

Scaling Up Graph Neural Networks to Large Graphs

CS224W: Machine Learning with Graphs

Jure Leskovec, Stanford University

<http://cs224w.stanford.edu>



EXAM: LOGISTICS

- Open from **Fri 11/19 10AM to Sat 11/20 9:59AM**;
you can take it in any 2-hour period
 - If you need an extension (OAE), please request now!
- If you have any clarifying questions, make a **private** Ed post about it
- Open-book, open-notes, but **do not discuss** the exam with any other students until after Saturday

CS224W: Machine Learning with Graphs

Jure Leskovec, Stanford University

<http://cs224w.stanford.edu>



EXAM: WHAT TO EXPECT

- **11 questions** with subparts, covering different topics
 - Each should take 5-15 min (aim for 10 min on average)
 - Submit answers on Gradescope: either write-in with LaTeX or take picture + upload
 - Read over our **Exam Information Ed announcement** and **Exam Prep OH slides** for details
- **Pace yourself:** if you find yourself stuck on a question, move on to the next one. You do not have to complete the exam to get a good grade.
- Good luck!!! You're going to do great!

CS224W: Machine Learning with Graphs

Jure Leskovec, Stanford University

<http://cs224w.stanford.edu>



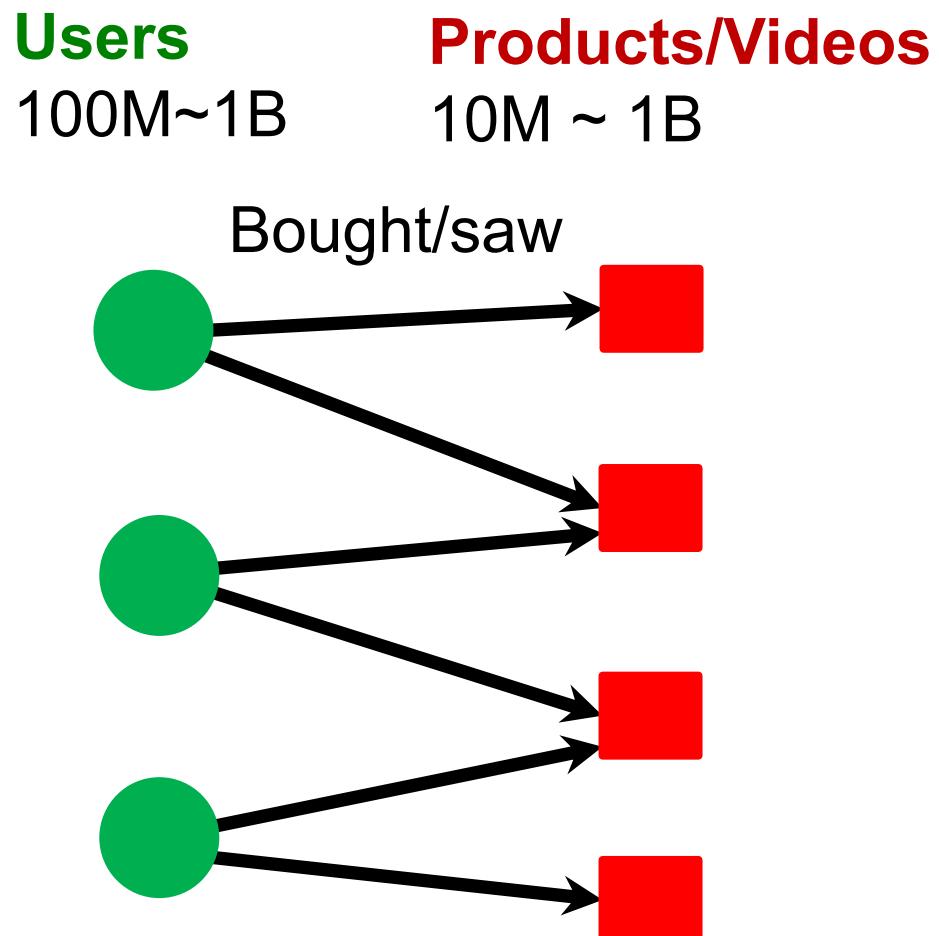
Graphs in Modern Applications

■ Recommender systems:

- Amazon
- YouTube
- Pinterest
- Etc.

■ ML tasks:

- Recommend items
(link prediction)
- Classify users/items
(node classification)



Graphs in Modern Applications

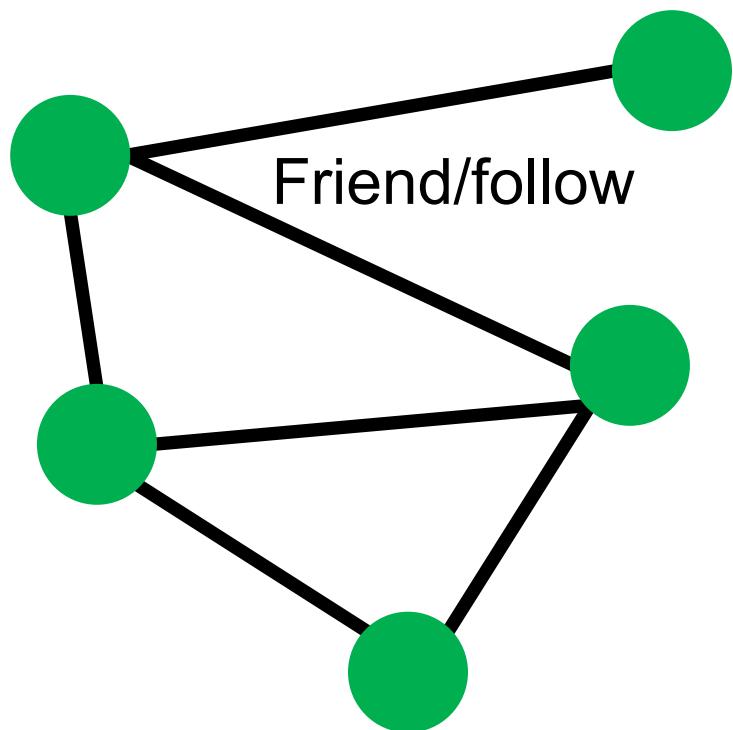
- **Social networks:**

- Facebook
- Twitter
- Instagram
- Etc.

- **ML tasks:**

- Friend recommendation
(link-level)
- User property prediction
(node-level)

Users
300M~3B



Graphs in Modern Applications

■ Academic graph:

- Microsoft Academic Graph

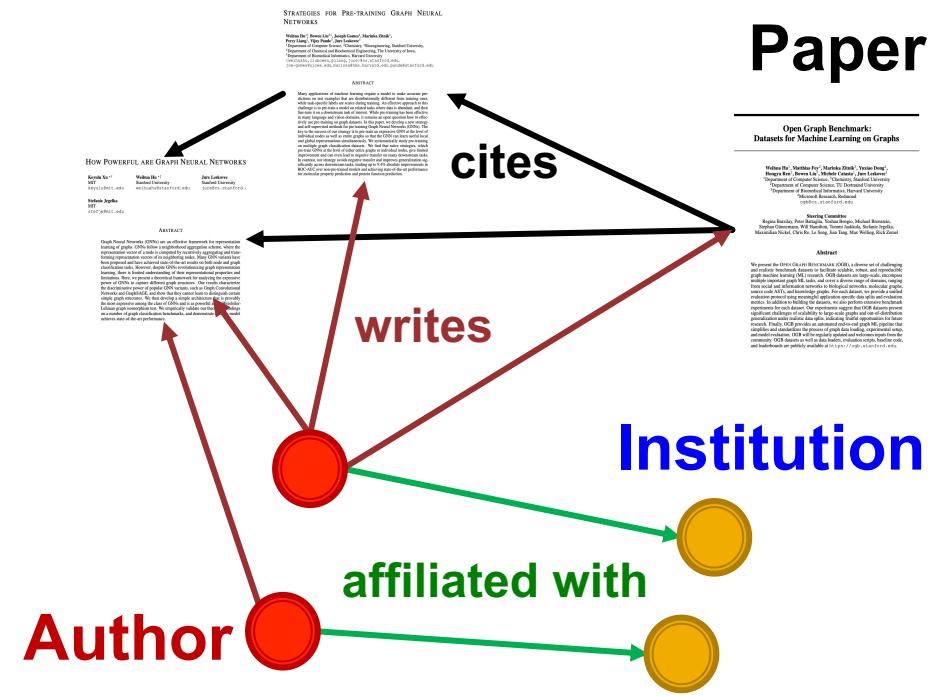
■ ML tasks:

- Paper categorization
(node classification)
- Author collaboration recommendation
- Paper citation recommendation
(link prediction)

Papers
120M

Authors
120M

Paper



Graphs in Modern Applications

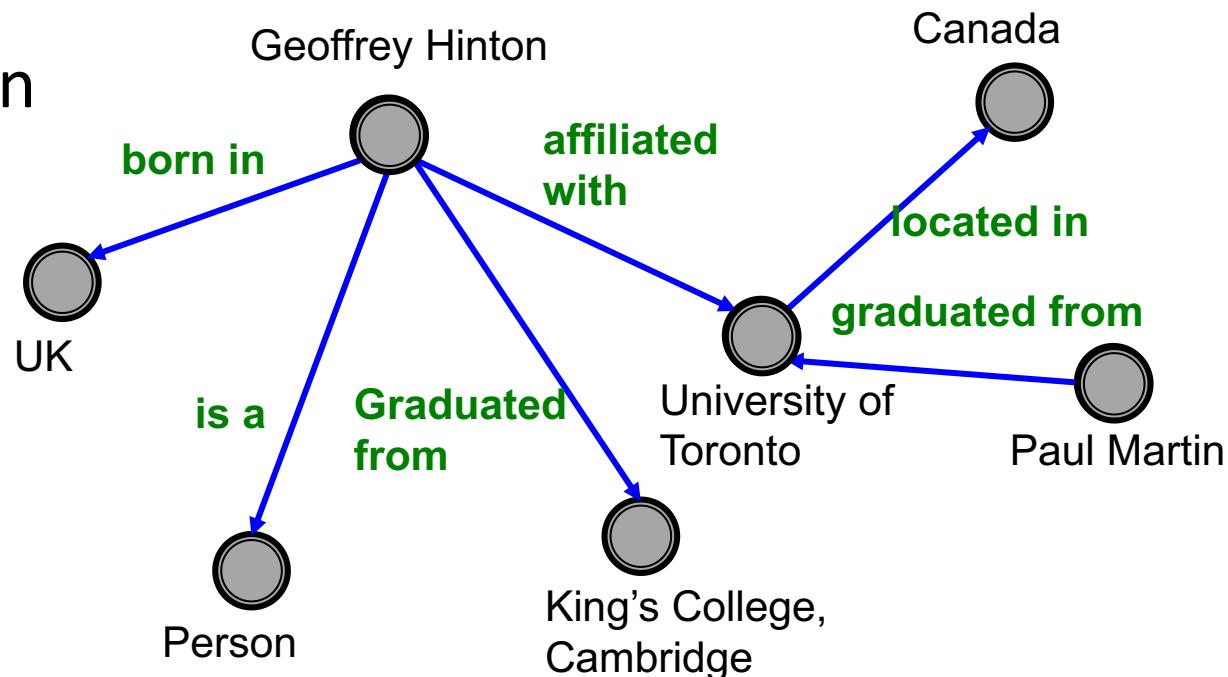
■ Knowledge Graphs (KGs):

- Wikidata
- Freebase

■ ML tasks:

- KG completion
- Reasoning

Entities
80M—90M



What is in Common?

- **Large-scale:**
 - #nodes ranges from 10M to 10B.
 - #edges ranges from 100M to 100B.
- **Tasks**
 - **Node-level:** User/item/paper classification.
 - **Link-level:** Recommendation, completion.
- **Todays' lecture**
 - **Scale up GNNs to large graphs!**

Why is it Hard?

- **Recall:** How we usually train an ML model on large data ($N=\#\text{data}$ is large)
- **Objective:** Minimize the averaged loss

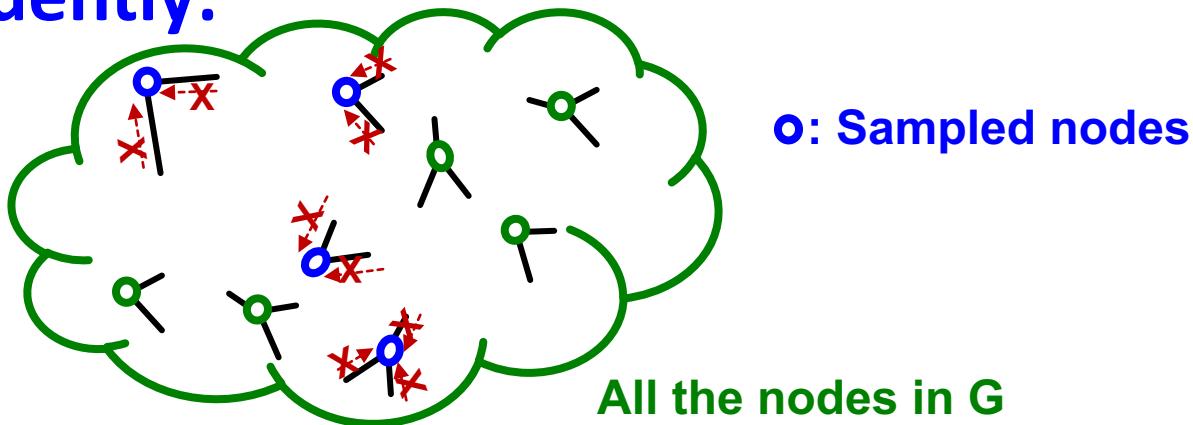
$$\ell(\boldsymbol{\theta}) = \frac{1}{N} \sum_{i=0}^{N-1} \ell_i(\boldsymbol{\theta})$$

- $\boldsymbol{\theta}$: model parameters, $\ell_i(\boldsymbol{\theta})$: loss for i -th data point.
- We perform **Stochastic Gradient Descent (SGD)**.
 - Randomly sample M ($\ll N$) data (**mini-batches**).
 - Compute the $\ell_{sub}(\boldsymbol{\theta})$ over the M data points.
 - Perform SGD: $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \nabla \ell_{sub}(\boldsymbol{\theta})$

Why is it Hard?

What if we were to use the standard SGD for GNN?

- In mini-batch, we sample M ($\ll N$) nodes independently:



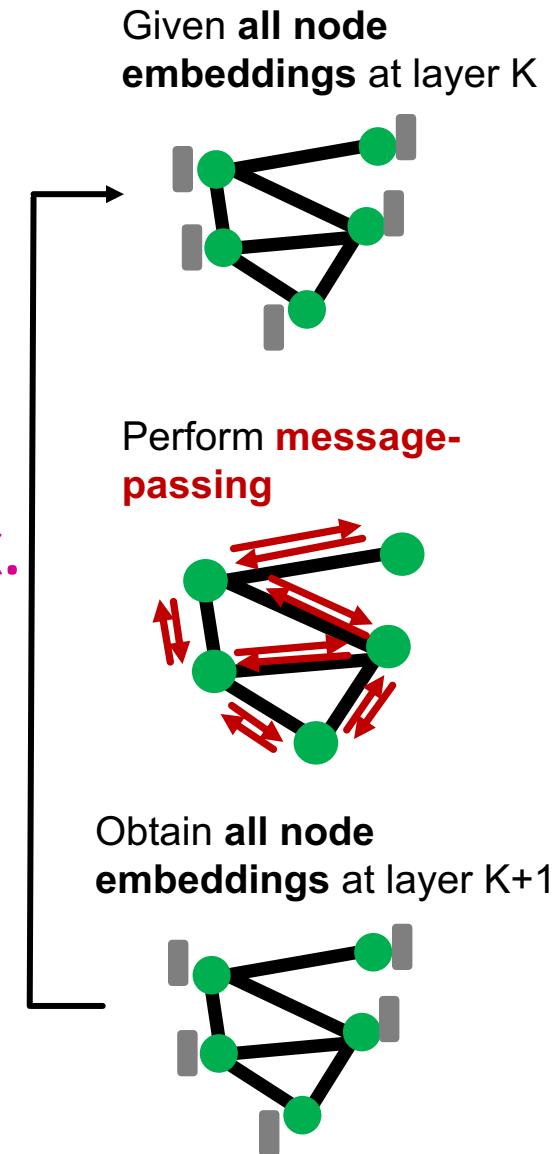
- Sampled nodes tend to be isolated from each other.
- Recall:** GNN generates node embeddings by aggregating neighboring node features.
 - GNN does not access to neighboring nodes within the mini-batch!
- Standard SGD cannot effectively train GNNs.**

Why is it Hard?

- **Naïve full-batch implementation:** Generate embeddings of all the nodes **at the same time**:

$$H^{(k+1)} = \sigma(\tilde{A}H^{(k)}W_k^T + H^{(k)}B_k^T)$$

- Load **the entire graph A and features X .**
Set $H^{(0)} = X$.
- **At each GNN layer:** Compute embeddings of all nodes using all the node embeddings from the previous layer.
- Compute the loss
- Perform gradient descent



Why is it Hard?

- However, **Full-batch** implementation is **not feasible** for a large graphs. **Why?**
- Because we want to use GPU for fast training, but GPU memory is extremely limited (only 10GB--20GB).
 - **The entire graph and the features cannot be loaded on GPU.**

Slow computation,
large memory

CPU
1TB—10TB

Fast computation,
limited memory

GPU
10GB—20GB

Today's Lecture

We introduce **three methods for scaling up GNNs**:

- Two methods perform message-passing over **small subgraphs in each mini-batch**; only the subgraphs need to be loaded on a GPU at a time.
 - **Neighbor Sampling** [Hamilton et al. NeurIPS 2017]
 - **Cluster-GCN** [Chiang et al. KDD 2019]
- One method **simplifies a GNN into feature-preprocessing operation** (can be efficiently performed even on a CPU)
 - **Simplified GCN** [Wu et al. ICML 2019]

GraphSAGE Neighbor Sampling: Scaling up GNNs

CS224W: Machine Learning with Graphs

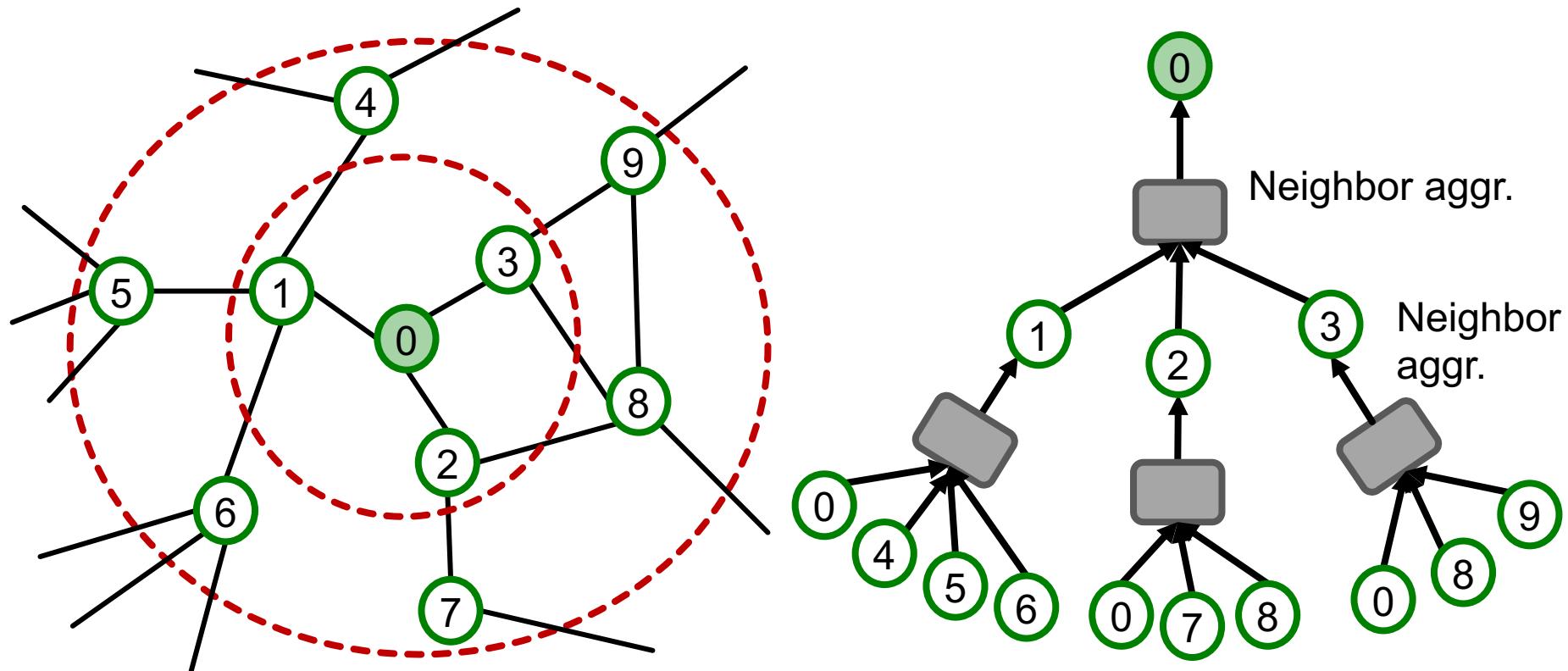
Jure Leskovec, Stanford University

<http://cs224w.stanford.edu>



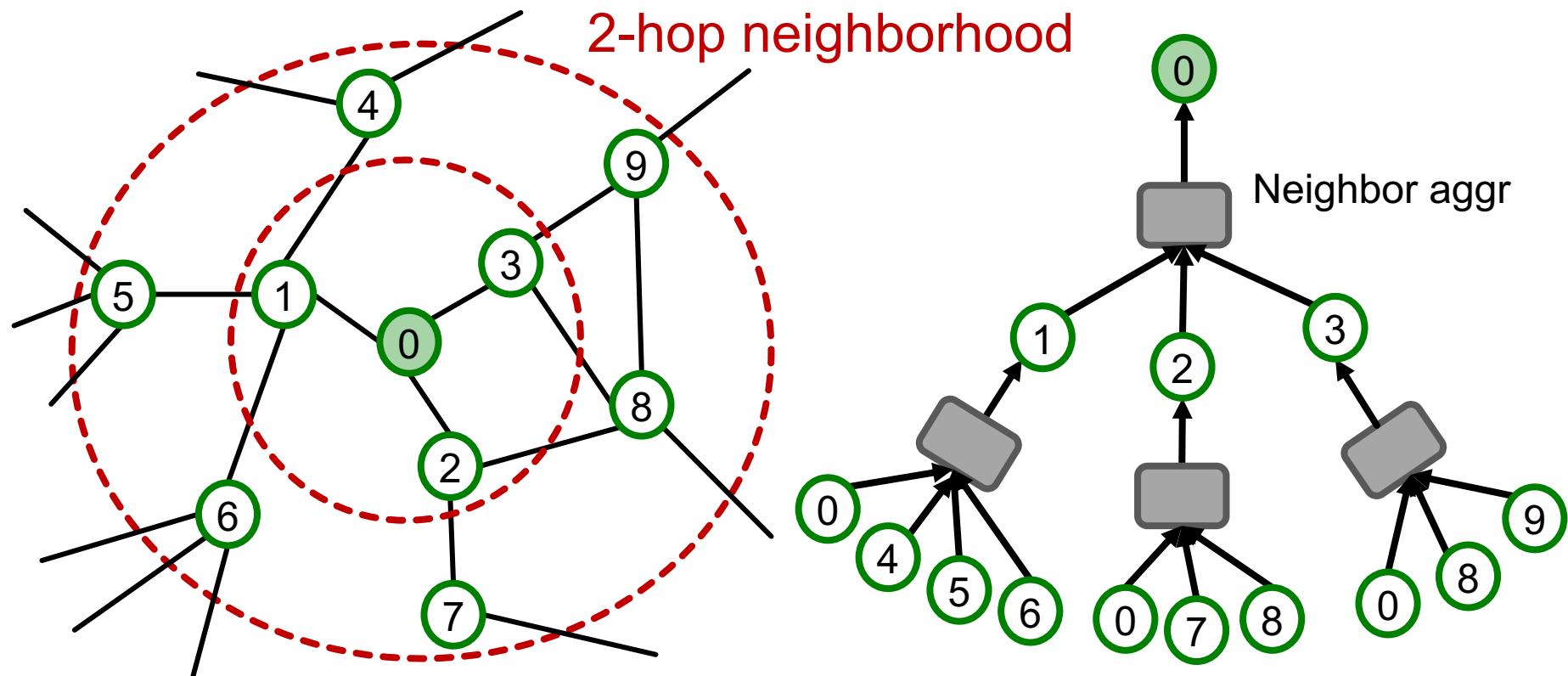
Recall: Computational Graph

- **Recall:** GNNs generate node embeddings via neighbor aggregation.
 - Represented as a computational graph (right).



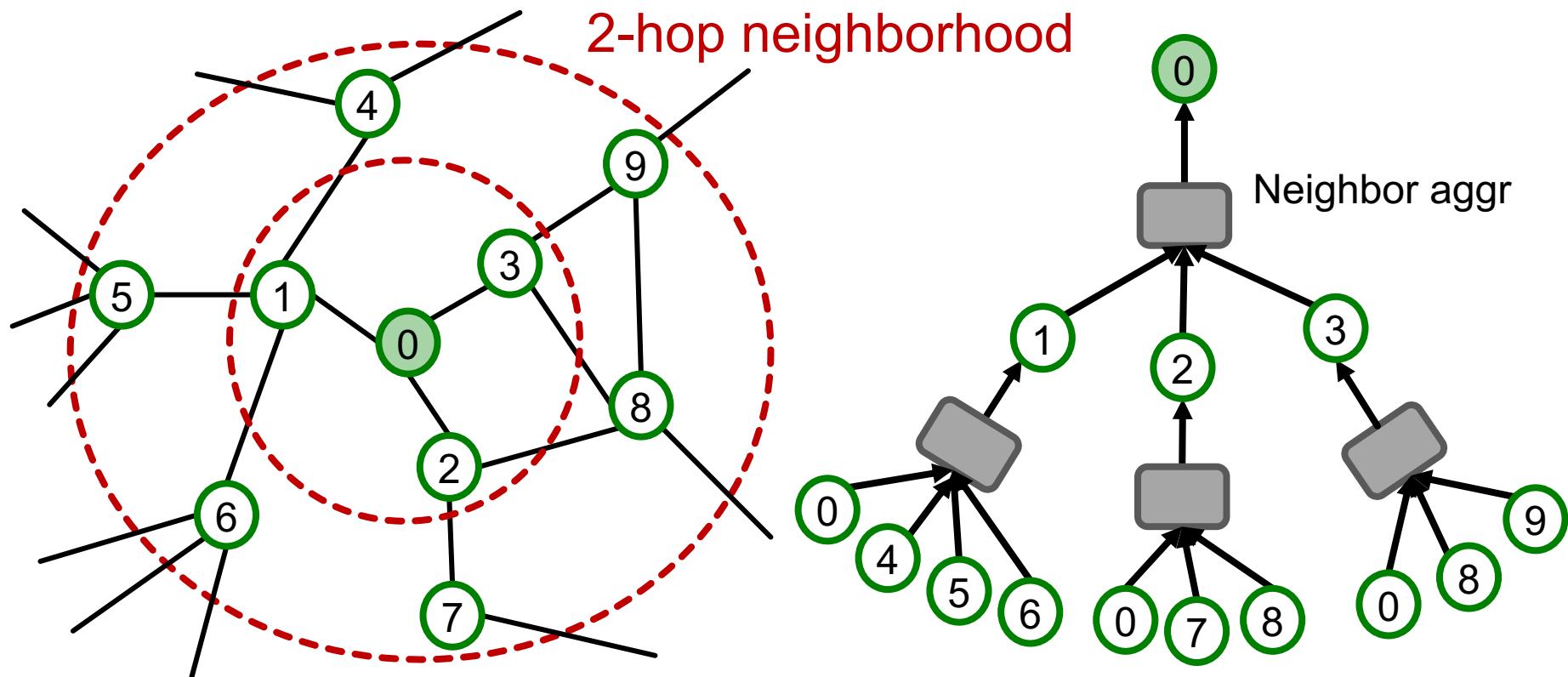
Recall: Computational Graph

- **Observation:** A 2-layer GNN generates embedding of node “0” using 2-hop neighborhood structure and features.



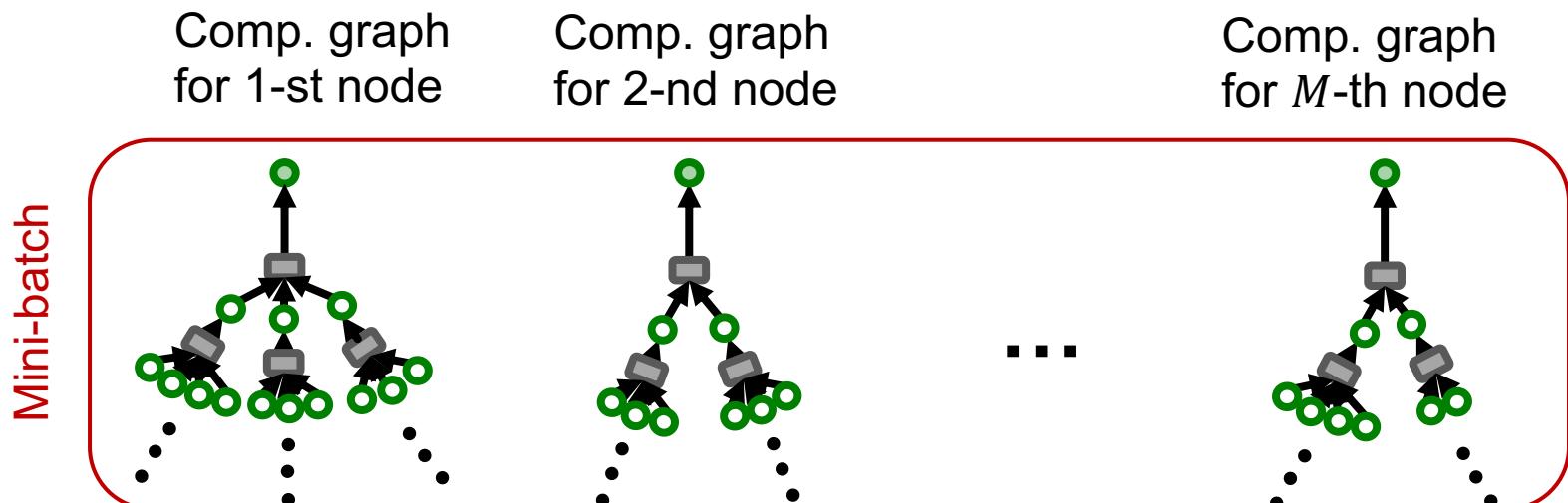
Recall: Computational Graph

- **Observation:** More generally, K -layer GNNs generate embedding of a node using K -hop neighborhood structure and features.



Computing Node Embeddings

- **Key insight:** To compute embedding of a single node, all we need is **the K -hop neighborhood** (which defines the computation graph).
- Given a set of M different nodes in a **mini-batch**, we can generate their embeddings using M computational graphs. **Can be computed on GPU!**

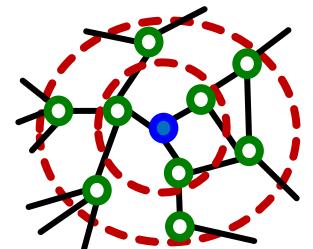


Stochastic Training of GNNs

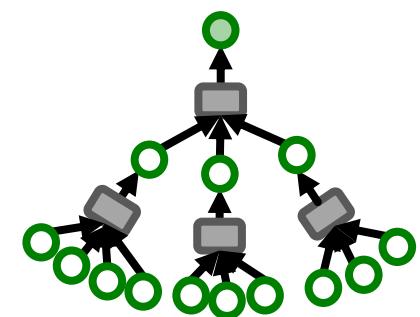
- We can now consider the following SGD strategy for training K -layer GNNs:

- Randomly sample M ($\ll N$) nodes.
- For each sampled node v :
 - Get **K -hop neighborhood**, and construct the **computation graph**.
 - Use the above to generate v 's embedding.
- Compute the loss $\ell_{sub}(\theta)$ averaged over the M nodes.
- Perform SGD: $\theta \leftarrow \theta - \nabla \ell_{sub}(\theta)$

K -hop neighborhood

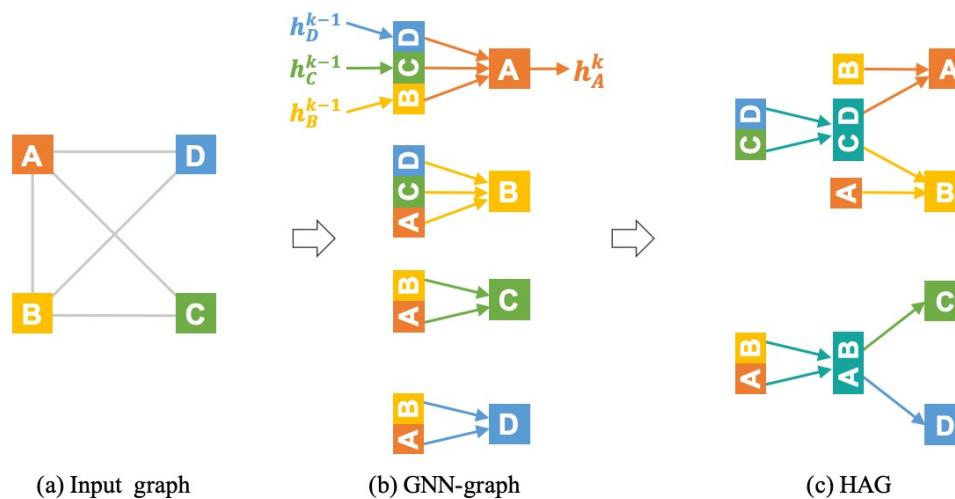


Computational graph



Issue with Stochastic Training (1)

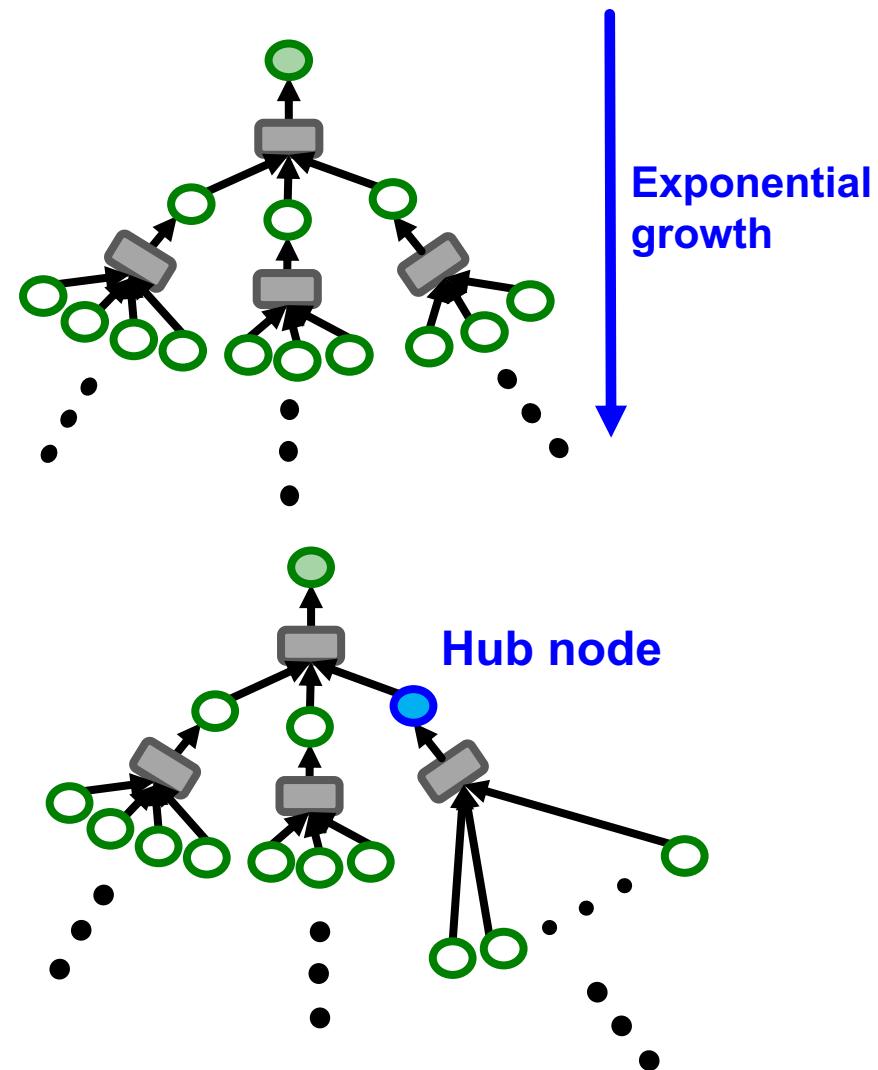
- For each node, we need to get the entire **K-hop neighborhood** and pass it through the computation graph.
- We need to aggregate lot of information just to compute one node embedding.
- Some computational redundancy:**



Issue with Stochastic Training (2)

■ 2nd issue:

- Computation graph becomes **exponentially large** with respect to the layer size K .
- Computation graph explodes when it hits a **hub node** (high-degree node).
- **Next:** Make the comp. graph more compact!



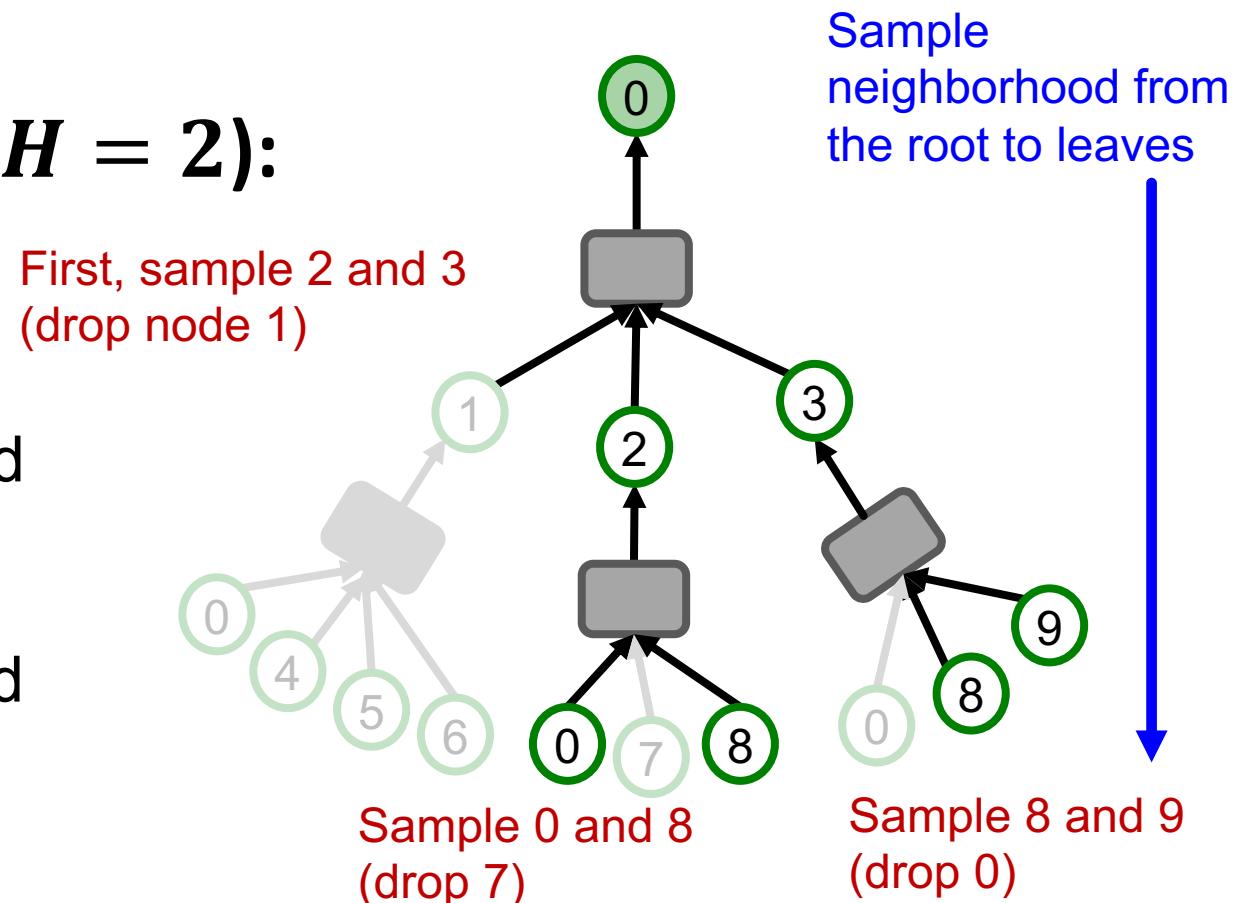
Neighborhood Sampling

Key idea: Construct the computational graph by (randomly) sampling at most H neighbors at each hop.

- Example ($H = 2$):

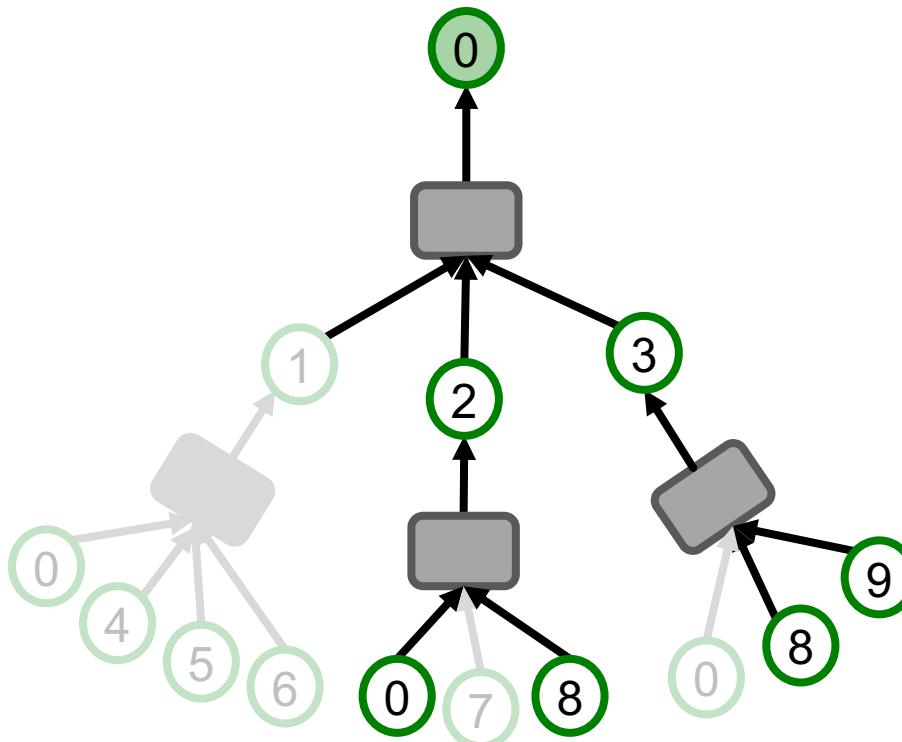
1st-hop neighborhood

2nd-hop neighborhood



Neighborhood Sampling

We can use the pruned computational graph to more efficiently compute node embeddings.



Neighborhood Sampling Algorithm

Neighbor sampling for K -layer GNN

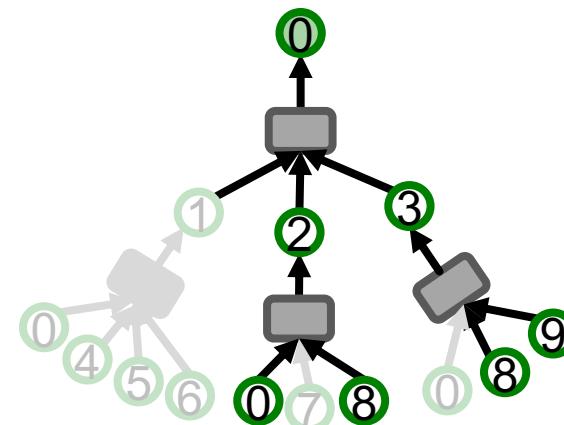
- For $k = 1, 2, \dots, K$:
 - For each node in k -hop neighborhood:
 - (Randomly) sample at most H_k neighbors:

1st-hop neighborhood

Sample $H_1 = 2$ neighbors

2nd-hop neighborhood

Sample $H_2 = 2$ neighbors



- K -layer GNN will at most involve $\prod_{k=1}^K H_k$ leaf nodes in comp. graph.

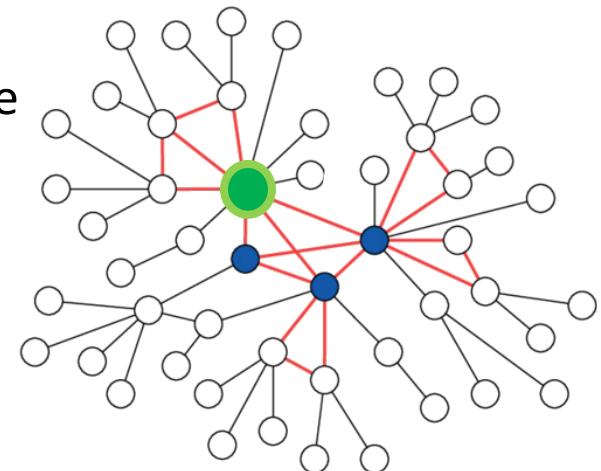
Remarks on Neighbor Sampling (1)

- **Remark 1: Trade-off in sampling number H**
 - Smaller H leads to more efficient neighbor aggregation, but results in more unstable training **due to the larger variance** in neighbor aggregation.
- **Remark 2: Computational time**
 - Even with neighbor sampling, **the size of the computational graph is still exponential with respect to number of GNN layers K .**
 - Adding one GNN layer would make computation H times more expensive.

Remarks on Neighbor Sampling (2)

■ Remark 3: How to sample the nodes

- **Random sampling:** fast but many times not optimal (may sample many “unimportant” nodes)
- **Random Walk with Restarts:**
 - Natural graphs are “scale free”, sampling random neighbors, samples many low degree “leaf” nodes.
 - Strategy to sample important nodes:
 - Compute Random Walk with Restarts score R_i starting at the **green** node
 - At each level sample H neighbors i with the highest R_i
 - This strategy works much better in practice.



Summary: Neighbor Sampling

- A computational graph is constructed for each node in a mini-batch.
- In neighbor sampling, the comp. graph is pruned/sub-sampled to increase computational efficiency.
- The pruned comp. graph is used to generate a node embedding.
- However, computational graphs can still become large, especially for GNNs with many message-passing layers.

Cluster-GCN: Scaling up GNNs

CS224W: Machine Learning with Graphs

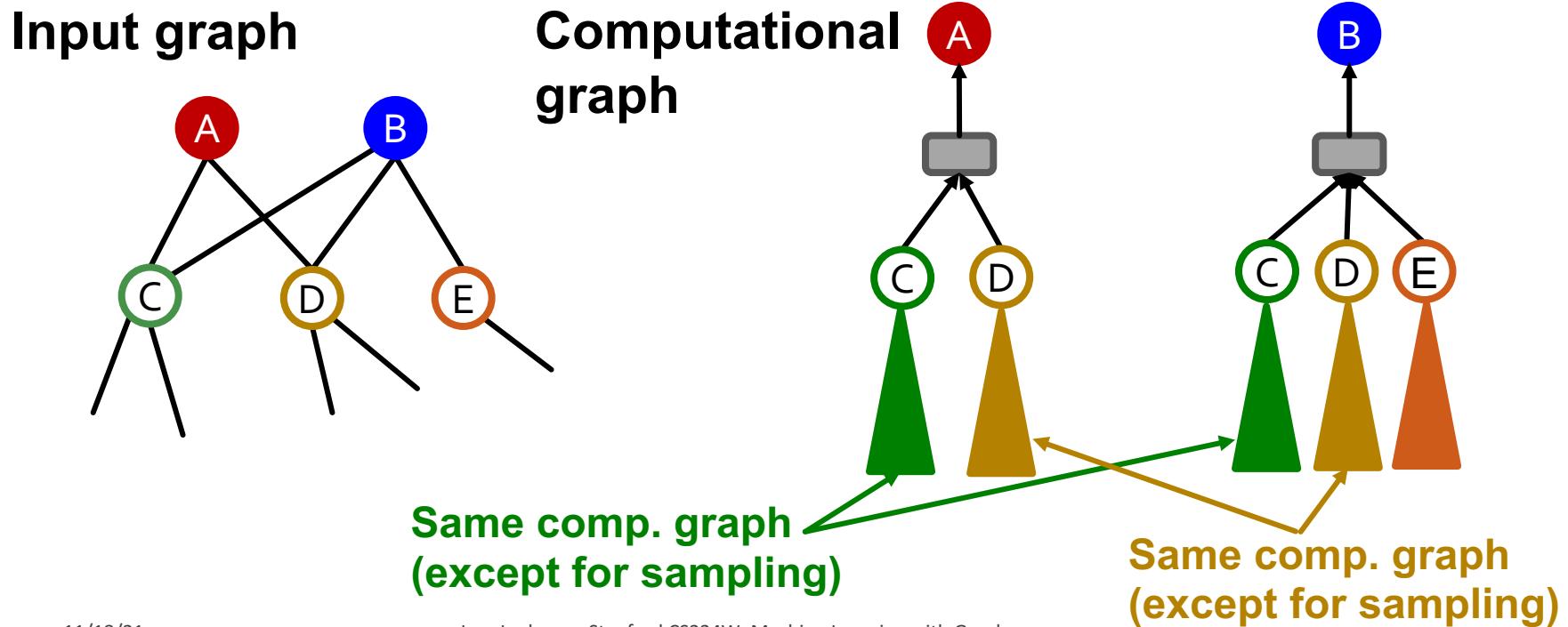
Jure Leskovec, Stanford University

<http://cs224w.stanford.edu>



Issues with Neighbor Sampling

- The size of computational graph becomes exponentially large w.r.t. the #GNN layers.
- Computation is redundant, especially when nodes in a mini-batch share many neighbors.



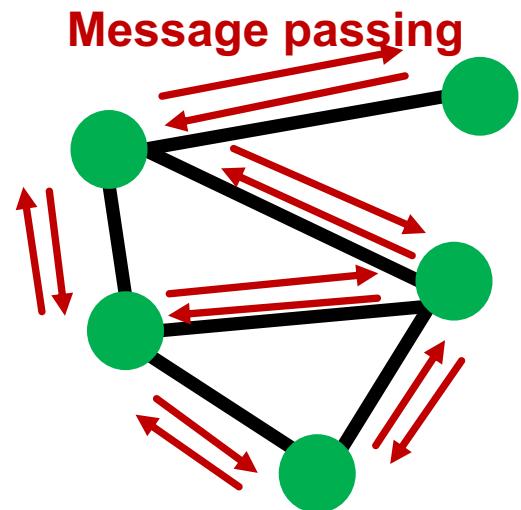
Recall: Full Batch GNN

- In full-batch GNN implementation, **all the node embeddings are updated together using embeddings of the previous layer.**

Update for all $v \in V$

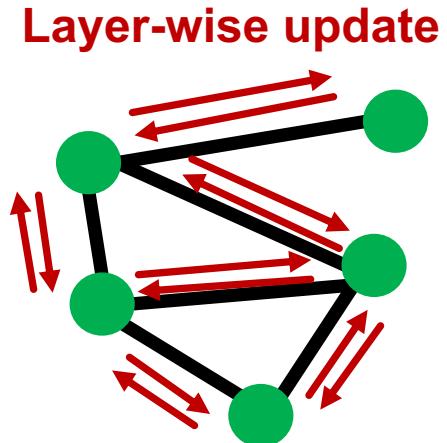
$$h_v^{(\ell)} = COMBINE \left(h_v^{(\ell-1)}, AGGR \left(\left\{ \mathbf{h}_u^{(\ell-1)} \right\}_{u \in N(v)} \right) \right)$$

- In each layer, only **2*(#edges) messages** need to be computed.
- For K -layer GNN, only $2K * \#(\text{edges})$ messages need to be computed.
- GNN's entire computation is only **linear** in $\#(\text{edges})$ and $\#(\text{GNN layers})$. **Fast!**



Insight from Full-batch GNN

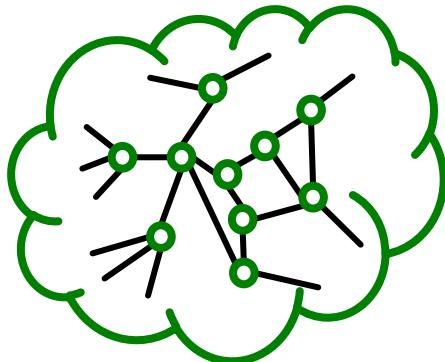
- The **layer-wise** node embedding update allows the re-use of embeddings from the previous layer.
- This significantly **reduces the computational redundancy of neighbor sampling.**
 - Of course, the **layer-wise** update is **not feasible** for a large graph due to **limited GPU memory**.
 - Requires putting the entire graph and features on GPU.



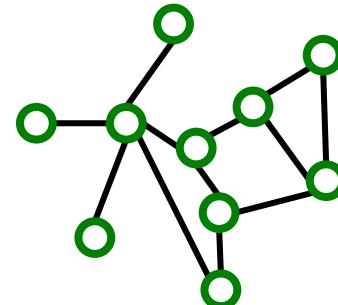
Subgraph Sampling

- **Key idea:** We can **sample a small subgraph of the large graph** and then perform the efficient **layer-wise** node embeddings update over the subgraph.

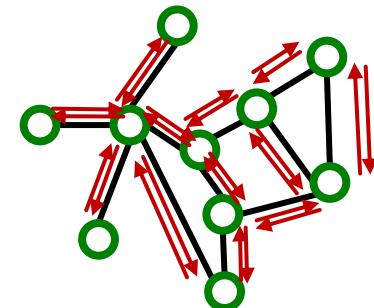
Large graph



Sampled subgraph
(small enough to
be put on a GPU)



Layer-wise
node embeddings
update on the GPU



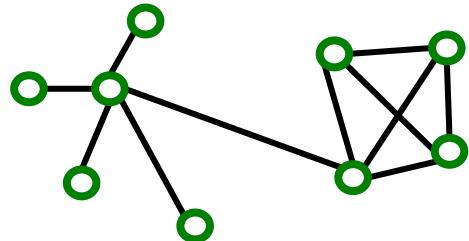
Subgraph Sampling

- **Key question:** What subgraphs are good for training GNNs?
 - Recall: GNN performs node embedding by passing messages **via the edges**.
 - Subgraphs **should retain edge connectivity structure of the original graph as much as possible.**
 - This way, the GNN over the subgraph generates embeddings closer to the GNN over the original graph.

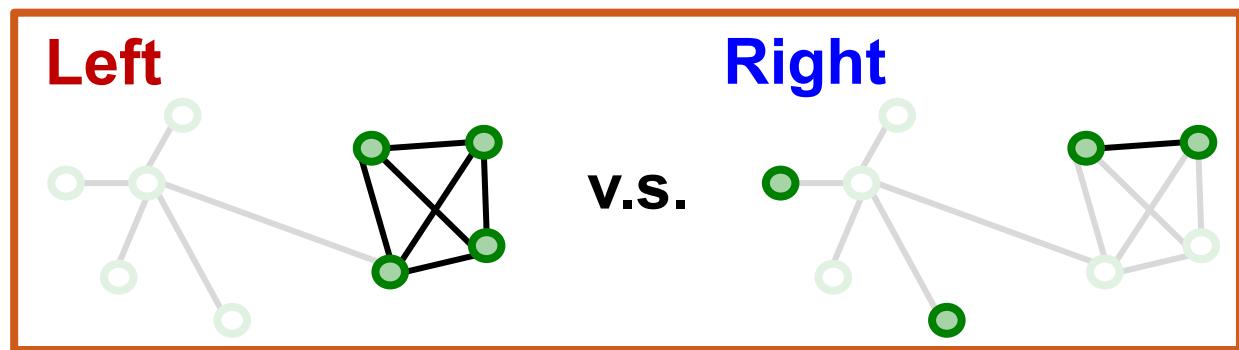
Subgraph Sampling: Case Study

- Which subgraph is good for training GNN?

Original graph



Subgraphs (both 4-node induced subgraph)

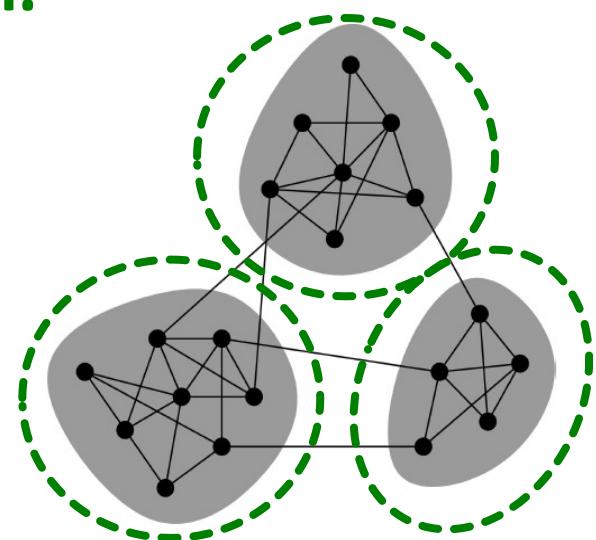


- **Left subgraph** retains the essential community structure among the 4 nodes → **Good**
- **Right subgraph** drops many connectivity patterns, even leading to isolated nodes → **Bad**

Exploiting Community Structure

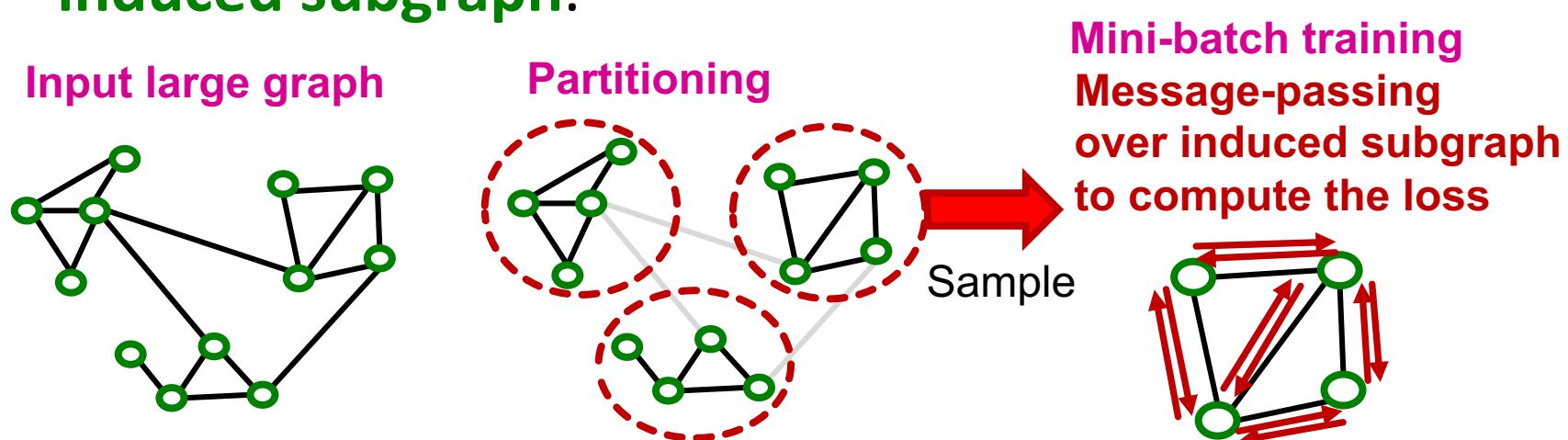
Real-world graph exhibits community structure

- A large graph can be decomposed into many small communities.
- **Key insight** [Chiang et al. KDD 2019]:
**Sample a community as a subgraph.
Each subgraph retains essential
local connectivity pattern of the
original graph.**



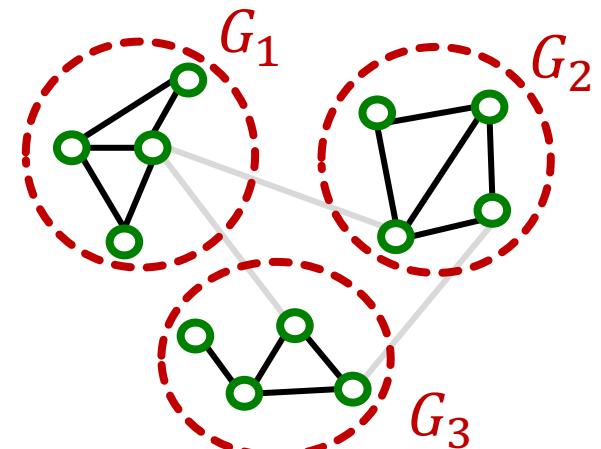
Cluster-GCN: Overview

- We first introduce “vanilla” Cluster-GCN.
- Cluster-GCN consists of two steps:
 - **Pre-processing**: Given a large graph, partition it into groups of nodes (i.e., subgraphs).
 - **Mini-batch training**: Sample one node group at a time. Apply GNN’s message passing over the **induced subgraph**.



Cluster-GCN: Pre-processing

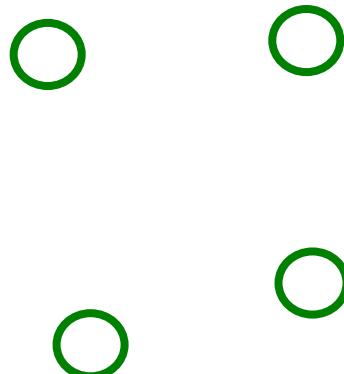
- Given a large graph $G = (V, E)$, **partition its nodes V into C groups:** V_1, \dots, V_C .
 - We can use any scalable community detection methods, e.g., Louvain, METIS [Karypis et al. SIAM 1998].
 - V_1, \dots, V_C induces C subgraphs, G_1, \dots, G_C ,
 - Recall: $G_c \equiv (V_c, E_c)$,
 - where $E_c = \{(u, v) | u, v \in V_c\}$
- Notice: Between-group edges are *not* included in G_1, \dots, G_C .**



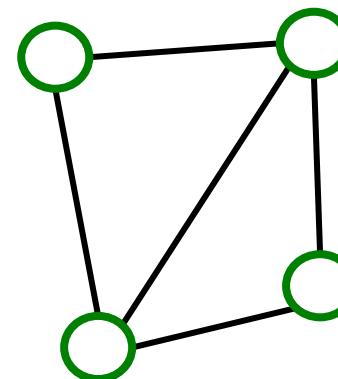
Cluster-GCN: Mini-batch Training

- For each mini-batch, **randomly sample a node group V_c .**
- Construct **induced subgraph $G_c = (V_c, E_c)$**

Sampled node group V_c



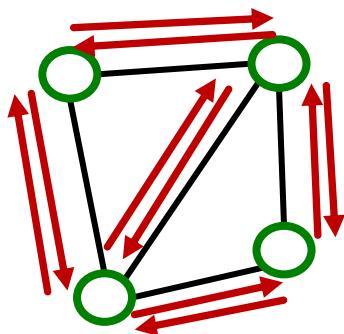
Induced subgraph G_c



Cluster-GCN: Mini-batch Training

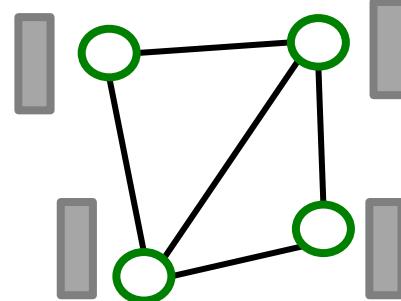
- Apply GNN's **layer-wise node update** over G_c to obtain embedding \mathbf{h}_v for each node $v \in V_c$.
- Compute the loss for each node $v \in V_c$ and take average: $\ell_{sub}(\theta) = (1/|V_c|) \cdot \sum_{v \in V_c} \ell_v(\theta)$
- Update params: $\theta \leftarrow \theta - \nabla \ell_{sub}(\theta)$

Induced subgraph G_c



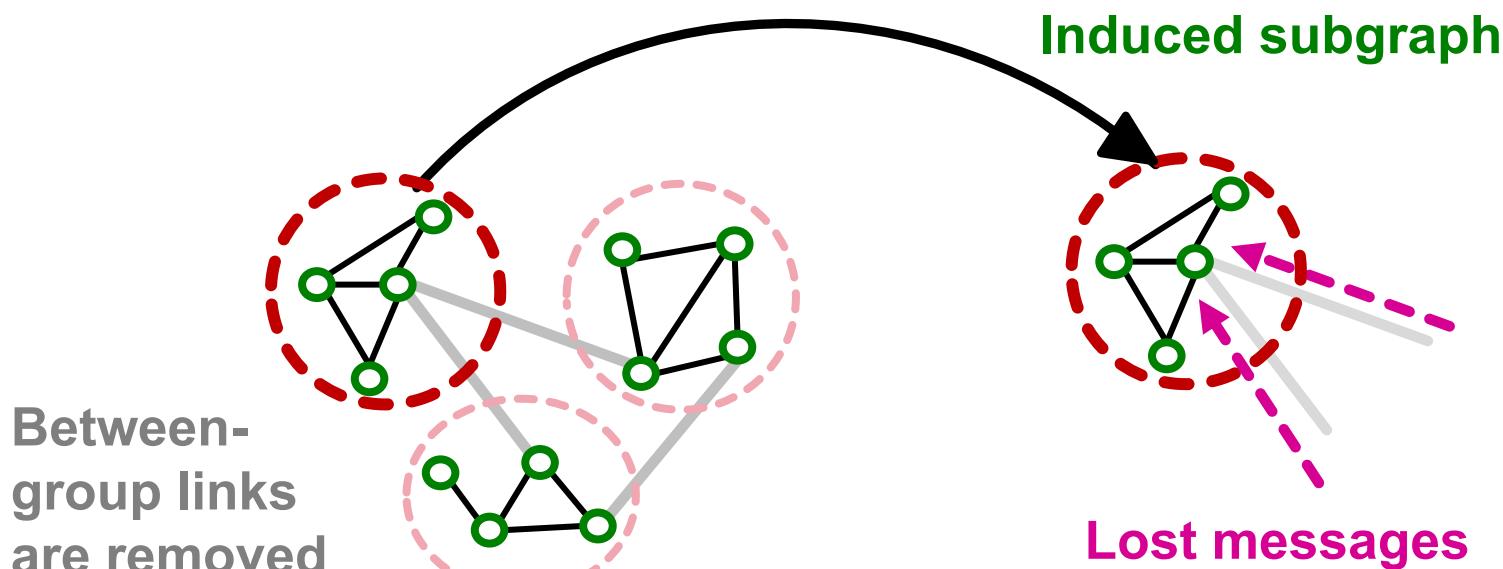
Layer-wise node
embedding update

Embedding



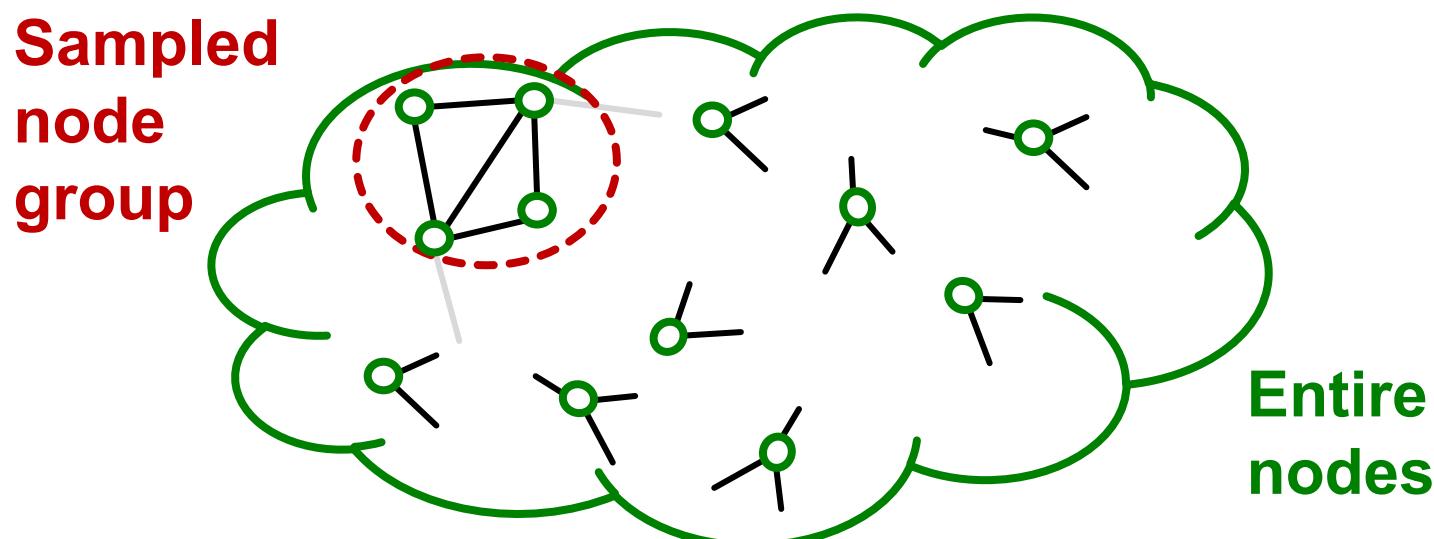
Issues with Cluster-GCN (1)

- The induced subgraph removes between-group links.
- As a result, messages from other groups will *be lost during message passing*, which could hurt the GNN's performance.



Issues with Cluster-GCN (2)

- Graph community detection algorithm **puts similar nodes together in the same group.**
- **Sampled node group** tends to only cover the small-concentrated portion of the **entire data**.



Issues with Cluster-GCN (3)

Sampled nodes are not diverse enough to be represent the entire graph structure:

- As a result, the gradient averaged over the sampled nodes, $\frac{1}{|V_c|} \sum_{v \in V_c} \ell_v(\theta)$, becomes unreliable.
 - Fluctuates a lot from a node group to another.
 - In other words, the gradient has high variance.
- Leads to slow convergence of SGD

Advanced Cluster-GCN: Overview

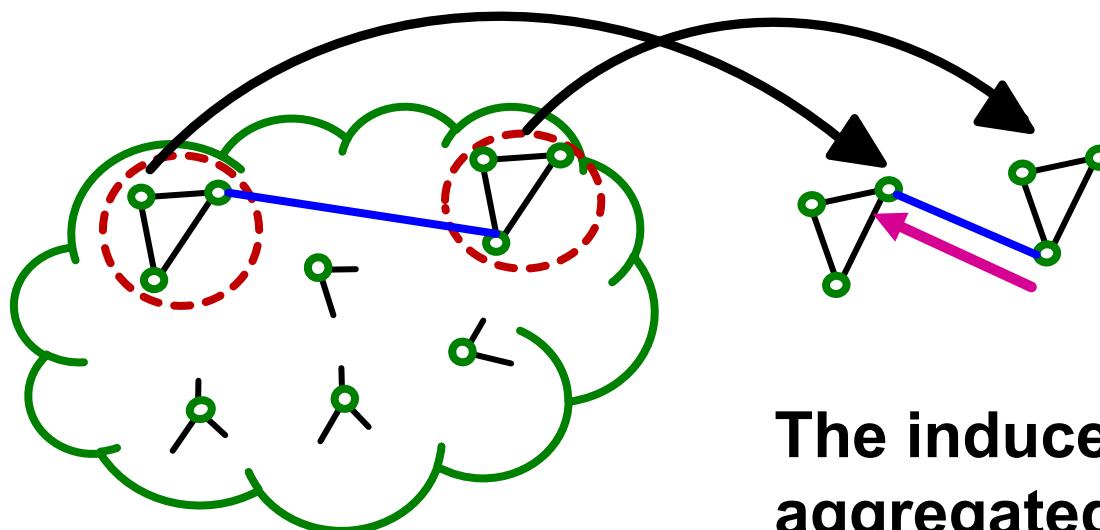
- **Solution: Aggregate multiple node groups per mini-batch.**
- Partition the graph into **relatively-small groups of nodes**.
- **For each mini-batch:**
 - Sample and aggregate **multiple node groups**.
 - **Construct the induced subgraph of the *aggregated node group*.**
 - The rest is the same as vanilla Cluster-GCN (compute node embeddings and the loss, update parameters)

Advanced Cluster-GCN: Overview

■ Why does the solution work?

Sampling **multiple node groups**

→ Makes the sampled nodes more representative of the entire nodes. Leads to less variance in gradient estimation.



The induced subgraph over aggregated node groups

→ Includes edges between groups
→ Message can flow across groups.

Advanced Cluster-GCN

Similar to vanilla Cluster-GCN, advanced Cluster-GCN also follows 2-step approaches.

Pre-processing step:

- Given a large graph $G = (V, E)$, partition its nodes V into C **relatively-small** groups:
 V_1, \dots, V_C .
 - V_1, \dots, V_C needs to be small so that even if multiple of them are aggregated, the resulting group would not be too large.

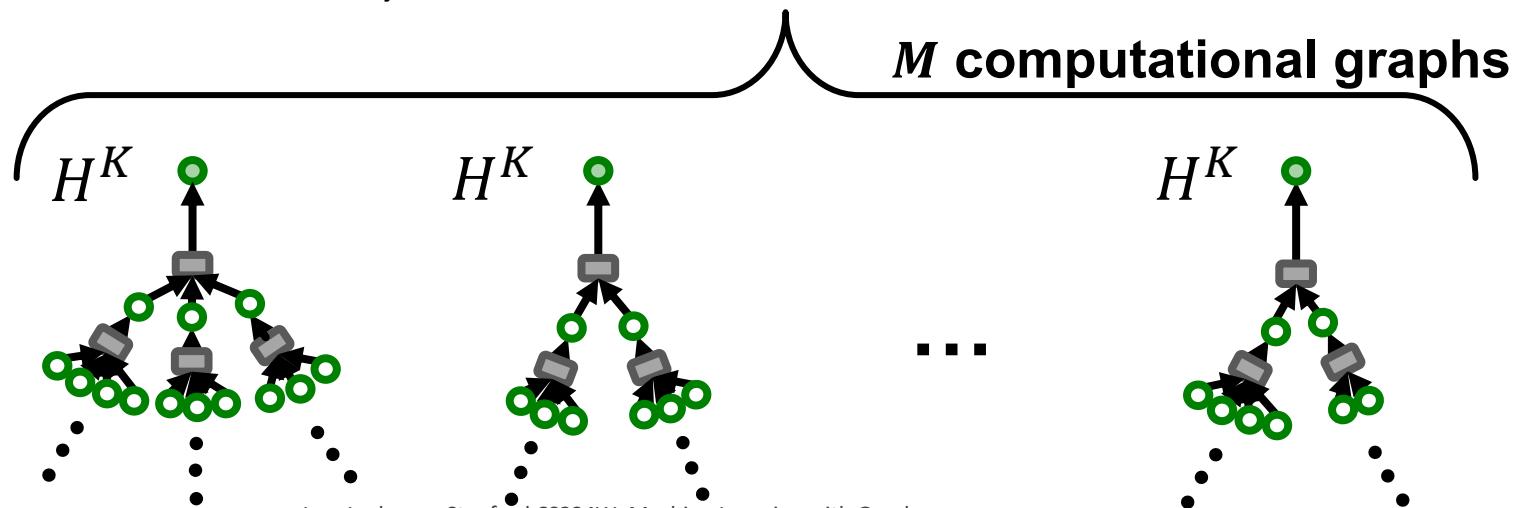
Advanced Cluster-GCN

Mini-batch training:

- For each mini-batch, **randomly sample a set of q node groups**: $\{V_{t_1}, \dots, V_{t_q}\} \subset \{V_1, \dots, V_C\}$.
- **Aggregate all nodes across the sampled node groups**: $V_{aggr} = V_{t_1} \cup \dots \cup V_{t_q}$
- Extract the **induced subgraph**
 $G_{aggr} = (V_{aggr}, E_{aggr})$,
where $E_{aggr} = \{(u, v) \mid u, v \in V_{aggr}\}$
 - E_{aggr} also includes between-group edges!

Comparison of Time Complexity

- Generate M ($\ll N$) node embeddings using K -layer GNN (N : #all nodes).
- Neighbor-sampling (sampling H nodes per layer):
 - For each node, the size of K -layer computational graph is H^K .
 - For M nodes, the cost is $M \cdot H^K$



Comparison of Time Complexity

- Generate M ($\ll N$) node embeddings using K -layer GNN (N : #all nodes).
- Cluster-GCN:
 - Perform message passing over a subgraph induced by the M nodes.
 - The subgraph contains $M \cdot D_{avg}$ edges, where D_{avg} is the average node degree.
 - K -layer message passing over the subgraph costs at most $K \cdot M \cdot D_{avg}$.

Comparison of Time Complexity

- In summary, the cost to generate embeddings for M nodes using K -layer GNN is:
 - **Neighbor-sampling (sample H nodes per layer):**
 $M \cdot H^K$
 - **Cluster-GCN:** $K \cdot M \cdot D_{avg}$
- Assume $H = D_{avg}/2$. In other words, 50% of neighbors are sampled.
 - Then, Cluster-GCN (cost: $2MHK$) is much more efficient than neighbor sampling (cost: MH^K).
 - Linear (instead of exponential) dependency w.r.t. K .

Cluster-GCN: Summary

- Cluster-GCN first **partitions the entire nodes into a set of small node groups.**
- At each mini-batch, multiple node groups are sampled, and their nodes are aggregated.
- **GNN performs layer-wise node embeddings update over the induced subgraph.**
- Cluster-GCN is more computationally efficient than neighbor sampling, especially when #(GNN layers) is large.
- But Cluster-GCN leads to systematically biased gradient estimates (due to missing cross-community edges)

Scaling up by Simplifying GNN Architecture

CS224W: Machine Learning with Graphs
Jure Leskovec, Stanford University
<http://cs224w.stanford.edu>



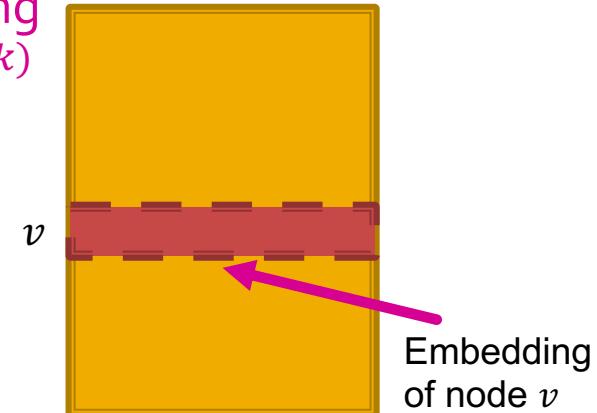
Roadmap of Simplifying GCN

- We start from Graph Convolutional Network (GCN) [Kipf & Welling ICLR 2017].
- We simplify GCN (LightGCN) by **removing the non-linear activation** from the GCN [Wu et al. ICML 2019].
 - Wu et al. demonstrated that the performance on benchmark is not much lower by the simplification.
 - Simplified GCN turns out to be extremely scalable by the model design.
 - **The simplification strategy is very similar to the one used by LightGCN for recommender systems.**

Quick Overview of LightGCN (1)

- Adjacency matrix: A
- Degree matrix: D
- Normalized adjacency matrix:
 $\tilde{A} \equiv D^{-1/2} A D^{-1/2}$
- Let $E^{(k)}$ be the embedding matrix at k -th layer.
- Let E be the input embedding matrix.
- GCN's aggregation in the matrix form
 - $E^{(k+1)} = \text{ReLU}(\tilde{A}E^{(k)}W^{(k)})$

Embedding
matrix $E^{(k)}$



Quick Overview of LightGCN (2)

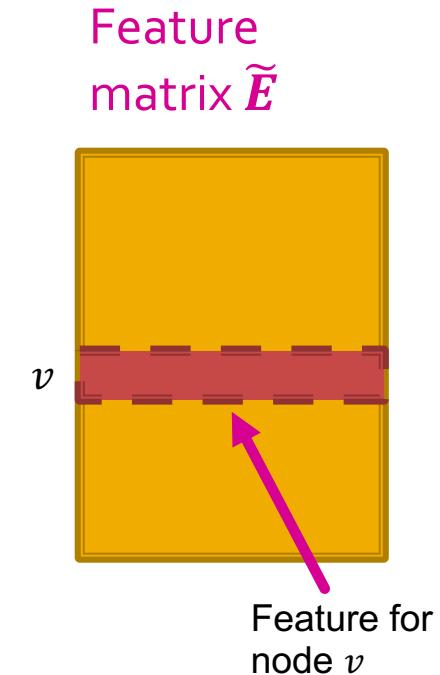
- Removing ReLU non-linearity gives us
 - $E^{(K)} = \tilde{A}^K E W$, where $W \equiv W^{(0)} \dots W^{(K-1)}$
**Diffusing node embeddings
along the graph**
- Efficient algorithm to obtain $\tilde{A}^K E$
 - Start from input embedding matrix E .
 - Apply $E \leftarrow \tilde{A} E$ for K times.
- Weight matrix W can be ignored for now.
 - W acts as a linear classifier over the diffused node embeddings $\tilde{A}^K E$.

Differences to LightGCN

- LightGCN adds **self-loops** to adjacency matrix A :
 - $A \leftarrow A + I$
 - Follows the original GCN by Kipf & Welling.
- LightGCN assumes input node embeddings E to be **given as features**.
 - Input embedding matrix E is **fixed** rather than learned.
 - **Important consequence:** $\tilde{A}^K E$ needs to be calculated **only once**.
 - Can be treated as a **pre-processing step**.

Simplified GCN

- Let $\tilde{E} = \tilde{A}^K E$ be pre-processed feature matrix.
 - Each row stores the pre-processed feature for each node.
 - \tilde{E} can be used as input to any scalable ML models (e.g., linear model, MLP).
- LightGCN empirically shows learning a linear model over \tilde{E} often gives performance comparable to GCN!



Comparison with Other Methods

- Compared to neighbor sampling and cluster-GCN, **simplified GCN is much more efficient.**
 - **Simplified GCN computes \tilde{E} only once at the beginning.**
 - The pre-processing (sparse matrix vector product, $E \leftarrow \tilde{A} E$) can be performed efficiently on CPU.
 - Once \tilde{E} is obtained, getting an embedding for node v only takes **constant time!**
 - Just look up a row for node v in \tilde{E} .
 - No need to build a computational graph or sample a subgraph.
 - But the model is **less expressive** (next).

Potential Issue of Simplified GCN

- Compared to the original GNN models,
**simplified GCN's expressive power is limited
due to the lack of non-linearity in generating
node embeddings.**

Performance of Simplified GCN

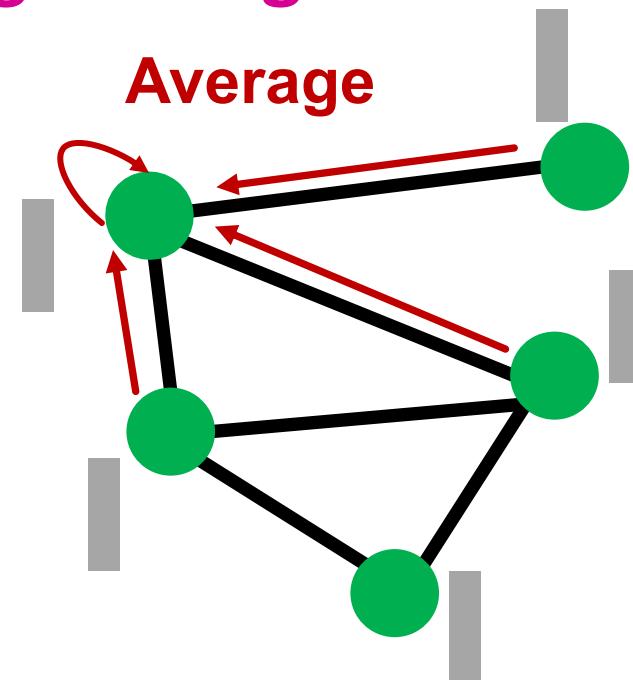
- Surprisingly, in semi-supervised node classification benchmark, **simplified GCN works comparably to the original GNNs despite being less expressive.**
- **Why?**

Graph Homophily

- Many node classification tasks exhibit homophily structure, i.e., **nodes connected by edges tend to share the same target labels.**
- **Examples:**
 - Paper category classification in paper-citation network
 - Two papers tend to share the same category if one cites another.
 - Movie recommendation for users in social networks
 - Two users tend to like the same movie if they are friends in a social network.

When does Simplified GCN Work?

- Recall the preprocessing step of the simplified GCN: **Do $X \leftarrow \tilde{A} X$ for K times.**
- Pre-processed features are obtained **by iteratively averaging their neighboring node features.**
- As a result, nodes connected by edges tend to have similar pre-processed features.



When does Simplified GCN Work?

- **Premise:** Model uses the pre-processed node features to make prediction.
- Nodes connected by edges tend to get similar pre-processed features.
→ **Nodes connected by edges tend to be predicted the same labels by the model**
- **Simplified SGC's prediction aligns well with the graph homophily in many node classification benchmark datasets.**

Simplified GCN: Summary

- **Simplified GCN removes non-linearity in GCN and reduces to the simple pre-processing of node features.**
- Once the pre-processed features are obtained, scalable mini-batch SGD can be directly applied to optimize the parameters.
- **Simplified GCN works surprisingly well in node classification benchmark.**
 - The feature pre-processing aligns well with graph homophily in real-world prediction tasks.