Physics 250 Homework 5 due Friday March 20, 4am 2009

To be graded: 1, and 1 of 2,3

1) Using a numerical experiment the find the distribution function of $\chi^2(N)$ and compare it to a Gaussian. Specifically write a program that will
   a) Calculate *M* random numbers x_i that are the sum of the squares of N random numbers with a normal/Gaussian distribution with zero average and unit variance, e.g. $x_i = \sum_{j=1}^{N} (\theta_{ij})^2$, θ_{ij} Gaussian, $\langle \theta_{ij} \rangle = 0, \langle (\theta_{ij})^2 \rangle = 1$. Such Gaussian random numbers can be found by calling the MatLab function randn, then square each random number and add these *N* squares together. Do this *M* times.
   b) "Bin" these random numbers. Specifically, let $C(i)$ be the number of times each of these numbers are between $i-1, i$ e.g. $i-1 < x_i < i$. Use bins from 0 to $2N$, for $i < 2N$ $C(i)$ is the number of times $i-1 < x_i < i$ and bin all $x_i > 2N-1$ together in $C(2N)$ so that $C(2N)$ is the number of times $x_i > 2N-1$ Also calculate $\mu = \sum_i x_i / M$ and $\sigma^2 = \sum_i (x_i - \mu)^2 / (M-1)$
   c) Run this program for $N = 10, M = 100$ and $N = 30, M = 1{,}000$, report μ, σ and plot (on the same graph), for each pair N, M (i) $C(i)/M$, as a function of $x = i - 1/2$ e.g. the center of the range. Also plot $\sigma^{-1}(2\pi)^{-1/2}\exp\left[-(x-\mu)^2/2\sigma^2\right]$, and $(4\pi N)^{-1/2}\exp\left[-(x-N)^2/(4N)\right]$, all in different colors. Optionally, also plot $\exp(-x/2)2^{-N/2}x^{(N/2)-1}/\Gamma(N/2)$.

2) Experiment with fitting curves polynomials. In particular, write a program that will call the Garcia program pollsf. This program should combine parts of the Garcia programs lsdemo and linreg. Specifically, it should
   a. Ask what the RMS deviation σ in a data point should be
   b. Ask what order M of polynomial the model should be meaning that the model is
      $$Y(x) = \sum_{j=0}^{M} a_j x^j$$
   c. Ask for the number of points N to be fitted and the maximum value of x, $x_{\max}$ to be fitted

d. Generate data $y_i = \theta_i + \sum_{j=0}^{M} a_j x_i^j$ where θ_i is a Gaussian random variable with $\langle\theta\rangle = 0, \left\langle\left(\theta - \langle\theta\rangle\right)^2\right\rangle = \sigma^2$ (see randn in the documentation and/or Garcia's lsfdemo or normrnd(mu,sigma) or random('gaussian',…)) for $x_i = (i-1)x_{\max}/N$ for $i = 1,2,\ldots N$

e. Use pollsf to fit the data

f. Plot, on a single graph all of (i) the data points with errorbars (see lsdemo e.g. to find out how to plot error bars) (ii) the actual curve (iii) the fitted curve

Then, run this program (and turn in graphs) *twice* with different random numbers for each of

a. $x_{\max} = 10 \quad N = 100, M = 1 \; Y = .5 + .9x$ and $\sigma = .5$

b. $x_{\max} = 10 \quad N = 10, M = 7 \; Y = 0 = 0x + 0x^2 + \ldots 0x^7$ and $\sigma = .5$

Comment on your results

3) This is a simplification of a problem I had to solve with a graduate student about a decade ago, and similar works are published in the literature routinely (with other "stuff", of course.) In designing an experiment you note that you can measure a result I_j as a function of a control variable θ for $\theta_j = 2\pi j/100$, $j = 1,2,\ldots 100$. Your model for these results is

$$I(\theta) = a_1\cos(b) + a_2\sin(b)\cos(\theta + c) + a_3\sin(b)\sin(\theta - c)$$

where a_1, a_2, a_3 are parameters you wish to determine and b, c depend on how you have set your apparatus, which is to say (for example) they are the angles at which you have set various polarizers in you apparatus. You also suppose that there is a *constant* noise in your data which is Gaussian, with zero mean and standard deviation σ. Try to design a good way to do this experiment. Specifically:

a. Write a program that will predict the inverse covariance matrix $A_{notes} = C_{Garcia}^{-1}$ for the a's as a function of b, c, σ. This program should also use the MatLab function eig to find the eigenvalues of this matrix. Finally, find which eigenvalue (of the three) is smallest.

b. Run your result above with $\sigma = .1$ and $b = \pi n/5, c = \pi m/10$ with $n, m = 1,2,3,4$ Make a table of the smallest eigenvalue for each pair n,m and say which of these values of b, c you would most prefer to do the experiment at, which is the values of *b,c* for which this smallest eigenvalue is largest.