LOGIC

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PROPOSITION

Definition

A proposition (p, q, r, …) is simply a statement (i.e., a declarative sentence) with a definite meaning, having a truth value that's either ***true (T)*** *or* ***false (F)*** *(**never** both, neither, or somewhere in between).*

- ***Example***
  - *p: Today is Sunday.*
  - *q: 1+1=9-8.*
  - *r: Bangkok is the capital of Thailand.*
  - *t: I got "A" in Math for IT I.*

PROPOSITION

- ***<u>Example: Not Proposition</u>***

  - *p: Let's drink.*
  - *q: a+b.*
  - *r: Take a rest.*
  - *S: A-B=3.*
  - *t: DO IT NOW!*
  - *u: The School of IT*

  WHY? *Because we cannot tell the truth of the given statements .*

PROPOSITION

Negative Operator (NOT)

- *Let **p** be a proposition. The Negation of **p**, denoted by* $\sim p$ **(or** $\neg p$ **),** *is the statement*
  "It is not the case that p.", "not p"

P	$\sim P$
F	T
T	F

Example: *Find the negation of the proposition*
"Today is Friday."

Solution: *The negation is*

"It is not the case that today is Friday."

*In simple English, **"Today is not Friday."** or **"It is not Friday today."***

CONJUNCTION

Conjunction

- *Let **p** and **q** be propositions. The conjunction of **p** and **q**, denoted by **p** $\wedge$ **q**, is the proposition "**p** and **q**."*
  - *Following shows the "Truth Table" for conjunction.*

P	Q	$P \wedge Q$
F	F	F
F	T	F
T	F	F
T	T	T

DISJUNCTION

Disjunction

- *Let **p** and **q** be propositions. The disjunction of **p** and **q**, denoted by **p** v **q**, is the proposition "**p** or **q**."*
  - *Following shows the "Truth Table" for disjunction.*

P	Q	$P \vee Q$
F	F	F
F	T	T
T	F	T
T	T	T

TRUTH TABLE

If the statement from S has three variables P_1, P_2, and P_3, then the setup of truth table will look like.

P_1	P_2	P_3
F	F	F
F	F	T
F	T	F
F	T	T
T	F	F
T	F	T
T	T	F
T	T	T

EXAMPLE

<u>Example</u>

- *When the following statement is false?*
  - *I am a singer OR the queen of Hollywood.*

EXAMPLE

Example

- *When the following statement is false?*
  - *I am a singer OR the queen of Hollywood.*

Sol

P: I am a singer

Q: I am the queen of Hollywood

- *The proposition is represented by* the Symbolic Notation $\boldsymbol{P \vee Q}$
- *The above statement is false if both P and Q are both false.*

EXAMPLE

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by a sentential form.

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- P: I will go to the movies on Friday.
- Q: I will go to the movies on Sunday.

EXAMPLE

- Represent the statement

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by a sentential form.

Sol

- P: I will go to the movies on Friday.
- Q: I will go to the movies on Sunday.

- The first clause of the given statement is represented by $P \vee Q$, and the last by $\sim(P \wedge Q)$, so the complete answer in Symbolic Notation is

$$(P \vee Q) \wedge \sim(P \wedge Q)$$

EXAMPLE

Let K be the following sentential form

$$\sim(P \wedge Q) \wedge (P \wedge (\sim Q \vee (\sim P \vee Q)))$$

Under what truth values for propositions *P* and *Q* is the proposition represented by *K* true?

EXAMPLE

Let K be the following sentential form

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Under what truth values for propositions *P* and *Q* is the proposition represented by *K* true?

Sol

Let $J = (P \wedge (\sim Q \vee (\sim P \vee Q)))$, construct the truth table for **K,** we have

P	Q	$P \wedge Q$	$\sim Q$	$\sim P$	$\sim P \vee Q$	$\sim Q \vee (\sim P \vee Q)$	J	$\sim (P \wedge Q)$	K
F	F	F	T	T	T	T	F	T	F
F	T	F	F	T	T	T	F	T	F
T	F	F	T	F	F	T	T	T	T
T	T	T	F	F	T	T	T	F	F

- We conclude that *K* is true only when *P* is true and *Q* is simultaneously false.

CONDITIONAL STATEMENT

Conditional Proposition or a Material Implication

P	Q	$P \implies Q$
F	F	T
F	T	T
T	F	F
T	T	T

Note: there are other common notations, e.g., $p \rightarrow q, q \leftarrow p, q \subset p$

CONDITIONAL STATEMENT

Example

Construct the truth table for $\mathbf{J} = \boldsymbol{P \Rightarrow ((\sim Q \Rightarrow P) \wedge (Q \vee \sim P))}$.

CONDITIONAL STATEMENT

Example

Construct the truth table for $\boldsymbol{J = P \Rightarrow ((\sim Q \Rightarrow P) \wedge (Q \vee \sim P))}$.

Sol

Let K= $\boldsymbol{((\sim Q \Rightarrow P) \wedge (Q \vee \sim P))}$, we have:

P	Q	$\sim P$	$\sim Q$	$\sim Q \Longrightarrow P$	$Q \vee \sim P$	K	J
F	F	T	T	F	T	F	T
F	T	T	F	T	T	T	T
T	F	F	T	T	F	F	F
T	T	F	F	T	T	T	T

BICONDITIONAL PROPOSITION

Biconditional Proposition or a **Material Equivalence,** $P \leftrightarrow Q$

P	Q	$P \Longrightarrow Q$	$P \Longleftarrow Q$	$P \Longleftrightarrow Q$
F	F	T	T	T
F	T	T	F	F
T	F	F	T	F
T	T	T	T	T

BICONDITIONAL PROPOSITION

Example Construct the truth table of

$$(P \wedge Q) \Rightarrow (((\sim P) \vee R) \leftrightarrow (R \Rightarrow S)).$$

TAUTOLOGY

Tautology

- A **tautology** is a sentential form that becomes a true proposition whenever the letters in the expression are replaced by actual propositions. In other words, a compound statement is a tautology if it is always true, regardless of the truth values of the simple statements from which it is constructed.
- A statement that is always false is called a ***contradiction***;
  - A very simple example is $p \wedge \sim p$.
- Other statements that do not fall into either category are called ***contingent***.
- the expressions $P \vee \sim P$ and $(P \vee Q) \leftrightarrow (Q \vee P)$ are both tautologies, for e.g.

TAUTOLOGY

Examples

- Use the truth table to determine if the following is tautology

$$P \rightarrow ((\sim P) \rightarrow Q)$$

- Determine if the following is tautology

$$P \vee \sim (P \wedge Q)$$

- Determine if the following is tautology

$$(P \wedge Q) \rightarrow (P \vee R)$$

LOGICALLY EQUIVALENT

Logically equivalent

- The compound propositions p and q are called logically equivalent if p $\boxed{\leftrightarrow}$ q is a tautology. The notation **p ≡ q** denotes that p and q are logically equivalent, e.g.
  - $\sim P \vee Q$ **and** $\sim P \wedge \sim Q$

- **Practice**
  - Determine if $((\sim P) \rightarrow Q)$ and **$P \wedge \sim Q$ is logically equivalent**

LOGICALLY EQUIVALENT

TABLE Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws