An empirical study of the statistics of phase drift of off-the-shelf oscillators for distributed MIMO applications

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***Abstract*—We report on the results of an empirical study of the phase drift of commercial RF oscillators, and the performance limitations implied by these drifts for distributed multi-input multi-output (DMIMO) wireless systems. The stochastic clock drifts of the oscillators at the nodes of the virtual array and the resulting synchronization errors limit the performance of DMIMO systems. Furthermore the synchronization requirements for DMIMO applications are subtly different from other synchronization problems. We examine two commercially available oscillators and present a series of measurements and calculations to characterize their short-term stability. Specifically we quantify the decrease in phase drift for a temperature controlled oscillator (TCXO) as compared to an uncompensated (VCXO) crystal oscillator. We also present the results of tracking the clock drifts of both oscillators using a simple algorithm based on an Extended Kalman Filtering framework. One key observation coming out of our study is that while the temperature compensated oscillators are an order of magnitude better than the uncompensated ones as measured by their frequency stability, the corresponding performance improvement for DMIMO applications is quite modest. This suggests the need for new figures of merit and novel oscillator designs specifically optimized for DMIMO applications.**

I. Introduction

This paper presents the initial results of our ongoing empirical study of the short-term stability of commercial off-the-shelf (COTS) RF oscillators, and the resulting performance limitations for distributed MIMO (DMIMO) techniques in wireless networks.

This study involved an extensive set of measurements of relative phase drifts between pairs of oscillators, a statistical characterization of these phase drifts using a standard stochastic model, and a practical algorithm for estimating and compensating these drifts using an Extended Kalman Filtering framework. We used two types of oscillators for this study, one temperature controlled (TCXO) and the other an uncompensated (VCXO) crystal oscillator and quantified the performance improvement in terms of synchronization accuracy for DMIMO applications, from the TCXO compared to the VCXO.

A. Motivation

Multi-input multi-output (MIMO) wireless techniques potentially allow dramatic increases in the data rates of wireless communication systems by allowing multiple concurrent transmissions over the same band of spectrum and the same region of space. These techniques work by carefully controlling the interference between concurrent transmissions in order to achieve orders of magnitude improvements in spectral efficiency and spatial reuse.

MIMO technology has obvious applications for cognitive radio and opportunistic spectrum access. For instance, a secondary wireless node with multiple antennas can arrange the signals from its multiple antennas to cancel each other at the primary receiver ("nullforming"), so that its transmission causes no interference to the primary user as shown in Fig. 1. However, the practical application of MIMO is limited by cost, power and form-factor constraints that restrict the number of antenna elements on individual devices. Distributed MIMO (DMIMO) refers to a class of techniques wherein multiple nodes in a wireless network organize themselves into a *virtual antenna array* and collaboratively use multi-antenna techniques using single-antenna nodes.

The key challenge in implementing distributed MIMO techniques is in synchronizing the RF signals on the nodes in the virtual array. DMIMO requires precise control over the amplitude and phase of the RF signal from each node in the array, and in order to achieve that, it is necessary to estimate and compensate for the unavoidable stochastic phase drifts between the oscillators from which each nodes derive their RF signals.

B. Background and Related Work

While the idea of cooperative communication has been studied for decades [1], synchronization issues have only recently attracted significant research interest. Much of the recent work on this topic has focused on the simple but important DMIMO technique of distributed transmit beamforming [2] for which many algorithms and prototypes have been developed [3]–[5]. However, these algorithms do not easily generalize to more general DMIMO techniques because, beamforming, unlike other DMIMO schemes e.g. nullforming, is highly robust and insensitive, to moderately large phase errors (upto about 30 degrees [6]). There has also been work on synchronizing Base-

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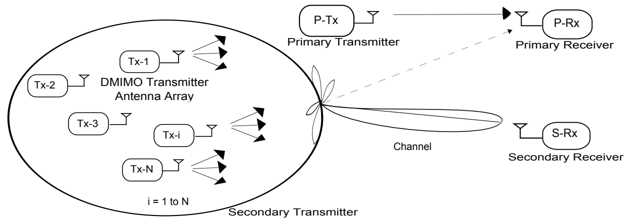

Fig. 1. Multi-antenna interference cancellation for cognitive radio.

Stations in cellular and WiFi networks [7] to operate as virtual arrays; however these techniques require high bandwidth, low-latency wired backhaul links for synchronizing the nodes.

The importance of synchronization for DMIMO is now widely recognized in the technical literature. This consensus can be stated colloquially as follows: *The performance of DMIMO systems will increase proportionally with the ability to achieve very high precision synchronization among the array nodes.* This statement, however, is imprecise in its use of ambiguous terms like "performance" and "very high precision". This paper attempts to quantify and clarify this statement. Specifically, this paper considers the following claims and presents empirical data to test and evaluate them.

1) It is possible to implement DMIMO using COTS oscillators at an overhead cost small compared to typical protocol overheads in present-day WiFi and cellular networks.
2) Long-term frequency stability is a poor measure of oscillator quality for DMIMO applications. Specifically, oscillators with excellent long-term frequency stability may not necessarily deliver good performance for DMIMO applications.
3) A simple tracking algorithm based on an Extended Kalman Filter is effective in estimating and correcting for typical oscillator drifts.

The rest of the paper is organized as follows. Section II defines the synchronization problem for DMIMO and identifies key differences from other synchronization problems. Section III introduces a stochastic model of oscillator phase drift and a reference master-slave architecture for synchronization of DMIMO arrays that is used in the rest of the paper. Measured statistics for the phase drifts of two COTS oscillators are presented and compared in Section IV. Section V concludes with a reexamination of the claims presented in Section I.

II. Synchronization for DMIMO

Modern day digital communication units almost universally come equipped with clocks derived from crystal oscillators, from which they derive the digital clock signals to drive the processing units as well as the RF signals for the wireless transceivers. Synchronization of clocks is, of course, a classic topic of scientific research with a history that goes back centuries alongside the larger history of time-keeping. Over the past decades, synchronization of clocks in a network has also been widely studied. DMIMO techniques, however, introduce a set of synchronization challenges that are different from what has been addressed in the previous network synchronization literature in two important ways.

1) **Precision.** DMIMO techniques generally require synchronization to within a small fraction of the period of the RF carrier signal, which translates to an extremely stringent requirement for the range of RF frequencies commonly used for wireless communication systems. For instance, in the 2.4 GHz band, DMIMO operation would require the cooperating nodes to be synchronized to within 1 picoseconds or so.
2) **Short-term stability.** DMIMO requires that when clocks get synchronized, they *stay* synchronized to within the necessary limits of precision for a reasonable interval of time (on the order of at least a few seconds). This, again, can be extremely stringent. For instance, clocks compliant with the IEEE 802.11 standard can drift by as much as 2.5 nanoseconds per second, which means that such a clock if perfectly synchronized at time $t = 0$ can drift out of synchrony in as little as $400\ \mu s$.

While there are applications other than DMIMO that require high precision synchronization, the combination of the two requirements above make synchronization for DMIMO uniquely challenging. For instance, the GPS system requires very high-precision clocks on the satellites [8]. However, the precision requirements for GPS are subtly different from that of DMIMO. In particular, GPS requires clocks with excellent long-term frequency stability; in other words, GPS requires clocks whose oscillation frequency deviates very little from the universal time reference over very long periods of time on the order of years. On the other hand, GPS systems are relatively insensitive to short-term deviations over time intervals on the order of seconds.

This is in sharp contrast with DMIMO, where the synchronization requirement is *relative* rather than with reference to a global standard. In other words, what is important is for the clocks to be synchronized to each other. Even large long-term frequency deviations are irrelevant if we can make sure that all the nodes in a DMIMO array *deviate by the same amount*. Conversely, it is vitally important for DMIMO applications to keep short-term phase drifts very small.

This leads to the key premise behind this work: that efficient, practical implementation of DMIMO requires a fundamental rethinking of the figures of merit for oscillators, and a large-scale, long-term effort to design new low-phase noise architectures for oscillators specifically optimized for DMIMO applications. The present paper is intended as a start in this direction where our main goal is to quantify the extent to which current oscillator technology and simple tracking schemes limit the performance of DMIMO systems.

III. System model

We consider the master-slave architecture shown in Fig. 2 for synchronization of the DMIMO array. The basic idea is master node periodically transmits a known reference signal to all the "slave" nodes; using these messages, each slave

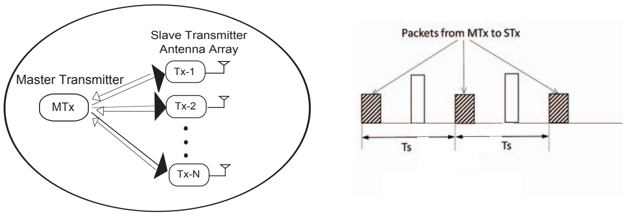

Fig. 2. Synchronization architecture for DMIMO array.

estimates its phase offset relative to the master node, its frequency offset (the first derivative of the phase) as well as higher order derivatives as needed. The slave nodes then use these relative offset estimates to predict and correct the future drift of its own clock signal and the resulting corrected clock signals can then used for DMIMO applications.

In order to keep synchronization overheads low, we would like to keep the reference signals short and the resynchronization times large. For concreteness, we assume that we want to resynchronize at intervals no smaller than every 50 milliseconds, and in between retransmissions, we want the average uncompensated phase drift of the slave node from the master to be no larger than $1°$. This is roughly the precision needed to form nulls of at least -20 dB [9]), and a reference packet transmissions every 50 ms would represent a small overhead on present-day WiFi or cellular networks.

A. Stochastic model of oscillator drift

Let us introduce some notation considering the synchronization process at one of the slave nodes. Let the quadrature RF signals at the master and slave nodes be respectively $s_m(t) = e^{j(2\pi f_c t + \phi_m(t))}$, and $s_s(t) = e^{j(2\pi f_c t + \phi_s(t))}$. Here, f_c is the nominal carrier frequency and $\phi_m(t), \phi_s(t)$ represent the phase drifts of the master and slave clock relative to some universal clock standard. The relative phase offset of the slave node with respect to the master node is $\phi(t) \doteq \phi_s(t) - \phi_m(t)$. If the slave node is able to measure or estimate $\phi(t)$ at all times, then it can easily construct a corrected RF signal $\tilde{(s)}_s(t) = s_s(t)e^{j-\phi(t)}$ which is perfectly synchronized to the master node. If all slave nodes are able to do this, then we have a perfectly synchronized DMIMO array. Note however, that these nodes are only relatively synchronized; they can still drift with respect to the universal clock reference, but such a drift is of no consequence for the performance of the DMIMO system.

We adopt the standard model for the dynamics of the oscillator phase drift $\phi(t)$ from [10]:

$$\mathbf{x}(t) = \begin{bmatrix} \phi(t) \\ \dot{\phi}(t) \end{bmatrix};\ \dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix} \quad (1)$$

where $w_1(t),\ w_2(t)$ are independent white Gaussian noise processes with variances $q_1^2,\ q_2^2$ respectively. The resulting mean squared phase drift over a time interval T can be shown to be

$$E\left(\left(\phi(t) - \phi(0)\right)^2\right) = 4\pi^2 f_c^2 \left(q_1^2 T + q_2^2 \frac{T^3}{3}\right) \quad (2)$$

In order to track these phase drifts over time using measurements from periodic reference packet transmissions from the master node, we adopt the Extended Kalman Filtering framework introduced in [5] which is based on the measurement model $\mathbf{z}_k = \mathbf{h}\left(\mathbf{x}(kTs)\right) + \mathbf{v}_k$ where $\mathbf{x}(kTs)$ is the relative phase drift of the slave node at the time of arrival of the k'th reference packet from the master, $\mathbf{h}\left(\mathbf{x}\right) = [\cos\phi,\ \sin\phi,\ \dot{\phi}]^T$ and $\mathbf{v}_k \sim \mathcal{N}(0, \sigma_m^2)$ is the additive white Gaussian measurement noise of variance σ_m^2. Note that this EKF is based on measurements of the *unwrapped* phase to account for 2π ambiguities.

IV. EMPIRICAL RESULTS

The experimental setup used to generate the required data consisted of two Ettus Research's USRP N200s. One acted as the Master transmitter(MTx) and the other as the Slave (STx).

Method. In order to measure the relative phase drift $\phi(t)$, an unmodulated carrier tone at the nominal frequency of 900 MHz was sent from the MTx. The STx node was also tuned to the nominal frequency of 900 MHz and samples of the received complex baseband signal were recorded at the STx for a period of time of several tens of seconds. This experiment was repeated first with the USRP's default uncompensated oscillator CVHD-950 [11], and then with the temperature controlled TCXO PPRO [12] on both nodes.

We then analyzed the measured phase drift data to determine the extent to which it is predictable using past observations. Neglecting noise, the received complex baseband signal is $r(t) = e^{j\phi(t)}$. The phase drift $\phi(t)$ is obtained by unwrapping $\angle r(t)$. We used a sampling rate of 200 ksps which was large enough to allow phase unwrapping without any ambiguities.

According to the phase drift model in (1), the drift process consists of a Brownian motion in phase proportional to the q_1 parameter, and a Brownian motion in frequency proportional to the q_2 parameter. Since this process is Markovian, knowledge of $\phi(0), \dot{\phi}(0)$ is sufficient to form the best possible estimate of the future evolution of $\phi(t),\ t > 0$; specifically, we can write $\phi(t) = \phi_p(t) + \phi_u(t),\ t > 0$, where the predictable part of the phase drift is $\phi_p(t) \doteq \phi(0) + \dot{\phi}(0)t$ and the unpredictable zero-mean part $\phi_u(t)$ consisting of the Brownian motion terms.

Any practical algorithm to predict the phase drift $\phi(T)$ at some future time $T > 0$ using past observations $\phi(t),\ t \leq 0$ will be limited by (a) its ability to accurately estimate the predictable terms $\phi(0),\ \dot{\phi}(0)$, and (b) the size of the unpredictable term as determined by $q_1,\ q_2$. In the DMIMO system described in Section III, the slave node needs to estimate $\phi(0),\ \dot{\phi}(0)$ using only measurements made on the bursty reference signal transmissions from the master node. However, in order to see just how large the unpredictable part of the drift is, we used the entire past history $\phi(t),\ t \leq 0$ in order to estimate $\phi(0),\ \dot{\phi}(0)$. More precisely, we created a best straight line fit to $\phi(t),\ \ -T_0 \leq t \leq 0$ and used this fit to predict $\phi(\tau)$, for various values of $\tau > 0$, and T_0 chosen by trial-and-error to give the minimum possible estimation error. In addition, we also repeated this procedure with quadratic and higher-order polynomial fits; note that according to the model of (1),

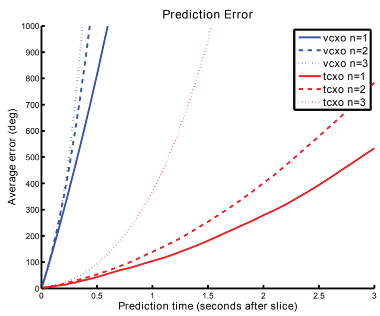

Fig. 3. Unpredictable part of oscillator phase drift vs prediction delay τ.

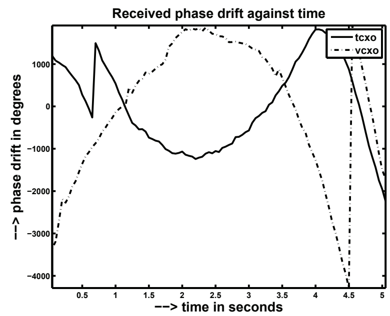

Fig. 4. Residual phase drift tracked using an EKF.

higher-order polynomial fits should not give any improvement in prediction accuracy, however the model in (1) is only an approximate representation of real-world oscillators and the performance of higher order polynomial fits can be taken as an indirect measure of just how good the model is.

The results are shown in Fig. 3. We can see that the phase drifts prediction errors for the VCXO are far larger than for the TCXO. Furthermore, we see the linear fits to the phase drift are significantly better than the quadratic fits suggesting that the frequency drift contrary to (1) is somewhat predictable, and not a pure Brownian motion.

Next we used the phase drifts calculated above along with (2) to estimate the q_1, q_2 parameters for both clocks. For the TCXO this gave ${q_1}^2 = 2 \times 10^{-16}$ sec, ${q_2}^2 = 2 \times 10^{-20}$ Hz, and for the VCXO ${q_1}^2 = 8.47 \times 10^{-19}$ sec, ${q_2}^2 = 8.95 \times 10^{-18}$ Hz. According to this calculation, the q_2 parameter is smaller for the TCXO suggesting that the temperature compensation is successful in limiting the size of the frequency deviations and this is consistent with the increased frequency stability as reported in the spec sheet of the TCXO. However, somewhat suprisingly, the q_1 parameter representing the Brownian motion phase drift is actually much *larger* for the TCXO than for the VCXO. This may explain why the phase drift of the TCXO is only modestly smaller than that of the VCXO.

We next looked at how a practical clock tracking algorithm that does not have the entire past history of $\phi(t)$ to work with, performs with the two clocks. Specifically, we used bursty reference signals from the master node consisting of GMSK modulated 7 msec long data packets repeated every 50 msec. We then used the EKF-tracking framework described in Section III. The residual portion of the resulting phase drifts (after removing the linear term) as estimated by the EKF are shown in Fig. 4. We note that the temperature controlled oscillators perform only about 2-3 times better than the uncompensated clocks, which is in contrast with Fig. 3. This suggests that the accuracy of the EKF itself may be a limiting factor in the tracking error in Fig. 4.

V. Conclusion

We can now reconsider the three key claims presented in Section I in light of the results of our study. Our tentative conclusion is that with present-day COTS oscillators, the unpredictable part of the phase drifts require resynchronization intervals on the order of 100 ms. While this level of resynchronization does not represent a prohibitive overhead for typical WiFi and cellular networks, it may limit the scalability of DMIMO to large arrays. Our most surprising conclusion is that good frequency stability is not a good measure for DMIMO. This opens the door for future work involving novel ideas for oscillator design and modeling specifically optimized for DMIMO applications.

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