🧩 *Math Prep – Exercise*

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Introductory Math Quiz

Problem 1

Consider the sequence defined by the approximation formula

$$a_n = 3 + n - \frac{n^2 + c^n}{n + 3}, \qquad n \in \mathbb{N},$$

where $c \in \mathbb{R}$ is a parameter.

a. Does the sequence converge for $c = 0$?

b. For which values of c does this sequence converge to a finite limit?

Problem 2

A newly formed political party named the "Cynically Democratic Party" is deciding where to position itself on the left-right political spectrum, represented by the variable $x \in [-1, 1]$. The party aims to balance the trade-offs between being close to the median voter, $\hat{x}$, and attracting radical supporters. The party's popularity is given by the function $V(x) = -|x - \hat{x}| + x^2$.

a. Where should the party position itself if $\hat{x} = 0$?

b. Discuss how the optimal position of the party varies with changes in $\hat{x}$.

Problem 3

A car manufacturer needs to decide how powerful an engine to equip their car with. The firm's profit, based on engine power, is given by the function $\Pi(x) = 240x - x^2$, where x represents the engine's power. Given the technology used, the car produces emissions according to the function $e(x) = 10x + x^2$. Regulations require that emissions must be strictly less than 24,000.

a. What engine power should the firm choose to maximize profit while complying with emission regulations?

b. If the profit function for the next car model is $\Pi(x) = 320x - x^2$, what should the firm choose for the engine power in this case?

Problem 4

A chef is preparing the Universal Brown Sauce. For each liter of sauce, he uses x grams of salt and y grams of concentrate. The chef's skill lies in choosing the right amounts to match consumers' preferences for saltiness u and density v, where the desired value for both is 100. Over time, the chef makes the following observations:

- The characteristics $u = u(x, y)$ and $v = v(x, y)$ are linear in x and y with no intercept, meaning that $u(0, 0) = 0$ and $v(0, 0) = 0$.
- $u(20, 80) = 140$, $v(20, 80) = 86$
- $u(0, 120) = 60$, $v(0, 120) = 114$

Given these observations:

a. Find the formulas for the functions $u(x, y)$ and $v(x, y)$.

b. What values of x and y should the chef use to achieve the desired saltiness and density?

Solutions

5 cells hidden