Math Prep – Exercise

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Problem Set 3

1. Consider the function $f(x) = 3 + 2x + x^2$. Which is true (use Taylor approximation, or the definition of the little–o and big–O to argue):
   a. $f(x) = 3 + o(1)$ as $x \to 0$
   b. $f(x) = 3 + o(x)$ as $x \to 0$
   c. $f(x) = 3 + O(x)$ as $x \to 0$
   d. $f(x) = 3 + 2x + o(x)$ as $x \to 0$
   e. $f(x) = 6 + o\big((x-1)\big)$ as $x \to 1$
   f. $f(x) = 6 + O\big((x-1)\big)$ as $x \to 1$
   g. $f(x) = 6 + 4(x-1) + O\big((x-1)^2\big)$ as $x \to 1$
   i. $f(x) = 6 + 4(x-1) + o\big((x-1)\big)$ as $x \to 1$
2. Consider the function $f(x) = 3 + 2x + |x|^{1.5}$. Which of the following statements are true:
   a. $f(x) = 3 + o(1)$ as $x \to 0$
   b. $f(x) = 3 + o(x)$ as $x \to 0$
   c. $f(x) = 3 + O(x)$ as $x \to 0$
   d. $f(x) = 3 + 2x + o(x)$ as $x \to 0$
   e. $f(x) = 6 + o\big((x-1)\big)$ as $x \to 1$
   f. $f(x) = 6 + O\big((x-1)\big)$ as $x \to 1$
   g. $f(x) = 6 + 3.5(x-1) + O\big((x-1)^2\big)$ as $x \to 1$
   i. $f(x) = 6 + 3.5(x-1) + o\big((x-1)\big)$ as $x \to 1$
3. Consider the functions $F(x,y) = 2x^2 + 2xy + y^2 - 2x + 2$ and $g(x,y) = 2x$:
   a. Find the minimum of F in $\mathbb{R}^2$
   b. Find the minimum of F subject to the constraint $g(x,y) = 4$
   c. Find the minimum of F subject to the constraint $g(x,y) \geq 4$
   d. Find the minimum of F subject to the constraint $g(x,y) \leq 4$