Discussion 4: Intro to Image Priors

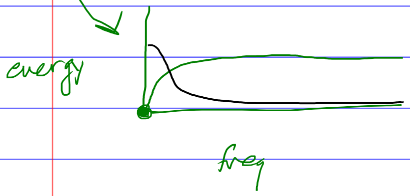
Example: Denoising

- "Real" photo $I(x,y)$
  + noise $n(x,y)$ =

$J(x,y)$

- $n(x,y)$ is independent, Gaussian noise

- How do we find $I(x,y)$?

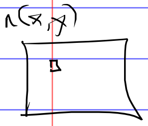


$$\underbrace{J}_{} = \underbrace{I}_{} + \underbrace{n}_{}$$

Approach: Maximum likelihood estimation

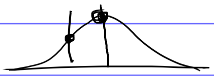
- We know the noise model: $n(x,y) \sim \mathcal{N}(0, \sigma^2)$ ← (known)

- Therefore, we know the likelihood function:



$$P(J \,|\, I) = \sum_{x,y} e^{-\underbrace{|J(x,y) - I(x,y)|^2}_{n(x,y)}/\sigma^2}$$

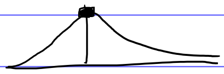
$n(x,y) = 0$



- Then the MLE is $I^* = \underset{I}{\arg\max}\, P(J \,|\, I)$

↳ What's wrong with this?

$$\boxed{I^* = J}$$



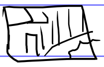
"Hey! Wjat's up?"

What's

1. Noise model → h ↔ j
2. Word model → Wjat's ✗ What's ✓

↓

Image prior

p() = ✓ p() = ✗

Approach: Maximum a posteriori estimation

- Some values of I are more plausible than others: Suppose we know $P(I)$, the probablity that any image I is a natural photograph.

- Then, using Bayesian model:

$$\underbrace{P(I|J)}_{\text{posterior}} = \underbrace{P(J|I)}_{\text{likelihood}} \cdot \underbrace{P(I)}_{\text{prior}}$$

$$P(J|I) \propto \frac{P(I|J)}{P(I)}$$

$$\cancel{P(I)}$$

- Then the MAP estimate is:

$$I^* = \underset{I}{\operatorname{argmax}}\ P(J|I) \cdot P(I)$$

$$= \underset{I}{\operatorname{argmax}} \left[\sum_{x,y} e^{-|J(x,y) - I(x,y)|^2/\sigma^2} \right] \cdot P(I)$$

Before we get to solving for the MAP, how can we possibly know $P(I)$?

→ $P(I)$ only needs to quantify

	Desirable	vs. Undesirable
Denoising	Noise-free	Noisy
Deblurring	Sharp edges	Smooth edges
Recoloring	Realism	Not

These are all <u>ill-posed problems</u>. They require external information to even be solvable. → $P(I)$

Solving problems with priors:

- Analytic solution: If $P(I|J)$ is well-behaved, solve it

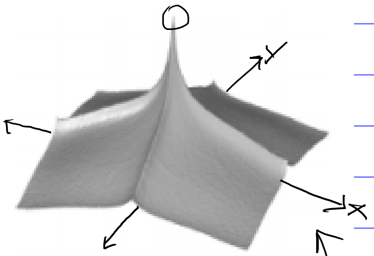
local optima

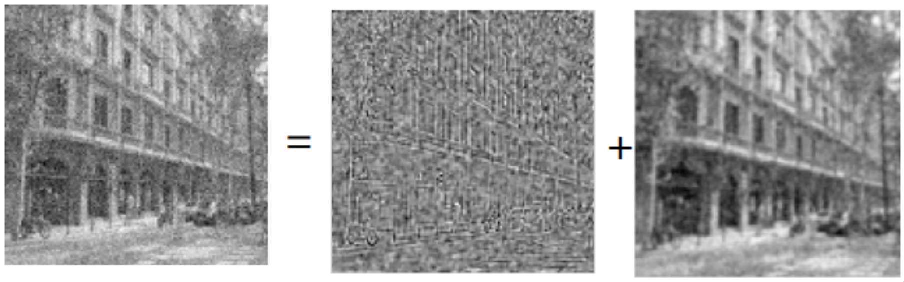
- Nonlinear optimization techniques: Cplex, Gurobi, other linear/nonlinear solvers

- Gradient-based algorithms: Catchall. Neural network methodology.

Denoising prior: Gradients

$I_x, I_y \longrightarrow$





I?