Part I: Please answer the following problems. Unless otherwise specified, you may use any valid method to solve a problem.

Problem 1. *Set*

Given

$U = \{green, red, white, black, yellow, pink, orange, blue\}$

$A = \{x|\ x$ is a color with the letter 'e' $\}$

$B = \{x|\ x$ is a color with 5 letters$\}$, and

$C = \{x|\ x$ is a color with the letter 'b'$\}$
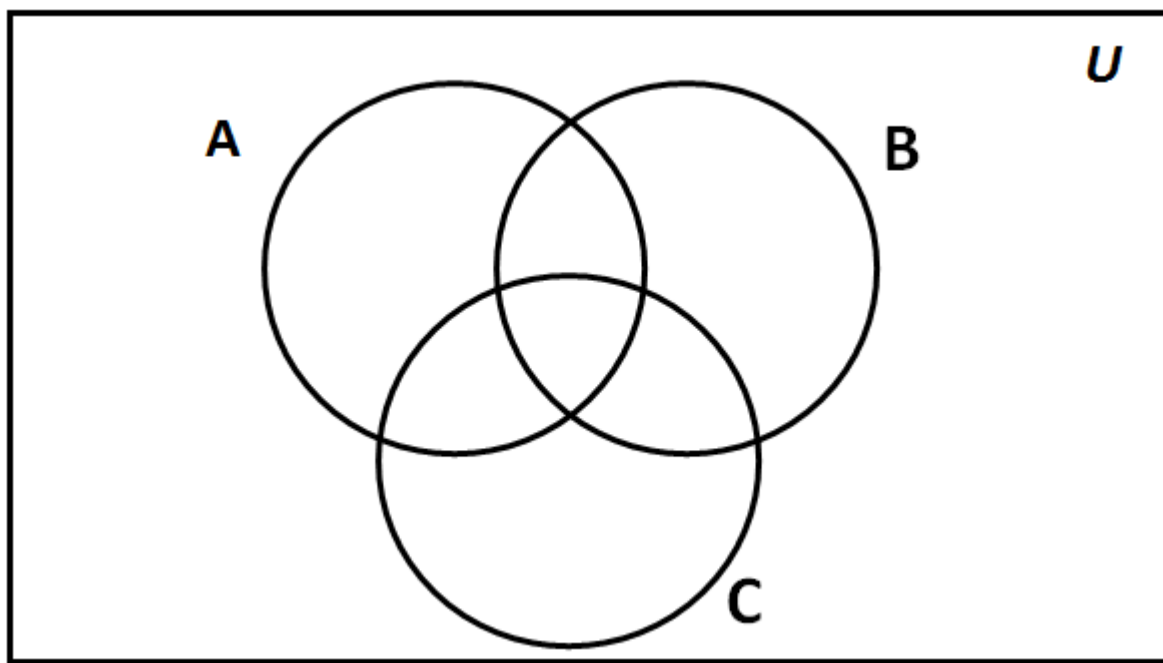
(a) *(3 Pts.)* Fill the Venn Diagram

Figure 1: Venn Diagram for Problem 1

(b) *(6 Pts.)* Write the following sets:

    i) $\bar{A}$

ii) $B - (A \cup C)$

iii) $\overline{A \cup B \cup C}$

Problem 2. (9 Pts.) *Venn Diagram*
Use a copy of the following diagram given in Figure 2 to shade the region corresponding to:

(a) *(2 Pts.)* $A - (B - C)$.

(b) *(2 Pts.)* $(A - B) - C$.

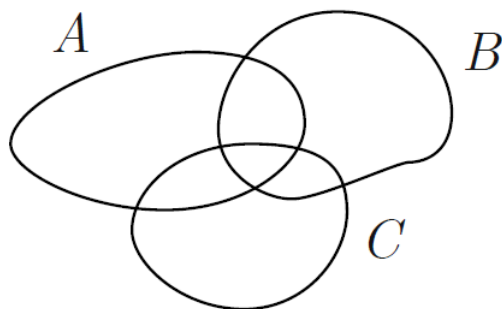(c) *(2 Pts.)* $(A - B) \cup (B - A) \cap C$.

Figure 2: Diagram for Problem 2

Problem 3. *Set*

Let the universal set be **R**, the set of all real numbers, and let $A = \{x \in 0 < x < 2\}$, $B = \{x \in 1 \le x < 4\}$, $C = \{x \in 3 \le x < 9\}$. Find the followings:

(a) *(2 Pts.)* $A \cup B$

(b) *(2 Pts.)* $A \cap B$

(c) *(2 Pts.)* $A \cup C$

(d) *(2 Pts.)* B'

(e) *(2 Pts.)* $(A \cap C)'$

(f) *(2 Pts.)* $(B \cap C)'$

(g) *(2 Pts.)* $(B - C)'$

(h) *(2 Pts.)* $(A \cap C')' - B$

Problem 4. *Set*

Let $C_i = i, -i$ for each nonnegative integer i.

(a) *(2 Pts.)* $\displaystyle\bigcup_{i=1}^{4} C_i =$?

(b) *(2 Pts.)* $\displaystyle\bigcap_{i=1}^{4} C_i =$?

(c) *(2 Pts.)* Are $C_0, C_1, C_2, ...$ disjoint? Explain

(d) *(2 Pts.)* $\displaystyle\bigcup_{i=1}^{n} C_i =$?

(e) *(2 Pts.)* $\displaystyle\bigcap_{i=1}^{n} C_i =$?

(f) *(2 Pts.)* $\displaystyle\bigcup_{i=1}^{\infty} C_i =$?

(g) *(2 Pts.)* $\displaystyle\bigcap_{i=1}^{\infty} C_i =$?

Problem 5. (18 Pts.) *True or False*

Which of the following are true and which are false:

(a) $\{\varnothing\} \subset A$ for all sets A.

(b) $\varnothing \subset A$ for all sets A.

(c) $\varnothing \subset P(A)$ for all sets A.

(d) $\{\varnothing\} \subset P(A)$ for all sets A.

(e) $\varnothing \in A$ for all sets A.

(f) $\varnothing \in P(A)$ for all sets A.

(g) $\{\{\varnothing\}\} \subset P(\varnothing)$.

(h) $\{\varnothing\} \subset \{\{\varnothing, \{\varnothing\}, \{\{\varnothing\}\}\}\}$.

(i) $P(\{\varnothing\}) = \{\varnothing, \{\{\varnothing\}\}\}$

Problem 6. (10 (4+6) Pts.) *Sets*
List all elements of the following sets.

(a) $P(P(\{\varnothing\}))$

(b) $P(\varnothing, P(\{\varnothing\}))$.

Problem 7. *(12 Pts.) Set*

Please proof the followings or make counterexample to disproof:

(a). For all A and B, if $A \subseteq B$, then $A \cap B' = \varnothing$ and

(b). $(A - B) \cup (B - C) \cup (A - C) = \varnothing$