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Problem Set 1

1. Prove that the $\sqrt{3}$ is irrational.
2. Find the minimum, maximum, infimum, and supremum (in case it exists) of the following sets:
 - a. $S = [-1, 1)$
 - b. $S = \mathbb{R}_-$
 - c. $S = \mathbb{R}_{--}$
 - d. $S = \left\{-\frac{1}{n+1} : n \in \{1, 2, \dots, 5\}\right\}$
 - e. $S = \left\{n + \frac{1}{n} : n \in \mathbb{N}\right\}$
 - f. $S = \{x^3 \in \mathbb{Q} : x^2 < 2\}$
 - g. $S = \left\{\frac{a}{b} : a < 10 \text{ and } b > 5\right\}$
 - h. $S = \mathbb{I} \cap [0, 1]$, where \mathbb{I} is the set of all irrational numbers
 - i. $S = [-10, 5) \cap (-5, 10]$
 - j. $S = [-10, 5) \cup (-5, 10]$
3. Consider the function $f : \mathbb{R} \rightarrow \mathbb{R}$ given by the formula $f(x) = 2x + x^2$:
 - a. What is its domain and codomain?
 - b. What is its range?
 - c. What is the image of the interval $(-1, 2)$
 - d. What is the preimage of the interval $(-10, 0]$.
 - e. Does the inverse function f^{-1} exist? If so, what is its formula?
4. Consider the function $f : [-5, 10] \rightarrow \mathbb{R}$ defined as the indicator function of rational numbers (i.e., $f(x) = \mathbf{1}_{\mathbb{Q}}(x)$):
 - a. What is its domain and codomain?

- b. What is its range?
 - c. What is the image of the interval $(-1, 2)$?
 - d. What is the preimage of the interval $(-10, 0]$.
 - e. Does the inverse function f^{-1} exist? If so, what is its formula?
5. Consider the (real valued) function $f(x) = 1 + \sqrt{5 - x^2}$:
- a. What is its domain? (When a function's domain is not specified, it is assumed to be the set of all real numbers on which the function is well-defined — i.e., the function's prescription makes sense.)
 - b. What is its range?
 - c. What is the image of the interval $(-1, 2)$?
 - d. What is the preimage of the interval $(-10, 0]$.
 - e. Does the inverse function f^{-1} exist? If so, what is its formula?