**Mathematics for DII**        **Examination 2**        **Summer Semester 2021**

**Part I:** Please answer the following problems. Unless otherwise specified, you may use any valid method to solve a problem.

**Problem 1.** (8 Pts.) *Truth Table*
Exhibit the truth tables for each of the following statements form:

(a) $(p \rightarrow q) \Leftrightarrow (\sim q \rightarrow \sim p)$

(b) $(p \rightarrow \sim q) \rightarrow (r \rightarrow (\sim q \vee p))$

**Problem 2.** (12 Pts.) *Tautology*
Determine which of the following are tautologies.

(a) $(p \wedge q) \rightarrow (p \vee r)$

(b) $p \rightarrow (q \rightarrow (q \rightarrow p))$

(c) $((p \rightarrow q) \Leftrightarrow q) \rightarrow p$

(d) $((p \longleftrightarrow q) \longleftrightarrow [(p \longrightarrow q) \wedge (q \longrightarrow p)]$

**Problem 3.**(10 Pts.) *Logic*

Find the negation of the following statement

"If you study hard, you will get a good grade."

$p$ : You study hard.

$q$ : You get a good grade.

(a) *(3 Pts.)* **Symbolic Notation:**

(b) *(5 Pts.)* **Negation (Symbolic):** *(Show your work step by step)*

(c) *(2 Pts.)* **Negation (English Statement):**

**Problem 4.** (8 Pts.) *Boolean Algebra*

    **Simplify** the following Boolean expression and also **draw** the circuits of the simplified expression.

$$\overline{(x + \bar{y}) + xy}$$

Also, find the truth value of the expression where $x = 0$ and $y = 1$.

**Problem 5.** (8 Pts.) *Boolean Algebra*

   **Simplify** the following Boolean expression and also **draw** the circuits of the simplified expression.
$$yx + y\overline{z} + \overline{y}xz + z$$
Also, find the truth value of the expression where $x = 1$, $y = 1$, and $z = 0$.

**Problem 6.** (10 Pts.) *The Logic Of Quantified Statements*
Let $D = E = \{-2, -1, 0, 1, 2\}$, write negations for each of the following statements and determine which is true, the given statement or its negation.

(a) $\forall x$ in $D$, $\exists y$ in $E$ such that $x + y = 1$.

(b) $\exists x$ in $D$ such that $\forall y$ in $E$ such that $x + y = -y$.

(c) $\forall x$ in $D$, $\exists y$ in $E$ such that $xy \geq y$.

(c) $\exists x$ in $D$ such that $\forall y$ in $E$ such that $x \leq -y$.

**Problem 7.** *(10 Pts.) Boolean Algebra*

Simplify the followings:

(a). $(w' + y')(y' + xy)(y' + x)$

(b). $ab' + a'b'd' + bc'd' + abcd'$