1. Executive summary

This report tries to construct ORP based on Treynor-Black model. Equation (1) shows the regression model. Though more assumptions are needed in this model, it surprisingly presents similar results to Markowitz model in constructing ORP.

The ORP based on Treynor-Black model presents the highest Sharpe ratio and holds a large position in active portfolio with a proportion of around 90% (see Table 2). Investors are recommended to short sell any security with a negative α. WMT could be largely held thanks to its relatively low embedded risk and satisfying α.

2. An Overview of Treynor-Black Model

2.1. Process

The construction of the model starts with a choice of market index and representation of risk-free return. The excess-return regression is then built based upon Equation (1).

$$R_i(t) = \alpha_i + \beta_i R_M(t) + e_i(t) \tag{1}$$

$R_i(t)$: Excess returns of analysed security i in month t

$R_M(t)$: Excess returns of the market index in month t

From above, the following estimates are obtainable:

Table 1 Results from Regression

Counts	n	n	n	1	1
Estimates	α_i: extra-market expected excess returns	β_i: sensitivity coefficients	$\sigma^2(e_i)$: Residual variance	$E(R_M)$: Market risk premium	σ_M^2: variance of the macroeconomic factor

An Optimal Risky Portfolio (ORP) could be calculated with values from Table 1 as the input.

2.2. Assumptions

Treynor-Black Model decomposes sources of risk of returns into macro-economy as well as the firm itself and they are assumed to be uncorrelated and present a mean of zero over time (Bodie, Kane, & Marcus, 2018). Individual assets are all related to the common factor, macro-economy, which has a β of 1 and is also where their correlation derive but they hold uncorrelated firm-specific risks (residual).

2.3. A Comparison to Markowitz ORP

Markowitz ORP requires an exponentially increasing number of estimates and could be meaningless if there are errors in estimation of correlation coefficients which is not necessarily needed in Treynor-Black Model (Bodie, Kane, & Marcus, 2018). Markowitz Model provides a wider efficient frontier and, but they present similar results including weights and Sharpe ratios in ORP construction. The difference partially arises from Markowitz considering the correlation among residuals.

3. ORP Recommendation

3.1. Composition of ORP

w_i^* of each individual security is listed in Table 2 where negative values indicate short selling is recommended for corresponding securities. An intuitive reason is they all return a negative α from either regression model or analysts' estimation so that including them into the portfolio is not profitable. The optimal weight of active portfolio is decided by Equation (2) and (3). Thanks to WMT's sufficiently high α and relatively low variance, the position in it is the largest (21.6%) among these assets. The active portfolio (A) with a higher Sharpe Ratio (S) of 0.335 ends with a weight of 89.7%

$$w_A^0 = \frac{\frac{\alpha_A}{\sigma_A^2}}{\frac{E(R_M)}{\sigma_M^2}} \tag{2}$$

$$w_A^* = \frac{w_A^0}{1 + (1 - \beta_A) w_A^0} \tag{3}$$

Table 2 Optimal weights of securities

Securities	Optimal Weight	α	$E(R)$	σ	S
Market PF (M)	10.3%		0.013	0.044	0.3000
Active PF(A)	89.7%	0.0034	0.016	0.047	0.3350
AMZN	19.9%	0.0037			
AAPL	14.0%	0.0026			
BA	7.7%	0.0019			
CAKE	8.7%	0.0021			
EBAY	3.5%	0.0012			
FB	6.6%	0.0013			
GME	-0.6%	-0.0002			
GS	0.8%	0.0002			
ILMN	2.9%	0.0007			
MCD	3.5%	0.0003			
MGM	-4.9%	-0.0015			
NVDA	1.3%	0.0005			
PFE	8.9%	0.0021			
RL	-10.6%	-0.0032			
RCL	-4.9%	-0.0008			
SAVE	0.5%	0.0002			
TSLA	4.9%	0.0034			
WMT	21.6%	0.0027			
YELP	4.4%	0.0015			
YUM	1.5%	0.0003			

3.2. Important indicators from ORP

Table 3 is a brief overview of ORP which generally benefits from the α contribution from the active portfolio and suffers from its corresponding variance. The optimal $E(R_p)$ and σ_P^2 are decided by Equation (4) and (5).

$$E(R_P) = (w_M^* + w_A^*\beta_A)E(R_M) + w_A^*\alpha_A \tag{4}$$

$$\sigma_P^2 = (w_M^* + w_A^*\beta_A)^2 + [w_A^*\sigma(e_A)]^2 \tag{5}$$

Table 3 The Optimal Risky Portfolio (ORP)

Optimal Risky Portfolio	
Item	**Value**
$E(R_P)$	0.0154
β	0.9380
σ_P^2	0.0021
σ_P	0.0460
Information Ratio	0.1499
S_P	0.3354

Considering that there exists such relationship as Equation (6), the Information Ratio, $\frac{\alpha}{\sigma(e_A)}$, is then 0.1499, which measures the return gained per unit of firm-specific risk.

$$S_{ORP}^2 = S_M^2 + \left[\frac{\alpha}{\sigma(e_A)}\right]^2 \tag{6}$$

4. Comparison of S_P, S_A and S_M

These ratios are reachable from Table 2 and Table 3 with S_P slightly larger[1] than that of active and market portfolio, which is understandable considering the objective of ORP is to maximize S_{OPR}.

Equation (6) suggests maximizing Information Ratio, which measures the contribution of active portfolio against its embedded volatility, is necessary before maximizing S_{ORP}.

The market portfolio benefits from a lower risk while the active portfolio takes advantage of a better expected return. The difference between S_P and S_A as well as S_M implies it is worth adding in some weights of active portfolio to improve returns even though it comes with more exposure to firm-specific risk.

5. Response to Nabi Kang

Some assumptions applied to each scenario Nabi expects are as follows:

[1] Note that the Sharpe Ratio results are obtained from the supporting excel file and all decimal points had been adjusted in excel which unavoidably leads to some minor differences in terms of calculating Sharpe Ratio using current entries in the table.

a. The change of each factor of interest will only affect factors directly related to this factor of interest. For instance, the changing of α_A can only alter w_A^0 which further brings changes to w_A^* and all inputs other than these two will remain constant.
b. All relevant inputs are from supporting work file.
c. Nabi is free from any investment constraints.
d. The change of information ratio is achieved by sampling α_A and σ_A from a uniform distribution at the same time.

5.1. How will w_A^* be affected when α_A changes?

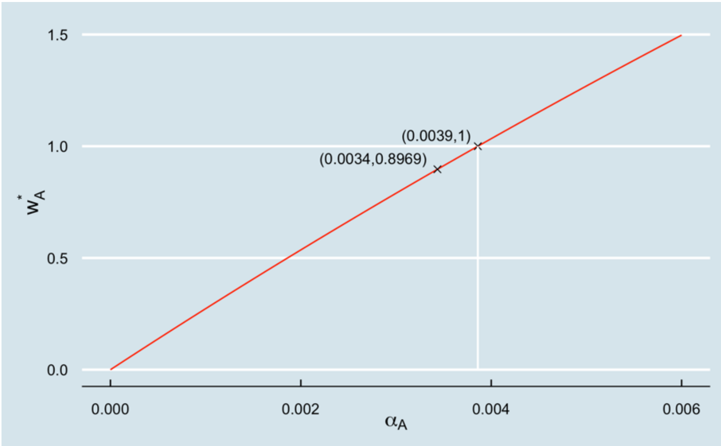

Figure 1 Calculated w_A^* based on changing α_A

w_A^* is decided by Equation (3), one component of which, w_A^0, is affected directly when α_A changes according to Equation (2). Figure 1 shows a positive linear relationship between α_A and w_A^*. As α_A is sufficiently large (around 0.0039), Nabi benefits from holding all its positions in the active portfolio.

5.2. How will w_A^* be affected when β_A changes?

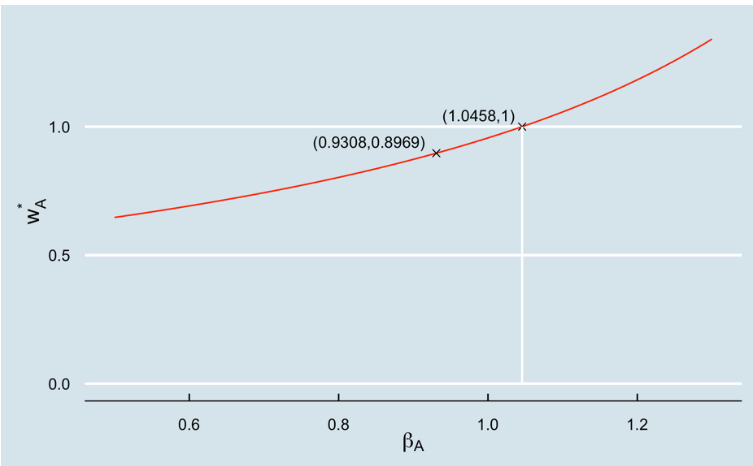

Figure 2 Calculated w_A^* based on changing β_A

Figure 2 is created based upon Equation (3) when β_A takes a value from 0.5 to 1.3. Such curve suggests w_A^* increases as β_A rises but it is not a linear relationship as above.

5.3. How will w_A^* be affected when the Information Ratio changes?

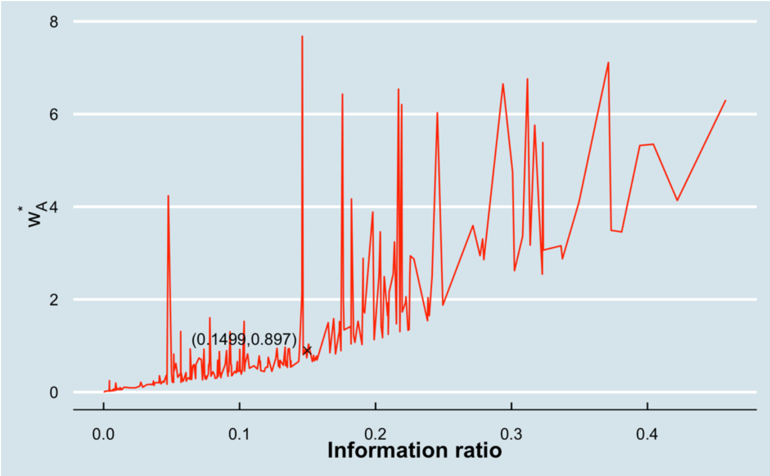

Figure 3 Calculated w_A^* based on changing Information ratio

Figure 3 presents how w_A^* varied with different Information ratio. It seems to imply that w_A^* becomes volatile as Information ratio gets increased but it presents an overall increasing trend.

6. Response to Amani Namir

6.1. A Recap of assumptions

As mentioned above in 2.2, the dichotomy rule on risks defined by Single Index Model (SIM) suggests risks between firms only arise from their correlation with a common factor, the market, and most importantly, firm-specific risks (residuals) are uncorrelated. Equation (7) explains it from a statistical perspective.

$$Corr(r_i, r_j) = \frac{\beta_i B_j \sigma_M^2}{\sigma_i \sigma_j} = Corr(r_i, r_M) \times Corr(r_j, r_M) \tag{7}$$

6.2. Explanation

Thus, if residual correlation between given securities is not neglectable, they cannot fit a satisfying SIM, which as shown in Table 4, is unfortunately true for our dataset. Since a half of our securities were selected from the same industry, they should be embedded with a high correlation in terms of firm-specific risks as expected, which in turn contradicts the previous assumption.

Table 4 Sector distribution in our dataset

Sector	Count	Proportion
Communication Services	2	10.0%
Consumer Discretionary	10	50.0%
Consumer Staples	1	5.0%
Financials	1	5.0%
Health Care	2	10.0%
Industrials	2	10.0%
Information Technology	2	10.0%

In addition, the classification of risk set by SIM disregards industry effects on firms. A couple of securities in our dataset come from industries that are complementary to each other, for instance, Consumer Discretionary and Consumer Staples. In such situation, some industry events can balance out each other while leaving the whole market unaffected, and SIM then fail to capture those changes (Bodie, Kane, & Marcus, Index Models, 2018).

6.3. What to need?

To quantify some critical indicators, residual correlation particularly, monthly excess return on the market index for each security will be required. Then a regression will be applied to those excess returns, which then returns a correlation matrix between each pair.

7. Response to Leon Olsen

There are three methods to get VaR, i.e., Variance-Covariance, Historical Simulation and Monte Carlo simulation. In this particular case, we take the first approach to calculating VaR. Main assumptions are as follows:

a. The monthly returns of our portfolio are normally distributed.
b. Fluctuations in returns are uncorrelated and are equally likely to happen in the future (EBRARY, 2021).

Table 5 Z score under different level of significance

A single tail with significance level of	1%	5%	10%
Z score	2.326	1.645	1.282

Given $E(R_P) = 0.0154$, $\sigma_P = 0.046$ in Table 3, Z scores in Table 5 and Equation (8),

$$VaR = [E(R_P) - z\sigma_P] \times V_P \tag{8}$$

Monthly 1%, 5% and 10% VaR are -$457,980, -$301,350 and -$217,860 respectively.

8. Response to Tito Parker

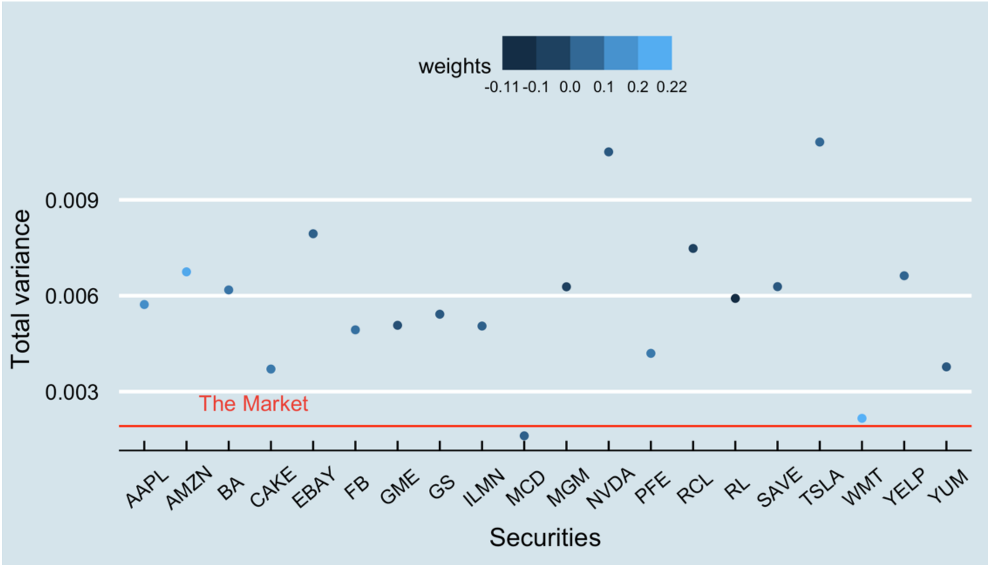

Figure 4 Weights allocated to each security in ORP and their total variance

8.1. Mutual Fund or Hedge Fund

90% of ORP is active portfolio, implying our portfolio is quite aggressive in contrast to the market (See Figure 4 and Table 3). Considering there are no limits on short selling, an equity-oriented hedge fund that takes either position in the market sounds viable (Bodie, Kane, & Marcus, Investments, 2017).

8.2. Fee Structure Recommended

Therefore, referring to the regular fee structure of hedge funds, an annual management fee, which is usually around 1% to 2% of NAV, and an incentive fee around 20% of profits from outperforming.

8.3. Assumptions

a. NAV is now \$100MM
b. 10-Year US Government Bond Yield 1.56% will be used as r_f.
c. Tax and any costs are excluded.
d. No high water mark.

8.4. A range of fees

Current monthly $E(R_p)$, 0.0016, could be annualized to $(1 + 0.016)^{12} - 1 = 20.98\%$. Presumably, the fund could earn us $100 \times (20.98\% + 1.56\%) = 22.54$ and we would end with \$122.54MM next year. The fee is summarised in Table 6.

Table 6 Fees to access the fund

Management Fee	Incentive Fee
$1\% \times 100 = \$1MM \sim 2\% \times 100 = \$2MM$	$20\% \times 22.54 - 2 - (1.56\% \times 100) = \$0.948MM$

Thus, a fee in the range from $1.948MM to $2.948MM is sensible.

9. Repose to Rosa Gatakis

9.1. Can we outperform the market?

I do think we can outperform the market especially in short to medium term. There has been a great amount of empirical research conducted to prove the market is not completely efficient (Bodie, Kane, & Marcus, Index Models, 2018). Bodie, Kane et al. suggested inefficiencies in the market and mispricing will have resources misallocated. This then creates an opportunity for people who can capture this misallocation and take advantage of it promptly. The index-tracking ETF provides equity investors low-cost way to include market indexes into their portfolio (Bodie, Kane, & Marcus, Investments, 2017). However, it can only allow investors to implement passive strategies.

9.2. Can alpha be realized?

α as the non-system premium depends on whether current security is over-priced or under-priced, indicating that in an efficient market where all assets are correctly priced, alpha would be 0. However, mispricing does occur, and such effect could potentially balance out within portfolio leaving the average of α being 0. Theoretically, if analysts are able to build a portfolio composed of all mispriced assets that return positive α, it could be realized but that is rarely the case practically.

9.3. Active or passive strategy specialization?

Personally, I would prefer to pursue a career that manages active portfolio, which is aligned with my previous belief that the market is not completely efficient, leaving us with some opportunities to outperform the market per se. Passive investment strategies focus on replicating the market. But if market information was not captured timely due to market inefficiencies, this strategy becomes worthless then. I personally believe market inefficiencies will get expanded in the next couple of years and Vasileiou (2020) did find S&P 500 Index was not efficient during the pandemic outbreak. This could be explained by behavioural finance somehow but one thing for sure is that active management strategies will come in handy under such circumstance.

10. Key Recommendations and Responses

The way Treynor-Black model treats risk simplifies inputs but also lose signals from industry side. The α of each security determines its position in ORP which is mostly composed of active portfolio (90%). S_{OPR} is higher than S_A and S_M as expected since the object of Treynor-Black model is to maximize S_{ORP}.

Given all aforementioned assumptions:

a. Nabi should find there is a positive linear relationship between α_A and w_A^* which also rises when β_A increases even though it is not a linear relationship. w_A^* fluctuates much when information grows but it presents an increasing trend overall.

b. For Leon, monthly 1%, 5% and 10% VaRs are -$457,980, -$301,350 and -$217,860 respectively and they are calculated using Variance-Covariance method.

c. Tito could expect a theoretical fee range from $1.948MM to $2.948MM per year.

d. Rosa should believe outperforming the market is possible but α is hard to get realized practically.

Bibliography

Bodie, Z., Kane, A., & Marcus, A. J. (2017). Investments. In Z. Bodie, A. Kane, & A. J. Marcus, *Hedge funds* (pp. 883-906). McGraw-Hill Education.

Bodie, Z., Kane, A., & Marcus, A. J. (2018). Index Models. In Z. Bodie, A. Kane, & A. J. Marcus, *Investments* (p. 247). New York: McGraw-Hill Education.

EBRARY. (2021, 11). *Value at Risk History, Characteristics, and Assumptions of VaR*. Retrieved from EBRARY.NET: https://ebrary.net/10033/management/value_risk

Vasileiou, E. (2020). Behavioral finance and market efficiency in the time of the COVID-19 pandemic: does fear drive the market? *International Review of Applied Economics*, 224-241.