
Improving Data Efficiency for LLM Reinforcement Fine-tuning Through Difficulty-targeted Online Data Selection and Rollout Replay

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Abstract

Reinforcement learning (RL) has become an effective approach for fine-tuning large language models (LLMs), particularly to enhance their reasoning capabilities. However, RL fine-tuning remains highly resource-intensive, and existing work has largely overlooked the problem of data efficiency. In this paper, we propose two techniques to improve data efficiency in LLM RL fine-tuning: *difficulty-targeted online data selection* and *rollout replay*. We introduce the notion of adaptive difficulty to guide online data selection, prioritizing questions of moderate difficulty that are more likely to yield informative learning signals. To estimate adaptive difficulty efficiently, we develop an attention-based framework that requires rollouts for *only* a small reference set of questions. The adaptive difficulty of the remaining questions is then estimated based on their similarity to this set. To further reduce rollout cost, we introduce a rollout replay mechanism that reuses recent rollouts, lowering per-step computation while maintaining stable updates. Extensive experiments across 6 LLM-dataset combinations show that our method reduces RL fine-tuning time by 25% to 65% to reach the same level of performance as the original GRPO algorithm.

1 Introduction

Reinforcement learning (RL) has emerged as a promising and increasingly adopted paradigm for fine-tuning large language models (LLMs) toward stronger reasoning capabilities [9, 27, 15, 50]. Despite a steady stream of algorithmic improvements [48, 25, 1, 47], relatively little attention has been paid to improving the *data efficiency* of LLM RL fine-tuning. This gap is particularly concerning given that RL fine-tuning for LLMs is notoriously computationally expensive¹.

In this paper, we present two simple yet effective techniques to improve the data efficiency for LLM RL fine-tuning: **difficulty-targeted online data selection** and **rollout replay**. Our goal is to

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¹For example, Luo et al. [27] report that training a relatively small 1.5B-parameter model on just 40K samples required over 3,800 A100 GPU hours—equivalent to approximately \$4,500 in compute cost—even before scaling to larger models or longer training horizons.

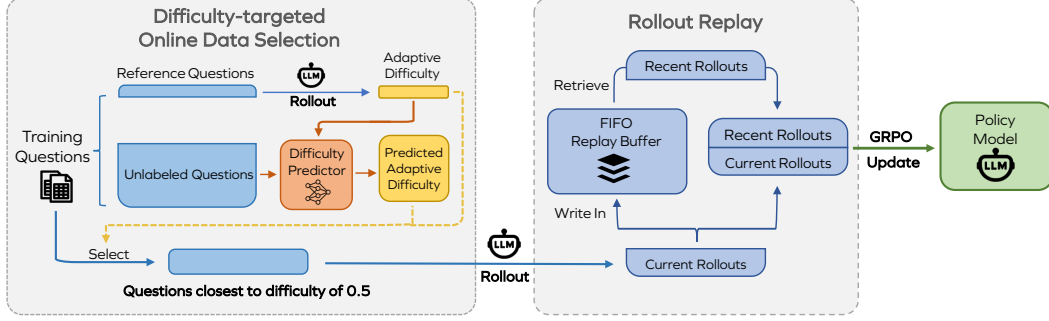


Figure 1: **Overview of our framework combining difficulty-targeted online data selection and rollout replay.** At each training step, the online data selection module selects training questions with adaptive difficulty near 0.5, requiring rollouts on only a small reference set (§4.1, §4.2). The rollout replay module combines current rollouts with retrieved recent rollouts from a FIFO buffer, and the current rollouts are stored into the buffer for future use (§4.3).

reduce both (1) the number of training steps required to match the performance of the original GRPO algorithm, and (2) the per-step computational cost.

$$\text{Total RL fine-tuning time} = \underbrace{\text{Number of training steps}}_{\text{Reduced by difficulty-targeted online data selection}} \times \underbrace{\text{Time per step}}_{\text{Reduced by rollout replay}}$$

Difficulty-targeted Online Data Selection (DOTS) In RL, tasks that are too easy or too difficult often provide limited learning signal [7, 17]. Moreover, since the policy evolves during training, it is crucial to adopt an online and adaptive mechanism for selecting informative data [31, 34]. To this end, we introduce the notion of *adaptive difficulty*, which measures how likely the current policy is to fail on a given question. At each training step, we prioritize questions of **moderate** adaptive difficulty, as these are most likely to yield meaningful learning signals.

However, computing adaptive difficulty exactly requires executing multiple rollouts per question, which is computationally expensive. To address this, we propose an *attention-based adaptive difficulty prediction framework* that efficiently estimates difficulty without generating full rollouts for all questions. At each training step, we generate rollouts only for a small reference set and compute their ground-truth adaptive difficulty. The difficulty of the remaining questions is estimated by comparing them to the reference set using similarity-based attention.

Rollout Replay (RR) To further reduce the cost of rollout generation, we introduce a simple rollout replay mechanism. At each training step, we generate a smaller number of new rollouts and reuse past rollouts from recent steps. A bounded First-In-First-Out (FIFO) buffer is maintained to store recent rollouts, from which we retrieve samples to complete each training batch. Although this makes GRPO slightly off-policy, our modified GRPO loss ensures stability, and RR effectively reduces per-step training time without degrading model performance.

Our key contributions are summarized as follows:

- We propose a novel attention-based adaptive difficulty prediction framework that efficiently estimates how likely a question will be answered incorrectly by the current policy, without requiring full rollouts for all questions.
- Guided by this difficulty prediction framework, we introduce an adaptive difficulty-targeted online data selection mechanism (DOTS) for RL fine-tuning, supported by theoretical justifications. DOTS prioritizes questions of moderate difficulty relative to the current policy, accelerating convergence.
- We develop a rollout replay (RR) mechanism that reuses recently generated rollouts. With a modified GRPO training loss, RR remains stable and effectively reduces per-step rollout cost.
- Extensive experiments on six LLM–dataset combinations show that our method reduces RL fine-tuning time by 25% to 65% while achieving the same performance as the original GRPO algorithm.

2 Related Work

Online Data Selection Data selection seeks to accelerate training by focusing computation on the most informative examples [2]. A key limitation of static data selection methods is their assumption that the importance of samples remains fixed throughout training. Online methods instead periodically reselect data during training to reflect the model’s evolving state [43, 49, 26, 16, 18]. Such adaptability is particularly important in RL, where non-stationary policy updates and environmental interactions necessitate continuous re-evaluation of data utility [31, 44, 33].

Experience Replay On-policy algorithms such as Proximal Policy Optimization (PPO) [38] and Group Relative Policy Optimization (GRPO) [39] have become standard in online RL for LLM reasoning tasks [10]. However, their reliance on freshly collected rollouts for each policy update leads to substantial data inefficiency and computational overhead [24, 4]. Experience replay mitigates this by maintaining a fixed-size buffer of recent transitions collected by the policy. Instead of discarding data after a single use, the buffer enables multiple passes over past rollouts, thereby improving sample efficiency and stabilizing training [6, 51, 36].

3 Problem Setup

GRPO We focus on GRPO algorithm [39] with verifiable rewards. For each question q , a group of G individual responses $\{o_i\}_{i=1}^G$ are sampled from the old policy π_{old} . The advantage of the i -th response is calculated by normalizing the group-level rewards $\{r_i\}_{i=1}^G$:

$$\hat{A}_i = r_i - \text{mean}(\{r_i\}_{i=1}^G). \quad (1)$$

Compared to the original formulation proposed by [39], we remove the standard deviation normalization, as it has been shown to introduce bias into the optimization process [25]. Based on this, the GRPO objective can be formulated as:

$$\mathcal{J}_{\text{GRPO}}(\theta) = \mathbb{E}_{q \sim \mathcal{D}, \{o_i\}_{i=1}^G \sim \pi_{\theta_{\text{old}}}(\cdot|q)} \left[\frac{1}{G} \sum_{i=1}^G \frac{1}{|o_i|} \sum_{t=1}^{|o_i|} \left(\min \left(r_{i,t}(\theta) \hat{A}_i, \text{clip}(r_{i,t}(\theta), 1 - \epsilon, 1 + \epsilon) \hat{A}_i \right) - \beta D_{\text{KL}}(\pi_{\theta} \parallel \pi_{\text{ref}}) \right) \right].$$

The first term represents a clipped policy update, where the ratio term $r_{i,t}(\theta) = \frac{\pi_{\theta}(o_{i,t}|q, o_{i,<t})}{\pi_{\theta_{\text{old}}}(o_{i,t}|q, o_{i,<t})}$ represents the probability ratio between the current and old policies. A KL penalty $D_{\text{KL}}(\pi_{\theta} \parallel \pi_{\text{ref}})$ is applied with respect to a fixed reference policy π_{ref} , weighted by a scalar coefficient β .

Problem Formulation Let $\mathcal{D} = \{q_i\}_{i=1}^N$ denote the full dataset of N questions. In standard GRPO, each policy update uses a batch of questions uniformly sampled from \mathcal{D} . However, not all questions contribute equally to learning progress. In particular, questions that are either too easy or too hard relative to the *current* policy’s capability may yield weak gradient signals, slowing convergence.

To address this, we consider an **online data selection** setting [49]. At each step t , a batch $\mathcal{B}_t \subset \mathcal{D}$ of fixed size B is selected based on the current policy π_t . Unlike static data pruning, this selection is repeated throughout training and adapts to the evolving policy. While more frequent selection allows better adaptation, it also increases computational overhead. In practice, when policy updates are relatively stable, it is often more efficient to perform data selection every μ (e.g., 2,4,8...) steps, selecting a sequence of μ batches to be used in the subsequent updates [26, 41].

4 Method

Our proposed method is two-fold: (1) **difficulty-targeted online data selection**, which reduces the number of training steps needed to achieve the same performance as the original GRPO algorithm by prioritizing questions of moderate difficulty, and (2) **rollout replay**, which reduces the per-step computational cost by reusing recent rollouts. Full pseudocode is provided in Algorithm 1.

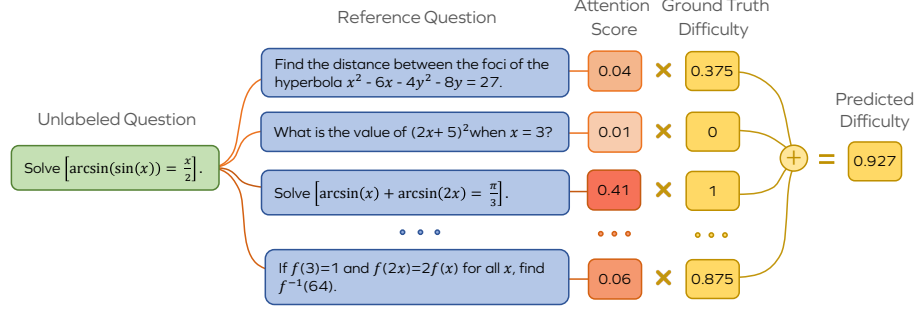


Figure 2: **Illustration of our attention-based adaptive difficulty prediction framework.** For each unlabeled question, we compute its embedding and attend to reference questions with known difficulty. The predicted difficulty is obtained by computing a similarity-weighted average over the reference difficulties. In this example, the unlabeled question involves inverse trigonometric functions. The model assigns high attention to a reference question that tests a closely related concept and has a difficulty of 1.0. As a result, the predicted difficulty is also close to 1.0. All difficulty values shown correspond to *adaptive* difficulty scores computed at the same step.

We propose using **adaptive difficulty** to guide online data selection. The adaptive difficulty of a question is defined with respect to the current policy and reflects how challenging the question is for the policy at the current stage of training. Formally, at step t , for each question q , we sample a group of G responses $\{o_i^{(t)}\}_{i=1}^G$ from the current policy and obtain their corresponding rewards $\{r_i^{(t)}\}_{i=1}^G$. The adaptive difficulty at step t is then computed as:

$$d_q^{(t)} = \frac{1}{G} \sum_{i=1}^G (1 - r_i^{(t)}). \quad (2)$$

This value represents the average failure rate under the current policy, with higher values indicating greater difficulty. Unlike static difficulty measures, adaptive difficulty evolves with the policy and provides a dynamic signal for selecting informative training samples.

Challenge: How to estimate adaptive difficulty efficiently? A key challenge in using adaptive difficulty is that computing it requires executing multiple rollouts to obtain rewards, which is one of the most expensive components in LLM RL fine-tuning². This raises the question: *can we estimate adaptive difficulty efficiently without generating rollouts for all questions?* To address this, we propose a lightweight attention-based adaptive difficulty prediction framework that generates rollouts for only a small reference subset of questions. The adaptive difficulty of the remaining questions is then estimated by comparing them to reference questions with known difficulty values using similarity-based attention, thereby avoiding full rollouts. See Fig. 2 for an illustration.

4.1 Attention-based Adaptive Difficulty Prediction Framework

At each step t , given the full training dataset \mathcal{D} , we first sample a small subset of K questions (e.g., 128 or 256) uniformly at random to form the **reference set** \mathcal{D}_{ref} . For each question in the reference set, we execute rollouts and compute its adaptive difficulty at step t , denoted by $\{d_i^{(t)}\}_{i=1}^K$ using Eq. 2.

For the remaining $N - K$ questions, we aim to estimate their adaptive difficulty without performing rollouts. To this end, we employ a lightweight embedding model E_θ to encode questions and capture similarity. We first compute the embeddings $\{z_i = E_\theta(q_i)\}_{i=1}^K$ for all reference questions. Then, for each unlabeled question q , we compute its embedding $z_q = E_\theta(q)$ and use similarity-weighted averaging to estimate its adaptive difficulty:

$$a_i = \frac{\exp(z_q^\top z_i / \sqrt{h})}{\sum_{j=1}^K \exp(z_q^\top z_j / \sqrt{h})}, \quad \hat{d}_q^{(t)} = \sum_{i=1}^K a_i d_i^{(t)},$$

²For instance, generating rollouts for a batch of 512 samples with maximum sequence length 3072 takes 109.83 seconds on 8 L40S GPUs, nearly half of the total step time.

where h is the embedding dimension.

Calibration To improve the prediction performance, we apply Platt scaling [32] that utilizes the information of mean and standard deviation of the reference set difficulties. Specifically, let $\mu^{(t)} = \frac{1}{K} \sum_{i=1}^K d_i^{(t)}$ and $\sigma^{(t)} = \sqrt{\frac{1}{K} \sum_{i=1}^K (d_i^{(t)} - \mu^{(t)})^2}$ denote the mean and standard deviation of the reference difficulties at step t . These two statistics are passed through a lightweight MLP to produce scale and bias parameters $(w^{(t)}, b^{(t)}) = \text{MLP}([\mu^{(t)}, \sigma^{(t)}])$. We then apply a calibrated transformation to the predicted difficulty:

$$\hat{d}_{q,\text{cal}}^{(t)} = \sigma \left(w^{(t)} \cdot \log \frac{\hat{d}_q^{(t)}}{1 - \hat{d}_q^{(t)}} + b^{(t)} \right),$$

where $\sigma(\cdot)$ denotes the sigmoid function. The MLP is optimized using binary cross-entropy loss. Full training details can be found in §5.1 and Appendix D.1.

4.2 Adaptive Difficulty-targeted Online Data Selection

At each training step, with the adaptive difficulty prediction framework, we now efficiently obtain the adaptive difficulty for all questions in the dataset. Inspired by prior work on goal curriculum in RL [7, 45], we prioritize questions whose predicted difficulty is closest to 0.5.

This selection strategy selects questions that are neither too easy nor too hard for the current policy, as these are intuitively the most informative for learning. Moreover, in GRPO, when all sampled rewards for a question are either 0 or 1, the group-normalized advantage becomes identically zero, resulting in no gradient signal. By focusing on questions with predicted difficulty near 0.5, we avoid such degenerate cases and ensure each update contributes meaningfully to policy gradients, thereby accelerating optimization convergence. We formalize this intuition in Theorem 1, which shows that the expected gradient magnitude is maximized when the reward success rate is 0.5 (*i.e.*, the adaptive difficulty is also 0.5). A complete proof is provided in Appendix C.

Theorem 1 (Maximal Gradient Signal at 50% Success Rate). *Consider a single question q , where G responses $\{o_i\}_{i=1}^G$ are sampled independently from the current policy $\pi_\theta(\cdot | q)$. Each response receives a binary reward $r_i \in \{0, 1\}$, sampled i.i.d. from a Bernoulli(p) distribution, where p represents the reward success rate. Define the group-relative advantage \hat{A}_i as in Eq. 1. We consider the unclipped policy gradient estimator for this question without KL penalty:*

$$g = \sum_{i=1}^G \hat{A}_i \nabla_\theta \log \pi_\theta(o_i | q).$$

Assume that the likelihood gradients $\nabla_\theta \log \pi_\theta(o_i | q)$ are independent of the reward distribution parameter p and have bounded variance. Then, the expected squared norm of the gradient satisfies:

$$\mathbb{E}[\|g\|^2] \propto p(1-p) \cdot (1 - 1/G),$$

and is maximized when $p = 0.5$.

4.3 Rollout Replay

To further improve data efficiency, we aim to reduce the **time cost of each training step**. Since rollout generation is one of the most expensive components, we adopt a *rollout replay* mechanism. Specifically, at each training step, we generate new rollouts for only a fraction δB of the batch, where $\delta \in (0, 1]$, and fill the remaining $(1 - \delta)B$ samples using recent rollouts sampled from a FIFO replay buffer $\mathcal{D}_{\text{replay}}$ with capacity C .

However, naively reusing past rollouts introduces bias into the policy gradient estimation, as the data is no longer drawn from the current policy. This mismatch can lead to unstable training and performance degradation [28]. Inspired by off-policy variants of PPO [35], we propose a modified GRPO loss using importance sampling with respect to the behavior policy π_{behavior} *under which the*

Algorithm 1 GRPO with DOTS and RR

Require: Initial policy model π_θ , reward model r_φ , math question dataset \mathcal{D} , target difficulty α , batch size B , total steps T , reference set size K , sampling temperature τ , adaptive difficulty prediction framework DIFFPRED (§4.1), fresh rollout fraction $\delta \in (0, 1]$, buffer capacity C

- 1: Initialize replay buffer $\mathcal{R} \leftarrow \emptyset$
 - 2: **for** step = 1, ..., T **do**
 // Adaptive difficulty prediction
 - 3: Sample reference set $\mathcal{D}_{\text{ref}} \subset \mathcal{D}$ uniformly at random, where $|\mathcal{D}_{\text{ref}}| = K$
 - 4: **for** each $q \in \mathcal{D}_{\text{ref}}$ **do**
 - 5: Generate G outputs $\{o_i^q\}_{i=1}^G \sim \pi_{\theta_{\text{old}}}(\cdot | q)$
 - 6: Compute rewards $r_i^q = r_\varphi(o_i^q)$ for $i \in [G]$ and difficulty score $d_q = \frac{1}{G} \sum_{i=1}^G (1 - r_i^q)$
 - 7: **end for**
 - 8: Predict adaptive difficulty $\hat{d}_{q'} = \text{DIFFPRED}(\mathcal{D}_{\text{ref}}, \{d_q\}, q')$ for all $q' \in \mathcal{D} \setminus \mathcal{D}_{\text{ref}}$.
 - 9: Sample rollout batch $\mathcal{B}_{\text{rollout}}$ of size δB from \mathcal{D} according to:
$$P(q) = \frac{\exp(-|\hat{d}_q - \alpha|/\tau)}{\sum_{q' \in \mathcal{D}} \exp(-|\hat{d}_{q'} - \alpha|/\tau)}$$
 - 10: **for** each $q \in \mathcal{B}_{\text{rollout}}$ **do**
 - 11: Generate G outputs $\{o_i^q\}_{i=1}^G \sim \pi_{\theta_{\text{old}}}(\cdot | q)$
 - 12: Compute rewards $r_i^q = r_\varphi(o_i^q)$ for $i \in [G]$ and group average reward $\bar{r}_q = \frac{1}{G} \sum_{i=1}^G r_i^q$
 - 13: Obtain advantages \hat{A}_i^q and policy probabilities $\pi_{\theta_{\text{old}}}(o_i^q | q)$ for $i \in [G]$
 - 14: **end for**
 // Rollout Replay and Update
 - 15: Sample $(1 - \delta)B$ samples from buffer \mathcal{R} to complete batch \mathcal{B}
 - 16: Update policy π_θ using **modified** GRPO objective on batch \mathcal{B}
 // Store informative rollouts in buffer
 - 17: Add $(q, o_i^q, \hat{A}_i^q, \pi_{\theta_{\text{old}}}(o_i^q | q))$ to \mathcal{R} for $q \in \mathcal{B}_{\text{rollout}}$, $\bar{r}_q \notin \{0, 1\}$, $i \in [G]$
 - 18: Remove the oldest samples from \mathcal{R} if buffer is full until $|\mathcal{R}| \leq C$
 - 19: **end for**
-

rollouts stored in the buffer were originally collected:

$$\mathcal{J}_{\text{GRPO-RR}}(\theta) = \mathbb{E}_{q \sim \mathcal{D}_{\text{replay}}, \{o_i\}_{i=1}^G \sim \pi_{\text{behavior}}(\cdot | q)} \left[\frac{1}{G} \sum_{i=1}^G \frac{1}{|o_i|} \sum_{t=1}^{|o_i|} (\min(\tilde{r}_{i,t}(\theta) A_i, \text{clip}(\tilde{r}_{i,t}(\theta), 1 - \epsilon, 1 + \epsilon) A_i) - \beta D_{\text{KL}}(\pi_\theta \| \pi_{\text{ref}})) \right],$$

where $\tilde{r}_{i,t}(\theta) = \frac{\pi_\theta(o_{i,t} | q, o_{i,<t})}{\pi_{\text{behavior}}(o_{i,t} | q, o_{i,<t})}$. By appropriately controlling the buffer size C , we empirically show that rollout replay improves sample efficiency while maintaining training stability.

For each newly generated rollout, if the group average reward is neither 0 nor 1 (*i.e.*, the sample yields a non-zero gradient signal), we store the question, its sampled rollouts, computed advantages, and policy probabilities into the buffer. When the buffer is full, the oldest samples are discarded.

5 Experiments

5.1 Experimental Setup

LLMs and RL training datasets We perform GRPO training on three model scales: Qwen2.5-Math-1.5B, Qwen2.5-3B, and Qwen2.5-Math-7B [46]. We adopt four open-source datasets for training: MATH [13], DeepScaleR-40K [27], Open-Reasoner-Zero-57K (ORZ) [15] and DeepMath-103K dataset [12]. For MATH, we include all level 3–5 questions. For the other three datasets, we sample 8K to 10K subsets to construct the training pools. These datasets span diverse mathematical domains and difficulty levels. In total, we experiment with six LLM-training dataset combinations to assess the effectiveness of our framework.

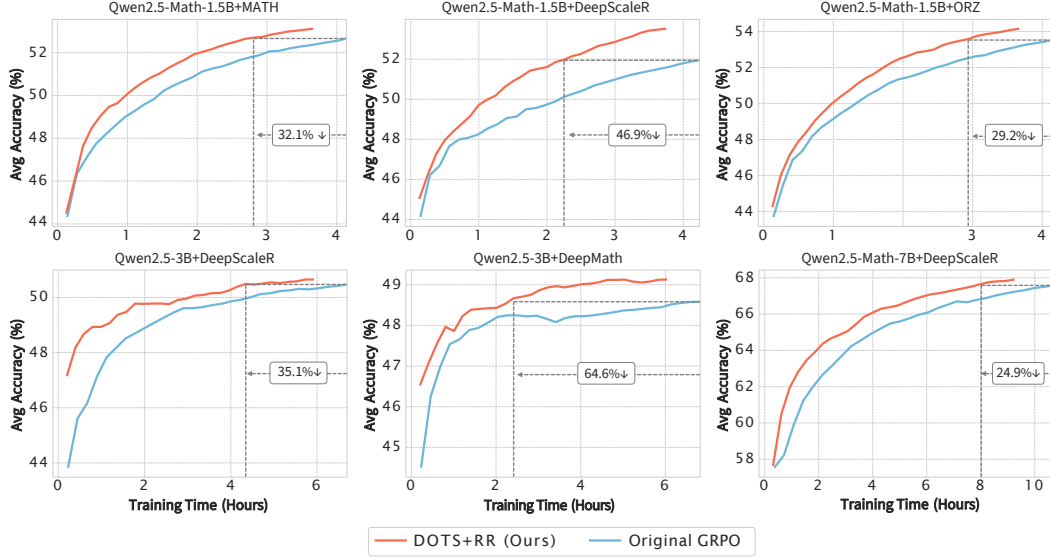


Figure 3: **Average accuracy curves of our method and original GRPO under various LLM-dataset combinations.** The curves show average performance aggregated over four benchmarks with exponential smoothing for visualization. Although both methods are trained for the same number of steps (60), our curve is shorter in duration because RR reduces the wall-clock time per step. Our method consistently outperforms the original GRPO throughout training and reduces the time required to match the original GRPO’s final accuracy after 60 training steps by an average of 38.8%.

Implementation details for adaptive difficulty prediction framework In practice, we observe that off-the-shelf pretrained embedding models struggle to capture fine-grained similarity between math questions. To address this, we freeze the Qwen2.5-Math-1.5B-Instruct backbone [46] and train a 3-layer MLP adapter with a calibration head, using binary cross-entropy loss. We fix the reference set size to 256. Additional implementation details are provided in Appendix D.1 and ablation results on key components are presented in Appendix F.1.

Implementation details for RL training We employ the verl framework [40] to perform GRPO training. We use a batch size of 512, generate 8 rollouts per prompt, and train for 60 steps in all experiments. The maximum rollout length is set to 3072 tokens for the Qwen2.5-Math series models (due to max position embedding limits) and 4096 tokens for Qwen2.5-3B. For reward computation, we use a simple rule-based function based solely on answer correctness, without incorporating any format-related signals. The 1.5B and 3B models are trained on 8 L40S GPUs, while the 7B model is trained on 8 A100 GPUs. For DOTS, data selection is performed every 2 steps. For RR, we choose the fresh rollout ratio δ as 0.5 and buffer capacity $C \in \{256, 512\}$. All RL training hyperparameters are detailed in Appendix E.2.

Evaluation We adopt the official Qwen2.5-Math evaluation implementation [46], setting the maximum generation length to 3072 tokens for Qwen2.5-Math series models and 4096 tokens for Qwen2.5-3B. Following [9, 27, 25], we evaluate RL model performance on standard mathematical reasoning benchmarks, including GSM8K [3], MATH500 [23], Minerva Math [20] and Olympiad-Bench [11]. Accuracy is measured using a sampling temperature of 0.6, top-p of 0.95, and the standard prompt template, consistent with [9]. We exclude benchmarks with very few questions, such as AIME 24 (30 questions) and AMC 23 (40 questions), as their small size results in high evaluation variance and unreliable performance comparisons on smaller models [14]. We report the final performance as the average accuracy across the four benchmarks to mitigate benchmark-specific variance. As the baseline, we use the original GRPO algorithm with uniform batch selection.

5.2 Main Results

The total training costs can be decomposed into two components: the number of steps required to reach a target performance and the average wall-clock time per step. Each training step consists of

Table 1: **Percentage of training steps saved, per-step time saved, and total training time saved.** All metrics are averaged over four mathematical reasoning benchmarks and reported relative to the original GRPO baseline. All timing measurements are conducted on the same computational devices.

Model	Dataset	Steps Saved (%)	Time Saved/Step (%)	Total Time Saved (%)
Qwen2.5-Math-1.5B	MATH	23.33	11.71	32.15
	DeepScaleR	40.00	11.69	46.90
	ORZ	20.00	11.66	29.20
Qwen2.5-3B	DeepScaleR	26.67	11.52	35.10
	DeepMath	60.00	11.35	64.60
Qwen2.5-Math-7B	DeepScaleR	13.33	13.39	24.94

processing a fixed-size batch, primarily including rollout generation and policy update. To ensure a fair comparison, each set of experiments is run on the same type and number of GPU devices.

Our method reaches the same performance as original GRPO with fewer steps. Tab. 1 reports the number of training steps required by DOTS+RR to match the final performance of the original GRPO at 60 steps. Across all LLM–dataset combinations, our method consistently reaches the same performance with substantially fewer steps, achieving reductions ranging from 13.33% to 60.00%. These results demonstrate that DOTS significantly accelerates convergence by prioritizing informative training samples.

Our method reduces per-step cost. In our experiments, rollout generation accounts for approximately 47%, 46%, and 54% of the total per-step time for the 1.5B, 3B, and 7B models, respectively, with the remaining time primarily spent on policy updates³. By reducing the number of rollouts per step, our RR strategy leads to a 11%–13% reduction in per-step training time, as shown in Tab. 1.

Our method significantly reduces total training cost. As shown in Fig. 3, DOTS+RR (orange) consistently outperforms the original GRPO (blue) throughout training, maintaining higher accuracy at almost every step. Across all six settings, DOTS+RR reduces total training time by an average of 38.8%, with the largest improvement observed on Qwen2.5-3B trained on DeepMath (64.6%).

5.3 Effectiveness of Adaptive Difficulty Prediction Framework

To better understand why our method accelerates training effectively, we examine whether the attention-based prediction framework can accurately estimate adaptive difficulty and consistently prioritize informative training signals throughout learning.

The adaptive difficulty prediction aligns with evolving training dynamics. To assess the fitness of online predictions, we collect ground-truth adaptive difficulty labels from training batches and compute the Pearson correlation between these labels and the predicted difficulty scores. As shown in Tab. 2, our framework consistently achieves strong Pearson correlation ($\rho > 0.7$) across settings, demonstrating its ability to effectively track policy behavior throughout training. Additional qualitative examples are provided in Appendix D.2 for further insight into our attention-based prediction mechanism.

Table 2: **Average Pearson correlation (ρ) between predicted and ground-truth adaptive difficulties.** Reported as mean \pm standard deviation over 60 training steps.

Model	Dataset	ρ
Qwen2.5-Math-1.5B	MATH	0.7843 \pm 0.0243
	DeepScaleR	0.7244 \pm 0.0318
	ORZ	0.7153 \pm 0.0257
Qwen2.5-3B	DeepScaleR	0.7789 \pm 0.0191
	DeepMath	0.7029 \pm 0.0082
Qwen2.5-Math-7B	DeepScaleR	0.7076 \pm 0.0195

Our prediction framework effectively filters out uninformative samples. Based on the discussion in §4.2, questions with adaptive difficulty 0 or 1 correspond to cases where all rollouts receive the same reward. In such cases, the group-normalized advantage becomes zero, yielding no gradient signal. We define *effective questions* as those with adaptive difficulty strictly between 0 and 1. As shown in Fig. 4, on average across all LLM-dataset combinations, DOTS selects 25.4% more effective

³In practice, for longer generation lengths, such as 8K and 16K, rollout time increases substantially, making it the dominant computational bottleneck. In such settings, our rollout replay mechanism can yield even greater wall-clock savings.

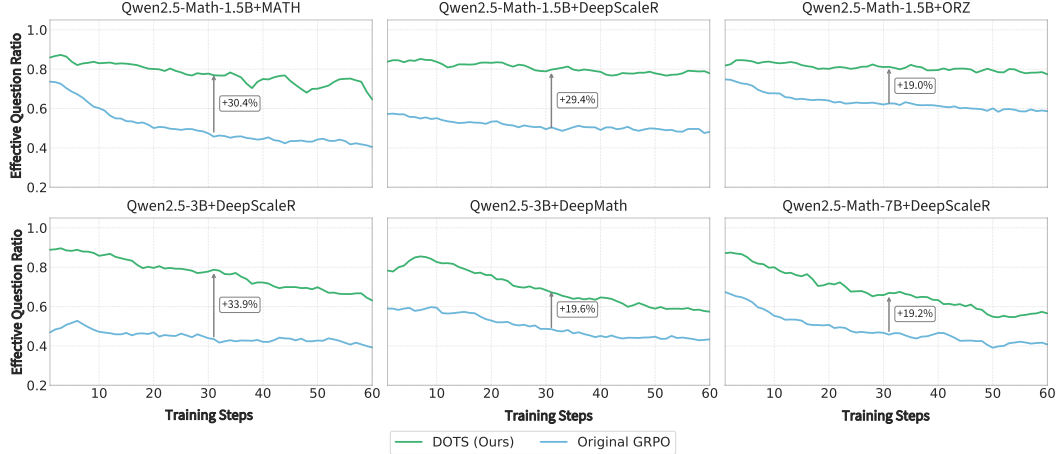


Figure 4: **Ratio of effective questions (i.e., questions with adaptive difficulties strictly between 0 and 1) during training across various LLM-training dataset combinations.** Annotated percentages indicate the per-step increase in effective question ratio achieved by DOTS compared to original GRPO, averaged over the training process. Our adaptive prediction framework consistently selects more informative samples throughout training.

questions than the original GRPO, demonstrating a clear advantage in selecting more informative questions throughout training, thereby accelerating convergence.

Our prediction framework incurs minimal computational overhead and scales efficiently to large datasets. By caching question embeddings and using a lightweight encoder, our prediction framework remains highly efficient—processing 10K samples in just 1.71 seconds at deployment.

5.4 Analysis and Discussion

We further investigate two important questions: **(Q1)** What are the *individual* contributions of DOTS and RR to training efficiency? **(Q2)** How does DOTS compare to an online data selection method based on *external* difficulty labels?

DOTS accelerates convergence, while RR reduces per-step cost. As shown in Fig. 5(a), training guided by DOTS alone yields a steeper learning curve compared to the original GRPO algorithm. Fig. 5(b) shows that incorporating RR further reduces total training time by approximately 20% without sacrificing performance. These results show that DOTS and RR improve RL training efficiency in *complementary* ways.

DOTS outperforms online data selection method based on external difficulty labels. We compare DOTS with an online data selection baseline that relies on external difficulty annotations (e.g., annotated by GPT-4o), where training questions are selected at different stages based on static difficulty labels, gradually shifting from easier to harder questions over time. Results in Appendix F.2 show that DOTS consistently outperforms this baseline. Moreover, such methods require expensive external labeling and offer limited adaptability, as they typically follow hand-crafted curricula that demand extensive manual design and tuning. In contrast, by leveraging adaptive difficulty, DOTS automatically adjusts to the model’s learning progress without relying on external supervision, enabling more scalable and efficient training.

6 Conclusion

In this paper, we propose two techniques to improve the data efficiency of LLM RL fine-tuning: difficulty-targeted online data selection and rollout replay. We hope these effective techniques encourage future work to explore data-centric approaches to improving LLM RL fine-tuning.

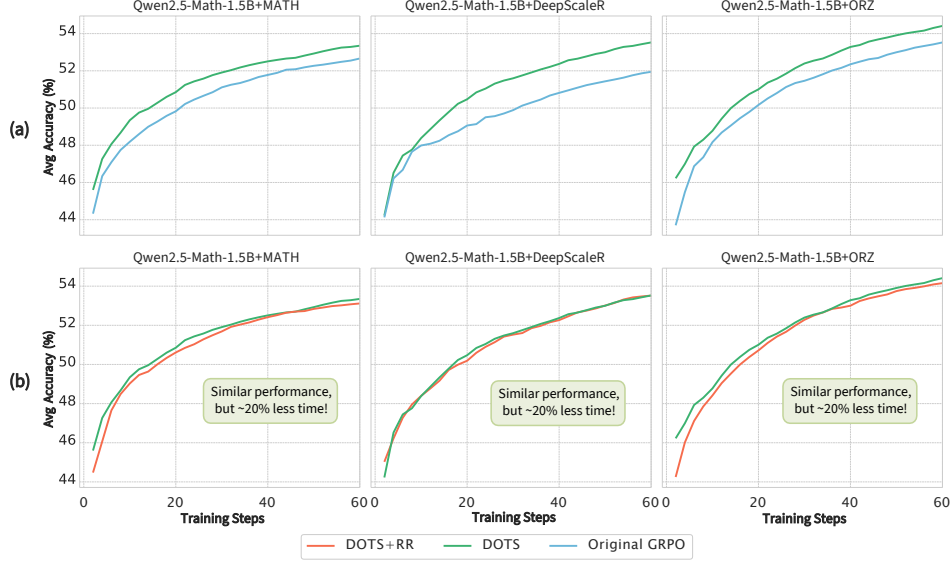


Figure 5: **Average accuracy curves of (a) DOTS vs. Original GRPO, and (b) DOTS+RR vs. DOTS on the Qwen2.5-Math-1.5B model.** The curves show average performance aggregated over four benchmarks with exponential smoothing. Note that the x-axis is the number of **steps** (rather than time). (a) DOTS consistently outperforms the original GRPO and leads to faster convergence. (b) Incorporating RR reduces training time by 20% while preserving the performance of DOTS.

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A Reproducibility

Our code repository is available at <https://github.com/ASTRAL-Group/data-efficient-llm-rl/>.

B Discussion

B.1 Limitations and Future Work

Our adaptive difficulty prediction framework currently relies on randomly sampling a reference set of K questions at each selection step. While effective, the quality of the reference set can influence prediction performance. In principle, one could improve prediction performance by selecting a more diverse reference set that better covers the dataset. Building on this idea, a natural extension is to fix a shared set of K reference questions (with sufficient coverage) across training, re-evaluating their adaptive difficulty at each selection step.

Moreover, while we demonstrate the effectiveness of experience replay in the GRPO setting, our current strategy is relatively straightforward: we randomly replay rollouts associated with questions whose average reward across all rollouts is neither 0 nor 1. A promising direction for further improving efficiency is to incorporate more principled strategies, such as drawing inspiration from prioritized experience replay [37, 51].

B.2 Extended Related Work

RL fine-tuning of LLMs (with verifiable rewards) has recently attracted significant attention, driven in part by the success of DeepSeek-R1 [9]. Compared to the original GRPO algorithm [39], recent work has proposed several algorithmic improvements: DAPO [48] introduces techniques such as clip-higher, dynamic sampling, token-level policy gradient loss, and overlong reward shaping; Dr. GRPO [25] removes the length and standard deviation normalization terms to improve stability. Beyond these algorithmic enhancements, Vojnovic and Yun [42] and Mroueh [30] provide theoretical insights into GRPO, while Zeng et al. [50] and Yeo et al. [47] conduct large-scale empirical studies across models, sharing key design choices that enable RL fine-tuning.

In contrast, relatively little attention has been paid to data-centric approaches. LIMR [22] explores a static data selection strategy for RL fine-tuning by prioritizing samples based on their alignment with the policy’s learning trajectory. However, it requires a full training run over the entire dataset beforehand, limiting its practicality. Our online data selection method DOTS is more efficient and applicable in realistic settings. In addition, prior work has not explored the use of rollout replay in GRPO, which we show can further reduce training costs.

C Proofs

Proof of Theorem 1. We restate Theorem 1 and provide a complete proof below.

Theorem 1 (Maximal Gradient Signal at 50% Success Rate). *Consider a single question q , where G responses $\{o_i\}_{i=1}^G$ are sampled independently from the current policy $\pi_\theta(\cdot | q)$. Each response receives a binary reward $r_i \in \{0, 1\}$, sampled i.i.d. from a Bernoulli(p) distribution, where p represents the reward success rate. Define the group-relative advantage \hat{A}_i as in Eq. 1. We consider the unclipped policy gradient estimator for this question without KL penalty:*

$$g = \sum_{i=1}^G \hat{A}_i \nabla_\theta \log \pi_\theta(o_i | q).$$

Assume that the likelihood gradients $\nabla_\theta \log \pi_\theta(o_i | q)$ are independent of the reward distribution parameter p and have bounded variance. Then, the expected squared norm of the gradient satisfies:

$$\mathbb{E}[\|g\|^2] \propto p(1-p) \cdot (1 - 1/G),$$

and is maximized when $p = 0.5$.

Proof. Let $r_i \in \{0, 1\}$ be the binary reward for response o_i , sampled i.i.d. from a Bernoulli(p) distribution. Define the group-relative advantage as:

$$\hat{A}_i = r_i - \frac{1}{G} \sum_{j=1}^G r_j.$$

We aim to analyze the expected squared norm of the gradient estimator

$$g = \sum_{i=1}^G \hat{A}_i \nabla_\theta \log \pi_\theta(o_i | q).$$

Assume that the gradients $\nabla_{\theta} \log \pi_{\theta}(o_i | q)$ are independent of the rewards $\{r_i\}$ and are independent across i , with bounded second moment:

$$\mathbb{E}[\|\nabla_{\theta} \log \pi_{\theta}(o_i | q)\|^2] \leq C < \infty.$$

Because \hat{A}_i and \hat{A}_j are correlated, we compute the full second moment, where the expectation is taken with respect to π_{θ} :

$$\mathbb{E}[\|g\|^2] = \sum_{i,j=1}^G \mathbb{E}[\hat{A}_i \hat{A}_j] \cdot \mathbb{E}[\nabla_{\theta} \log \pi_{\theta}(o_i | q)^{\top} \nabla_{\theta} \log \pi_{\theta}(o_j | q)].$$

By assumption, the log-likelihood gradients are zero-mean, independent, and identically distributed across i :

$$\mathbb{E}[\nabla_{\theta} \log \pi_{\theta}(o_i | q)^{\top} \nabla_{\theta} \log \pi_{\theta}(o_j | q)] = \begin{cases} V, & i = j, \\ 0, & i \neq j. \end{cases}$$

So,

$$\mathbb{E}[\|g\|^2] = V \cdot \sum_{i=1}^G \mathbb{E}[\hat{A}_i^2].$$

We now compute $\mathbb{E}[\hat{A}_i^2]$. Let $\bar{r} := \frac{1}{G} \sum_{j=1}^G r_j$, then:

$$\mathbb{E}[\hat{A}_i^2] = \mathbb{E}[(r_i - \bar{r})^2] = \text{Var}(r_i - \bar{r}) = \text{Var}(r_i) + \text{Var}(\bar{r}) - 2 \text{Cov}(r_i, \bar{r}).$$

Since $r_i \sim \text{Bernoulli}(p)$ and r_j are i.i.d.,

$$\text{Var}(r_i) = p(1-p), \quad \text{Var}(\bar{r}) = \frac{p(1-p)}{G}, \quad \text{Cov}(r_i, \bar{r}) = \frac{p(1-p)}{G}.$$

Substitute in:

$$\mathbb{E}[\hat{A}_i^2] = p(1-p) + \frac{p(1-p)}{G} - 2 \cdot \frac{p(1-p)}{G} = p(1-p) \left(1 - \frac{1}{G}\right).$$

Therefore,

$$\mathbb{E}[\|g\|^2] = V \cdot G \cdot p(1-p) \left(1 - \frac{1}{G}\right),$$

which is maximized when $p = 0.5$.

□

D Details of Adaptive Difficulty Prediction Framework

D.1 Design and Implementation Details

The core of our adaptive difficulty prediction framework lies in obtaining proper embeddings to enable attention-based weighted prediction, as described in Section 4.1. To achieve this efficiently, we freeze the Qwen2.5-Math-1.5B-Instruct model as the backbone and augment it with a lightweight adapter and a calibration head.

The adapter is a GELU-activated MLP with three hidden layers, each containing 896 units and a dropout rate of 0.1. A LayerNorm is applied to the projection output to stabilize training. The calibration head is a two-layer MLP that takes the mean and standard deviation of reference set difficulties as input. The first output passes through a Softplus activation to yield the scale parameter $w^{(t)}$, while the second is transformed by a Tanh activation to produce a bounded bias term $b^{(t)}$, as defined in Section 4.1.

We collect training data from a set of LLMs that are disjoint from our policy models. These include Qwen2.5-Instruct and Qwen2.5-Math-Instruct series [46], Euror-2-7B-PRIME [5], Mathstral-7B-v0.1⁴, DeepSeek-R1-Distill-Qwen-1.5B [9], DeepScaleR-1.5B-Preview [27], and Qwen2.5-7B-SimpleRL-Zoo [50]. For each model, we sample query questions and reference questions from math datasets and compute their adaptive difficulty as supervision labels. Each training instance consists of a query question q , a reference set $\{(q_i, d_i)\}_{i=1}^K$ with known difficulty scores, and a ground-truth difficulty label d_q . Repeating this procedure across models yields the training dataset $\mathcal{D}_{\text{pred-train}}$.

We train the adapter and calibration head using the standard binary cross-entropy loss:

$$\mathcal{L}_{\text{BCE}} = -\frac{1}{|\mathcal{D}_{\text{pred-train}}|} \sum_{(q, \{(q_i, d_i)\}, d_q) \in \mathcal{D}_{\text{pred-train}}} \left[d_q \log \hat{d}_{q, \text{cal}} + (1 - d_q) \log(1 - \hat{d}_{q, \text{cal}}) \right],$$

where $\hat{d}_{q, \text{cal}}$ is the calibrated predicted difficulty for the query question.

⁴<https://huggingface.co/mistralai/Mathstral-7B-v0.1>

Table 3: **Qualitative example illustrating similarity-based attention mechanism in adaptive difficulty prediction.** The table shows one unlabeled question and its top- and bottom-ranked reference questions by attention score. High-attention references (red) typically share similar concepts and difficulty with the target question (e.g., rhombus and incircle geometry), while low-attention references (blue) diverge in topic and are significantly easier.

Data Source: DeepScaleR

Unlabeled Question

[adaptive diff. = 1.000, predicted score = 0.907]

In the rhombus $ABCD$, point Q divides side BC in the ratio 1 : 3 starting from vertex B , and point E is the midpoint of side AB . It is known that the median CF of triangle CEQ is equal to $2\sqrt{2}$, and $EQ = \sqrt{2}$. Find the radius of the circle inscribed in rhombus $ABCD$.

#	Attention Score	Adaptive Diff.	Reference Question
1	0.487	1.000	Rhombus $ABCD$ has $\angle BAD < 90^\circ$. There is a point P on the incircle of the rhombus such that the distances from P to the lines DA , AB , and BC are 9, 5, and 16, respectively. Find the perimeter of $ABCD$.
2	0.093	1.000	Circle ω_1 with radius 3 is inscribed in a strip S having border lines a and b . Circle ω_2 within S with radius 2 is tangent externally to circle ω_1 and is also tangent to line a . Circle ω_3 within S is tangent externally to both circles ω_1 and ω_2 , and is also tangent to line b . Compute the radius of circle ω_3 .
...			
255	0.000	0.125	A package of milk with a volume of 1 liter cost 60 rubles. Recently, for the purpose of economy, the manufacturer reduced the package volume to 0.9 liters and increased its price to 81 rubles. By what percentage did the manufacturer's revenue increase?
256	0.000	0.125	Given $\tan\left(\alpha - \frac{\pi}{4}\right) = 2$, find the value of $\sin\left(2\alpha - \frac{\pi}{4}\right)$.

D.2 Qualitative Examples

Tab. 3 presents a qualitative example from the Qwen2.5-3B model, showing one unlabeled question alongside reference questions with the highest and lowest attention scores. The example demonstrates that our difficulty prediction framework assigns higher attention to reference questions that share key mathematical topics and structures (e.g., rhombus, incircle), while down-weighting unrelated questions.

E Implementation Details

E.1 Training Datasets and Models

Our experiments involve three model sizes: Qwen2.5-Math-1.5B, Qwen2.5-3B, and Qwen2.5-Math-7B [46]. We adopt four open-source datasets of mathematical reasoning for RL fine-tuning:

- **MATH** [13]: This dataset contains 12,500 competition-level problems from sources such as AMC and AIME, spanning seven mathematical subjects and five difficulty levels. Following [22, 50], we merge the train and test splits and retain only Level 3–5 questions. These are guaranteed to have no overlap with the MATH500 benchmark to prevent data contamination.
- **DeepScaleR-40K** [27]: A collection of approximately 40,000 curated mathematical problems from AMC (pre-2023), AIME (1984–2023), Omni-MATH [8], and Still [29]. Deduplication is performed using embedding-based retrieval, and ungradable problems are filtered to ensure high-quality reward signals. We randomly sample 10,240 problems for training.
- **Open-Reasoner-Zero-57K (ORZ)** [15]: This dataset includes 57,000 high-quality reasoning problems sourced from AIME (up to 2023), AMC, MATH, Numina-MATH [21], and Tulu3 MATH [19]. Extensive cleaning via rule-based and LLM-based filters ensures evaluability and difficulty balance. We sample 8,192 problems for training.

- **DeepMath-103K** [12]: A large-scale dataset focused on high-difficulty mathematical problems, constructed with rigorous data decontamination procedures to support reliable benchmark evaluation. We also sample 8,192 problems for training.

E.2 RL Fine-tuning Details

Tab. 4 summarizes the hyperparameters used in our GRPO training. We adopt the same configuration across all experiments. Following [48, 15], we remove the KL regularization terms. For reward computation, we use a simple rule-based function based solely on answer correctness, without incorporating any format-related signals. Specifically, a reward of 1 is assigned for exact matches with the reference answer, and 0 otherwise. Answer matching is implemented using the Math-Verify library⁵. We adopt a standard chain-of-thought (CoT) prompt template, provided in Tab. 5.

Table 4: **Detailed RL fine-tuning recipes.**

Optimizer	AdamW
Total Batch Size	512
Learning Rate	1e-6
LR Schedule	Constant
Weight Decay	0
Warm-up Ratio	0
Number of Steps	60
Max Prompt Length	1024
Max Rollout Length	3072/4096
Number of Rollouts Per Prompt	8
Rollout Sampling Temperature	0.6
Rollout Sampling Top-p	0.95
GPU Hardware	8x NVIDIA L40S/8x NVIDIA A100

Table 5: **Prompt template used for RL fine-tuning and evaluation.** The placeholder `<question>` is replaced with the actual mathematical question during fine-tuning and evaluation. Special tokens "`<lim_start!>`" and "`<lim_end!>`" are omitted for clarity.

system
Let's think step by step and output the final answer within <code>\boxed{}</code> .
user
<code><question></code>
assistant

E.3 Implementation Details of DOTS and RR

We present the detailed hyperparameter settings of Algorithm 1 in Tab. 6. For DOTS, data selection is performed every two steps during RL fine-tuning.

Table 6: **Hyperparameters of DOTS and RR.**

Target Difficulty α	0.5
Reference Set Size K	256
Data Sampling Temperature τ	1e-3
Fresh Rollout Fraction δ	0.5
Buffer Capacity C	256/512

E.4 Evaluation Details

Consistent with RL fine-tuning, we use a sampling temperature of 0.6, top-p of 0.95, and the same prompt template. We evaluate model performance on four commonly-used mathematical reasoning benchmarks and report the average accuracy to mitigate benchmark-specific variance. We exclude benchmarks with very few questions, such as AIME 24 (30 questions) and AMC 23 (40 questions), as their limited size leads to high evaluation variance and unreliable performance comparisons for smaller models [14].

⁵<https://github.com/huggingface/Math-Verify>

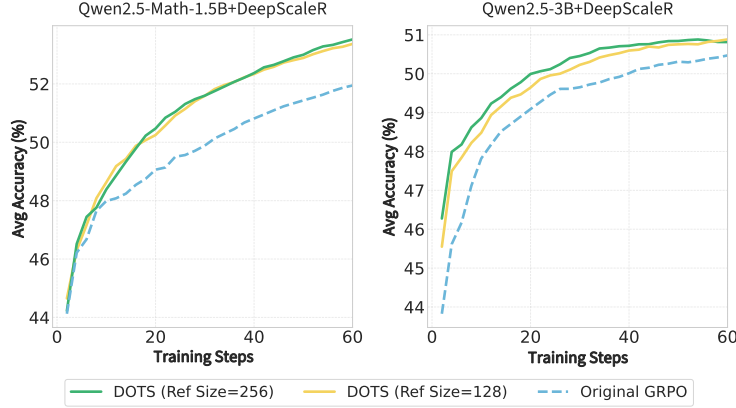


Figure 6: **Average accuracy curves of DOTS (Ref Size = 256), DOTS (Ref Size = 128), and Original GRPO on Qwen2.5-Math-1.5B and Qwen2.5-3B.** The curves show average performance aggregated over four benchmarks with exponential smoothing for visualization. Note that the x-axis is the number of **steps** (rather than time). The results show that a reference set size of 128 achieves performance comparable to that of 256, indicating the robustness of our method to smaller reference sets.

- **GSM8K** [3]: A test set of 1,319 grade school math word problems from the GSM8K dataset, requiring multi-step arithmetic reasoning.
- **MATH500** [23]: A widely used subset of the MATH test split [13]. These problems are excluded from our MATH training data.
- **Minerva Math** [20]: A set of 272 undergraduate-level science and math questions from MIT OpenCourseWare.
- **OlympiadBench** [11]: A benchmark of 675 problems from international math olympiads and physics contests.

F Additional Experimental Results

F.1 Ablation Study on the Adaptive Difficulty Prediction Framework

Off-the-shelf embeddings fail to capture difficulty structure. We evaluate a baseline that directly uses frozen embeddings from the Qwen2.5-Math-1.5B-Instruct model without any training or calibration. In contrast, our framework incorporates trained adapter layers and a calibration head. As shown in Tab. 7, our framework consistently achieves significantly higher Pearson correlation with ground-truth adaptive difficulty across all settings. The poor performance of the off-the-shelf baseline highlights the necessity of further adapter layers and calibration for accurately predicting question difficulty.

Table 7: **Ablation study on training with adapter and calibration.** Comparison of average Pearson correlation (ρ) between predicted scores and ground-truth adaptive difficulties, reported as mean \pm standard deviation over 60 training steps. Results show that training with adapter layers and calibration significantly improves prediction performance.

Model	Dataset	Off-the-shelf Embedding	Our Framework (With Adapter Layers + Calibration)
Qwen2.5-Math-1.5B	MATH	0.2682 ± 0.0207	0.7843 ± 0.0243
	DeepScaleR	0.2064 ± 0.0518	0.7244 ± 0.0318
	ORZ	0.1598 ± 0.0266	0.7153 ± 0.0257
Qwen2.5-3B	DeepScaleR	0.2688 ± 0.0369	0.7789 ± 0.0191
	DeepMath	0.0671 ± 0.0168	0.7029 ± 0.0082
Qwen2.5-Math-7B	DeepScaleR	0.1983 ± 0.0254	0.7076 ± 0.0195

DOTS is robust to the size of reference set. We further investigate the impact of the reference set size K in RL fine-tuning. Fig. 6 compares the performance of the original GRPO and the DOTS method under reference set sizes of 128 and 256, trained with Qwen2.5-Math-1.5B and Qwen2.5-3B on the DeepScaleR dataset. The results show that a reference set size of 128 yields RL performance comparable to that of 256. This indicates that our approach is robust to smaller reference sets, enabling more efficient rollout collection without sacrificing RL fine-tuning quality.

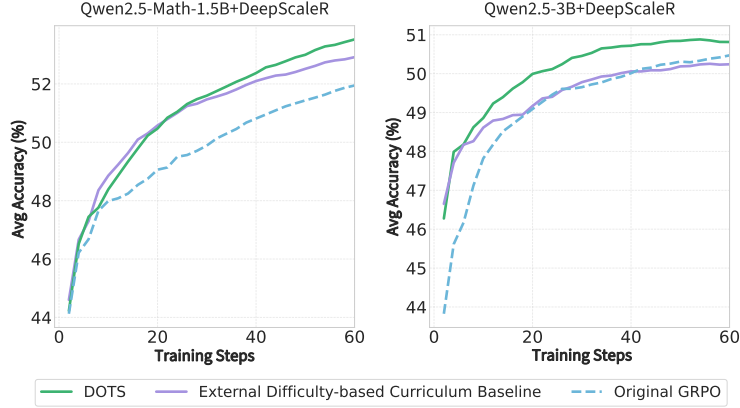


Figure 7: **Comparison between DOTS (ours) and an external difficulty-based curriculum baseline.** The curves show average performance aggregated over four benchmarks with exponential smoothing for visualization. Note that the x-axis is the number of **steps** (rather than time). Our method consistently outperforms the baseline.

F.2 Case Study: Online Data Selection Via External Difficulty-based Curriculum

We additionally implement an online data selection baseline that relies on external difficulty annotations. Specifically, we use the DeepScaleR dataset and label each question with GPT-4o-mini, following the difficulty annotation prompt introduced in Luo et al. [27]. Each question is annotated 32 times, and the average score is used as its final difficulty.

We then follow a staged curriculum: in the first third of training steps, batches are sampled from the easiest third of the dataset; in the middle third, from the medium-difficulty third; and in the final third, from the hardest third. To ensure a fair comparison of online data selection strategies, we compare this baseline with DOTS (without RR). As shown in Fig. 7, our DOTS method consistently outperforms this baseline on both Qwen2.5-Math-1.5B and Qwen2.5-3B. We further discuss in the main text the limitations of such approaches, including the high cost of annotation and limited adaptability due to their reliance on fixed, hand-crafted curricula.