Assignment 7

120100093

h=0.1

	1	2	3	4
1	0.0129	0.0039	0.0039	-0.0119
2	0.0047	0.0014	0.0014	-0.0043
3	0.0016	5.2945e-04	5.2262e-04	-0.0016
4	5.5296e-04	1.9574e-04	1.9224e-04	-5.8632e-04
5	1.6009e-04	7.2788e-05	7.0702e-05	-2.1574e-04

Column 1 = RK 2, 2 = RK 3, 3 = RK 4, 4 = AB 4

Row 1 = (x = 1), 2 = (x = 2), 3 = (x = 3), 4 = (x = 4), 5 = (x=5)

h=0.05

	1	2	3	4
1	0.0061	0.0018	0.0018	-0.0043
2	0.0022	6.6539e-04	6.6366e-04	-0.0016
3	8.0481e-04	2.4494e-04	2.4415e-04	-5.7790e-04
4	2.8316e-04	9.0227e-05	8.9815e-05	-2.1260e-04
5	9.3583e-05	3.3288e-05	3.3040e-05	-7.8215e-05

Column 1 = RK 2, 2 = RK 3, 3 = RK 4, 4 = AB 4

Row 1 = (x = 1), 2 = (x = 2), 3 = (x = 3), 4 = (x = 4), 5 = (x=5)

h=0.025

	1	2	3	4
1	0.0029	8.4259e-04	8.4212e-04	-0.0017
2	0.0011	3.1000e-04	3.0980e-04	-6.4142e-04
3	3.8447e-04	1.1406e-04	1.1397e-04	-2.3597e-04
4	1.3824e-04	4.1976e-05	4.1927e-05	-8.6808e-05
5	4.8238e-05	1.5454e-05	1.5424e-05	-3.1935e-05

Column 1 = RK 2, 2 = RK 3, 3 = RK 4, 4 = AB 4

Row 1 = (x = 1), 2 = (x = 2), 3 = (x = 3), 4 = (x = 4), 5 = (x=5)

h=0.0125

	1	2	3	4
1	0.0014	3.9304e-04	3.9298e-04	-7.5930e-04
2	4.9648e-04	1.4459e-04	1.4457e-04	-2.7933e-04
3	1.8161e-04	5.3196e-05	5.3184e-05	-1.0276e-04
4	6.6015e-05	1.9571e-05	1.9565e-05	-3.7803e-05
5	2.3635e-05	7.2014e-06	7.1977e-06	-1.3907e-05

Column 1 = RK 2, 2 = RK 3, 3 = RK 4, 4 = AB 4

Row 1 = (x = 1), 2 = (x = 2), 3 = (x = 3), 4 = (x = 4), 5 = (x=5)

h=0.00625

	1	2	3	4
1	6.3315e-04	1.8337e-04	1.8336e-04	-3.4203e-04
2	2.3255e-04	6.7457e-05	6.7454e-05	-1.2583e-04
3	8.5294e-05	2.4816e-05	2.4815e-05	-4.6289e-05
4	3.1179e-05	9.1297e-06	9.1289e-06	-1.7029e-05
5	1.1308e-05	3.3588e-06	3.3584e-06	-6.2645e-06

Column 1 = RK 2, 2 = RK 3, 3 = RK 4, 4 = AB 4

Row 1 = (x = 1), 2 = (x = 2), 3 = (x = 3), 4 = (x = 4), 5 = (x=5)

Computational Time

	1	2	3	4	5
1	4.3385e-05	8.4438e-05	1.5628e-04	2.7897e-04	4.9917e-04
2	5.5981e-05	9.3302e-05	1.7447e-04	3.4102e-04	5.8874e-04
3	6.6711e-05	1.2642e-04	2.0760e-04	3.9280e-04	7.3102e-04
4	7.2309e-05	1.2409e-04	2.3512e-04	4.2826e-04	7.8934e-04
5					

Row 1 = RK 2, 2 = RK 3, 3 = RK 4, 4 = AB 4

Column 1 = (h=0.1), 2 = (h=0.05), 3 = (h=0.025), 4 = (h=0.0125), 5 = (h=0.00625)

Error Using Richardson Extrapolation

h=0.1

	1	2	3	4
1	-0.0038	-0.0038	-0.0038	-0.0038
2	-0.0014	-0.0014	-0.0014	-0.0014
3	-5.1020e-04	-5.1020e-04	-5.1020e-04	-5.1020e-04
4	-1.8769e-04	-1.8769e-04	-1.8769e-04	-1.8769e-04
5	-6.9047e-05	-6.9047e-05	-6.9047e-05	-6.9047e-05

Comparison abs (error_richardson) - abs (error)

	1	2	3	4
1	-0.0091	-1.3376e-04	-9.8810e-05	-0.0081
2	-0.0033	-4.8782e-05	-3.3829e-05	-0.0029
3	-0.0011	-1.9250e-05	-1.2418e-05	-0.0011
4	-3.6527e-04	-8.0534e-06	-4.5471e-06	-3.9863e-04
5	-9.1048e-05	-3.7413e-06	-1.6549e-06	-1.4670e-04

Column 1 = RK 2, 2 = RK 3, 3 = RK 4, 4 = AB 4

Row 1 = (x = 1), 2 = (x = 2), 3 = (x = 3), 4 = (x = 4), 5 = (x=5)

h=0.05

	1	2	3	4
1		-0.0016	-0.0016	-0.0016
2	-5.7946e-04	-5.7946e-04	-5.7946e-04	-5.7946e-04
3	-2.1317e-04	-2.1317e-04	-2.1317e-04	-2.1317e-04
4	-7.8422e-05	-7.8422e-05	-7.8422e-05	-7.8422e-05
5	-2.8850e-05	-2.8850e-05	-2.8850e-05	-2.8850e-05
6				

Comparison abs (error_richardson) - abs (error)

	1	2	3	4
1	-0.0061	-2.3287e-04	-2.2888e-04	-0.0027
2	-0.0017	-8.5923e-05	-8.4199e-05	-9.9137e-04
3	-5.9163e-04	-3.1769e-05	-3.0973e-05	-3.6473e-04
4	-2.0474e-04	-1.1806e-05	-1.1393e-05	-1.3418e-04
5	-6.4733e-05	-4.4378e-06	-4.1902e-06	-4.9366e-05
6				

Column 1 = RK 2, 2 = RK 3, 3 = RK 4, 4 = AB 4

Row 1 = (x = 1), 2 = (x = 2), 3 = (x = 3), 4 = (x = 4), 5 = (x=5)

h=0.025

	1	2	3	4
1	-6.9368e-04	-6.9368e-04	-6.9368e-04	-6.9368e-04
2	-2.5519e-04	-2.5519e-04	-2.5519e-04	-2.5519e-04
3	-9.3879e-05	-9.3879e-05	-9.3879e-05	-9.3879e-05
4	-3.4536e-05	-3.4536e-05	-3.4536e-05	-3.4536e-05
5	-1.2705e-05	-1.2705e-05	-1.2705e-05	-1.2705e-05

Comparison abs (error_richardson) - abs (error)

	1	2	3	4
1	-0.0022	-1.4891e-04	-1.4844e-04	-0.0010
2	-8.0115e-04	-5.4813e-05	-5.4608e-05	-3.8623e-04
3	-2.9059e-04	-2.0184e-05	-2.0089e-05	-1.4209e-04
4	-1.0370e-04	-7.4401e-06	-7.3903e-06	-5.2272e-05
5	-3.5533e-05	-2.7487e-06	-2.7187e-06	-1.9230e-05
6				

Column 1 = RK 2, 2 = RK 3, 3 = RK 4, 4 = AB 4

Row 1 = (x = 1), 2 = (x = 2), 3 = (x = 3), 4 = (x = 4), 5 = (x=5)

h=0.0125

	1	2	3	4
1	-3.1421e-04	-3.1421e-04	-3.1421e-04	-3.1421e-04
2	-1.1559e-04	-1.1559e-04	-1.1559e-04	-1.1559e-04
3	-4.2524e-05	-4.2524e-05	-4.2524e-05	-4.2524e-05
4	-1.5644e-05	-1.5644e-05	-1.5644e-05	-1.5644e-05
5	-5.7550e-06	-5.7550e-06	-5.7550e-06	-5.7550e-06
6				

Comparison abs (error_richardson) - abs (error)

	1	2	3	4
1	-0.0010	-7.8826e-05	-7.8769e-05	-4.4508e-04
2	-3.8089e-04	-2.9002e-05	-2.8978e-05	-1.6374e-04
3	-1.3909e-04	-1.0672e-05	-1.0660e-05	-6.0236e-05
4	-5.0372e-05	-3.9278e-06	-3.9217e-06	-2.2160e-05
5	-1.7880e-05	-1.4464e-06	-1.4427e-06	-8.1521e-06
6				

Column 1 = RK 2, 2 = RK 3, 3 = RK 4, 4 = AB 4

Row 1 = (x = 1), 2 = (x = 2), 3 = (x = 3), 4 = (x = 4), 5 = (x=5)

Comparison between RK 3 and RK 4

	1	2	3	4	5
1	3.4948e-05	3.9941e-06	4.7187e-07	5.6698e-08	6.8736e-09
2	1.4953e-05	1.7241e-06	2.0496e-07	2.4750e-08	3.0131e-09
3	6.8322e-06	7.9548e-07	9.5224e-08	1.1562e-08	1.4143e-09
4	3.5062e-06	4.1242e-07	4.9736e-08	6.0748e-09	7.4691e-10
5	2.0863e-06	2.4763e-07	3.0061e-08	3.6915e-09	4.5600e-10
6					

Column 1 = (x = 1), 2 = (x = 2), 3 = (x = 3), 4 = (x = 4), 5 = (x=5)

Row 1 = (h=0.1), 2 = (h=0.05), 3 = (h=0.025), 4 = (h=0.0125), 5 = (h=0.00625)

Comparison of time for RK 4 and AB 4

H	Time difference (RK4-AB4)	Error (RK4-AB4) X=1	Error (RK4-AB4) X=2	Error (RK4-AB4) X=3	Error (RK4-AB4) X=3	Error (RK4-AB4) X=3
0.1:0.025	-0.000170276211340302	0	0.054881074575435	0.126793837152719	0.202431824457189	0.274252283397570
0.05:0.0125	-0.000346150544697272	0	0.017337908635123	0.025383340121259	0.093883374235619	0.127266044199696
0.025:0.00625	-0.000518292796709797	0	0.011788508893888	0.027306631240297	0.043652660592401	0.059186707074498

Conclusions and Interpretation

1) For all methods accuracy is increasing with decreasing value of h
2) RK2 method is least accurate of all the method
3) The error in case of RK3 and RK4 method is not very much for all values of h and the difference in error for the methods is further decreasing as value of h is decreasing
4) The computational time is minimum for RK2 method and increasing with the order of the method and also as h is decreasing
5) The error the estimated using Richardson extrapolation method is less as compared to tabulated errors for all the methods and all the values of h.
6) The computational time in case of AB4 method is very slightly more as compared to the RK4 method, however the step size in case of AB 4 method is $1/4^{th}$ of RK 4.