QMS Journal Club

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QMS Journal Club 2nd meeting: Postulates of quantum mechanics and Bloch sphere

Postulate 1: Associated to any isolated physical system is a complex vector space with inner product, i.e. a Hilbert space, known as the state space of the system. The system is completely described by its state vector, which is a unit vector in the systems' state space.

Postulate 2: The evolution of a closed quantum system is described by a unitary transformation. That is, the state $|\psi\rangle$ of the system at time t_1 is related to the state $|\psi'\rangle$ of the system at time t_2 by a unitary operator U which depends only on the times t_1 and t_2,

$$|\psi'\rangle = U|\psi\rangle. \tag{1}$$

Explicitly, it can be described by the Schrodinger equation,

$$i\hbar\frac{d}{dt}|\psi\rangle = \hat{H}|\psi\rangle. \tag{2}$$

Or,

$$|\psi(t_2)\rangle = e^{\frac{-i\hat{H}(t_2-t_1)}{\hbar}}|\psi(t_1)\rangle. \tag{3}$$

Question: Suppose A and B are commuting Hermitian operators. Show that $e^A e^B = e^{(A+B)}$ by expanding the left and right hand sides until $n = 3$.

Question: Show that e^{iK} is unitary for some Hermitian matrix K. You only need to expand until $n = 6$.

Note that the matrix exponential is given by

$$e^A = \sum_{n=0}^{\infty} \frac{A^n}{n!}. \tag{4}$$

Postulate 3: Quantum measurements are described by a collection $\{M_m\}$ of measurement operators. These are operators acting on the state space of the system being measured. The index m refers to the measurement outcomes that may occur in the experiment. If the state of the quantum system is $|\psi\rangle$ immediately before the measurement then the probability that result m occurs is given by

$$p(m) = \langle\psi|M_m^\dagger M_m|\psi\rangle, \tag{5}$$

and the state of the system after the measurement is

$$\frac{M_m|\psi\rangle}{\sqrt{\langle\psi|M_m^\dagger M_m|\psi\rangle}}. \tag{6}$$

The measurement operators satisfies the completeness equation,

$$\sum_m M_m^\dagger M_m = I. \tag{7}$$

A projective measurement is described by an observable, M, a Hermitian operator on the state space of the system being observed. The observable can be written as

$$M = \sum_m m|m\rangle\langle m|, \tag{8}$$

where $P_m = |m\rangle\langle m|$ is the projector onto the eigenspace of M with eigenvalue m, satisfying $P_m P_m^\dagger = \delta_{mm'} P_m$.
The expected value of the measurement is

$$\begin{aligned} E(M) = \langle M\rangle &= \sum_m m p(m) \\ &= \sum_m m\langle\psi|P_m|\psi\rangle \\ &= \langle\psi|\sum_m m|m\rangle\langle m||\psi\rangle \\ &= \langle\psi|M|\psi\rangle \end{aligned} \tag{9}$$

Question: The variance associated to observations of M, $[\Delta(M)]^2$ is given by

$$[\Delta(M)]^2 = \left\langle (M - \langle M \rangle)^2 \right\rangle . \tag{10}$$

Show that it is equivalent to $[\Delta(M)]^2 = \left\langle M^2 \right\rangle - \langle M \rangle^2$.

Question: Suppose A and B are two Hermitian operators, and $\langle\psi|AB|\psi\rangle = x + iy$. Show that $\langle\psi|[A, B]|\psi\rangle = 2iy$, $\langle\psi|\{A, B\}|\psi\rangle = 2x$.

The above implies that

$$|\langle\psi|[A, B]|\psi\rangle|^2 + |\langle\psi|\{A, B\}|\psi\rangle|^2 = 4|\langle\psi|AB|\psi\rangle|^2. \tag{11}$$

By the Cauchy-Schwarz inequality,

$$|\langle\psi|AB|\psi\rangle|^2 \leq \langle\psi|A^2|\psi\rangle\langle\psi|B^2|\psi\rangle. \tag{12}$$

Hence,

$$|\langle\psi|[A, B]|\psi\rangle|^2 \leq 4\langle\psi|A^2|\psi\rangle\langle\psi|B^2|\psi\rangle. \tag{13}$$

Question: Suppose C and D are two observables, and $A = C - \langle C \rangle$, $B = D - \langle D \rangle$. Obtain the Heisenberg's uncertainty principle,

$$\Delta(C)\Delta(D) \geq \frac{|\langle \psi | [C, D] | \psi \rangle|}{2}. \tag{14}$$