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FD3447: Assignment 3

Problem 1 Derivations of the Importance Weights

Show that:

$$w_n = \frac{p(y_{1:n}|x_{1:n})p(x_{1:n})}{q(x_{1:n})} = \alpha(x_n)w_{n-1}$$

where:

$$\begin{aligned} x_n &\sim q(x_n|x_{n-1}) \\ \alpha(x_n) &= \frac{p(y_n|x_n)p(x_n|x_{n-1})}{q(x_n|x_{n-1})} \\ w_{n-1} &= \frac{p(y_{1:n-1}|x_{1:n-1})p(x_{1:n-1})}{q(x_{1:n-1})} \end{aligned}$$

Answer: Using the following simplifying assumptions:

$$\begin{aligned} q(x_{1:n}) &= q(x_1)\prod_{i=2}^{n} q(x_i|x_{i-1}) && \text{i) Markov's property} \\ p(x_{1:n}) &= p(x_1)\prod_{i=2}^{n} p(x_i|x_{i-1}) && \text{ii) Markov's property} \\ p(y_{i:n}|x_{1:n}) &= \prod_{i=1}^{n} p(y_i|x_i) && \text{iii) Conditional independence of } y_i's \text{ given } x_i's \end{aligned}$$

Then,

$$\begin{aligned} w_n &= \frac{p(y_{1:n}|x_{1:n})p(x_{1:n})}{q(x_{1:n})} \\ &= \frac{p(y_n|x_n)\prod_{i=1}^{n-1} p(y_i|x_i) \times p(x_1)p(x_n|x_{n-1})\prod_{i=2}^{n-1} p(x_i|x_{i-1})}{q(x_1)q(x_n|x_{n-1})\prod_{i=2}^{n-1} q(x_i|x_{i-1})} \\ &= \frac{p(y_n|x_n)p(x_n|x_{n-1})}{q(x_n|x_{n-1})} \times \frac{\prod_{i=1}^{n-1} p(y_i|x_i) \times p(x_1)\prod_{i=2}^{n-1} p(x_i|x_{i-1})}{q(x_1)\prod_{i=2}^{n-1} q(x_i|x_{i-1})} \\ &= \frac{p(y_n|x_n)p(x_n|x_{n-1})}{q(x_n|x_{n-1})} \times \frac{p(y_{1:n-1}|x_{1:n-1})p(x_{1:n-1})}{q(x_{1:n-1})} \\ &= \alpha(x_n) \times w_{n-1} \end{aligned}$$