Set

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SET

- A **set** is a collection of objects, things or symbols which are clearly defined.
- The individual objects in a set are called the members or elements of the set.
- A set must be properly defined so that we can find out whether an object is a member of the set.

There are two ways of declaring set

1. Listing the elements

The set can be defined by listing all its elements, separated by commas and enclosed within braces.

Example:

$B = \{2, 4, 6, 8, 10\}$

$X = \{a, b, c, d, e\}$

There are two ways of declaring set. (Cont.)

2. Describing the elements

The set can be defined, where possible, by describing the elements.

Example:

$C = \{x : x \text{ is an integer, } x > -3\}$

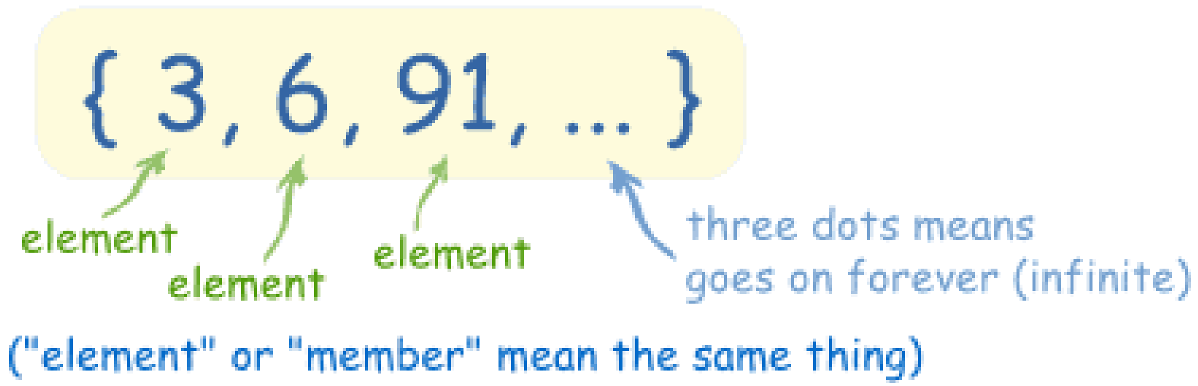
This is read as: "C is the set of elements x such that x is an integer greater than –3."

There are two ways of declaring set (Cont.)

- We relate a member and a set using the symbol $\in$.
- If an object x is an element of set A, we write $x \in A$. If an object z is not an element of set A, we write $z \notin A$.
- $\in$ denotes "is an element of' or "is a member of" or "belongs to"
- $\notin$ denotes "is not an element of" or "is not a member of" or "does not belong to"
- Example:
  If $A = \{1, 3, 5\}$ then $1 \in A$ and $2 \notin A$

SET Notation

There is a fairly simple notation for sets. You simply list each element, separated by a comma, and then put some curly brackets around the whole thing.



Finite SET

- **Finite sets** are sets that have a finite number of members. If the elements of a finite set are listed one after another, the process will eventually "run out" of elements to list.

Example:

$A = \{0, 2, 4, 6, 8, \ldots, 100\}$

$C = \{x : x \text{ is an integer}, 1 < x < 10\}$

The **number of elements** in a finite set A is denoted by **n(*A*).**

Infinite SET

- An **infinite set** is a set which is not finite. It is not possible to explicitly list out all the elements of an infinite set.

Example:

$T = \{x : x \text{ is a triangle}\}$

N is the set of natural numbers

A is the set of fractions

Example

- If A is the set of positive integers less than 12 then
  $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$ and $n(A) = 11$
- If C is the set of numbers which are also multiples of 3 then
  $C = \{3, 6, 9, \ldots\}$ and C is an infinite set
- If D is the set of integers x defined by $-3 < x < 6$ then
  $D = \{-2, -1, 0, 1, 2, 3, 4, 5\}$ and $n(D) = 8$
- If Q is the set of letters in the word *'HELLO'* then
  $Q = \{H, E, L, O\}$, $n(Q) = 4 \leftarrow$ 'L' is not repeated

Empty set or Null set

- There are some sets that do not contain any element at all. For example, the set of months with 32 days. We call a set with no elements the null or empty set. It is represented by the symbol { } or Ø .

Example:

- The set of squares with 5 sides.
- The set of integers which are both even and odd.

Set Equality

- P ={Tom, Rose, Harry, John}
- Q = {Rose, Harry, John, Tom}
- Since P and Q contain exactly the same number of members and the members are the same, we say that P is equal to Q, and we write $P = Q$. The order in which the members appear in the set is not important.

Set Equality (Cont.)

- $R = \{2, 4, 6, 8\}$ $S = \{2, 4, 6, 8, 10\}$
- Since R and S do not contain exactly the same members, we say that R is not equal to S and we write $R \neq S$.

Venn Diagrams

- We can also represent sets using **Venn diagrams**. In a Venn diagram, the sets are represented by shapes; usually circles or ovals. The elements of a set are labeled within the circle.

Example:

Given the set P is the set of even numbers between 15 and 25. Draw and label a Venn diagram to represent the set P and indicate all the elements of set P in the Venn diagram

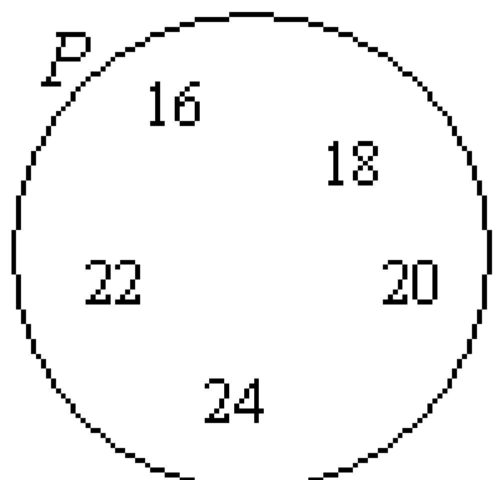
Venn Diagrams (Cont.)

Solution:

List out the elements of P.

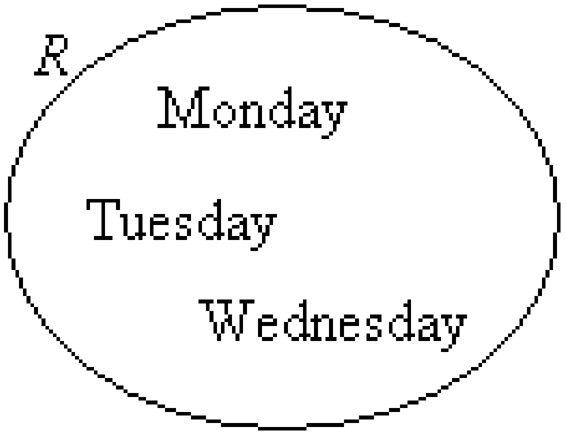
$P = \{16, 18, 20, 22, 24\} \leftarrow$ 'between' does not include 15 and 25

Draw a circle or oval. Label it P. Put the elements in P.



Example 1

- Draw and label a Venn diagram to represent the set.
- R = {Monday, Tuesday, Wednesday}.
- Solution: Draw a circle or oval. Label it R . Put the elements in R.

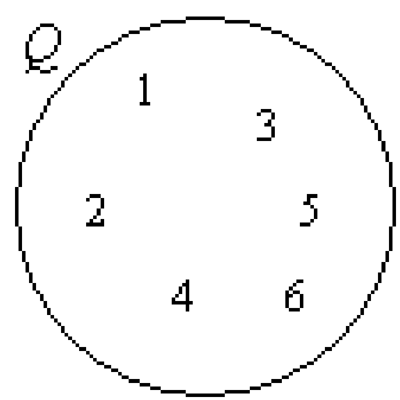


Example 2

Given the set $Q = \{x : 2x - 3 < 11,$ x is a positive integer $\}$. Draw and label a Venn diagram to represent the set Q.

Solution: Since an equation is given, we need to first solve for x. $2x - 3 < 11 \Rightarrow 2x < 14 \Rightarrow x < 7$

So, $Q = \{1, 2, 3, 4, 5, 6\}$



Subset

- If every element of a set B is also a member of a set A, then we say B is a **subset** of A. We use the symbol $\subset$ to mean “is a subset of” and the symbol $\not\subset$ to mean “is not a subset of.

Example:

$$A = \{1, 3, 5\},\ B = \{1, 2, 3, 4, 5\}$$

So, $A \subset B$ because every element in A is also in B.

$$X = \{1, 3, 5\},\ Y = \{2, 3, 4, 5, 6\}.$$

$X \not\subset Y$ because 1 is in X but not in Y.

Subset (Cont.)

- **Note:**
  - Every set is a subset of itself, i.e., for any set A, $A \subset A$
  - The empty set is a subset of any set A i.e. $\emptyset \subset A$
  - For any two sets A and B, if $A \subset B$ and $B \subset A$ then $A = B$
  - The number of subsets for a finite set A is given by the formula:

$$\text{Number of subsets} = 2^{n(A)}$$

  - where $n(A)$ = number of elements in the finite set A

Example

1. List all the subsets of the set $Q = \{x, y, z\}$

The subsets of Q are

$\{\ \}, \{x\}, \{y\}, \{z\}, \{x, y\}, \{x, z\}, \{y, z\}$ and $\{x, y, z\}$

2. $Q = \{x, y, z\}$. How many subsets will Q have?

$n(Q) = 3$

Number of subsets $= 2^3 = 8$

Example (Cont.)

Draw a Venn diagram to represent the relationship between the sets:

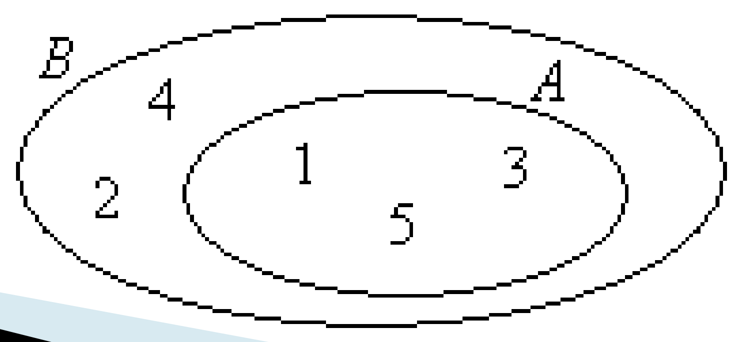
$A = \{1, 3, 5\}$ and $B = \{1, 2, 3, 4, 5\}$

Since A is a subset of B:

Step 1: Draw circle A within the circle B

Step 2 : Write down the elements in circle A.

Step 3 : Write down the remaining elements in circle B.



Universal Set

- A universal set is the set of all elements under consideration, denoted by capital U or sometimes capital E.

Example: Given that U = {5, 6, 7, 8, 9, 10, 11, 12}, list the elements of the following sets.

a) $A = \{x : x \text{ is a factor of } 60\}$

b) $B = \{x : x \text{ is a prime number}\}$

Solution: The elements of sets A and B can only be selected from the given universal set U .

a) $A = \{5, 6, 10, 12\}$

b) $B = \{5, 7, 11\}$

Universal Set (Cont.)

Example: Draw a Venn diagram to represent the following sets: U = {1, 2, 3, 4, 5, 6, 7, 8, 9},

$$A = \{1, 2, 5, 6\},\ B = \{3, 9\}$$

Solution:

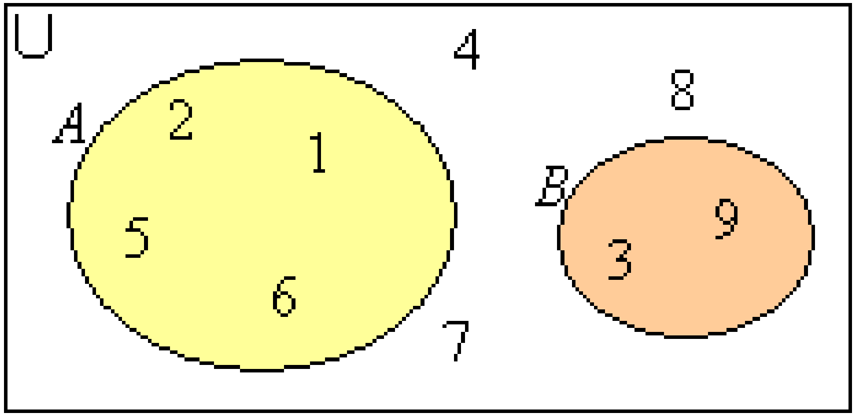
Step 1 : Draw a rectangle and label it U to represent the universal set.

Step 2 : Draw circles within the rectangle to represent the other sets. Label the circles and write the relevant elements in each circle.

Step 3 : Write the remaining elements outside the circles but within the rectangle.

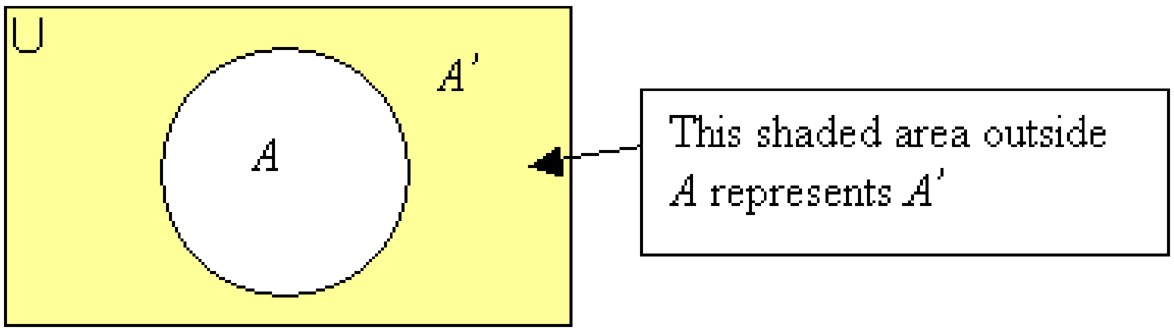
Universal Set (Cont.)

In Venn diagrams, the universal set is usually represented by a rectangle and labeled U.



Complement of A Set

- The **complement** of set A, denoted by A', is the set of all elements in the universal set that are not in A.



- The number of elements of A and the number of elements of A' make up the total number of elements in U .
- **$n(A) + n(A') = n(U)$**

Complement of A Set (Cont.)

Example: Let U = {x : x is an integer, $-4 \le x \le 7$},

P = {-4, -2, 0, 2, 4, 5, 6}

and Q' = {-3, -2, -1, 2, 3}.

a) List the elements of set P'

b) Draw a Venn diagram to display the sets U ,

P and P'

c) Find n(Q)

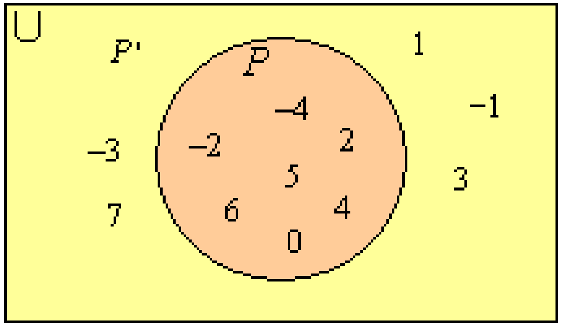
Complement of A Set (Cont.)

a) First, list out the members of U.

$U = \{-4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7\}$

$P' = \{-3, -1, 1, 3, 7\} \leftarrow$ in U but not in P

b) Draw a Venn diagram to display the sets U , P and P'



c) Find n(Q)

$n(U) = 12$, $n(Q') = 5$

Use the formula: $n(Q) + n(Q') = n(U)$

$n(Q) = n(U) - n(Q') = 12 - 5 = 7$

Set Operations: Intersection

- The **intersection** of two sets X and Y is the set of elements that are common to both set X **and** set Y. It is denoted by $X \cap Y$ and is read 'X intersection Y'.

Example:

Draw a Venn diagram to represent the relationship between the sets

$X = \{1, 2, 5, 6, 7, 9, 10\}$ and

$Y = \{1, 3, 4, 5, 6, 8, 10\}$

Set Operations: Intersection (Cont.)

Solution:

We find that $X \cap Y = \{1, 5, 6, 10\} \leftarrow$ in both X and Y

For the Venn diagram,

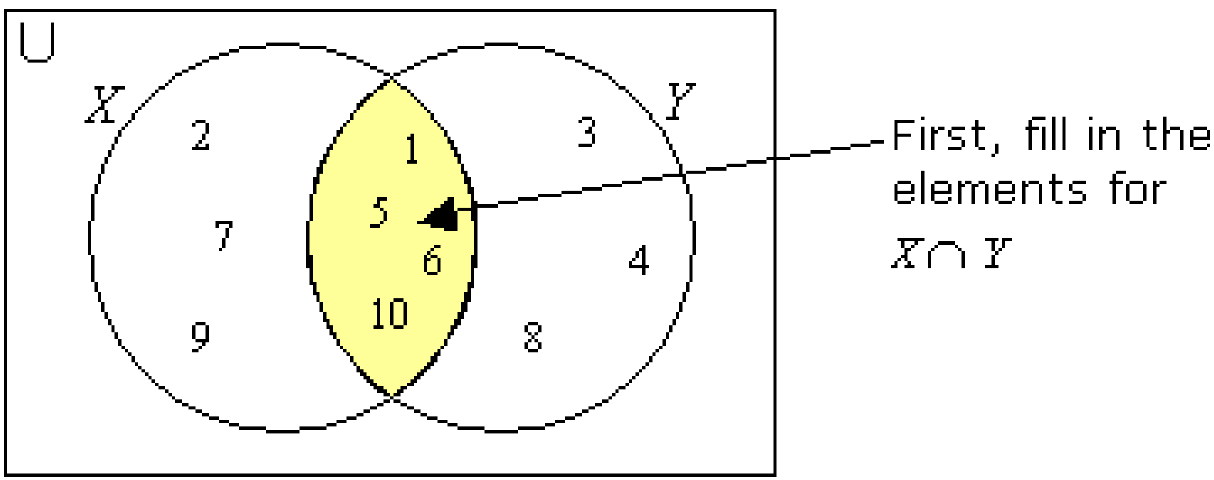
Step 1 : Draw two overlapping circles to represent the two sets.

Step 2 : Write down the elements in the intersection.

Step 3 : Write down the remaining elements in the respective sets.

Set Operations: Intersection (Cont.)

Notice that you start filling the Venn diagram from the elements in the intersection first.



Set Operations: Intersection (Cont.)

Example: If $X \subset Y$ then $X \cap Y = X$.

- We will illustrate this relationship in the following example.
- Draw a Venn diagram to represent the relationship between the sets

  $X = \{1, 6, 9\}$ and $Y = \{1, 3, 5, 6, 8, 9\}$
- We find that $X \cap Y = \{1, 6, 9\}$ which is equal to the set X

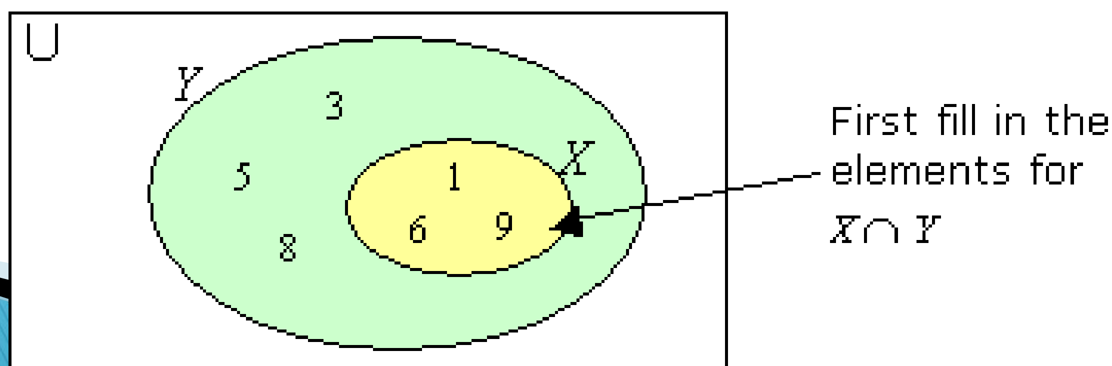
Set Operations: Intersection (Cont.)

For the Venn diagram,

Step 1 : Draw one circle within another circle

Step 2 : Write down the elements in the inner circle.

Step 3 : Write down the remaining elements in the outer circle.



Intersection of Three Sets

- The **intersection** of three sets X, Y and Z is the set of elements that are common to sets X, Y and Z. It is denoted by $X \cap Y \cap Z$

Example: Draw a Venn diagram to represent the relationship between the sets

$X = \{1, 2, 5, 6, 7, 9\}$, $Y = \{1, 3, 4, 5, 6, 8\}$ and

$Z = \{3, 5, 6, 7, 8, 10\}$

Solution: We find that $X \cap Y \cap Z = \{5, 6\}$, $X \cap Y = \{1, 5, 6\}$, $Y \cap Z = \{3, 5, 6, 8\}$ and $X \cap Z = \{5, 6, 7\}$

Intersection of Three Sets (Cont.)

For the Venn diagram:

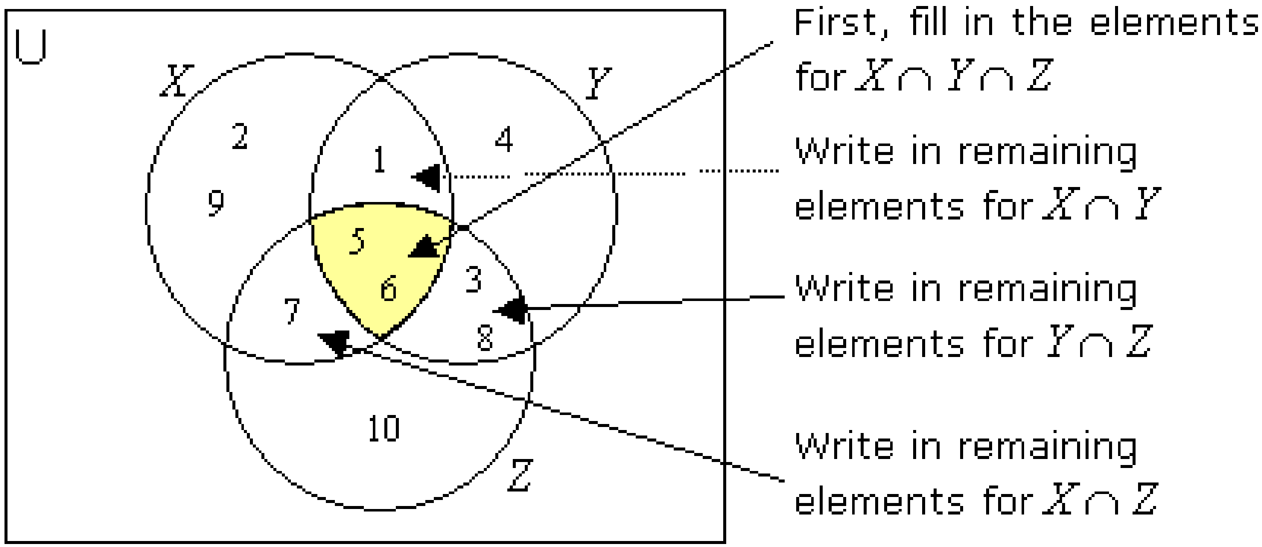
Step 1 : Draw three overlapping circles to represent the three sets.

Step 2 : Write down the elements in the intersection $X \cap Y \cap Z$

Step 3 : Write down the remaining elements in the intersections: $X \cap Y$, $Y \cap Z$ and $X \cap Z$

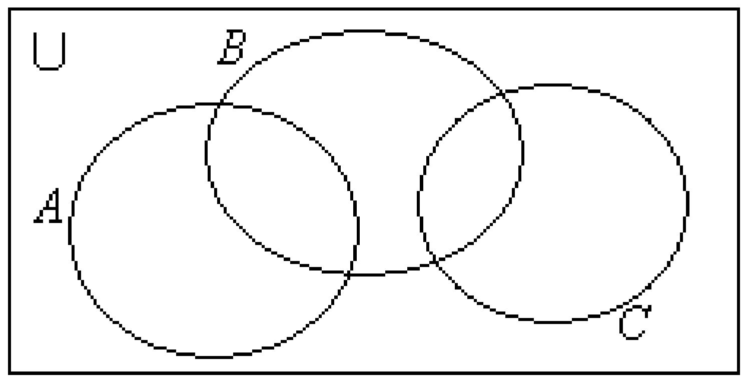
Step 4 : Write down the remaining elements in the respective sets.

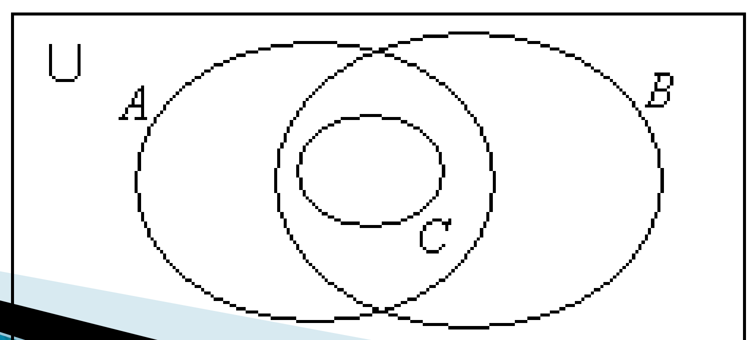
Intersection of Three Sets (Cont.)



Intersection of Three Sets (Cont.)

In general, there are many ways that 3 sets may intersect. Some examples are shown below.





Complement of The Intersection Of Sets

- The complement of the set $X \cap Y$ is the set of elements that are members of the universal set U but not members of $X \cap Y$. It is denoted by

  $(X \cap Y)'$

Example: Suppose U = set of positive integers less than 10,

$X = \{1, 2, 5, 6, 7\}$ and $Y = \{1, 3, 4, 5, 6, 8\}$.

a) Draw a Venn diagram to illustrate $(X \cap Y)'$

b) Find $(X \cap Y)'$

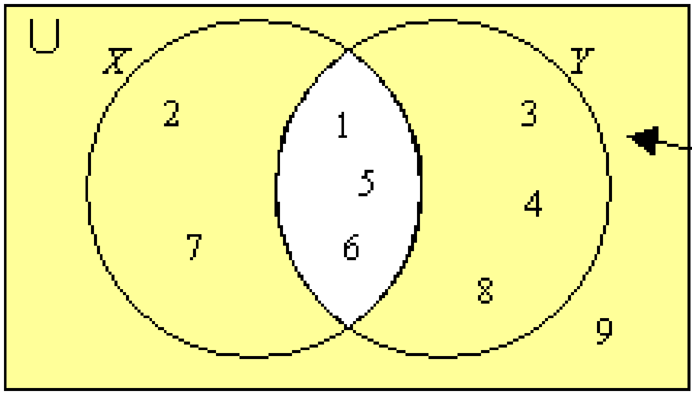
Complement of The Intersection Of Sets (Cont.)

Solution:

a) First, fill in the elements for $X \cap Y = \{1, 5, 6\}$

Fill in the other elements for X and Y and for U

Shade the region outside $X \cap Y$ to indicate $(X \cap Y)'$



The shaded region indicates $(X \cap Y)'$

b) We can see from the Venn diagram that

$(X \cap Y)' = \{2, 3, 4, 7, 8, 9\}$

Or we find that $X \cap Y = \{1, 5, 6\}$ and so

$(X \cap Y)' = \{2, 3, 4, 7, 8, 9\}$

Union of Sets

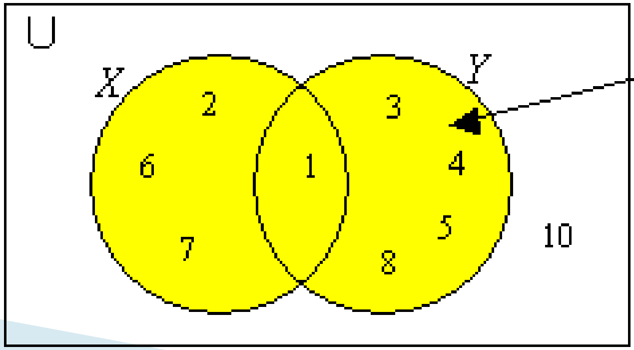
- The **union** of two sets A and B is the set of elements, which are in A **or** in B **or** in both. It is denoted by $A \cup B$ and is read 'A union B'

Example : Given U $= \{1, 2, 3, 4, 5, 6, 7, 8, 10\}$

$X = \{1, 2, 6, 7\}$ and $Y = \{1, 3, 4, 5, 8\}$

Find $X \cup Y$ and draw a Venn diagram to illustrate $X \cup Y$.

$X \cup Y = \{1, 2, 3, 4, 5, 6, 7, 8\} \leftarrow 1$ is written only once.



The shaded region indicates $X \cup Y$

Union of Sets (Cont.)

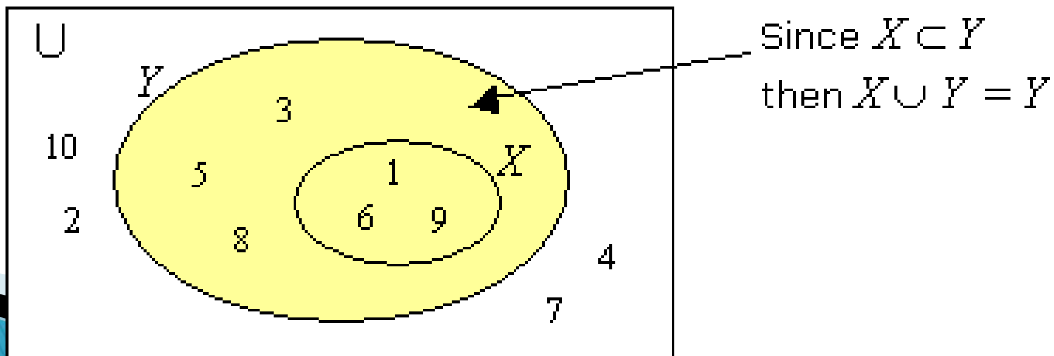
- If $X \subset Y$ then $X \cup Y = Y$. We will illustrate this relationship in the following example.

Example: Given U $= \{1, 2, 3, 4, 5, 6, 7, 8, 10\}$

$X = \{1, 6, 9\}$ and $Y = \{1, 3, 5, 6, 8, 9\}$

Find $X \cup Y$ and draw a Venn diagram to illustrate $X \cup Y$.

Solution: $X \cup Y = \{1, 3, 5, 6, 8, 9\}$



Complement of The Union of Sets

- The **complement of the set** $X \cup Y$ is the set of elements that are members of the universal set U but are not in $X \cup Y$. It is denoted by $(X \cup Y)'$

Example: Given: U $= \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$X = \{1, 2, 6, 7\}$ and $Y = \{1, 3, 4, 5, 8\}$

a) Draw a Venn diagram to illustrate $(X \cup Y)'$

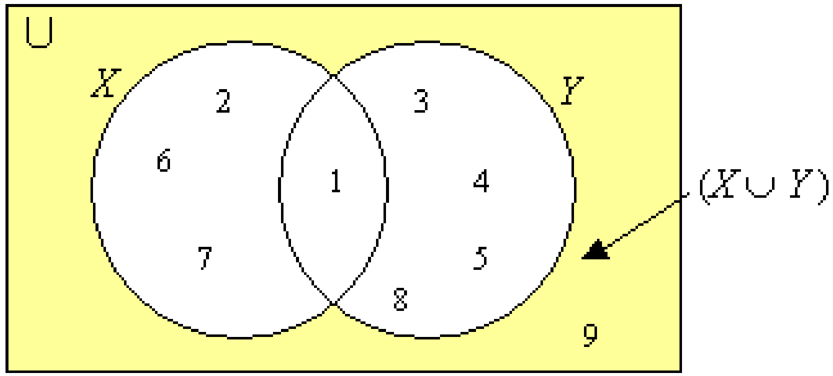
b) Find $(X \cup Y)'$

Solution: a) First, fill in the elements for $X \cap Y = \{1\}$

Fill in the other elements for X and Y and for U

Shade the region outside $X \cup Y$ to indicate $(X \cup Y)'$

Complement of The Union of Sets (Cont.)



b) We can see from the Venn diagram that

$$(X \cup Y)' = \{9\}$$

Or we find that $X \cup Y = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and so $(X \cup Y)' = \{9\}$

Complement of The Union of Sets (Cont.)

Example: Given U = $\{x : 1 \leq x \leq 10, x \text{ is an integer}\}$, A = The set of odd numbers, B = The set of factors of 24 and $C = \{3, 10\}$.

a) Draw a Venn diagram to show the relationship.

b) Using the Venn diagram or otherwise, find:

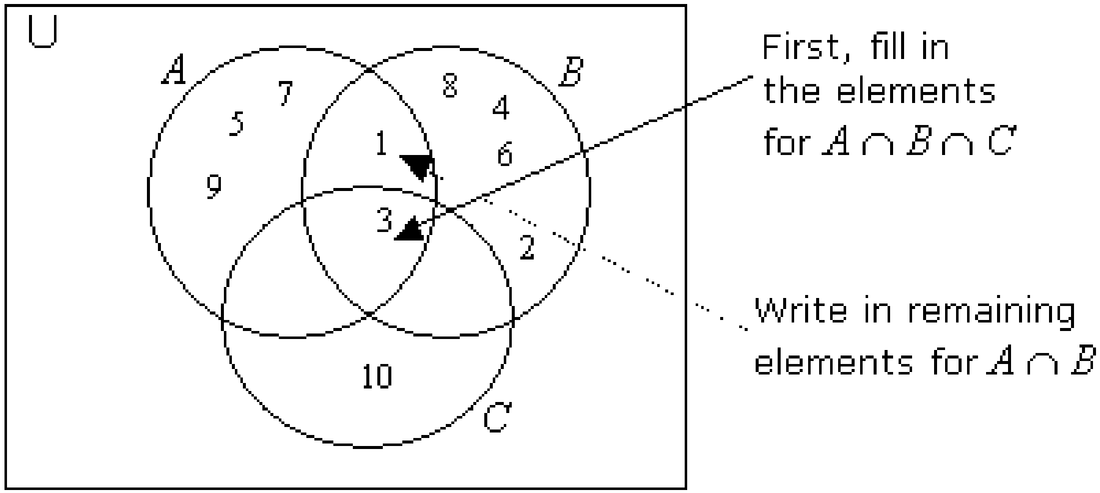
i) $(A \cup B)'$ ii) $(A \cup C)'$ iii) $(A \cup B \cup C)'$

Solution:

$A = \{1, 3, 5, 7, 9\}$, $B = \{1, 2, 3, 4, 6, 8\}$ and $C = \{3, 10\}$

a) First, fill in the elements for $A \cap B \cap C = \{3\}$, $A \cap B \{1, 3\}$, $A \cap C = \{3\}$, $B \cap C = \{3\}$ and then the other elements.

Complement of The Union of Sets (Cont.)



b) We can see from the Venn diagram that

i) $(A \cup B)' = \{10\}$

ii) $(A \cup C)' = \{2, 4, 6, 8\}$

iii) $(A \cup B \cup C)' = \{\ \}$

Combination Operation

- Combined operations involve the intersection, union and complement of sets. Perform the operations within brackets first. Other operations are performed from left to right.

Example: Given that $U = \{x : 1 \leq x \leq 10, x \text{ is an integer}\}$,

$G = \{x : x \text{ is a prime number}\}$,

$H = \{x : x \text{ is an even number}\}$,

$P = \{1, 2, 3, 4, 5\}$.

- List the elements of:

  a) $G \cap H \cup P$

  b) $(G \cap P)' \cup H$

  c) $H' \cap (G \cup P)$

  d) $(P \cup H \cup G)' \cap (G \cap H)$

Combination Operation (Cont.)

Solution:

$G = \{2, 3, 5, 7\}$, $H = \{2, 4, 6, 8, 10\}$

a) $G \cap H \cup P = \{2\} \cup P \leftarrow G \cap H = \{2\}$

$= \{1, 2, 3, 4, 5\}$

b) $(G \cap P)' \cup H = \{2, 3, 5\} \cup H$

$= \{2, 3, 4, 5, 6, 8, 10\}$

c) $H' \cap (G \cup P) = H' \cap \{1, 2, 3, 4, 5, 7\}$

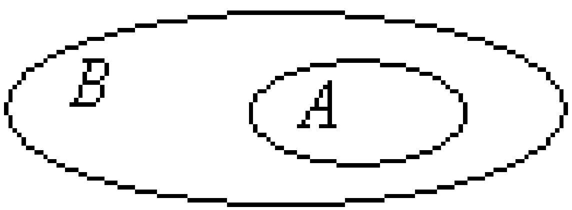
$= \{1, 3, 5, 7\}$

d) $(P \cup H \cup G)' \cap (G \cap H) = \{9\} \cap (G \cap H)$

$= \{9\} \cap \{2\} = \{\ \}$

Drawing Venn Diagrams

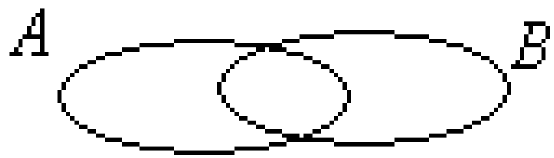
- First, we need to determine the relationships between the sets such as subsets and intersections. There could be several ways to describe the relationships.
- We would draw A **within** B if we know that:



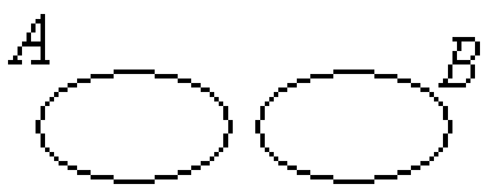
- All members of A belongs to B or $A \subset B$ or $A \cup B = B$ *or* $A \cap B = A$ or $n(A \cap B) = n(A)$

Drawing Venn Diagrams (Cont.)

- We would draw A **overlap** B if we know that: Some members of A belongs to B or $A \cap B \neq \emptyset$ or $n(A \cap B) \neq 0$



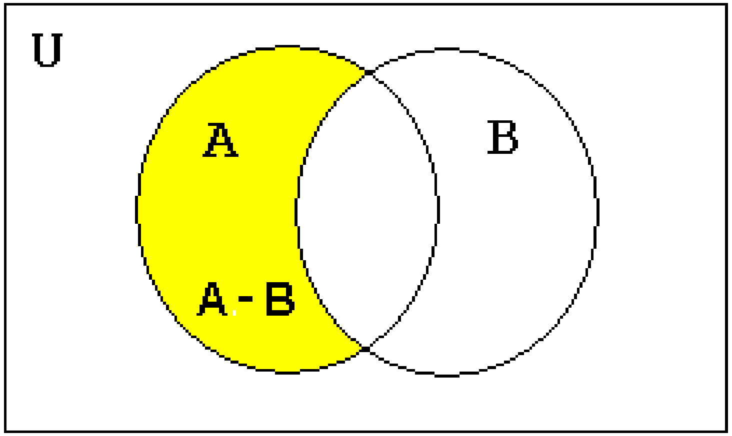
- We would draw **disjoint** sets A and B if we know that no members of A belongs to B or $A \cap B = \emptyset$ or $n(A \cap B) = 0$



Set Difference

- The difference of sets A and B is the set of all elements of A which are not also elements of B.
- Symbolically,

$$A - B = \{x | x \in A \wedge x \neg\text{in } B\}$$



Important Properties

- Distributive Laws

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

- De Morgan's Laws

$$\overline{(A \cup B)} = \bar{A} \cap \bar{B}$$

$$\overline{(A \cap B)} = \bar{A} \cup \bar{B}$$

The Power Set

- Given the set S, the power set of S is the set of all subsets.
- The power of set S is denoted by P(S).
- Example: What is the power set of the set

$$\{0,1,2\}$$

Solution:

$$P(\{0,1,2\}) = \{\phi, \{0\}, \{1\}, \{2\}, \{0,1\}, \{0,2\}, \{1,2\}, \{0,1,2\}\}$$

Cartesian Product

- Let A and B be sets. The Cartesian product of A and B is denoted by $A \times B$, is the set of all ordered pairs (a,b) where $a \in A$ and $b \in B$, Hence,

$$A \times B = \{(a,b) \mid a \in A \wedge b \in B\}$$

Cartesian Product (Cont.)

Example: What is the Cartesian product of $A = \{1,2\}$ and $B = \{a,b,c\}$?

Solution:

$$A \times B = \{(1,a),(1,b),(1,c),(2,a),(2,b),(2,c)\}$$

END

Q & A