

## Problem Set 2

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1. Consider the sequence  $a_n = (-1)^n \log n$ . Which of the following statements are true and why?
  - a. For any  $K \in \mathbb{R}$ , eventually  $a_n > K$ .
  - b. For any  $K \in \mathbb{R}$ , frequently  $a_n > K$ .
  - c. The sequence  $a_n$  diverges.
2. Can you find a convergent sequence  $a_n$  such that:
  - a. the sequence  $b_n = na_n$  diverges?
  - b. the sequence  $c_n = \frac{a_n}{n}$  diverges?
  - c. the sequence  $d_n = a_n + n$  converges?
3. Find the limits of the following sequences:
  - a.  $a_n = \frac{\sin(n)}{n + \frac{1}{n}}$
  - b.  $b_n = \frac{\log(n+1)}{\log(n)}$
4. Consider the set  $S = [0, 1) \cup (a, a + 1]$ :
  - a. For what values of  $a$  the set  $S$  is open?
  - b. For what values of  $a$  the set  $S$  is closed?
  - c. Determine the boundary  $\partial S$  for  $a = 1$ .
5. Consider the function

$$f(x) = \frac{x}{(x-1)(x+1)}.$$

- a. What is the largest subset of  $\mathbb{R}$  on which the function can be defined?
- b. Determine the largest intervals on which the function is continuous.
- c. Show that  $f(x) = \frac{1}{2}$  at some  $x \in (2, 3)$ .