Physics 250 Computational Methods in Physics
Homework 6
Due 3 of 4 problems due 4am Friday April 10, 2009

1) Modify the programs that I have posted that allow fitting to a background plus an exponential decay to automatically find the x% confidence limit region for the decay rate τ and to plot the curves that would correspond to these "limit of confidence" fits – e.g. the "theory" with the values of a, b, τ that correspond to the values of τ that are on the upper and lower limits of the confidence region - on the same graph as the data and the best fit. Using 10000 decays, 40 bins, $\tau = 1.1$, a time step of 0.1 and 0.2 background counts per decay run this program twice with different random numbers for each of 90%, and 95% confidence limits. I recommend using the scheme I used to calculate the confidence limits as a function of τ and / but then using a moderately clever scheme such as regula falsi (if you don't remember this, consult the web http://en.wikipedia.org/wiki/Regula_falsi) to find the actual values of τ, a, b that are the "limit" fits.

2) Modify the programs I have posted that allow fitting to a background plus an exponential decay so that there is no fixed background and/but there are two distinct exponential decays (corresponding e.g. to two different decaying particles) with a different number of decays (N_1, N_2) with each exponential decay rate (τ_1, τ_2). Then arrange your program to allow for *either* fitting to a single exponential *or* fitting to two exponentials. You should assume Poisson statistics (so that σ^2 is the number of counts in a bin). You should also report the reduced $\chi^{(2)}$'s for each type of fit and the appropriate f statistic for determining if adding in the additional two parameters associated with the second exponential decay is justified statistically. You should also plot the data and *both* fits to it.
   a) Then, run this program with $N_1 = N_2 = 10,000$ $\tau_1 = 1.3, \tau_2 = 4$ with 100 time bins, each 0.1 wide. Report if you believe that you could reliably find that there were two distinct decays with these parameters.
   b) Using $N_1 = N_2 = 10,000$ and 100 time bins, each 0.1 wide, run this program for $\tau_1 = 2 - \Delta\tau, \tau_2 = 2 + \Delta\tau$ for $\Delta\tau = 1,, 0.1,, 0.01$. Report the value of the relevant f tests and reduced $\chi^{(2)}$'s for each of these runs, and say if you would believe a claim that there were two differently decaying particles in each of these cases.

3) Modify the program you wrote for problem 2 to find the confidence limits on τ_1 and τ_2 - separately, not together. You need not make this automatic, as suggested in problem 1: it is enough to do this as my posted program does, simply computing the probability of a variety of τ's. Run this program for the conditions of (2a) above.

4) The programs I posted result in a variety of errors. There are the statistical errors we have discussed until now, there are also "binning" errors. In effect, if we put counts in "bins" we "forget" exactly when they occurred, which will inevitably imply that we

get a “wrong” answer for the decay time. Possibly the best way to be sure you are assessing the binning error correctly is to modify the programs I posted so that two problems can be run, with the same data: one with bins of some size, the other with bins some fraction (say ½) that size. Modify your program of (1) to do this and report the change in the best-fit value of τ under when the bin size is halved and the number of bins is doubled. State whether “bin-smearing” is a problem in this condition. If not, bearing in mind that the statistical error in τ is approximately *inversely* proportional to the square root of the decays that are observed, estimate the number of decays you would expect to have to observe before you needed to halve the bin size and double the number of bins to keep the statistical error bigger than the binning error. Also estimate the size you would expect for the bin-smearing error on the basis of mid-point rule integration and if your result is consistent with this.