Physics 250 Homework 5 due Friday April 24, 4am 2009

To be graded: 2 of three

1) A way to test the distribution of a random variable is to "bin it". The best way to do this, if you have a total of *N* samplings of the random variable is to make M bins with the size of the bins adjusted to so that the probability that any sampling of the random variable has a probability of $1/M$ of being in each (and every) bin. You can then calculate $\chi^{(2)}(M-1)=\sum_i (C_i - N/M)^2/(N/M)$, which you can think of as being a least squares fit with the "theory" $C_i = A = N/M$ and $\sigma_i^2 = A$. There are $M-1$ parameters because of the M bins and the one parameter, A, which is (rather easily but you need not) shown to be N/M. I want you to use Garcia's DSMCeq program to verify that, in equilibrium, the distribution function of speeds is $P(v)=cv^2\exp\left(-mv^2/(2k_bT)\right)$. Garcia chooses to use specific (numerical) values / units for m, k_bT which I may get wrong, so copy them from him, not from me.
   a) Write a program that makes M bins for v, each of which has probability $1/M$. To do this, realize / take my word that $mv^2/(k_bT) = m\left(v_x^2+v_y^2+v_z^2\right)/(k_bT)$ has the $\chi^{(2)}$ distribution with $\nu = 3$ variables – each of the scaled components is a Gaussian. These should be "simple" bins of adjacent values e.g. the i^{th} bin should be of the form $\chi_i < mv^2/(k_bT) < \chi_{i+1}$ with $\chi_1 = 0, \chi_{M+1} = \infty$ and χ_i is the inverse of the cumulative probability distribution function for a $\chi^{(2)}$ distribution with 3 degrees of freedom e.g. $P_{\chi^{(2)}}(\chi_i, 3) = (i+1)/M$ and can be easily calculated by MatLab.
   b) Run Garcia's dsmceq program with $20,000$ particles and arrange for Garcia to want to plot 10 different histograms. At each time that Garcia would plot the histogram, make and plot the histogram suggested above with $M = 15$. Calculate $\chi^{(2)}(M-1)=\sum_i (C_i - N/M)^2/(N/M)$ and say if the results (at each time) can with a confidence limit of 95% be said to be inconsistent with the equilibrium distribution.

2) Write a Monte-Carlo program that calculates an integral and estimates the error in this integral in the hypercube $0 < x_i < 1$ for $i = 1, 2, 3, \ldots N$ by choosing points randomly and uniformly in this region. Then use this, and both 10,000 and 100,000 attempts, to calculate following two integrals. Do *not* use importance sampling the VEGAS program, or anything similar. Report both the answer you get for the integral and the expected error. Give both the "1 σ" and the 95% confidence limits for this error.

   a) $\int dxdydzdw \exp(x^2\cos\left(y^2+zw\right)/(w+2))$

b) $\int dxdydzdwdv \cos\left[(x+y^2+z^3+vw)^2\right]\exp(2x)$

3) Write a very simple "nuclear reactor" simulation. Specifically, suppose that there are neutrons in the reactor are either "in the center" or "in the outside" of the reactor. There are N_c neutrons in the core and N_o neutrons in the surface of the reactor. At each time step each neutron in the reactor has a probability p_r of reacting to form two neutrons. Each neutron in the center has probability p_d of moving to the outside. If it does *neither* of these things e.g. with probability $1-p_r-p_d$ the neutron simply stays in the center. A neutron in the outside of the reactor can, with probability p_r react, with probability p_i move to the center, with probability p_e escape from the reactor and with probability $1-p_r-p_i-p_e$ just stay in the outside. The program should start from one neutron in the center and should end when either there are more than $2^{14}=16,384$ neutrons *or* there are none. Moreover, it should keep track of the number of time steps it takes to get to this goal. Run the program 20 times for $p_d=.3, p_e=.1, p_i=.07$ and (each of) $p_r=0.025,0.05,.1,.2,.4$. For each set of parameters report the number of times the chain reaction succeeds (you get to more than 16,000 neutrons) and the number of times the chain reaction "fizzles" e.g. you go to zero total neutrons. Also report the doubling time and the final ratio of neutrons in the center and outside of the reactor when the reaction "goes". The doubling time should be the average over all the times the reaction "goes" of $\log_2(N)/t$ where N is the final number of neutrons (the first number exceeding 2^{14}) and t is the number of time steps it takes to get to this number. The ratio of the number of neutrons in the core to that in the outside is (also) the average of that ratio over the "last step" of reactions that "go". If the reaction never "goes" report "not applicable" for the doubling time and ratio of neutrons in the center and outside.