MATH2070 Assignment

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Abstract

MATH2070 Assignment

Covariance and Correlation

Justify the use of the log return rate. What are the advantages of using it?

Based on the explanation and examples from investmentcache:

1. Log returns can be added across time periods. In the case where our target price/index increases and then decreases by same amount, simple returns would present a **positive** total return while log returns would present a 0 return which is much more in accord with our cognizance.
2. Log returns follows normal distribution. In some certain areas, the stock price is assumed to follow a log normal distribution. Also, considering the fact that log requires that its corresponding value should never be less than 0, which is exactly one property of assets price, log return is even more preferable.

Rebasing

As was shown in Figure1, the original indices have been adjusted based on the value on Mar 1, 2007.

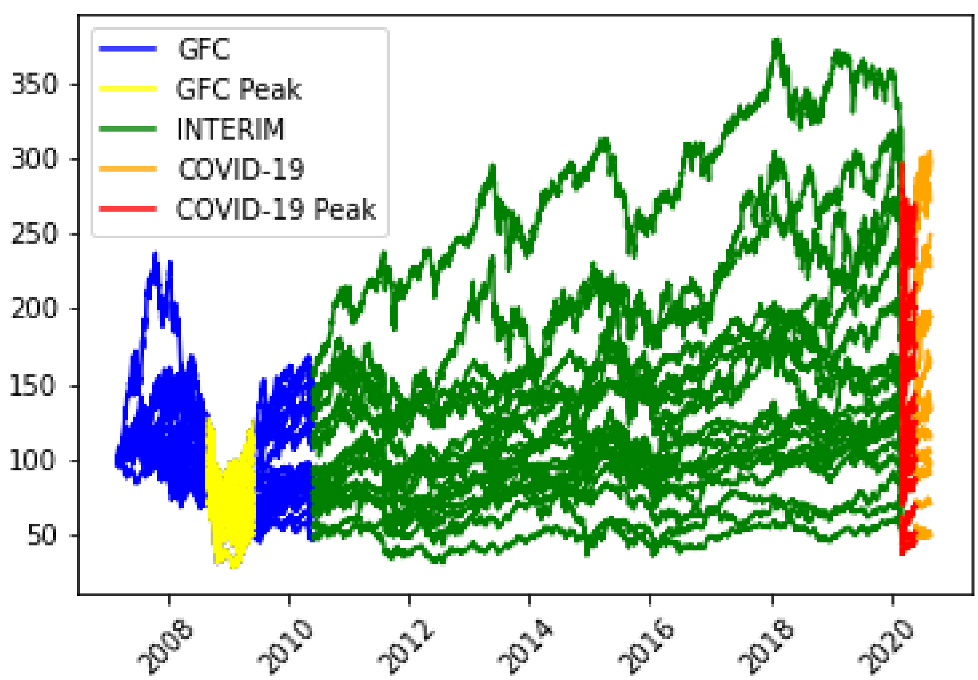
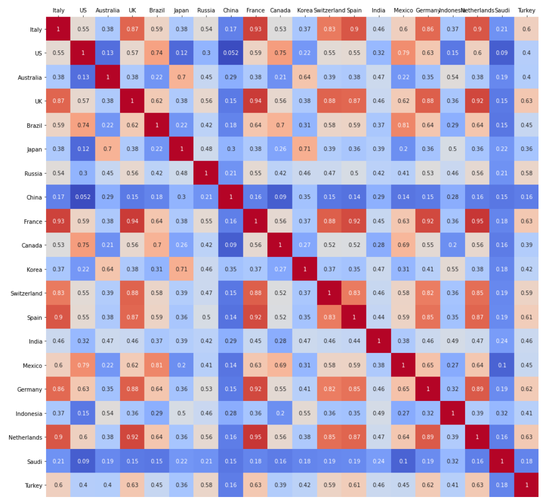
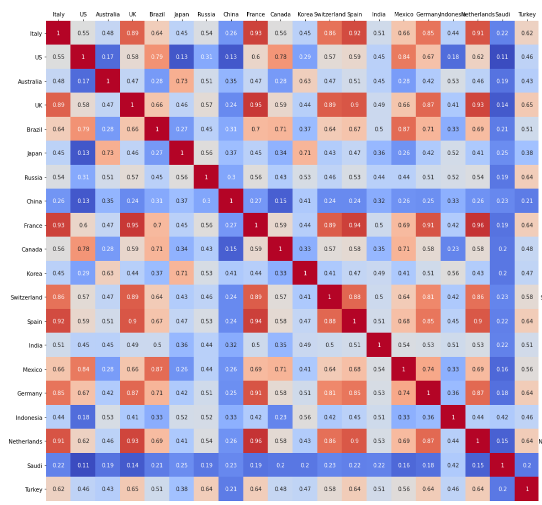


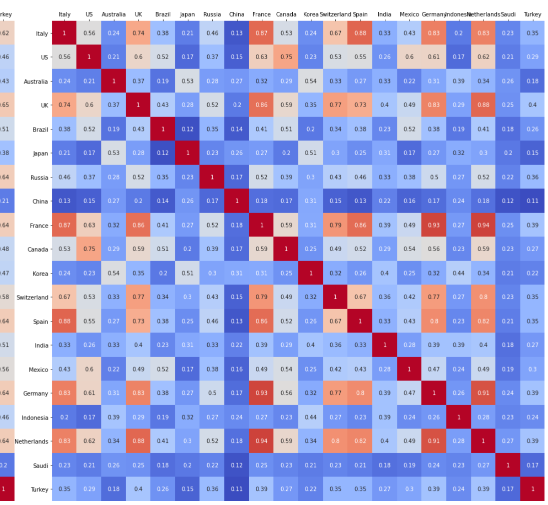
Figure 1: Rebased index values of each country from 2007 to 2020



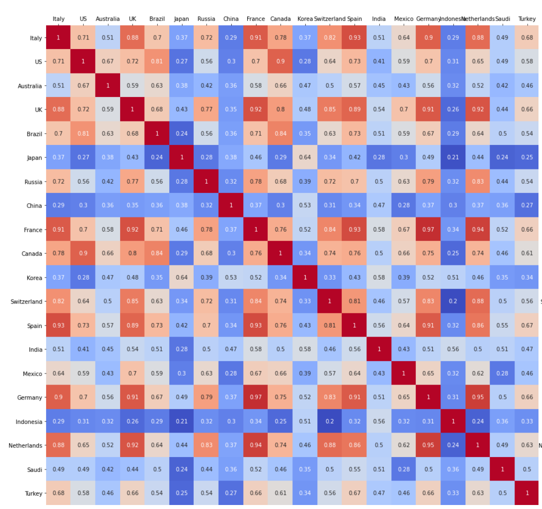
(a) GFC(03/01/2007 - 31/05/2010)



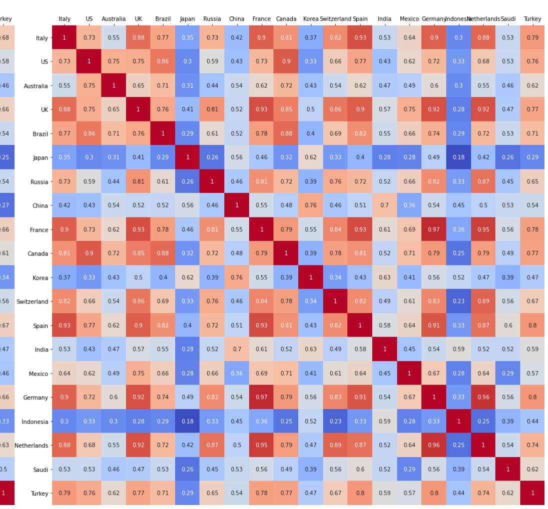
(b) GFC-peak(02/09/2008 – 01/06/2009)



(c) INTERIM(01/06/2010 – 10/03/2020)



(d) COVID-19(11/03/2020 – 31/08/2020)



(e) COVID-19-peak(11/03/2020 – 29/05/2020)

Figure 2: Correlation matriices of each country for 5 periods

Correlation matrices

Figure2 presents a **heatmap** based on the correlation between each market over the five periods.

The **heatmap** basically indicates that, during the peak of COVID—19, the 20 markets are relatively correlated with each other with *orange* and even *red* dominating the plot. Conversely, in the **INTERIM** period, most blue squares show that the interaction among these markets is quite low.

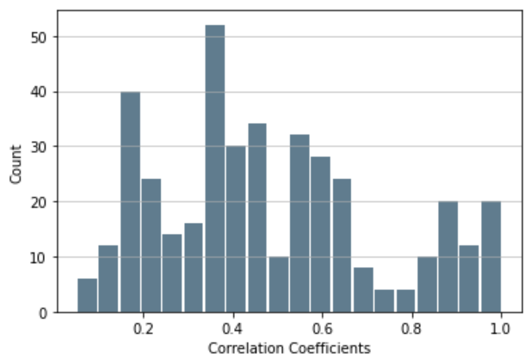
The Global Financial Crisis (GFC) and even its peak time did not bring much expected interaction to these markets. However, similar to what we have described above, COVID-19 and its peak did tie them together, probably more closely than ever before.

Histograms of the correlation coefficients

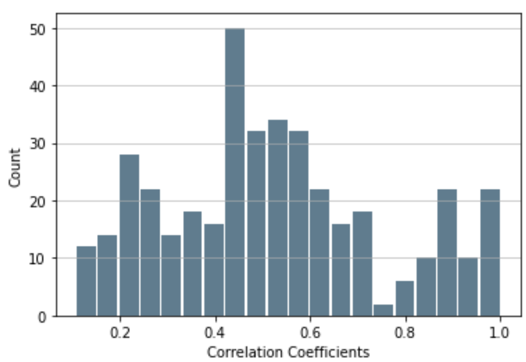
Figure3 presents the histogram of ρ_{ij} of each market. Note: we did include same ρ_{ij} twice. For example, ρ_{12} and ρ_{21} are both plotted into each histogram. But this does not affect the overall trend and relative quantity.

Similar to what we have found in Figure2, here the right skewed ρ_{ij} histograms is less significant during COVID-19 and its peak period, which is not a pleasant plot at all because that means the markets are more and more correlated with each other.

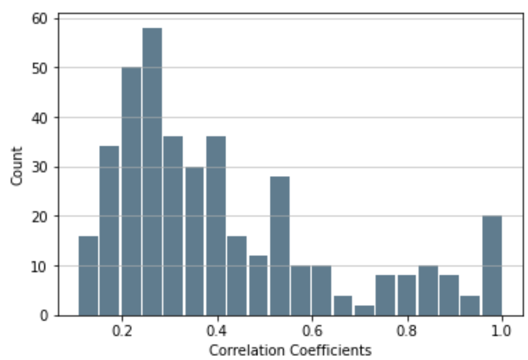
Also, we could not see significant difference as for the histogram of GFC and its following peak.



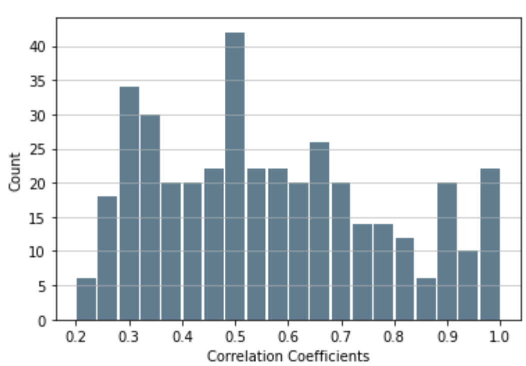
(a) GFC(03/01/2007 - 31/05/2010)



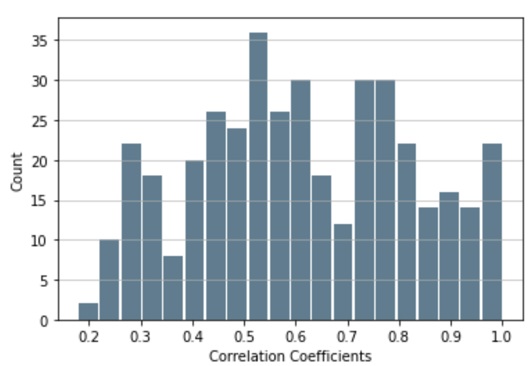
(b) GFC-peak(02/09/2008 – 01/06/2009)



(c) INTERIM(01/06/2010 – 10/03/2020)



(d) COVID-19(11/03/2020 – 31/08/2020)



(e) COVID-19-peak(11/03/2020 – 29/05/2020)

Figure 3: Correlation Coefficients ρ_{ij} for $1 < i, j < 20$

Portfolio Theory 2

Dollar amount invested in each country's index and optimal portfolio P*

	Country	Investment
1	Italy	-228000.00
2	US	1890000.00
3	Australia	126000.00
4	UK	-972000.00
5	Brazil	26700.00
6	Japan	311000.00
7	Russia	-260000.00
8	China	28800.00
9	France	-569000.00
10	Canada	-889000.00
11	Korea	-492000.00
12	Switzerland	48500.00
13	Spain	-815000.00
14	India	634000.00
15	Mexico	-389000.00
16	Germany	958000.00
17	Indonesia	435000.00
18	Netherlands	1060000.00
19	Saudi	-117000.00
20	Turkey	209000.00

Table 1: Investment in each market with 1,000,000 dollors.

Based on formulas from the textbook, we firstly calculated **a**, **b**, **c**, **d** and then with $t = 0.2$, the optimal portfolio $P^\star$ including the weight of each asset, its $\mu^\star$ and $\sigma^\star$ are available.

Thus, we should invest as below with negative values meaning we should short sell the corresponding market indices and positive values meaning we should take a long position in terms of the corresponding market indices. We get $\mu^\star = 0.00138432$ and $\sigma^\star = 0.0166244$

Partial codes are as below:

```
a = np.dot(ones_array_trans, returns_3_full_variance_annualized_inverse)
a = np.dot(a, ones_array)

b = np.dot(ones_array_trans, returns_3_full_variance_annualized_inverse)
b = np.dot(b, returns_3_full_mean_annualized)

c = np.dot(returns_3_full_mean_annualized_trans,
returns_3_full_variance_annualized_inverse)
c = np.dot(c, returns_3_full_mean_annualized)

d=np.dot(a, c)
d = np.subtract(d, b**2)

alpha = (1/a) * returns_3_full_variance_annualized_inverse
alpha = np.dot(alpha, ones_array)

beta_real = np.dot(returns_3_full_variance_annualized_inverse,
np.subtract(returns_3_full_mean_annualized, (b/a)*ones_array))

x_weight  = np.add(alpha, 0.2*beta_real)
money_invested = 1000000 * x_weight
miu_star = (b+d*0.2)/a
varian = (1+d*0.2**2)/a
sigma_star = np.sqrt(varian)
```

Plot

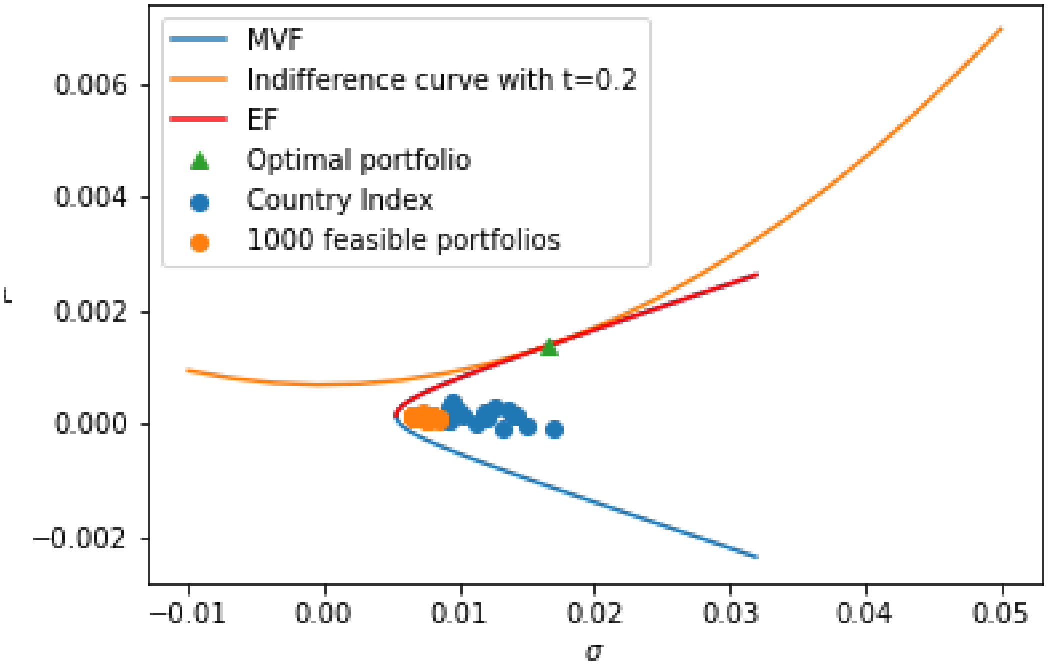


Figure 4: $\mu\sigma$-plane

Figure4 plots things as below:

- All 20 country indices;
- **MVF** (Minimum Variance Frontier) and **EF** (Efficient Frontier);
- 1000 random feasible portfolios;
- Indifference curve of an investor with t = 0.2 and their optimal portfolio.

We should notice that some of the randomly generated portfolios are away from EF even MVF. This *inefficient* random portfolios derive from the fact that our **"Randomly"** generated portfolios are not **random** at all, at least not random enough. That may be caused by the simplicity of the codes used to generate these portfolios:

```
while port_count < 1000:
    weights = np.random.random(num_assets)
    weights = weights/np.sum(weights)
    weights = weights.reshape((20, 1))
    # print(weights)
    varian_port_loop = np.dot(weights.transpose(), returns_3_full_variance_annualized)
    varian_port_loop = np.dot(varian_port_loop, weights)
    sigma_i = np.sqrt(varian_port_loop)
    if np.all([np.abs(weights) < 20*one_array_loop]) and (sigma_i < 0.1):
        p_weights.append(weights)
        p_vol.append(sigma_i)
        port_count = port_count + 1
```

where, although we did have a constraint $| x_{nj} | \leq 20$ for each of the i = 1, 2, . . . , 20 country indices, the random weights generated through above codes are rarely over 1 (test by uncomment codes `print(weights)` above and comment the codes below). This means that the short-selling is not even considered by the algorithm which is partly not what happens in the real market and particularly not what happens to the portfolios on EF. Therefore, some of these *randomly* portfolios are far away from the efficient frontier.

Portfolio Theory 3

INTERIM

	Country	Left	Right
1	Italy	-0.06	Inf
2	US	-Inf	0.00
3	Australia	2.24	Inf
4	UK	0.02	Inf
5	Brazil	-Inf	0.12
6	Japan	-Inf	0.01
7	Russia	-0.05	Inf
8	China	0.66	Inf
9	France	-0.19	Inf
10	Canada	0.04	Inf
11	Korea	0.03	Inf
12	Switzerland	0.27	Inf
13	Spain	0.02	Inf
14	India	-Inf	-0.05
15	Mexico	0.06	Inf
16	Germany	-Inf	0.01
17	Indonesia	-Inf	-0.04
18	Netherlands	-Inf	-0.01
19	Saudi	0.1	Inf
20	Turkey	-Inf	-0.05

Table 2: T range of short-selling for each country

For this question, we need to consider:

- Which investors short sell?
- Which indices they short sell?
- Are there any country indices which no investors short sell or which all investors will short sell?

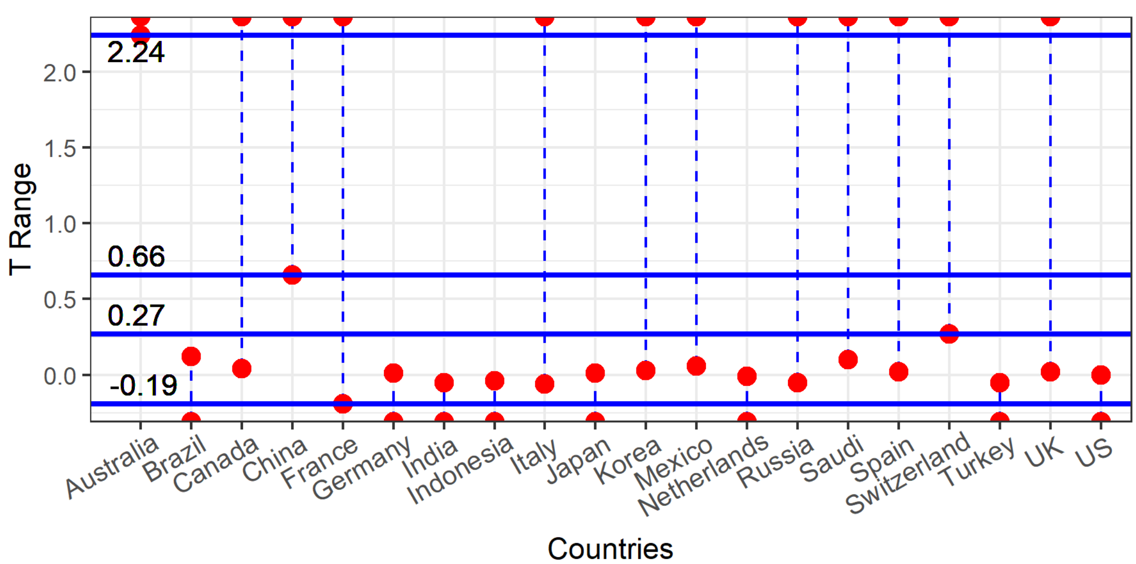


Figure 5: T-range for short-selling

Figure5 implies that investors with $0.27 \leq t < 2.24$ will shortsell most of its assets at hand except for the assets in **Australia**, **Germany**, **India**, **Indonesia**, **Japan**, **Netherlands**, **Turkey** and **US**. As > 2.24, investors will also take a short position at assets in **Australia**. Also, investors with $t < -0.19$ would decide to take a short position at the eight assets mentioned above while taking a long position at other assets. Based on our research and plot, **if only investors have a sufficiently high or low t, they would short sell some certain indices.**

COVID-19 period

	Country	Left	Right
1	Italy	-Inf	0.00
2	US	-Inf	0.00
3	Australia	0.01	Inf
4	UK	0.00	Inf
5	Brazil	-Inf	0.26
6	Japan	-Inf	-0.18
7	Russia	-Inf	0.06
8	China	-Inf	-0.07
9	France	-0.01	Inf
10	Canada	0.00	Inf
11	Korea	-Inf	0.04
12	Switzerland	-Inf	-0.05
13	Spain	0.00	Inf
14	India	-Inf	0.02
15	Mexico	0.04	Inf
16	Germany	-Inf	0.01
17	Indonesia	0.03	Inf
18	Netherlands	0.03	Inf
19	Saudi	-Inf	-0.06
20	Turkey	0.09	Inf

Table 3: T range of short-selling for each country (Covid-19 period)

The detailed values are available in Table**??** and similarly based on this table, we get the following plot.

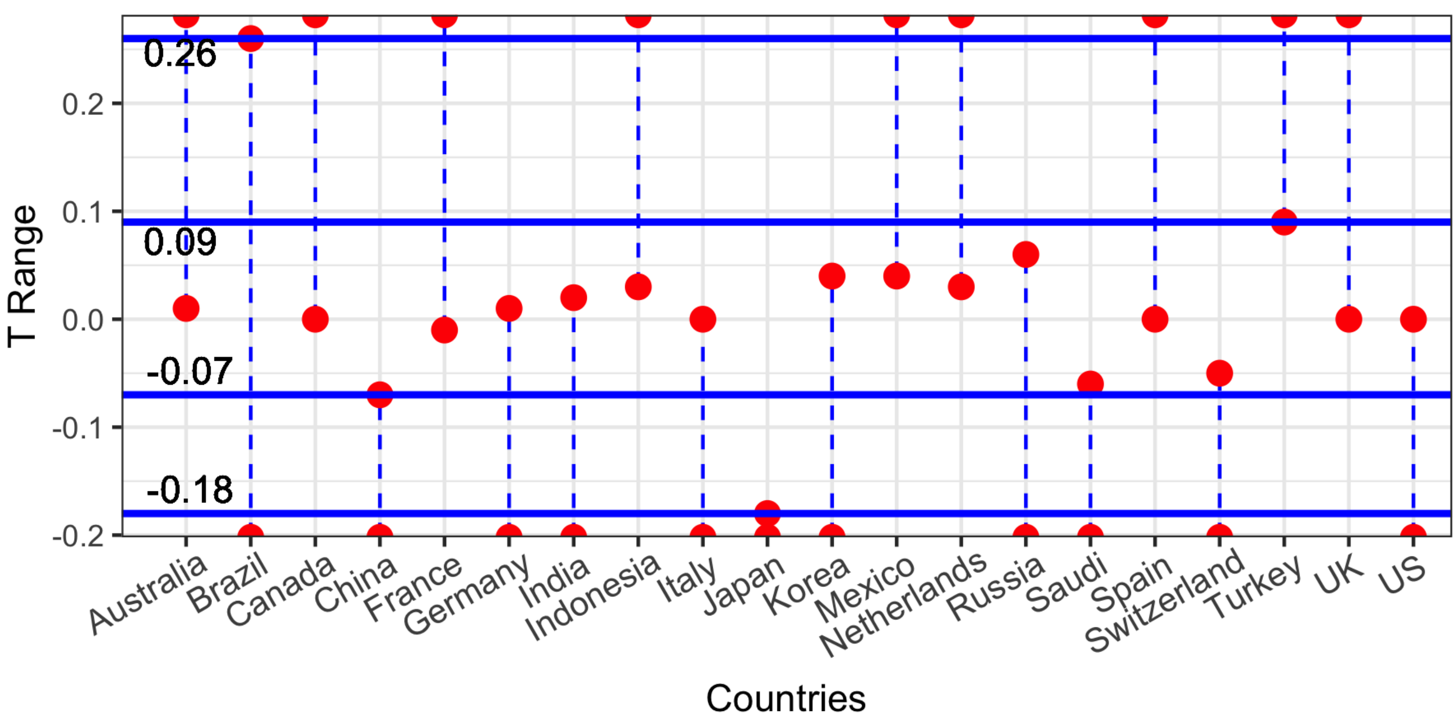


Figure 6: T-range for short-selling (Covid-19 period)

As was shown in Figure6, as $0.09 \leq t \leq 0.26$, investors short sell most of its indices except for it in **China**, **Italy**, **Japan**, **Korea**, **Russia** , **Saudi**, **Switzerland** and **US**. Conversely, investors with $-0.18 < t \leq -0.07$ would decide to take a long position at assets mentioned above.

Compared to the first plot (See Figure5), there are some significant distinctions. For example, investors with $t \geq 2.24$ would shortsell Australian index but during Covid-19, $\mathrm{t} = 0.09$ would be enough to urge them to take a short position. Similar situation happens to **China** and **France**.

Also, some investors actually took reversed position during Covid-19. For example, previously, investors with $t > 0.66$ would take a short position at Chinese market index whereas they would only do the same thing with $t < -0.07$ during Covid-19. Similar things also happen to **Indonesia**, **Italy**, **Korea**, **Netherlands**, **Russia**, **Saudi**, **Switzerland** and **Turkey**. Notably, Figure5 and Figure6 indicate that even during Covid-19, investors did not changes its attitude to US market index significantly with both t around less than 0.

Adding a Riskless Cash Fund and Constructing the Market Portfolio

New allocation of their investment

We got the daily return of this Riskless Cash Fund using `r0/250`. Then based on formulas from the textbook, we calculate the weight of each risky asset and the cash fund in the portfolio by

```
returns_3_full_mean_annualized_tilde = np.subtract(returns_3_full_mean_annualized,
r0*ones_array)
x_weight_tilde = 0.2 * np.dot(returns_3_full_variance_annualized_inverse,
returns_3_full_mean_annualized_tilde)
x_weight_0 = 1- np.dot(np.transpose(x_weight_tilde), ones_array)
```

where we obtain the following Table**??**. The negative weight means that investors with $t = 0.2$ should short sell the corresponding asset while the other way around for positive weight. This riskless cash fund would approximately account for 5% of our portfolio.

	Country	Weight
1	Italy	-0.22
2	US	1.89
3	Australia	0.12
4	UK	-0.98
5	Brazil	0.03
6	Japan	0.31
7	Russia	-0.26
8	China	0.03
9	France	-0.55
10	Canada	-0.90
11	Korea	-0.50
12	Switzerland	0.04
13	Spain	-0.82
14	India	0.63
15	Mexico	-0.40
16	Germany	0.96
17	Indonesia	0.43
18	Netherlands	1.06
19	Saudi	-0.12
20	Turkey	0.21
21	Cash Fund	0.05

Table 4: Weight of each asset including cash fund

Capital Market Line and the tangency portfolio

According to Buchen and Ivers (2020), CML (Capital Market Line is), which is denoted by $\hat{\sigma} = \frac{\hat{\mu} - r_0}{\sigma_0 \sqrt{d}}$, is the "degenerate form of the efficient frontier". It is the line P_0M as is shown in Figure7 from the textbook.

The fact that CML is tangential to EF has been proved in the textbook. Based on the summary of Buchen and Ivers, we have the following observations:

- $r_{riskless-asset} < \frac{b}{a}$ is the most practical case where CML has a positive slope and M is the *Tangency Portfolio*. An investor would not take any risky assets into consideration except for riskless asset at $\hat{\sigma} = 0$ which is understandable because they did not have risky assets in their portfolio while risk less asset offers 0 σ. Similarly, riskless assets will be dropped totally in our *Tangency Portfolio M*.

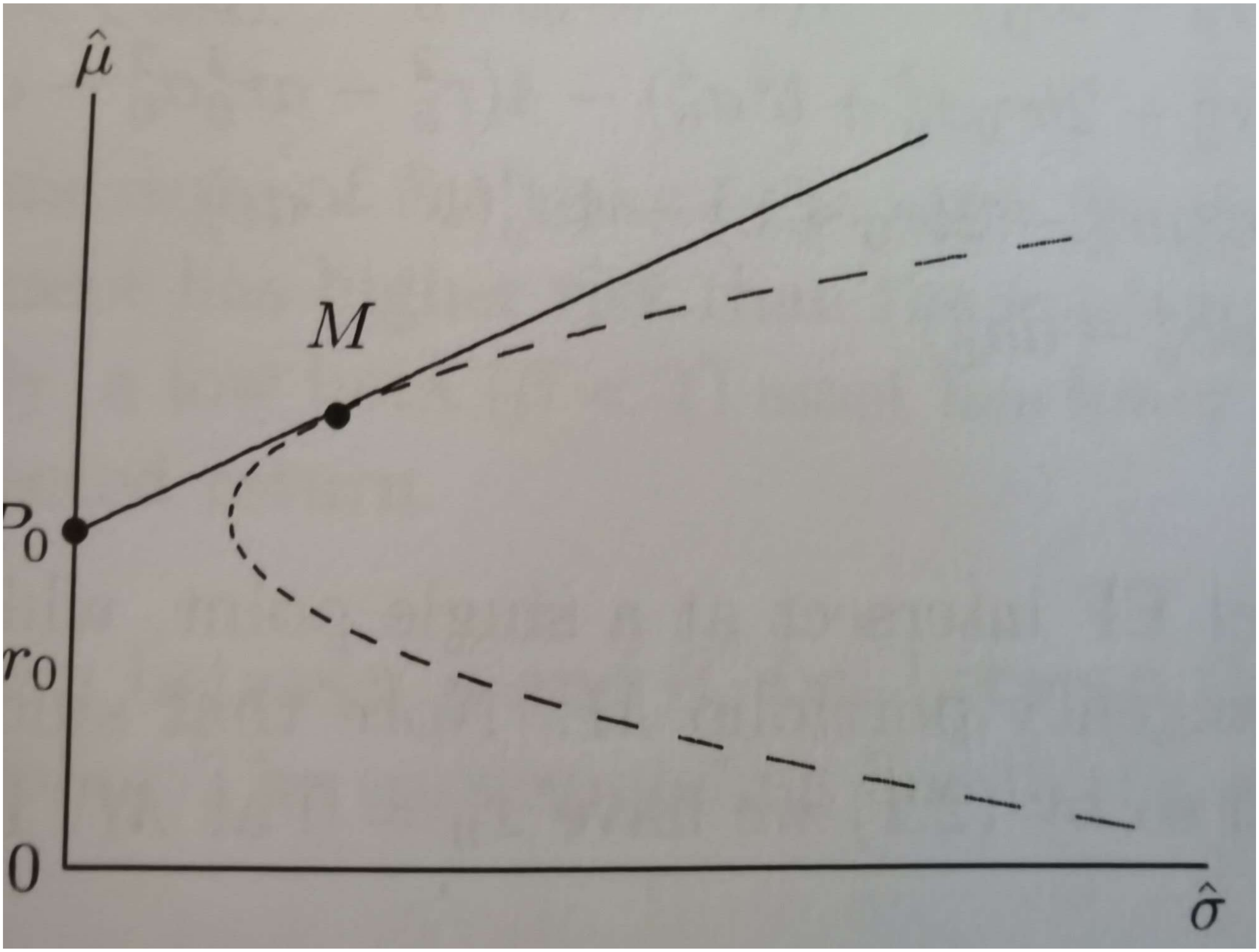


Figure 7: CML and MVF

- $r_{riskless-asset} > \frac{b}{a}$ represents a CML with negative slope and the *Tangency Portfolio M* is at the lower part of MVF, a symmetrical point of M at the upper part.
- In the remaining case, Buchen and Ivers (2020) found that CML is the asymptote to MVF.

In our case, where $r_0 = 0.00001$ and $\frac{b}{a} = 0.00014524$, we got a practical case at hand. Our *Tangency Portfolio M* has $\sigma_M = 0.0174574$ and $\mu_M = 0.00145337$ which would be plotted in the next part of this assignment.

GDP

We retrieve **Current Price Gross Domestic Product** of each country with seasonally adjusted from Federal Reserve Bank of St. Louis. We decided to use **Index(Scale value to the chosen date)** as the original data and set **2009-06-01** as the end of US recession date and the value of that day is the base value. All of these is done on Federal Reserve Bank of St. Louis.

Note that we did not get the quarterly indicator of **Saudi Arabia**. After getting its annual data, we calculate its log returns and using the equation below

$$(1 + r_{quarterly})^4 = (1 + r_{yearly}) \tag{1}$$

to get the quarterly return.

Since we have collected the original data, similar steps had been executed in our python code. Partial codes are as below:

```
one_array_20 = np.ones((20, 1), dtype=np.int32)
one_array_20_tran = np.transpose(one_array_20)
a_gdp = np.dot(one_array_20_tran, returns_quaterly_full_variance_inv)
a_gdp = np.dot(a_gdp, one_array_20)

b_gdp = np.dot(one_array_20_tran, returns_quaterly_full_variance_inv)
b_gdp = np.dot(b_gdp, returns_quaterly_full_mean)

c_gdp = np.dot(returns_quaterly_full_mean_tran, returns_quaterly_full_variance_inv)
c_gdp = np.dot(c_gdp, returns_quaterly_full_mean)

d_gdp = np.dot(a_gdp, c_gdp)
d_gdp = np.subtract(d_gdp, b_gdp**2)

alpha_gdp = (1/a_gdp) * returns_quaterly_full_variance_inv
alpha_gdp = np.dot(alpha_gdp, one_array_20)

beta_gdp = np.dot(returns_quaterly_full_variance_inv, np.subtract(returns_quaterly_full_mean,
(b_gdp/a_gdp)*one_array_20))

x_weight_gdp =np.add(alpha_gdp, beta_gdp * 0.2)

miu_gdp = (c_gdp-b_gdp*r0)/(b_gdp-a_gdp*r0)
```

We are required to derive a market portfolio from this new dataset. Within the following few steps, we use the formulas from textbook to get the *Tangency Portfolio* of this quarterly gdp dataset. The reasons are as follows:

- Buchen and Ivers (2020) suggest it is irresistible to equate the *Tangency Portfolio* with the *Market Portfolio* which, as they stated, is the key assumption of *Capital Asset Pricing Model*. Therefore, firstly we got the theoretical support for this equivalent relationship.
- Also, as what we have discussed in , at Point M, *Tangency Portfolio* actually holds 0 **riskeless asset** so that we could confidently believe this point can kind of represent our target **Market Portfolio**.

Capital Market Line

The final plot is shown in Figure8.

Basically, CML was plotted using Equation2 and Equation3 with $t \in [-0.4, 1]$

$$\hat{\mu} = r_0 + d\sigma_0^2 t \tag{2}$$

$$\hat{\sigma} = \frac{\hat{\mu} - r_0}{\sigma_0 \sqrt{d}} \tag{3}$$

where σ_0 is the volatility of the portfolio with r_0 return on EF for **risky assets only** and a, b, c and d are exactly what we have calculated before in order to get Figure4.

The **Market Portfolio** comes from the previous calculation. Note that we have change it into daily basis by using codes `sigma_gdp/(250/4)` and `miu_gdp/(250/4)`

If we have 100 million dollars available, considering there exists the riskless cash fund, the new weight of each asset is similar to the **x__weight__tilde**, which we have calculated here (Refer to).

Therefore, we have the following table:

Security Market Line

As was shown in Figure9, all assets are on Security Market Line. And Figure10 indicates that only $\beta_{US} > 1$ while β_{Japan} and β_{India} are very close to 1. Notably, Russia and Spain both presented a negative β.

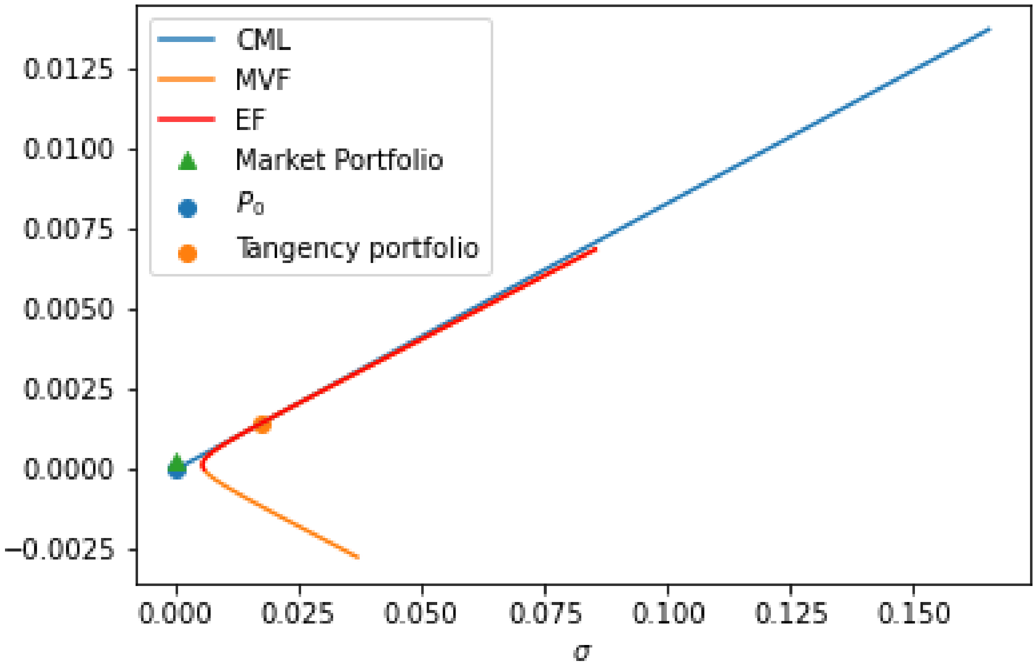


Figure 8: CML+Tangency Portfolio+Market Portfolio+P_0+EF

	Country	Investment
1	Italy	-22.5
2	US	189.0
3	Australia	11.9
4	UK	-97.9
5	Brazil	2.9
6	Japan	31.3
7	Russia	-25.7
8	China	2.7
9	France	-55.4
10	Canada	-90.2
11	Korea	-49.6
12	Switzerland	3.8
13	Spain	-81.9
14	India	62.7
15	Mexico	-39.7
16	Germany	96.0
17	Indonesia	43.1
18	Netherlands	106.0
19	Saudi	-12.3
20	Turkey	20.6

Table 5: Investment in each index. [a]Negative values mean that we short sell the corresponding assets.

Buchen and Ivers suggested in 2020 that *Portfolio Theory* is "about managing risk". β is one measure of the risk of assets relative to the market portfolio (Buchen & Ivers, 2020).

A higher β_{US} means a higher return as was stated by Table6 whereas, concurrently, an investor has to undergo the extra volatility.

However, a relatively low β would probably offer a negative return (such as **Italy**, **Russia** and **Spain**).

Then, based on the *Portfolio Theory*, risk management does not suggest we should pursue 0 risk. On the contrary, an appropriate portfolio taking both return and risk into consideration would be preferable. For example, **Saudi** with the lowest β but offers almost the same return as UK might benefit our portfolio. Undoubtedly, there are many other reasons in the real world affecting the return of our portfolio so that an investor with *Portfolio Theory* and a global vision might outperform its counterparts.

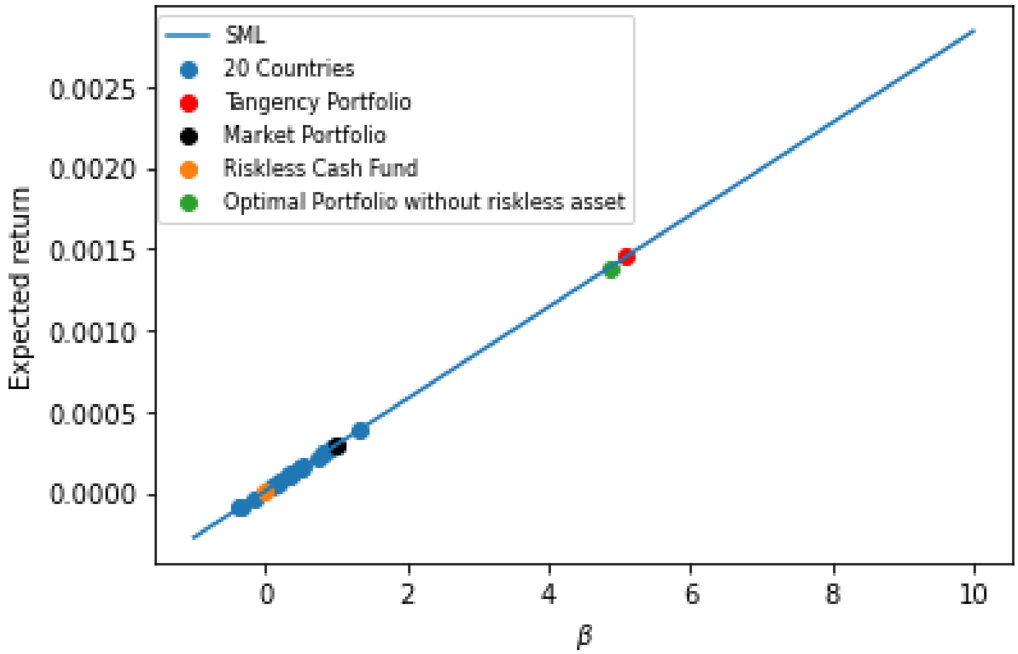


Figure 9: Security Market Line

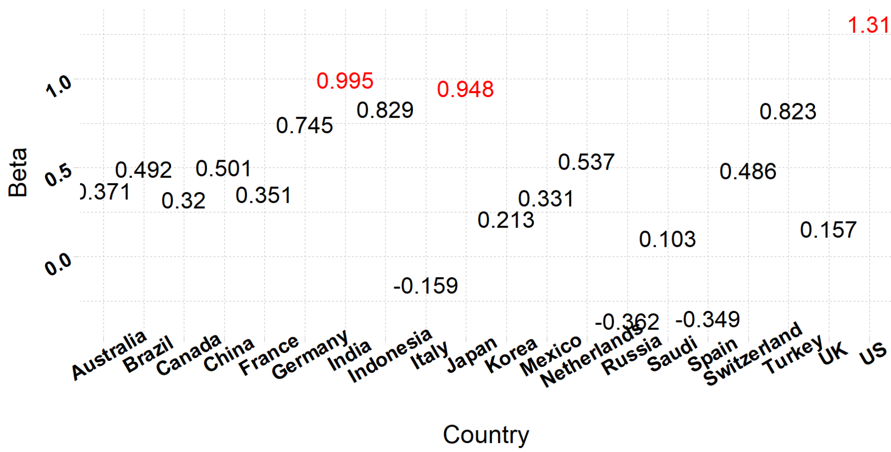


Figure 10: β of 20 countries

References

Buchen, P. W., & Ivers, D. J. (2020). *MATH2070 and MATH2970 Optimization and Financial Mathematics Lecture Notes.* Sydney: The University of Sydney School of Mathematics and Statistics.

E.G. (2018, October 4). *Magic of Log Returns: Concept – Part 1.* Retrieved from Investment Cache: https://investmentcache.com/magic-of-log-returns-concept-part-1/

Table 6: β, r_{daily} and r_{yearly}

	Country	Beta	Yearly Return	Daily Return
1	Italy	-0.16	-0.01	-0.00
2	US	1.31	0.10	0.00
3	Australia	0.37	0.03	0.00
4	UK	0.16	0.01	0.00
5	Brazil	0.49	0.04	0.00
6	Japan	0.95	0.07	0.00
7	Russia	-0.36	-0.02	-0.00
8	China	0.50	0.04	0.00
9	France	0.35	0.03	0.00
10	Canada	0.32	0.02	0.00
11	Korea	0.21	0.02	0.00
12	Switzerland	0.49	0.04	0.00
13	Spain	-0.35	-0.02	-0.00
14	India	0.99	0.07	0.00
15	Mexico	0.33	0.03	0.00
16	Germany	0.74	0.06	0.00
17	Indonesia	0.83	0.06	0.00
18	Netherlands	0.54	0.04	0.00
19	Saudi	0.10	0.01	0.00
20	Turkey	0.82	0.06	0.00