Math Prep – Exercise

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Problem Set 2

1. Consider the sequence $a_n = (-1)^n \log n$. Which of the following statements are true and why?

   a. For any $K \in \mathbb{R}$, eventually $a_n > K$.

   b. For any $K \in \mathbb{R}$, frequently $a_n > K$.

   c. The sequence a_n diverges.

2. Can you find a convergent sequence a_n such that:

   a. the sequence $b_n = na_n$ diverges?

   b. the sequence $c_n = \frac{a_n}{n}$ diverges?

   c. the sequence $d_n = a_n + n$ converges?

3. Find the limits of the following sequences:

   a. $a_n = \frac{\sin(n)}{n+\frac{1}{n}}$

   b. $b_n = \frac{\log(n+1)}{\log(n)}$

4. Consider the set $S = [0, 1) \cup (a, a+1]$:

   a. For what values of a the set S is open?

   b. For what values of a the set S is closed?

   c. Determine the boundary ∂S for $a = 1$.

5. Consider the function

$$f(x) = \frac{x}{(x-1)(x+1)}.$$

   a. What is the largest subset of $\mathbb{R}$ on which the function can be defined?

   b. Determine intervals on which the function is continuous.

   c. Show that $f(x) = \frac{1}{2}$ at some $x \in (2, 3)$.