to other methods, the difference is greater, although inside the

range of 1 SD. Most of the [SBM](#) features proposed in [[Martinez-Murcia2015](#)] perform better than our DARTEL paths, and the case of [[Martinez-MurciaVRLBP](#)] even surpass the barrier of 90% accuracy. However, there is a significant difference with these approaches, and it is that these measures used segmented [GM](#) and [WM](#) images, instead of using the whole MRI. Segmentation, thus, enhances the detection and extraction of features from the images, whereas the tracing of paths over the whole images is a more complex operation. When compared to LVQ-SVM or SCA, the difference in performance is even smaller and still inside the range of 1 SD, which gives us an idea of the ability of our methodology to detect patterns with a significant feature reduction.

Finally, one might argue if a different approach to the path tracing, such as tracing the set of paths in each subject individually might be of use. This strategy would still characterize the individual brain structure; however the way this structure is defined would be different: the spatial location of the nodes and topology of the paths instead of the intensity distribution. Given the time our algorithm takes to model one single MRI (around 2 hours) it can be extraordinarily computationally expensive, although faster than other methodology like DTI fiber tracing or Freesurfer surface extraction. Consequently, it would be an interesting option to explore in future works.

Part III
INCREASING THE SAMPLE SIZE

7

SIGNIFICANCE WEIGHTED PRINCIPAL COMPONENT ANALYSIS

Multicentre studies with structural (sMRI) and functional Magnetic Resonance Imaging (fMRI) are increasingly common, allowing for recruitment of larger samples in shorter periods of time. However, the use of images acquired at different sites still poses a major challenge. In addition to logistical difficulties, such as regulatory approvals and data protection, a number of technical and methodological issues can potentially affect the resulting maps, introducing undesired intensity and geometric variance. This issue has been addressed in other neurological conditions, such as Alzheimer's Disease (Jovicich, et al., 2006; Stonnington, et al., 2008), where group differences are well known, and demonstrating that the impact of a correction for site on the resulting neurobiological differences is relatively small. However, these effects have a stronger impact in psychiatric conditions where the atypical radiological signs on MRI are often subtle and require large samples of patients to observe on-average differences relative to control samples. Recent meta-analyses point to differences being inconsistently reported in schizophrenia (Friedman and Glover, 2006; Turner, et al., 2013), psychosis (Clementz, et al., 2016; Wang, et al., 2015), and ASD (using the multi-centre ABIDE database) [haar2014anatomical]

These inconsistencies can arise from a variety of variance sources, ranging from the multi-level (phenotypic, neurobiological, and etiological) heterogeneities of the conditions to technical issues that include differences in scanner make, model, manufacturer, static field strength, field inhomogeneities, slew rates and image reconstruction (Van Horn and Toga, 2009), as well as acquisition problems such as within-acquisition participant head motion. Field inhomogeneities are a source of misinterpretation of the data even when the same MRI system manufacturer and model are used (Van Horn and Toga, 2009). Furthermore, results in (Pearlson, 2009) demonstrate that a single scanner can change with time, which makes some widely used strategies, for example collecting controls first and patients later, a flawed approach. Recent neuroimaging research on ASD (Haar, et al., 2014) has shown that, while analyses performed on a particular database (acquired on a single platform) could yield coherent regions, the atypical structures are often inconsistent across the wider literature using different databases. Therefore, new methodologies focused on reducing multi-

site variance may be potentially helpful in increasing the power to identify the characteristic neurobiological signature of autism, should there be one.

[Martinez-Murcia2016a]

7.1 SIGNIFICANCE WEIGHTED PRINCIPAL COMPONENT ANALYSIS

The Significance Weighted Principal Component Analysis ([SWPCA](#)) is an algorithm to reduce, in this case, undesired intensity variance introduced by multi-site image acquisition. [SWPCA](#) takes any dataset of pre-processed images, spatially normalized, and decomposes them into their variance components to then provide a corrected dataset where these undesired variance components have been reduced. To do so, [PCA](#) was applied to each modality in turn to obtain the component scores and component loadings. Since [PCA](#) is a data-driven approach, it was only used to decompose the source images, and after this procedure, a one-way [ANOVA](#) estimated the relation between each variance component and a given categorical variable, in our case, the acquisition site. The between-site variability in the variance component was then identified by its corresponding *p*-value. Finally, these *p*-values were transformed into a weighting matrix Λ that weighted the influence of each variance component in a final [PCA](#) reconstruction of the corrected maps. The procedure is summarized in Figure 7.1.

7.1.1 Principal Component Analysis

The first step in the [SWPCA](#) algorithm was to perform a [PCA](#) decomposition of the dataset into a set of orthogonal components that model the variance present in the images.

[PCA](#) is a statistical procedure that uses an orthogonal transformation to convert a set of observations \mathbf{X} of possibly correlated variables, where \mathbf{X} is a $K \times N$ matrix, with K participants (in this case, with one image per participant) and N the number of voxels, into a set of N linearly uncorrelated variables called Principal Components (PC, also known as component loadings or the mixing matrix) \mathbf{W} of size $N \times N$ whose linear combination using a vector of component scores s_K can perfectly recompose each image. The set of these component scores \mathbf{S} (size $K \times N$) was estimated as:

$$\mathbf{S} = \mathbf{X}\mathbf{W}^T \quad (7.1)$$

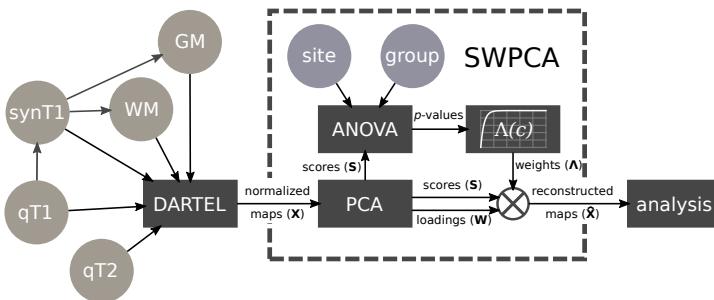


Figure 7.1: Summary of the [SWPCA](#) algorithm, along with its context in the pipeline used in this article. Circles represent the input data, both images (green shading) and class (group and acquisition site, purple shading). Rectangles represent the different procedures applied, comprising the DARTEL normalization and registration, the different steps contained in [SWPCA](#), ANOVA and obtaining the weighting function $\Lambda(c)$ - and the subsequent analysis.

This transformation computes a sequence of PCs, maximally explaining the variability of the data while maintaining orthogonality between components. [PCA](#) was computed using Singular Value Decomposition ([SVD](#)):

$$\mathbf{X} = \mathbf{U}\Sigma\mathbf{V}^* \quad (7.2)$$

where \mathbf{U} is an $K \times K$ orthogonal matrix, Σ is a $K \times N$ diagonal matrix with non-negative real numbers on the diagonal, and the $N \times N$ unitary matrix \mathbf{V}^* denotes the conjugate transpose of the $N \times N$ unitary matrix \mathbf{V} . With this decomposition both the component scores and estimates of the set of components loadings \mathbf{W} were obtained. In this work the truncated form of [SVD](#) was used such that only the first C components were considered, where most of the variability of the data was concentrated:

$$\mathbf{S}_C = \mathbf{U}_C \Sigma_C = \mathbf{X} \mathbf{W}_C \quad (7.3)$$

where \mathbf{S}_C is the set of component scores using the first C components (size $K \times C$). To achieve reasonable performance with minimal information loss, it was assumed that the number of components was the same as the number of images, $C = K$. Thus, a partial reconstruction of the original signal could be undertaken:

$$\hat{\mathbf{X}} = \mathbf{S}_C \mathbf{A}_C \quad (7.4)$$

where \mathbf{A}_C is the pseudoinverse of the truncated matrix of component loadings \mathbf{W}_C , and $\hat{\mathbf{X}}$ is the reconstructed set of images.

7.1.2 One-Way Analysis of Variance

The estimated PCs effectively model the variability of the image dataset. The next step was to assess each PC as a source of inter-site variance with one-way Analysis Of Variance ([ANOVA](#)). [ANOVA](#) estimates the F-statistic, defined as the ratio between the estimated variance within groups and the variance between groups:

$$F = \frac{MS_{within}}{MS_{between}} = \frac{SS_{within}/(G - 1)}{SS_{between}/(K - G)} = \frac{\sum_i n_i (\bar{Y}_i - \bar{Y})^2 / (G - 1)}{\sum_{ij} (Y_{ij} - \bar{Y}_i)^2 / (K - G)} \quad (7.5)$$

Where MS_{within} and $MS_{between}$ are the mean squares within- and between-groups respectively, G is the number of separate groups (in our case, two), \bar{Y} is the sample mean of a certain feature (in our case, the sample mean of all K values of a given component score), \bar{Y}_i is the sample mean of the features belonging to group $i = 1 \dots G$, Y_{ij} is the j_{th} observation of a feature belonging to group i and n_i is the number of participants in the i_{th} group. The F-distribution allows an easy computation of p-values, given the number of groups and degrees of freedom. The F-statistic and p-values were computed independently for each component score and acquisition site, and then used in the [SWPCA](#) algorithm.

7.1.3 Weighting Function

To obtain a set of corrected maps, a new signal matrix of all maps of the same modality, \hat{X} , was estimated with the influence of the PCs with variance related to acquisition site, assessed via the p-values, reduced. To do so, equation [7.4](#) was modified to include a square matrix Λ (dimension $C \times C$) whose diagonal contains a weight λ_c for each component that depends on its p-value; that is,

$$\hat{X} = S \Lambda A \quad (7.6)$$

The computation of each λ_c , for each component, was performed using the Laplace distribution, modified so that the weights were on the interval $[0, 1]$:

$$\Lambda_c(p_c, p_{th}) = 1 - e^{\frac{-p_c}{p_{th}}} \quad \forall p_c \in [0, 1] \quad (7.7)$$

where p_c is the statistical significance of the c_{th} component with respect to the acquisition site and p_{th} is the statistical threshold for significance; that is, $p_{th}=0.05$. A plot of the univariate weighting function $\Lambda_c(p_c, p_{th})$ can be found in Figure [7.2](#). This weighting ensured that most of the components of variance that are not related to the acquisition site are kept unchanged, while at the same

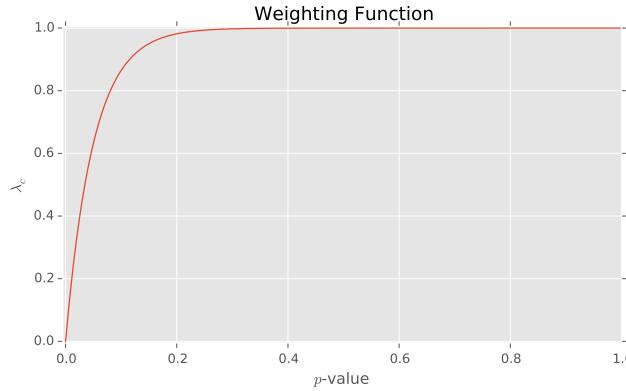


Figure 7.2: Weighting function $\Lambda_c(p_c, p_{th})$ used in SWPCA.

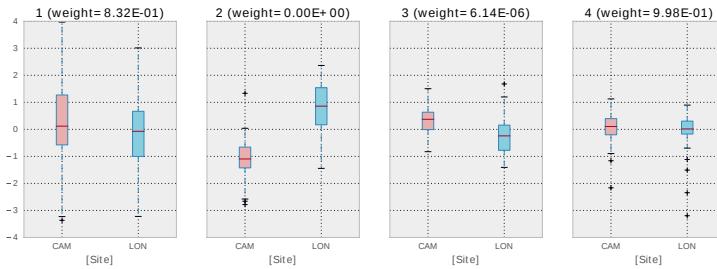


Figure 7.3: Box-plot of the distribution of the component scores at each site of the AIMS-MRI dataset (see Sections 7.2 and A.1.2) in the four first components. We assume that bigger differences between distributions imply a bigger influence of the acquisition site on the portion of variance modelled by that component and therefore, to parse out those differences, the resulting weight will be smaller.

time it strongly reduces the influence of components with p-values less than the threshold.

This procedure is illustrated in Figure 7.3, where a boxplot of the distribution of the first four principal component scores is shown. Since we have assumed that substantial differences imply a bigger influence of the acquisition site on the portion of variance modelled by that component, the resulting weight is reduced, and the contribution of that component to the reconstructed signal will be smaller. After computing all weights, most of the sources that are related to the acquisition site (for example, the second and third components) have been parsed out while keeping all other sources of variance.

7.2 RESULTS FOR AIMS-MRI DATASET

To validate the effects of the [SWPCA](#) algorithm on the inter-site variance, experiments were undertaken to assess the reduction of the undesired site variance in the original datasets, and its impact on the between-group signal. Two kind of analysis were performed: a characterization of voxel-wise differences, and a classification analysis.

Voxel-wise differences between groups were characterized using Voxel Based Morphometry ([VBM](#)) [[Ashburner2000](#)], comprising preprocessing (registration, smoothing) and mass-univariate t-test on the smoothed maps from each modality. [SWPCA](#) is included (when needed) in this pipeline as a plug in, after the smoothing and before the computation of the test. Permutation testing assessed the significance of the relationship between the tested and target variables. A max-type procedure was used to obtain family-wise, whole-brain corrected p -values (Freedman and Lane, 1983). Additionally, a Component Based Morphometry ([CBM](#)), based on Source Based Morphometry (SBM) [[xu2009source](#)] was used. This procedure provided Z-maps for visual inspection comparable to those obtained in [VBM](#), by selecting component loadings W , scaling them to unit standard deviation and weighting their contribution to the final map with their statistical significance, computed using the same permutation inference as in [VBM](#).

A classification analysis was undertaken using a common classification pipeline (Khedher, et al., 2015; López, et al., 2009) consisting of preprocessing, feature extraction and classification. [SWPCA](#) is used as a plug-in here as well, after the pre-processing and before the feature extraction step. We used [PCA](#) on the images for feature reduction and a Support Vector Classifier ([SVC](#)) with linear kernel, as implemented in LIBSVM [[Chang2001](#)], to classify the component scores in both corrected and uncorrected datasets (i.e. with and without [SWPCA](#)).

The classification was validated using stratified 10-fold cross-validation [[Kohavi1995a](#)]. In brief, 9 subsets of the dataset were used for extraction of the PCs and training of the classifier with the remaining subset used for testing. This procedure was repeated for each subset, repeated 10 times to avoid possible bias and random effects of the partitions. The average and standard deviation of the accuracy (acc), sensitivity (sens) and specificity (spec) values for each repetition were recorded.

For each modality independently, the following experiments were performed:

- **Experiment 1:** To demonstrate the ability of the [SWPCA](#) algorithm to reduce undesired effects due to acquisition site, the [PCA + SVC](#) pipeline was applied to the datasets labelled by acquisition site. Classification accuracy was compared to datasets with and without [SWPCA](#). [VBM](#) was then ap-

plied to identify the spatial location of the between-site differences. This was undertaken on the whole database (ALL), and subgroups containing only **ASD** or **ASD** participants.

- **Experiment 2:** The discrimination ability of each modality, acquired at different sites was assessed by classification performance of individuals from London (LON) and Cambridge (CAM) was separately assessed, using group (**ASD** and **CTL**) as the labels.
- **Experiment 3:** To assess the impact of **SWPCA** on the datasets when characterizing the differences between **ASD** and **CTL** groups, the classification pipeline comprising **PCA + SVC**, as well as **VBM** and **CBM**, were applied to all participants with group as the labels.

7.2.1 Experiment 1: Effect of Acquisition Site

The first experiment was to demonstrate the ability of SWPCA to reduce the intensity variance related to acquisition site. To do so, we first performed a **VBM** analysis in all five modalities (**qT₁**, **qT₂**, simulated T₁ - weighted Inversion Recovery (**synT₁**), **GM** and **WM**) separately, with the uncorrected (without applying SWPCA) and the corrected (after applying SWPCA) maps, using the acquisition site as labels.

To illustrate where the sources of variance of the acquisition sites are located, Figure 7.4 shows a brain t-map of significant ($p < 0.01$, $|t| > 2.57$) **GM** and **WM** between-site differences. The biggest reductions in variance were found in **qT₁** and **synT₁** maps, where high variability between acquisition sites, especially in the right hemisphere, was substantially reduced after the application of SWPCA. The reduction in the **qT₂**, **GM** and **WM** maps was smaller, although noticeable.

To quantify the impact of this variance reduction on the between-groups effects, the classification analysis was undertaken. Higher accuracy values imply that the maps contain site-related patterns that were significant, whereas accuracy close to 0.5 indicates that the site-related variance was low. The test was applied to ALL, and also to the ASD and CTL subgroups. The classification results are presented in Table 7.1.

Performance results indicate clear advantages of using SWPCA, in particular in the case of **qT₁** and **synT₁** which were associated with strong site-dependent variance. These results are also consistent with the reduction of significant between-group areas observed in Figure 7.4.

The between-site differences were smaller for **GM** and **WM** maps, possibly due their reduced sensitivity. Since fractional occupancy values are abstract, unit-

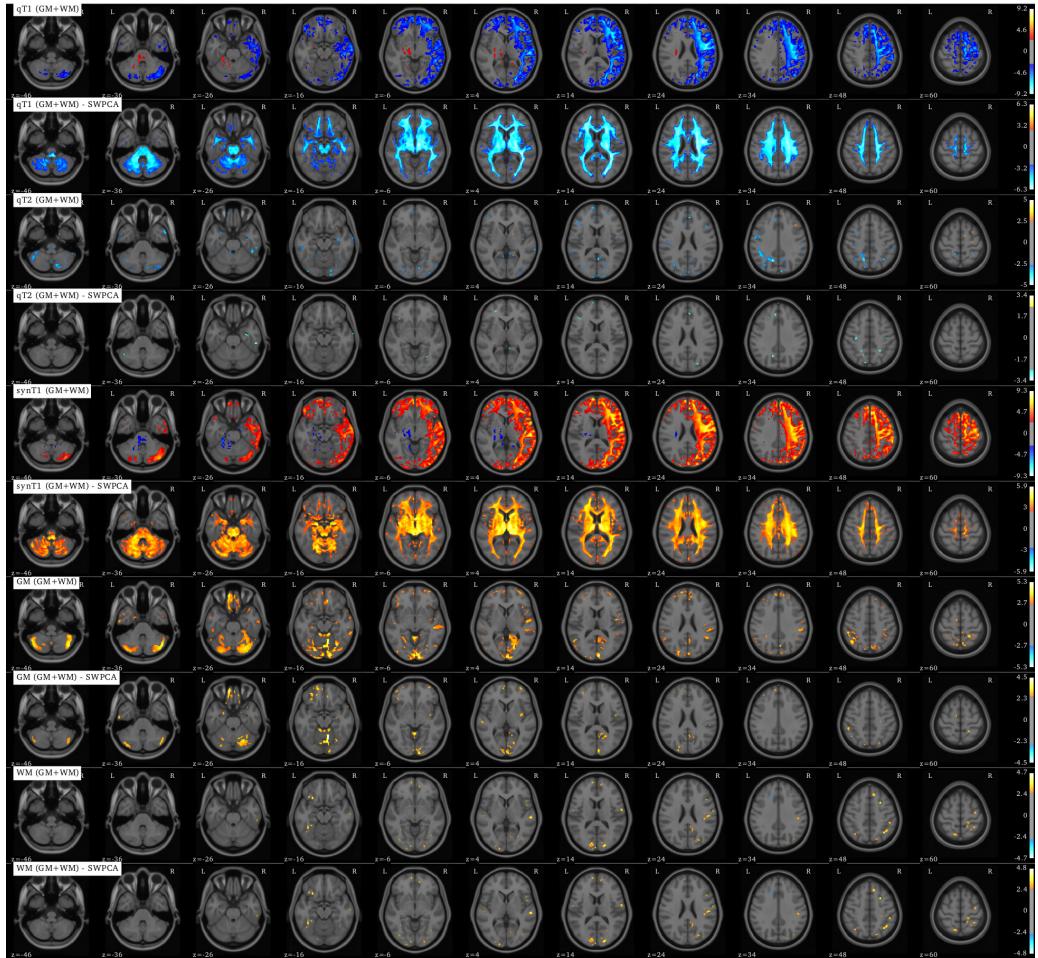


Figure 7.4: Brain t-map (VBM) of significant ($p < 0.01, |t| > 2.57$) GM and WM between-group differences using qT₁, qT₂, synT₁, GM and WM modalities after applying SWPCA to remove site effects.

less values derived from each image they are less influenced by the acquisition site effects. For qT_2 maps, the site-related differences were greater for the CTL participants than ASD where, according to the classification accuracy, they were nearly indistinguishable. Acquisition site differences were therefore noticeably reduced in the CTL and ALL databases, but not in the ASD.

7.2.2 Experiment 2: Within-site Between-Group Differences

In this second experiment, accuracy, sensitivity and specificity in the between-group comparison were recorded for images acquired from each site. This is an estimation of the discrimination ability of the different modalities without the influence of the site effects; Table 7.2. For all modalities, most of the values are close to a random classifier (~50%), indicative of having either no significant differences between groups, or having spatially heterogeneous patterns of sMRI measures across individuals where mass-univariate approaches are sub-optimal in detecting group differences. It is interesting to note that the London sample contained more between-group differences than those acquired in Cambridge.

7.2.3 Experiment 3: Effect of SWPCA on Group Differences

Finally, group differences were characterised with and without applying site-effects reduction via SWPCA to the five modalities.

Whole-brain VBM analysis was performed on the corrected and uncorrected maps from each modality. Figure 4 depicts the brain t-maps of significant ($p < 0.01$, $|t| > 2.57$) qT_1 , qT_2 , $synT_1$, GM and WM between-group differences, using ALL, with the GM+WM mask, before and after applying SWPCA, so that the reduction of site-related variability can be observed. Some of the highlighted areas after applying SWPCA are inconsistent across modalities, with spurious peaks and noise, including a large area around the ventricles in the qT_1 and $synT_1$ modalities related to some abnormal participants that will be discussed later. However, there were some areas that were consistent across modalities. Significant areas found across at least 4 of the 5 modalities correspond to the Advanced Automated Labelling (AAL) (Tzourio-Mazoyer, et al., 2002) areas of: A) right superior frontal gyrus, Brodmann areas 6 ($z=60$); B) the pars opercularis of the left inferior frontal gyrus, Brodmann areas 44; C) the pars triangularis of the left inferior frontal gyrus, Brodmann areas 45; D) the posterior part of the left middle temporal gyrus ($z=24$); CSF filled spaces on the margins of the ventricles ($z=-6,4,14,24$); and the left crus I of cerebellar hemisphere ($z=-26$).

MODALITY	MASK	ALL		CTL		ASD	
		NO-SWP PCA	SWPCA	NO-SWP PCA	SWPCA	NO-SWP PCA	SWPCA
qT_1	GM+WM	0.875 \pm 0.083	0.530 \pm 0.130	0.847 \pm 0.141	0.543 \pm 0.115	0.769 \pm 0.145	0.553 \pm 0.093
	GM	0.849 \pm 0.085	0.535 \pm 0.107	0.835 \pm 0.154	0.501 \pm 0.090	0.712 \pm 0.161	0.575 \pm 0.084
	WM	0.865 \pm 0.082	0.447 \pm 0.071	0.876 \pm 0.128	0.441 \pm 0.058	0.813 \pm 0.127	0.575 \pm 0.153
qT_2	GM+WM	0.596 \pm 0.128	0.503 \pm 0.093	0.615 \pm 0.196	0.454 \pm 0.075	0.506 \pm 0.192	0.476 \pm 0.103
	GM	0.596 \pm 0.126	0.493 \pm 0.097	0.549 \pm 0.187	0.478 \pm 0.108	0.497 \pm 0.197	0.425 \pm 0.091
	WM	0.612 \pm 0.131	0.560 \pm 0.128	0.576 \pm 0.195	0.550 \pm 0.146	0.541 \pm 0.185	0.575 \pm 0.172
synt $_1$	GM+WM	0.904 \pm 0.073	0.563 \pm 0.060	0.919 \pm 0.100	0.440 \pm 0.057	0.807 \pm 0.151	0.631 \pm 0.098
	GM	0.879 \pm 0.090	0.576 \pm 0.035	0.899 \pm 0.108	0.526 \pm 0.079	0.800 \pm 0.145	0.587 \pm 0.042
	WM	0.904 \pm 0.076	0.582 \pm 0.047	0.894 \pm 0.111	0.574 \pm 0.038	0.859 \pm 0.112	0.468 \pm 0.101
GM	GM+WM	0.595 \pm 0.133	0.586 \pm 0.141	0.582 \pm 0.192	0.566 \pm 0.093	0.481 \pm 0.169	0.468 \pm 0.152
	GM	0.620 \pm 0.141	0.585 \pm 0.078	0.604 \pm 0.227	0.574 \pm 0.038	0.499 \pm 0.188	0.525 \pm 0.114
	WM	0.659 \pm 0.139	0.448 \pm 0.066	0.635 \pm 0.180	0.507 \pm 0.144	0.522 \pm 0.206	0.525 \pm 0.198
WM	WM	0.639 \pm 0.124	0.549 \pm 0.072	0.578 \pm 0.194	0.516 \pm 0.126	0.549 \pm 0.160	0.526 \pm 0.136

Table 7.1: Between-site classification accuracy (\pm standard deviation) for different modalities and masks without and with [SWPCA](#) correction.

MODALITY	MASK	LONDON			CAMBRIDGE		
		ACC.	SENS.	SPEC.	ACC.	SENS.	SPEC.
qT₁	GM+WM	0.603 ± 0.175	0.512 ± 0.260	0.692 ± 0.237	0.504 ± 0.193	0.492 ± 0.276	0.515 ± 0.307
	GM	0.501 ± 0.157	0.440 ± 0.244	0.565 ± 0.245	0.484 ± 0.201	0.488 ± 0.300	0.480 ± 0.327
	WM	0.505 ± 0.174	0.485 ± 0.248	0.526 ± 0.242	0.451 ± 0.197	0.465 ± 0.297	0.435 ± 0.296
qT₂	GM+WM	0.628 ± 0.168	0.535 ± 0.246	0.719 ± 0.237	0.467 ± 0.181	0.527 ± 0.307	0.417 ± 0.314
	GM	0.539 ± 0.149	0.425 ± 0.220	0.654 ± 0.222	0.491 ± 0.196	0.548 ± 0.316	0.430 ± 0.298
	WM	0.619 ± 0.194	0.585 ± 0.262	0.655 ± 0.250	0.472 ± 0.195	0.448 ± 0.283	0.492 ± 0.290
synTr₁	GM+WM	0.665 ± 0.158	0.578 ± 0.224	0.755 ± 0.238	0.479 ± 0.201	0.478 ± 0.318	0.475 ± 0.316
	GM	0.547 ± 0.159	0.475 ± 0.237	0.622 ± 0.252	0.514 ± 0.218	0.477 ± 0.322	0.555 ± 0.342
	WM	0.515 ± 0.185	0.520 ± 0.288	0.506 ± 0.254	0.509 ± 0.209	0.472 ± 0.317	0.542 ± 0.316
GM	GM+WM	0.513 ± 0.171	0.507 ± 0.252	0.518 ± 0.245	0.488 ± 0.202	0.445 ± 0.318	0.528 ± 0.285
	GM	0.586 ± 0.174	0.610 ± 0.247	0.564 ± 0.270	0.521 ± 0.187	0.522 ± 0.303	0.535 ± 0.289
WM	GM+WM	0.471 ± 0.181	0.455 ± 0.245	0.488 ± 0.278	0.489 ± 0.206	0.502 ± 0.319	0.483 ± 0.314
	WM	0.465 ± 0.174	0.445 ± 0.243	0.484 ± 0.268	0.468 ± 0.210	0.488 ± 0.292	0.448 ± 0.305

Table 7.2: Classification accuracy (Acc), sensitivity (Sen) and specificity (Spec) ± standard deviation for each modality and mask using the participants acquired at the LON and CAM sites.

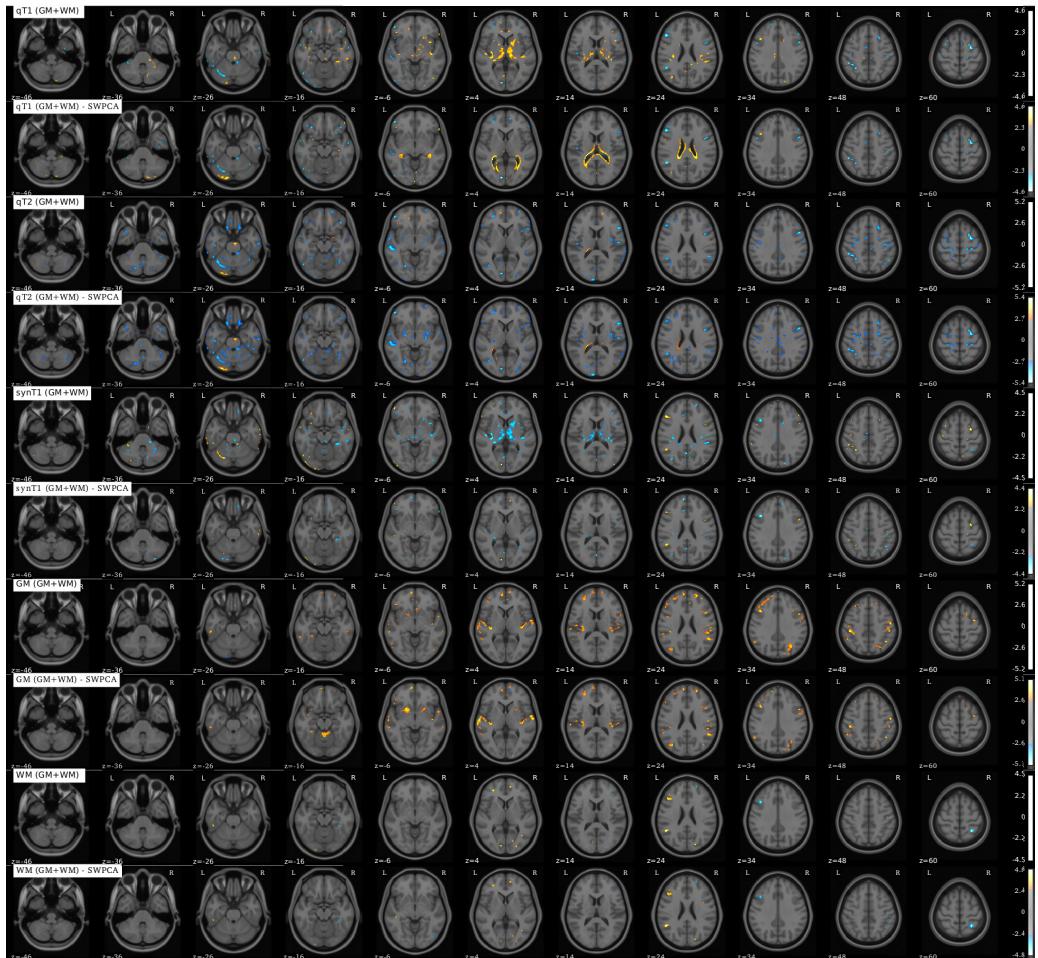


Figure 7.5: Brain t-map (VBM) of significant ($p < 0.01, |t| > 2.57$) GM and WM differences in ASD using **qT1**, **qT2**, **synT1**, **GM** and **WM** maps before and after applying **SWPCA** to remove site effects.

The complementary CBM (Section 2.4) analysis was performed on the most significant components. The resulting regions, statistically thresholded with $Z > 2.57$ (corresponding to $p < 0.01$), were superimposed on the MNI template, and are depicted in Figure 7.6. A reduction of significant between-group areas after applying SWPCA is evident in most modalities, but particularly noticeable in the qT₁ and qT₂. In WM no significant regions were observed, neither before nor after SWPCA. The significant regions identified in any modality corresponded to the AAL areas of the CSF filled areas around the ventricles (planes $z=-6, 4, 14, 24$), the right middle temporal gyrus (plane $z=14$) and the left crus I of cerebellar hemisphere (plane $z=-26$). However, none of these regions were repeated over more than two of the modalities, except for the large areas around ventricles that were caused by abnormalities in three participants, which will be discussed later.

Performance results for the classification analysis applied to ALL are shown in Table 7.3. Between-group results were quite similar before or after applying SWPCA, although reducing between-site variance generally reduced the performance towards a random classifier. The results in this table match the overall effects that were found in Figure 4, where most spurious significance peaks disappeared after applying SWPCA, but some regions were highlighted. These regions, where SWPCA did not seem to eliminate the significant areas but enhanced them, could be responsible for the accuracy increment in the analysis of the qT₂ modality, and the GM with GM mask.

7.2.4 Discussion

Brain anatomical and functional differences between ASD participants and controls have been explored by a number of previous studies (Di Martino, et al., 2014; Ecker, et al., 2015; Hernandez, et al., 2015; Lenroot and Yeung, 2013; Zürcher, et al., 2015). Many affected structures have been proposed in each of these studies, however as a recent large-scale study points out (Haar, et al., 2014), these are frequently inconsistent throughout the literature. Researchers argue that most of these structures are database-dependent, and since many studies use multi-site acquisition procedures, the variance introduced by each acquisition site is a probable source of Type I errors.

The technical and logistical drawbacks of multicentre studies are widely documented, including participant recruitment procedures (Pearlson, 2009) and technical effects that range from the usage of different equipment or acquisition parameters (Van Horn and Toga, 2009) to physical changes that affect the performance of MRI scanners across time (Pearlson, 2009). There is general recognition

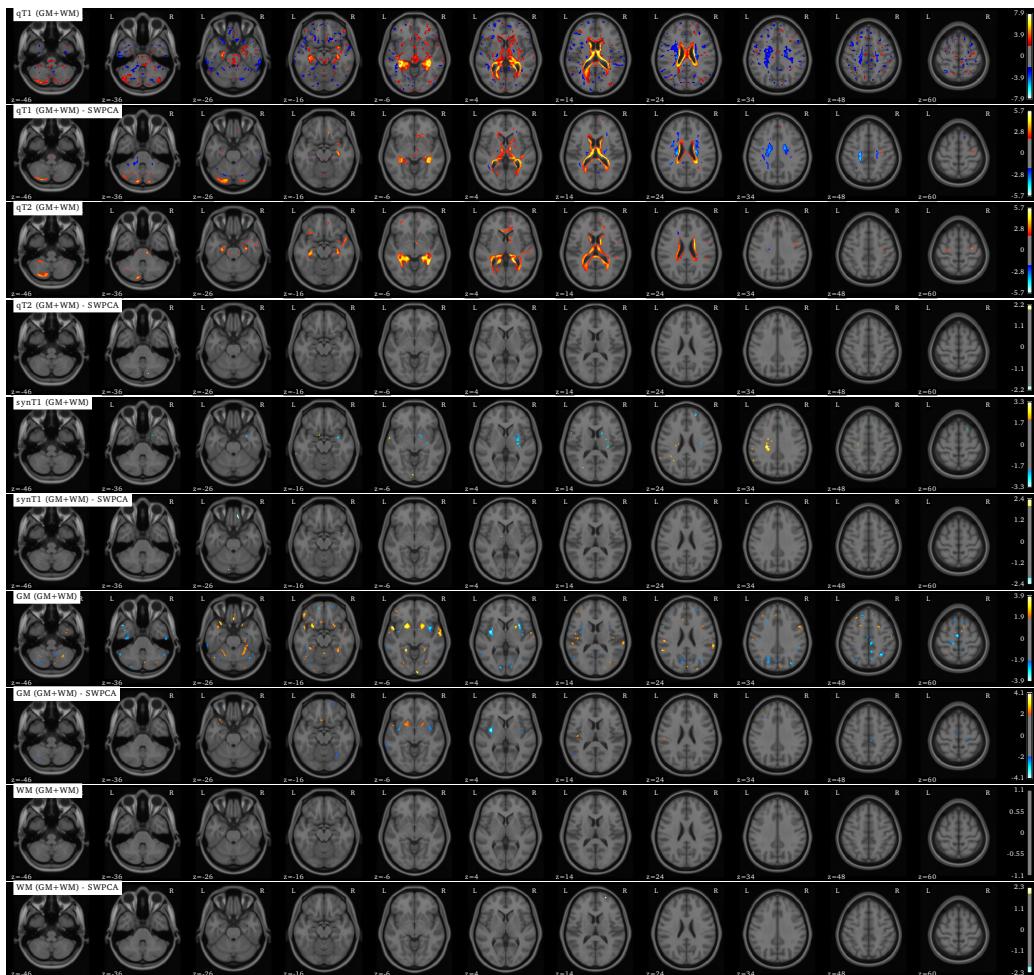


Figure 7.6: Brain Z-map (CBM) of significant ($p < 0.01, |t| > 2.57$) GM and WM differences using qT₁, qT₂, synT₁, GM and WM maps before and after applying SWPCA to remove site effects.

MODALITY	MASK	NO-SWPCA			SWPCA		
		ACC.	SENS.	SPEC.	ACC.	SENS.	SPEC.
qT_1	GM+WM	0.564 ± 0.123	0.503 ± 0.179	0.625 ± 0.177	0.435 ± 0.123	0.499 ± 0.181	0.371 ± 0.178
	GM	0.523 ± 0.112	0.468 ± 0.162	0.580 ± 0.192	0.458 ± 0.120	0.477 ± 0.187	0.441 ± 0.210
	WM	0.504 ± 0.131	0.475 ± 0.191	0.533 ± 0.194	0.484 ± 0.123	0.511 ± 0.179	0.456 ± 0.194
qT_2	GM+WM	0.578 ± 0.115	0.487 ± 0.208	0.669 ± 0.178	0.593 ± 0.136	0.546 ± 0.206	0.640 ± 0.194
	GM	0.554 ± 0.135	0.492 ± 0.194	0.614 ± 0.181	0.526 ± 0.144	0.512 ± 0.209	0.543 ± 0.222
	WM	0.516 ± 0.138	0.508 ± 0.198	0.522 ± 0.216	0.499 ± 0.137	0.477 ± 0.209	0.521 ± 0.196
$synT_1$	GM+WM	0.596 ± 0.132	0.509 ± 0.194	0.680 ± 0.172	0.577 ± 0.130	0.479 ± 0.208	0.676 ± 0.183
	GM	0.587 ± 0.139	0.509 ± 0.210	0.665 ± 0.169	0.483 ± 0.136	0.489 ± 0.218	0.480 ± 0.200
	WM	0.496 ± 0.139	0.500 ± 0.189	0.492 ± 0.194	0.487 ± 0.134	0.513 ± 0.189	0.461 ± 0.211
GM	GM+WM	0.498 ± 0.120	0.486 ± 0.197	0.507 ± 0.203	0.490 ± 0.123	0.514 ± 0.197	0.465 ± 0.182
	GM	0.574 ± 0.121	0.571 ± 0.189	0.579 ± 0.163	0.593 ± 0.127	0.602 ± 0.172	0.587 ± 0.190
WM	GM+WM	0.499 ± 0.132	0.506 ± 0.189	0.487 ± 0.181	0.521 ± 0.129	0.510 ± 0.209	0.532 ± 0.180
	WM	0.506 ± 0.143	0.488 ± 0.219	0.526 ± 0.197	0.507 ± 0.122	0.521 ± 0.165	0.492 ± 0.193

Table 7.3: Classification accuracy (Acc), sensitivity (Sen), and specificity (Spec) ± STD for the different modalities and masks using ALL, before and after applying SWPCA.

that standardization is needed to ensure the uniformity of the acquired maps. Different approaches have been used in large-scale studies, such as [ADNI](#) where human “phantoms” were used to perform a preparatory optimisation of [MRI](#) scanning platforms (Friedman and Glover, 2006).

There are two major types of site effects, regardless of their source: geometric distortions and intensity inhomogeneities. In this work, we focused on the latter, since much of the geometric distortion has been eliminated during acquisition (see Section 2.1), and the DARTEL normalization and registration acts as a homogenizing step, reducing both between-site and between-subject geometric differences, substantially reducing the impact of the site-related geometric differences.

Regarding intensity correction, in the MRC AIMS database used in this study (Ecker, et al., 2013; Ecker, et al., 2012), a standardization procedure based on quantitative imaging (Deoni, et al., 2008) was used to minimize inter-site variance and improve the signal-to-noise contrast. However, as the between-site analysis in Section 7.2.1 suggests, this strategy still results in variance that makes it easier to distinguish scanning sites than diagnostic groups. For example, when using [qT₁](#) the accuracy for LON vs. CAM classification was >80%, whilst when classifying ASD vs. CTL it was 52%. This marks the substantial effect of site variance on the maps’ intensity distribution, even when the multi-site study employs quantitative imaging protocol on the same model of scanner platform across sites. However, with the inclusion of [GM](#) and [WM](#) maps, we can observe that the inhomogeneities found on [qT₁](#) or [synT₁](#) barely affected the segmentation procedure.

In this work, the approach we have taken is to perform a multivariate decomposition of each dataset into a number of components that explain different portions of variance. The following step was to identify the components of variance that are due to multi-site acquisition and reduce them. Decomposition was completed using PCA and then, to identify which of the components were linked to acquisition site, we performed an ANOVA on the component scores. Finally, using the weighting function defined in Sec. 7.2.3, we reconstructed the original signal reducing the undesired variance, in what we called Significance Weighted PCA (SWPCA). The method has proven its ability in reducing undesired variance, quantifiable by means of the accuracy obtained in a site vs. site classification. In this case, SWPCA reduced the accuracy from >0.8 to approximately ~0.5, a random classifier, suggesting that most site-related variance was eliminated.

A simpler approach such as applying a voxel-by-voxel ANOVA would also be useful to reduce the acquisition site effects (Suckling, et al., 2012). However, SWPCA is a multivariate approach that still offers major advantages over this

voxel-wise algorithm, and similar algorithms have found utility in text document searches (Kriegel, et al., 2008; Tavoli, et al., 2013; Zhang and Nguyen, 2005). First, PCA models the different sources of variance of the dataset, whereas a simple voxel-wise ANOVA only removes mean site differences, which might result in less statistical power. Secondly, SWPCA is multivariate in nature, where each component contains information that potentially affects all voxels. Together, these two features allow SWPCA to identify the components linked to the undesired effects, and reduce their impact with a weighted reconstruction approach, reducing the general variance related to the acquisition site. However, this increased power reveals a major drawback: SWPCA needs at least a moderate number of participants to work properly. That is the reason why we cannot apply SWPCA to databases such as ADNI (Friedman and Glover, 2006) or ABIDE (Di Martino, et al., 2014), where the number of participants acquired at each site is small, or to the six travelling phantoms used in the calibration of the MRC AIMS study.

There exist a number of similar multivariate methods that model the influence of categorical variables, such as the well-known Partial Least Squares (PLS) algorithm (Vinzi, et al., 2010) or Surrogate Variable Analysis (SVA) (Leek and Storey, 2007). In the first case, both PLS and SWPCA take categorical variables \mathbf{Y} along with the data \mathbf{X} as inputs to partition the influence of these into components. However, the most significant difference is the underlying model. Whilst SWPCA estimates the principal components blindly using their variance, which is what we aim to reduce, and performs an ANOVA afterwards, PLS uses the categorical variable in the computation of the covariance matrix and then estimates the components.

On the other hand, SVA, used for gene expression studies (Leek and Storey, 2007), is more comparable to SWPCA. The SVA algorithm uses a number of decomposition and significance estimation steps to construct a set of surrogate variables; that is, variables that account for the unmodeled variance and expression heterogeneity. While similar to SWPCA in the steps used (i.e. SVD decomposition and significance estimation), their approaches are fundamentally different. SVA constructs a higher complexity model that starts by eliminating the contribution of primary variables to produce a number of unknown hidden (surrogate) variables, whereas SWPCA is intended to reduce complexity by producing variance-reduced maps to reduce the influence of previously known, but unconsidered, variables and facilitate a subsequent analysis focused only on the relevant variables.

Focusing on the VBM results, after performing the site-effects removal by SWPCA significant between-group differences were noted in five areas: A) the right superior frontal gyrus; B) the pars opercularis of the left inferior frontal gyrus;

C) the pars triangularis of the left inferior frontal gyrus; D) the posterior part of the left middle temporal gyrus; and E) the left crus I of cerebellar hemisphere. The first three regions are within Brodmann areas 6, 44 and 45. However, when examining the projection of the region D onto the MNI template (see Figure 6), it is also located in the posterior part of the left superior temporal gyrus. Therefore, D corresponds closely with the region between Brodmann areas 22 and 39, the Temporo-Parietal Junction (TPJ), with negative t-value at the left side (containing Wernicke's area) and positive t-value at the right side.

The role of these regions in autism has received much attention. Brodmann areas 44 and 45, that together make the Broca's Area (of importance in speech production and a proposed part of the human mirror neuron system (Nishitani, et al., 2005)), is a region where mirror neuron dysfunction has been consistently reported in ASD-affected children (Dapretto, et al., 2006) and adults (Hadjikhani, et al., 2006; Lopez-Hurtado and Prieto, 2008; Verly, et al., 2014). Wernicke's area, contained in the left TPJ, is also linked to language, and has been associated with ASD in several works (Hadjikhani, et al., 2006; Kriegel, et al., 2008; Verly, et al., 2014). Additionally, the right TPJ has been proposed as related to mentalizing and has been repeatedly implicated in autism (Barnea-Goraly, et al., 2004), including a fMRI study of a subsample of this same AIMS dataset (Lombardo, et al., 2011). The right superior frontal gyrus (region A) is more equivocal, with some studies (Ecker, et al., 2010; Ecker, et al., 2012) reporting abnormalities in this area, while others (Hadjikhani, et al., 2006; Segovia, et al., 2014) report no significant differences. Our analyses reveal no differences in the insula and amygdala, brain structures frequently linked to autism.

Some regions, particularly in qT₂, synT₁ and segmented GM maps show potentially spurious significance peaks around the ventricles and especially in the left crus I of cerebellar hemisphere (region E). After examining the database, two individuals had appreciable structural abnormalities in the form of abnormal ventricle size and cerebellar atrophy, as can be seen in Figure 7. It is possible that these participants influenced the computation of the t-maps, and therefore are responsible for the significance in region E and areas surrounding the ventricles and, since they are part of the LON subdataset, could also be responsible for the increased classification accuracy of the quantitative T₁ and T₂, and the synthetic T₁ maps in this sub-dataset.

After observing the influence of these participants on the computation of the t-maps, we can assume that most of the structural differences in ASD are so subtle that the influence of just one or two images can impact on the final results. This, along with the poor performance of the classification pipeline presented in Section 3, dramatically reduces the significance of the aforementioned t-maps. Therefore, the existing evidence leads to the conclusion that ASD presents as ei-

ther undetectable structural differences or, more likely, with such heterogeneous differences that are difficult to establish a common pattern even after reducing the variance introduced by acquisition site.

It may be the case that cohorts of individuals examined at different sites are somehow systematically biased towards a specific type of patient (in ways that we cannot see simply based on phenotypic information), then site-related intensity variability is also enriched with important variability about nested autism subgroups. So with any technique trying to remove the site-related inhomogeneity, the subgroup information could also be removed. Together, the evidence supports the claim that defining meaningful subgroups based on different measures, such as genetic profiling, clinical co-morbidities or sensory sensitivities, is the most urgent next step for ASD research (Haar, et al., 2014).

7.3 RESULTS FOR DATSCAN DATASETS

SWPCA	NORM.	PERFORMANCE		
		ACC.	SENS.	SPEC.
no	max	0.883 ± 0.030	0.855 ± 0.058	0.915 ± 0.058
	int	0.877 ± 0.035	0.849 ± 0.073	0.908 ± 0.079
	stable	0.898 ± 0.033	0.883 ± 0.057	0.915 ± 0.079
yes	max	0.539 ± 0.100	0.527 ± 0.373	0.550 ± 0.337
	int	±	±	±
	stable	0.361 ± 0.102	0.394 ± 0.295	0.322 ± 0.270

Table 7.4: Performance measures for the combined DaTSCAN dataset found before and after applying SWPCA.

8

SIMULATION OF FUNCTIONAL BRAIN IMAGES

8.1 SIMULATION PROCEDURE

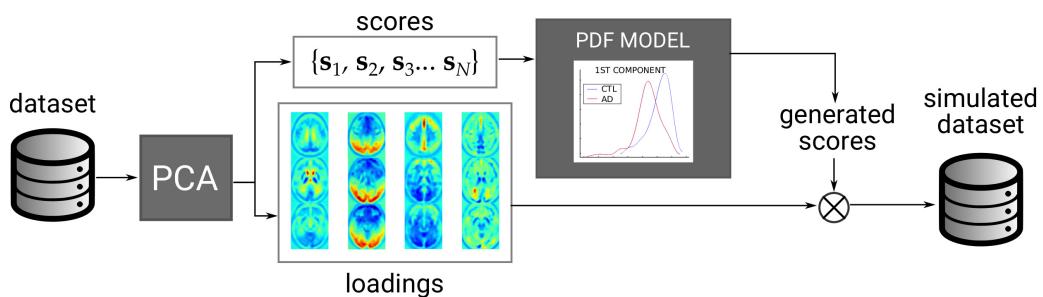


Figure 8.1: Schema of the brain image synthesis algorithm.

8.1.1 Decomposition via PCA

The first step in our simulation algorithm is to project the original dataset to a new space defined by the principal components of the set; that is, the eigenbrain space. In this space, each subject from the original dataset is projected to a point, and we can afterwards use the space basis (the principal components) to reconstruct that particular subject. In this work we will use the first N components for performance, where N is the number of subjects that are used in the computation of [PCA](#). For more details about [PCA](#), see Section [7.1.1](#).

8.1.2 Probability Density Modelling using Kernel Density Estimation

Kernel Density Estimation ([KDE](#)) is used here to model the statistical distribution of the projected subjects in the eigenbrain space, and it is applied independently to each [AD](#), [MCI](#) and [CTL](#) class. The [KDE](#) estimates the probability density

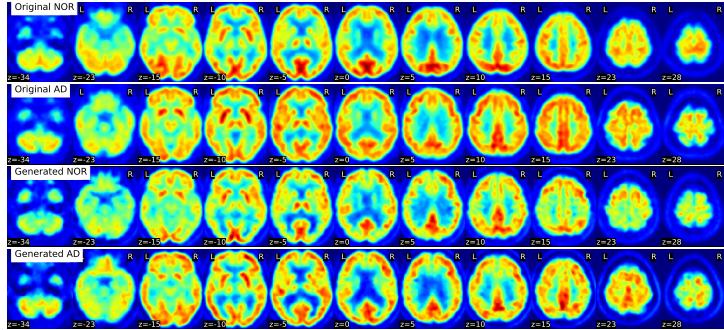


Figure 8.2: Comparison between simulated and original images from [AD](#) and [CTL](#) classes.

function f from a number of independent and identically distributed samples (x_1, x_2, \dots, x_n) , in the following manner:

$$\hat{f}_h(x) = \frac{1}{n} \sum_{i=1}^n K_h(x - x_i) = \frac{1}{nh} \sum_{i=1}^n K\left(\frac{x - x_i}{h}\right), \quad (8.1)$$

where $h > 0$ is the bandwidth, a smoothing parameter. The [KDE](#) via diffusion [[Botev2010](#)] used in this article uses a data-driven automatic estimation of the bandwidth, which unlike most methods, does not rely on arbitrary normal reference rules.

8.1.3 Probability Density Modelling using Multivariate Gaussian

8.1.4 Random Number Generation

8.1.5 Brain Image Synthesis

8.2 EXPERIMENTAL SETUP

To validate the simulated dataset, we have performed two different experiments:

- **Exp. 1:** We have estimated the predictive power of the simulated images by generating new images from the original training set in each cross-validation iteration and using them to predict the original test set.
- **Exp. 2:** We tested that the simulated images are independent from the original ones, although preserving similar characteristics. To do so, following a Voxel as Features (VAF) approach [[Stoeckel04](#)], we extract a small subset

(10 AD and 10 NOR) from the original dataset. Then, we trick the classifier, training it with the whole subset -instead of the training set only-, and testing it against the test set. Therefore, the performance of the tricked system must be close to 1. Then, we generate a new set of simulated images (100 AD and 100 NOR) from the reduced subset and proceed similarly. If our simulated images are independent from the originals, the performance of the system should decrease substantially.

Classification is performed using a Support Vector Machine (SVM) classifier with linear kernel. Estimation of parameter C is performed in an inner cross-validation loop within the training set. Values of accuracy (acc), sensitivity (sens) and specificity (spec) and their standard deviation (SD) are estimated.

8.3 RESULTS FOR ADNI-PET DATASET

8.3.1 Experiment 1

The performance results for the proposed experiments are shown in Table 8.1. Exp. 1 is applied to three different scenarios: only AD vs NOR (95 vs 101 subjects), and after incorporating MCI subjects, using them as NOR or AD.

SCENARIO	ACC (\pm SD)	SENS (\pm SD)	SPEC (\pm SD)
AD vs NOR	0.882 ± 0.012	0.865 ± 0.091	0.901 ± 0.118
MCI as NOR	0.727 ± 0.119	0.769 ± 0.155	0.789 ± 0.151
MCI as AD	0.739 ± 0.126	0.747 ± 0.147	0.845 ± 0.146

Table 8.1: Baseline performance of the set, using the original dataset.

SCENARIO	ACC (\pm SD)	SENS (\pm SD)	SPEC (\pm SD)
AD vs NOR	0.801 ± 0.095	0.782 ± 0.202	0.821 ± 0.191
MCI as NOR	0.751 ± 0.078	0.433 ± 0.201	0.851 ± 0.262
MCI as AD	0.712 ± 0.048	0.821 ± 0.062	0.382 ± 0.248

Table 8.2: Performance of Exp 1, demonstrating the predictive ability of the simulated images over the real dataset.

SCENARIO	ACC (\pm SD)	SENS (\pm SD)	SPEC (\pm SD)
Original	1.000 ± 0.000	1.000 ± 0.000	1.000 ± 0.000
Simulated	0.839 ± 0.094	0.830 ± 0.228	0.849 ± 0.206

Table 8.3: Performance of the Exp 3 proves the independence of the simulated images with respect to the originals.

8.4 RESULTS FOR DATSCAN DATASETS

Part IV

GENERAL DISCUSSION AND CONCLUSIONS

9

GENERAL DISCUSSION AND CONCLUSIONS

9.1 GENERAL DISCUSSION

9.1.1 Discussion on the algorithms

9.1.2 Discussion on the diseases

9.2 CONCLUSIONS

9.3 FUTURE WORK

asdf

Part V
APPENDIX

A | DATASETS

Many dataset are used in this thesis, covering three imaging modalities and three disorders. A summary of these can be found on Table A.1, folowed by a longer description of each one.

ACRONYM	ORIGIN	DISEASE	MODALITY	DRUG
ADNI-MRI	ADNI	AD	MRI	-
AIMS-MRI	MRC-AIMS	ASD	MRI	-
ADNI-PET	ADNI	AD	PET	HMPAO!
VDLN-HMPAO	VDLN	AD	SPECT	HMPAO!
VDLN-DAT	VDLN	PKS	SPECT	DaTSCAN
VDLV-DAT	VDLV	PKS	SPECT	DaTSCAN
PPMI-DAT	PPMI	PKS	SPECT	DaTSCAN

Table A.1: Summary of the datasets used in this thesis.

A.1 MAGNETIC RESONANCE IMAGING

A.1.1 ADNI-MRI, Alzheimer's Disease Neuroimaging Initiative

AD

A.1.2 AIMS-MRI, MRC-AIMS Consortium

Structural MRI were analysed from 136 adult, right-handed males (68 with ASD and 68 matched controls) with no significant mean differences in age and full-scale IQ, acquired from the centres contributing to the UK Medical Research Council Autism Imaging Multi-centre Study (MRC AIMS) (Ecker, et al., 2013; Ecker, et al., 2012) and recruited by advertisement. In this work, only participants recruited at the Institute of Psychiatry, King's College London (LON) and

the Autism Research Centre, University of Cambridge (CAM) were included where an equivalent set of images were acquired from each participant.

Participants were excluded from the study if they had a history of major psychiatric disorder or medical illness affecting brain function (e.g. psychosis or epilepsy), or current drug misuse (including alcohol), or were taking antipsychotic medication, mood stabilizers or benzodiazepines.

All participants with ASD were diagnosed according to International Classification of Diseases, 10th Revision (ICD-10) research criteria, and confirmed using the Autism Diagnostic Interview-Revised (ADI-R) (Lord, et al., 1994). Autism Diagnostic Observation Schedule (ADOS) (Lord, et al., 2000) was performed, but the score was not considered as an inclusion criteria. ASD participants, to be included, must have scored above the ADI-R cut-off in the three domains of impaired reciprocal social interaction, communication and repetitive behaviours and stereotyped patterns, although failure to reach cut-off in one of the domains by one point was permitted. Intellectual ability was assessed using the Wechsler Abbreviated Scale of Intelligence (WASI) (Wechsler, 1999), ensuring the participants fell within the high-functioning range on the spectrum defined by a full-scale IQ > 70. The demographics of the participants are shown in detail in Table A.2.

DATABASE	GROUP	N	AGE ($\mu \pm \sigma$ YEARS)	IQ ($\mu \pm \sigma$)
LON	ASD	39	28.74 ± 6.52	111.28 ± 13.13
	CTL	40	25.30 ± 6.62	104.67 ± 11.16
CAM	ASD	29	26.83 ± 4.64	115.83 ± 11.88
	CTL	28	26.75 ± 7.32	115.25 ± 13.67
ALL	ASD	68	25.90 ± 6.95	109.03 ± 13.31
	CTL	68	27.93 ± 5.87	113.22 ± 12.81

Table A.2: Demographics of the AIMS-MRI dataset.

Structural MRI were obtained using Driven Equilibrium Single Pulse Observation of T₁ and T₂ (DESPOT₁, DESPOT₂) (Deoni, et al., 2008) at King's College London and University of Cambridge, both with 3T GE Medical Systems HDx scanners. Using multiple Spoilt Gradient Recall (SPGR) acquisitions in the DESPOT₁ sequence and Steady State Free Procession (SSPF) acquisitions in the DESPOT₂ sequence, with different flip angles and repetition times, qT₁ and qT₂ maps were calculated with a custom ImageJ plug-in package. Correction of main and transmit magnetic field (B₀ and B₁) inhomogeneity effects was performed during the estimation of T₁ and T₂.

For accurate registration to the standard stereotatic space of the [MNI](#), a [synT₁](#) images were created based on the [qT₁](#) maps (Ecker, et al., 2013; Ecker, et al., 2012; Lai, et al., 2012). The [synT₁](#) images were then segmented using New Segment into [GM](#) and [WM](#) maps, and normalized to the [MNI](#) space using DARTEL in SPM8 (Friston, et al., 2007), with modulation (preserve volume) to retain information of regional/local [GM](#) and [WM](#) volumes, and smoothed with a 3mm FWHM Gaussian Kernel to account for inter-subject mis-registration. The [synT₁](#), [qT₁](#) and [qT₂](#) maps were also registered to the standard [MNI](#) space using the same DARTEL flow fields, but without modulation (preserve concentration) to retain information of regional/local T₁ contrast, T₁ relaxation time, and T₂ relaxation times, and smoothed with a 3mm FWHM Gaussian kernel. Therefore, there were five different modalities: [qT₁](#), [qT₂](#), [synT₁](#) map, [GM](#) and [WM](#) maps, for each every participant, which allows us to observe the impact of our [SWPCA](#) correction of site-related undesired variance on quantitative ([qT₁](#) and [qT₂](#)), simulated ([synT₁](#)) images and probability maps ([GM](#) and [WM](#)).

During the pre-processing of the images, several procedures targeted the reduction of inter-subject and inter-site geometric distortion, amongst them the correction of Bo and B₁ field inhomogeneity effects and the registration to [MNI](#) space. Many other algorithms have been proposed to help in this task. However, the study of their relative performance lies beyond the scope of this article. Following image registration, it was assumed that only the intensity of the maps was affected between sites.

A.2 POSITRON EMISSION TOMOGRAPHY

A.2.1 ADNI-PET, Alzheimer's Disease Neuroimaging Initiative

Data used in the preparation of this article were obtained from the Alzheimer's Disease Neuroimaging Initiative (ADNI) database (adni.loni.usc.edu). The ADNI was launched in 2003 as a public-private partnership, led by Principal Investigator Michael W. Weiner, MD. The primary goal of ADNI has been to test whether serial MRI, PET, other biological markers, and clinical and neuropsychological assessment can be combined to measure the progression of MCI and AD. For up-to-date information, see www.adni-info.org.

Data used in the preparation of this article were obtained from the Alzheimer's Disease Neuroimaging Initiative (ADNI) database (adni.loni.usc.edu). In this work, the ¹⁸F-FDG PET images, used to estimate the metabolic activity of the brain, are used to generate and validate the simulated images. 95 PET images from AD affected subjects, 207 images from Mild Cognitive Impairment (MCI)

affected subjects and 101 images from Normal Controls (NOR) have been used to construct the original $N = 403$ set from which the simulation parameters will be obtained.

A.3 SINGLE PHOTON EMISSION COMPUTED TOMOGRAPHY

A.3.1 VDLN-HMPAO, Virgen de las Nieves

The database is built up of imaging studies of subjects following the protocol of an hospital-based service. First, the neurologist evaluated the cognitive function, and those patients with findings of memory loss or dementia were referred to the nuclear medicine department in the “Virgen de las Nieves” hospital (Granada, Spain), in order to acquire complementary screening information for diagnosis¹. Experienced physicians evaluated the images visually. The images were assessed using 4 different labels: [CTL](#) for subjects without scintigraphic abnormalities and mild perfusion deficit (AD1), moderate deficit (AD2) and severe deficit (AD3), to distinguish between different levels of presence of hypo-perfusion patterns compatible with AD. In total, the database consists of $n = 97$ subjects: 41 [CTRL](#), 30 AD1, 22 AD2 and 4 AD3 (see table A.3 for demographic details). Since the patients are not pathologically confirmed, the subject’s labels possesses some degree of uncertainty, as the pattern of hypo-perfusion may not reflect the underlying pathology of AD, nor the different classification of scans necessarily reflect the severity of the patients symptoms. However, when pathological information is available, visual assessments by experts have been shown to be very sensitive and specific labelling methods, in contrast to neuropsychological tests [[jobst_accurate_1998](#), [dougall_systematic_2004](#)]. Given that this is an inherent limitation of ‘in vivo’ studies, our working-assumption is that the labels are true, considering the subject label positive when belonging to any of the AD classes, and negative otherwise.

A.3.2 VDLN-DAT, Virgen de las Nieves

SPECT DATSCAN

¹ Clinical information is unfortunately not available for privacy reasons, but only demographic information

	#samples	Sex(M/F)(%)	μ [range/ σ]
CTRL	41	32.95/12.19	71.51[46-85/7.99]
AD1	29	10.97/18.29	65.29[23-81/13.36]
AD2	22	13.41/9.76	65.73[46-86/8.25]
AD3	4	0/2.43	76[69-83/9.90]

Table A.3: Demographic details of the ADNI-PET dataset. CTRL = Normal Controls, AD 1 = possible AD, AD 2 = probable AD, AD 3 = certain AD. μ and σ stands for population mean and standard deviation respectively.

A.3.3 VDLV-DAT, Virgen de la Victoria Hospital

The images were obtained after a period of between 3 and 4 hours after the intravenous injection of 185 MBq (5 mCi) of DaTSCAN, with prior thyroid blocking with Lugol's solution. The tomographic study (SPECT) with Ioflupane/FP-CIT-I-123 was performed using a General Electric gamma camera, Millennium model, equipped with a dual head and general purpose collimator. A 360-degree circular orbit was made around the cranium, at 3-degree intervals, 60 images with a duration of 35 seconds per interval, 128×128 matrix. Image reconstruction was carried out using filtered back-projection algorithms without attenuation correction [Shepp82, Vardi1985], application of a Hanning filter (frequency 0.7) and images were obtained with transaxial cuts, following the method proposed in [Ramirez2009].

The images were interpreted by three Nuclear Medicine specialists, with masking of the clinical orientation. Visual assessment was established by exclusively considering the normal/abnormal criterion and after arriving at a consensus report between the three specialists, i.e. whether the FP-CIT SPECT allowed differentiation of a group of conditions with presynaptic involvement from others in which their integrity is assumed, without trying to assign them to different clinical groups within the set of pathological studies. A study was considered to be normal when bilateral, symmetrical uptake appeared in caudate and putamen nuclei, and abnormal when there were areas of qualitatively reduced uptake in any of the striatal structures.

A total of 208 subjects (100 patients and 108 controls), randomly selected from the total studies performed in this center until December 2008 and referred to it because of a movement disorder, were included in the study. Mean age was 70.2 years (41-87) with a standard deviation of 10.2 years (a detailed description of the database can be found in [Lozano2007]). Clinical diagnosis, a parameter used as 'gold Standard' to establish the existence of PS, was made using the

diagnostic criteria established previously, with an established minimum follow-up period of 18 months. Those patients who were receiving treatment with drugs that had known or suspected effect on the level of the dopaminergic transporters through direct competitive mechanism were excluded. Although PD is the most representative pathology of PS, there are other medical conditions which, though they differ clinically from this, are also expressed by this set of symptoms. Some of them are multisystem atrophy (MSA), progressive supra-nuclear palsy (PSP) and corticobasal degeneration (CBD), in which, unlike PD, as well as involvement of the presynaptic terminal, there is involvement at the post-synaptic level of the nigrostriatal pathway.

A.3.4 PPMI-DAT, Parkinson's Progression Markers Initiative

Data used in the preparation of this article were obtained from the Parkinson's Progression Markers Initiative (PPMI) database (www.ppmi-info.org/data). For up-to-date information on the study, visit www.ppmi-info.org.

The images in this database were imaged 4 + 0.5 hours after the injection of between 111 and 185 MBq of DaTSCAN. Subjects were also pretreated with saturated iodine solution (10 drops in water) or perchlorate (1000 mg) prior to the injection. All subjects had a supplied ^{57}Co line marker affixed along the canthomeatal line, which will facilitate subsequent image processing and allow the core lab to accurately distinguish left and right in the face of multiple image file transfers. These markers are only evident in the ^{57}Co window and hence do not contaminate the ^{123}I -DaTSCAN brain data [PPMI, [Initiative2010](#)].

Raw projection data are acquired into a 128×128 matrix stepping each 3 degrees for a total of 120 projection into two 20% symmetric photopeak windows centered on 159 KeV and 122 KeV with a total scan duration of approximately 30 - 45 minutes. Other scan parameters (collimation, acquisition mode, etc.) are selected for each site. The images of both the subject's data and the cobalt striatal phantom are reconstructed and attenuation corrected, implementing either filtered back-projection or an iterative reconstruction algorithm using standardized approaches [[Initiative2010](#)]. After the processing, the database contains 289 spatially normalized images, 114 from Normal Control subjects and 175 from PD patients, and of a $91 \times 109 \times 91$ size.

B

BACKGROUND ON SUPPORT VECTOR MACHINES

Support Vector Machine ([SVM](#)), introduced in the late 70s [[Vapnik1982](#)], are a set of related supervised learning methods widely used in pattern recognition, voice activity detection (VAD), classification and regression analysis.

We suppose the data to be linearly separable. In this case, a data point is viewed as a p-dimensional vector. Our objective is to separate a set of binary labelled training data with a hyperplane that is maximally distant from the two classes (known as the maximal margin hyper-plane). To do so, we build a function $f : \Re^n \rightarrow \{\pm 1\}$ using training data that is, p-dimensional patterns x_i and class labels y_i :

$$(x_1, y_1), (x_2, y_2), \dots, (x_l, y_l) \in \Re^n \times \{\pm 1\} \quad (\text{B.1})$$

so that f will correctly classify new examples (x, y) .

Linear discriminant functions define decision hypersurfaces or hyperplanes in a multidimensional feature space, that is:

$$g(x) = \mathbf{w}^T x + \omega_0 = 0, \quad (\text{B.2})$$

where \mathbf{w} is known as the weight vector and ω_0 as the threshold. The weight vector \mathbf{w} is orthogonal to the decision hyperplane and the optimization task consists of finding the unknown parameters $\omega_i, i = 1, \dots, n$ defining the decision hyperplane.

Let $x_i, i = 1, 2, \dots, n$ be the feature vectors of the training set, X . These belong to either of the two classes, ω_1 or ω_2 . If the classes were linearly separable, the objective would be to design a hyperplane that classifies correctly all the training vectors. The hyperplane is not unique, and the selection process is focused on maximizing the generalization performance of the classifier, that is, the ability of the classifier, designed using the training set, to operate satisfactorily with new data. Among the different design criteria, the maximal margin hyperplane is usually selected since it leaves the maximum margin of separation between the two classes. Since the distance from a point x to the hyperplane is given by $z = |g(x)|/\|\mathbf{w}\|$, scaling w and w_0 so that the value of $g(x)$ is $+1$ for the nearest point in ω_1 and -1 for the nearest points in ω_2 , reduces the optimization problem to maximizing the margin: $2/\|\mathbf{w}\|$ with the constraints:

$$\mathbf{w}^T \mathbf{x} + \mathbf{w}_0 \geq 1, \forall \mathbf{x} \in \omega_1 \quad (\text{B.3})$$

$$\mathbf{w}^T \mathbf{x} + \mathbf{w}_0 \leq 1, \forall \mathbf{x} \in \omega_2 \quad (\text{B.4})$$