Diagonal Batching Unlocks Parallelism in Recurrent Memory Transformers for Long Contexts

Danil Sivtsov[1,2] **Ivan Rodkin**[3,4] **Gleb Kuzmin**[1,5] **Yuri Kuratov**[1,3] **Ivan Oseledets**[1,2]
[1]AIRI, Moscow, Russia
[2]Skoltech, Moscow, Russia
[3]Neural Networks and Deep Learning Lab, MIPT, Dolgoprudny, Russia
[4]MBZUAI, Abu Dhabi, UAE
[5]FRC CSC RAS, Moscow, Russia

Abstract

Transformer models struggle with long-context inference due to their quadratic time and linear memory complexity. Recurrent Memory Transformers (RMTs) offer a solution by reducing the asymptotic cost to linear time and constant memory usage. However, their memory update mechanism leads to sequential execution, causing a performance bottleneck.

We introduce Diagonal Batching, a scheduling scheme that unlocks parallelism across segments in RMTs while preserving exact recurrence. This approach eliminates the sequential constraint, enabling efficient GPU inference even for single long-context inputs without complex batching and pipelining techniques. Because the technique is purely a run-time computation reordering, existing RMT models adopt it with no retraining.

Applied to a LLaMA-1B ARMT model, Diagonal Batching yields a 3.3x speedup over standard full-attention LLaMA-1B and a 1.8x speedup over the sequential RMT implementation on 131,072-token sequences. By removing sequential bottleneck, Diagonal Batching reduces inference cost and latency, thereby strengthening RMTs as a practical solution for real-world, long-context applications.

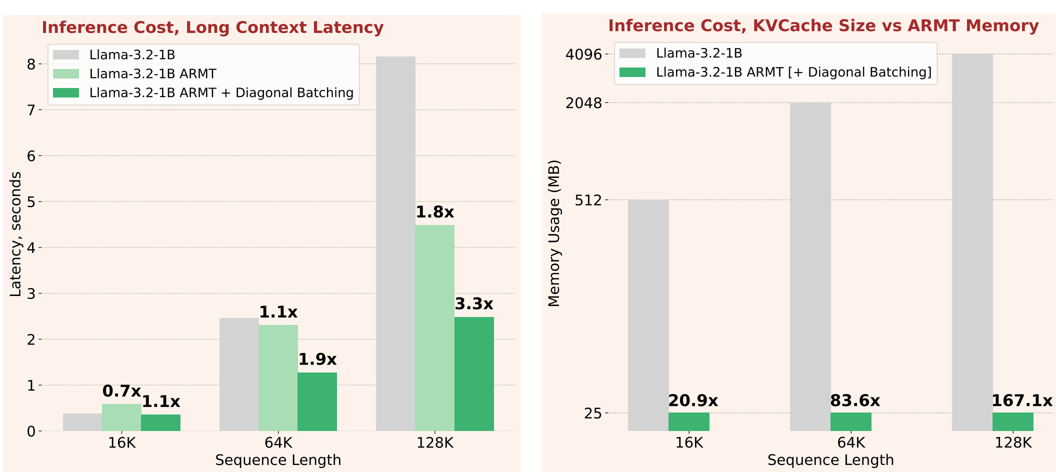


Figure 1: **Diagonal Batching enables the Recurrent Memory Transformers (ARMT) to process 128k tokens sequences 3.3x faster than the LLama-3.2-1B model, with 167.1x memory savings.** These results were obtained using an A100 GPU, and the segment size for the ARMT was set to 1,024 tokens.

1 Introduction

Transformer-based language models have not only revolutionized natural language processing (NLP) [34, 8, 25], but also catalyzed the development of intelligent agents that can solve complex, multi-step problems in various domains by scaling up to large language models (LLMs) [23, 27, 9]. However, these transformer-based models have quadratic time complexity and a linear memory footprint with respect to the length of the input sequence. Consequently, real-world applications are limited by the context window size of standard transformers that can fit within hardware constraints.

From an engineering perspective, numerous optimizations have been proposed to improve attention efficiency and manage GPU memory more effectively. Optimized attention kernels, such as FlashAttention [6, 5] and the xFormers library [18] focus on reducing memory access overhead and maximizing throughput. Memory-saving attention modifications like Multi-Query Attention (MQA) [30], Grouped Query Attention (GQA) [1], and Multi-head Latent Attention (MLA) [20] lower GPU RAM usage by sharing and optimizing KV-cache. For distributed long-context training, methods like Ring Attention [21] and Microsoft DeepSpeed's Ulysses [15] partition sequence data across multiple devices to scale beyond single-GPU memory limits.

Along with these engineering optimizations, alternative architectures to the standard Transformer have been explored. Recently, linear recurrent models, such as S4 [13], RWKV [24], RetNet [33], and Mamba [12, 7] have replaced the softmax attention with alternative read-write operations. These models offer efficient parallel training, like transformers, and require constant memory during inference, like RNNs. However, these approaches often suffer from reduced memory capacity [16] and decreased accuracy in read-write operations [28]. Furthermore, both state-space models and Transformers face theoretical limits, such as the TC^0 complexity bound on the class of functions computable in a single forward pass [22, 31], constraining their expressivity despite massive parallelism.

Memory-augmented models [35, 32], especially memory-augmented transformers with segment-level recurrence [4, 26, 3, 14] offer an alternative approach by compressing history into fixed-size memory states and propagating them across segments. In Recurrent Memory Transformers (RMT) [3], special memory tokens carry state between segments, and each Transformer block acts as a recurrent cell. This approach reduces inference complexity to linear time and constant memory, supporting arbitrarily long contexts [2]. However, the recurrent nature of RMT makes it not fully parallelizable; all subsequent layers have recurrent dependencies, and all segments must be processed sequentially.

Parallel Recurrent Memory Transformers (PRMTs) [28] are a broader class of architectures in which each layer maintains its own memory state. PRMTs localize recurrence within layers and eliminate all inter-layer memory flow. The Associative Recurrent Memory Transformer (ARMT) [28] belongs to this family and demonstrates exceptional scalability. It maintains high quality on sequences of up to 50 million tokens, which is far beyond the capacity of RMT and Mamba [28, 17]. Models such as RWKV, Mamba, and other linear-recurrent architectures can also be considered members of the PRMT family due to their layer-level memory design. In practice, however, these methods only exploit parallelism within individual segments. This parallelism is limited by RAM and compute bounds. Therefore, when processing extremely long sequences, these methods fall back to processing sequential segments, or even to token-level recurrence. This leaves true inter-segment parallelism unaddressed.

In this work, we introduce *Diagonal Batching*, a scheduling scheme that unlocks inter-segment parallelism in PRMTs inference without altering their exact recurrence. By reorganizing the 2D grid of layer and segment computations into independent "diagonals" our method enables concurrent execution of up to N_Layers operations per GPU kernel launch. Diagonal Batching fully encapsulates transformer block computations across segments, thus *eliminating the layer- and segment-level synchronization barriers* present in previous RMT implementations.

We implement diagonal batching in the ARMT framework and evaluate its performance on a LLaMA-1B, 3B, and 8B models with sequence lengths up to 131,072 tokens on an NVIDIA A100/H100 GPUs. Our experiments demonstrate a $3.3\times$ speedup over standard full-attention inference and a $1.8\times$ improvement relative to a sequential ARMT baseline for 1B models. These results demonstrate that diagonal batching is a practical solution for exact, linear-time inference on extremely long contexts. Diagonal Batching code and experiments are publicly available.[1]

[1] github.com/svtdanny/diagonal-batching

Our contributions are:

- We identify the key bottlenecks in existing implementations of RMTs and PRMTs, that limit efficient long-context inference.
- We introduce a novel *Diagonal Batching* technique that maximizes GPU utilization, preserves exact recurrence, and efficiently handles recurrent dependencies in PRMTs, enabling practical parallel execution.
- We empirically demonstrate that our diagonal batching method allows RMTs to achieve long-context scaling performance matching to the batch size scaling of their underlying transformer architectures.
- Our approach utilizes GPU with one long context request at a time, simplifying load balancing for production deployment.

2 Background

2.1 Recurrent Memory Transformers

Recurrent Memory Transformer (RMT) extends standard Transformer architectures by introducing segment-level recurrence. Specifically, the hidden representations corresponding to a segment s are conditioned on a recurrent state M—referred to as the *memory*—propagated from the previous segment $s-1$.

In the original RMT formulation, the memory state is implemented as a sequence of embeddings (Figure 2, left). The memory update mechanism can be formally expressed as:

$$[\_, \_, M_s] = \text{Transformer}([M_{s-1}, H_{s-1}, M_{s-1}]), \tag{1}$$

where M_s denotes the memory state associated with segment s, and H_{s-1} represents the input embeddings from segment $s-1$. The square brackets indicate concatenation of the input sequences.

Associative Recurrent Memory Transformer (ARMT) introduces a parallel memory mechanism designed to support a hierarchical memory structure. Unlike the original RMT, ARMT maintains distinct memory states across different layers. This design facilitates a more expressive memory representation by allowing each layer to store and update its own memory.

The memory update rule in ARMT is formulated as follows:

$$[\_, M_s^l] = \text{TransformerLayer}(\text{AssociativeLayer}([H_{s-1}^{l-1}, M_s^{l-1}])) \tag{2}$$

$$k_i, v_i = W_K m_i, W_V m_i; \quad \beta_i = \sigma(W_\beta m_i); \quad A_0^l = \vec{0}; \quad z_0^l = \vec{0}; \tag{3}$$

$$\overline{v}_i = \frac{A_{s-1}^l \phi(k_i)}{(z_{s-1})^T \phi(k_i)}; \quad \gamma_i = 1 - \frac{(z_{s-1})^T \phi(k_i)}{\|\phi(k_i)\|^2}; \tag{4}$$

$$A_s^l = A_{s-1}^l + \sum_i \beta_i (v_i - \overline{v}_i) \otimes \phi(k_i); \quad z_s^l = z_{s-1}^l + \sum_i \gamma_i \phi(k_i). \tag{5}$$

$$\text{AssociativeLayer}(x_i) = \frac{A_{s-1}^l \phi(W_Q x_i)}{(z_{s-1}^l)^T \phi(W_Q x_i)}, \tag{6}$$

where m_i is the vector from M_s^l, $A_s^l \in \mathbb{R}^{d_{\text{model}} \times 6 d_{\text{mem}}}$, $z_s^l \in \mathbb{R}^{6 d_{\text{mem}}}$, ϕ is the untrained nonlinearity DPFP-3 [29], x_i. is the vector from $[H_{s-1}^{l-1}, M_s^{l-1}]$.

This mechanism in fact implements quasi-linear attention with delta-rule for segment-level recurrence.

2.2 Layer-level Recurrent Models

Our method is primarily applicable to layer-level recurrent architectures, wherein the output of each segment (timestep) depends solely on the input and output of the preceding segment (timestep) within

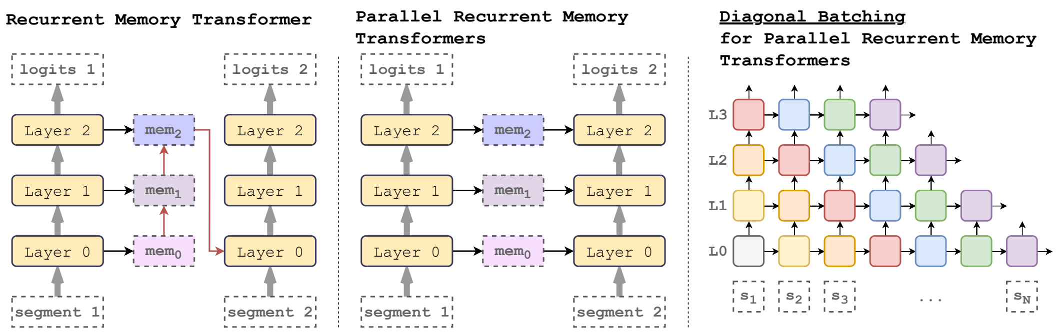


Figure 2: **Unlocking Parallelism in Recurrent Memory Transformers (RMT) with Diagonal Batching. Left:** Standard RMT splits long sequences and processes segments sequentially. Each layer updates a memory state ($\mathrm{mem}_0, \mathrm{mem}_1, \ldots$) and the final-layer memory state is fed as input to the next segment; red arrows highlight the recurrent dependencies that force strictly sequential execution. **Center:** Parallel RMT generalizes a family of models with *layer-level memory*. Each layer maintains its own memory state and passes it horizontally to the same layer in the next segment. This eliminates inter-layer memory flow, yet still requires processing segments in order within each layer, thereby creating layer-wise recurrence. **Right:** Diagonal Batching rearranges the 2D grid of layers (rows) and segments (columns) into independent "diagonals" (same colored blocks). This allows all operations on one diagonal (up to N_Layers) to execute concurrently on the GPU, thus eliminating the sequential bottleneck while preserving all layer-level recurrence.

the same layer. We broadly refer to models that satisfy this assumption as Parallel Recurrent Memory Transformers (PRMTs, Figure 2, center): Associative Recurrent Memory Transformer (ARMT) [28], RWKV [24], Mamba [12, 7], and other linear-recurrent models [38].

In ARMT, each layer l has its own memory state that consists of associative matrix A^l. Memory state is updated by special associative block that takes as input outputs of the transformer layer H^l_{t-1} on previous segment $t-1$ and memory update is defined as $A^l_t = \mathrm{AssociativeBlock}(A^l_{t-1}, H^l_{t-1})$. Inside the Associative Block, A^l_t is updated by delta rule, in a simplified form: $A^l_t = A^l_{t-1} + v^l_t \otimes k^l_t$, where v^l_t and k^l_t are obtained by linear transformations of H^l_{t-1}. Each memory update in each layer is made once per segment.

This per-layer memory allows us to optimize the scheduling of which segments can be computed in parallel and at which layers.

There also exists a class of models that do not satisfy these assumptions. For instance, in RMT [3], the output of a given layer at segment t additionally depends on the output of the final layer from the previous segment (Figure 2, left).

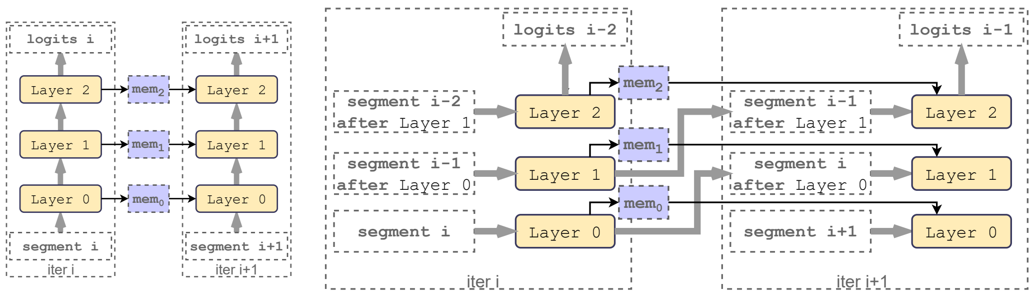
2.3 Existing inference optimizations techniques for transformer models

Several techniques are proposed to speed up the inference of transformer models, such as FlashAttention [6, 5], speculative decoding [37], quantization techniques [10, 19], and many others.

Therefore, any new approach should be compatible with these optimizations to be useful in practice. Diagonal Batching is independent of these methods and integrates with them seamlessly. It employs FlashAttention to group segments and achieve highly efficient attention computation.

2.4 Hardware utilization

Effectiveness of individual operations often analyzed via the roofline model, which characterizes the performance limits of hardware based on computational intensity and memory bandwidth [36]. Transformer architecture mostly consists of matrix multiplication - compute bound operation. Matrix multiplication's computational intensity don't depends on batch size. However, the total achievable floating-point operations per second (FLOPS) improves significantly, as larger batch sizes enable better parallel workload distribution across GPU cores, optimizing hardware utilization [6].



(a) Baseline compute scheme.

(b) Diagonal Batching: grouped compute scheme.

Figure 3: Baseline compute schedule in PRMTs leads to n_layers x n_segments sequential operations. Diagonal Batching reduces this value to n_layers + n_segments by grouped computations.

Despite these benefits, large batch size introduces significant memory demand. It mostly comes from intermediate activations computations and storing output logits, which scales linearly with batch size and sequence length. This limits practical usage of batching, as large language transformers often use almost all available GPU memory.

3 Diagonal Batching method

3.1 Intuition and dependency graph

In the naive approach, we must perform many forward operations (`n_segments` $\times$ `n_layers`) using inputs of shape (`segment_size`, `hidden_size`).

Due to parallel memory usage, each `(segment, layer)` pair only depends on the preceding pairs: `(segment, layer-1)` and `(segment-1, layer)`.

Given this dependency, all pairs where `segment + layer = i` can be computed in parallel during the i-th iteration. Each iteration can be visualized as a diagonal in the forward-pass computation graph, as shown in Figure 2, right.

If the execution is not compute-bound, this diagonal execution approach can yield significant speedup. Note that this property holds only for parallel memory models. In recursive memory models, each `(segment, layer)` depends on all previous `(segment-k, layer-n)` pairs, making diagonal batching not applicable.

3.2 Batching

Simplified description of the algorithm is given for ARMT in Algorithm 1. For parallel RMT, the algorithm is the same, but without memory association and update.

Lemma 3.1. *Diagonal Batching completes the DAG in the minimum possible number of groups,* $N_{\text{segments}} + N_{\text{layers}} - 1$*, and schedules each node* (i, j) *in its earliest feasible group* $i + j$*.*

Proof. Topologically sort the DAG by the key (i, j) with root being $(0, 0)$. In this ordering, each node (i, j) appears at level $i + j$, which is therefore the earliest group it can occupy, and the longest path has length $N_{\text{segment}} + N_{\text{layers}} - 1$ vertices. Hence, any schedule needs at least $N_{\text{segment}} + N_{\text{layers}} - 1$ groups. Diagonal batching uses precisely those levels as its groups, achieving both bounds. $\square$

3.3 Implementation details

To efficiently implement grouped layer computations, we modify the base model architecture. All layers are replaced with a single grouped layer, as shown in Figure 3. Using the initial layer of the model as the basis, we implement the following adjustments:

1. Replace the linear layers with a `GroupedMatmul` operation. The weights and biases are constructed by stacking those from the original layers.

Algorithm 1 GROUPED ARMT EXECUTION

Require: input sequence $\mathcal{I}$, number of layers L, grouped layer $\mathcal{G}$
ZEROGROUPEDMEMORY($\mathcal{M}$)
segments $\leftarrow$ SEGMENT($\mathcal{G}, \mathcal{I}$) ▷ token ids to segments with memory tokens
GInput $\leftarrow$ [], *Out* $\leftarrow$ []
for $i = 0$ **to** $L + |segments| - 1$ **do**
    if $i < |segments|$ **then**
        prepend *segments*[i] to *GInput* ▷ ingest new segment
    end if
    $X \leftarrow$ STACK(*GInput*)
    if $i > 0$ **then**
        $X_{0:|X|-1} \leftarrow$ ASSOCIATE($\mathcal{G}, X_{0:|X|-1}$) ▷ memory association operation between consecutive segments
    end if
    $Y \leftarrow$ GROUPEDFORWARD($\mathcal{G}, X$) ▷ multi-layer grouped call
    UPDATEMEM($\mathcal{G}, Y_{:,-num\_mem\_tokens:}$) ▷ memory update for next segment
    GInput $\leftarrow$ list of segments in Y
    if $i \geq L - 1$ **then**
        $O \leftarrow$ *GInput*.POPLAST ▷ segment went through all layers
        append O to *Out*
    end if
end for
return CONCAT(*Out*) ▷ final logits

2. Layer normalization weights are also replaced by stacking parameters across all layers. Additionally, the forward pass is adapted to ensure correct broadcasting behavior.

3. All other operations remain unchanged. However, they operate as though they handle significantly larger batch sizes, contributing to parallel execution.

For the grouped matrix multiplication, we utilize the `GroupedGEMM` function from the CUTLASS library with a minor optimization: the output tensor is pre-allocated as a single large tensor, which is subsequently partitioned into individual submatrices without additional overhead.

4 Experiments

In experiment section, we address two main questions regarding diagonal batching method:

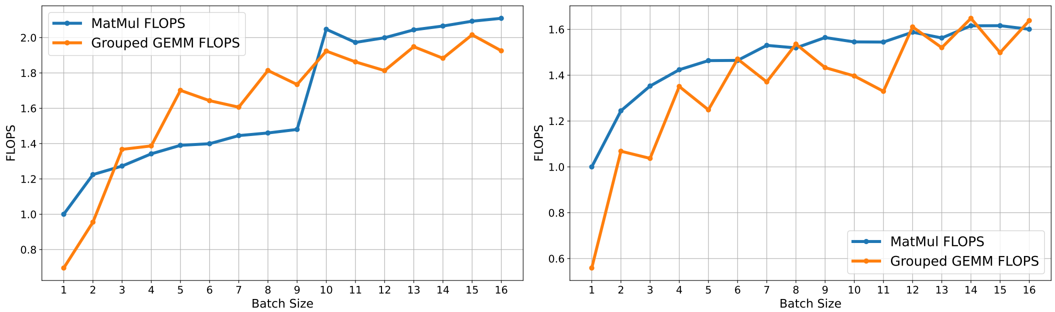
- How much speedup we can get compared to naive ARMT setup in single request inferences.
- How the proposed method compares with batching strategies.

We start from showing efficiency grows for individual bottleneck operations inside network - linear layers and attention. Then we show the resulting scaling for the transformer models with ARMT of different sizes. We conducted all experiments with the models from the Llama-3 family [11].

4.1 Linear layer efficiency

The only change from base model is that we substitute linear layer with matrix multiplication to layers with grouped GEMM with group equal to all linear layers weights. In Figure 4 we show, that grouped GEMM FLOPS scales similar throw group size to GEMM with corresponding batch size. This gives the basis that our method should scale similar to underlying model with batch size as all other operations basically the same (but in different order).

Second, we have group size equal to the number of layers in the model. This way, we move grouped GEMM operation to peak GEMM flops for a100 and h100 GPUs, ensuring high utilization. Corresponding FLOPS improvement shown in Figure 4.

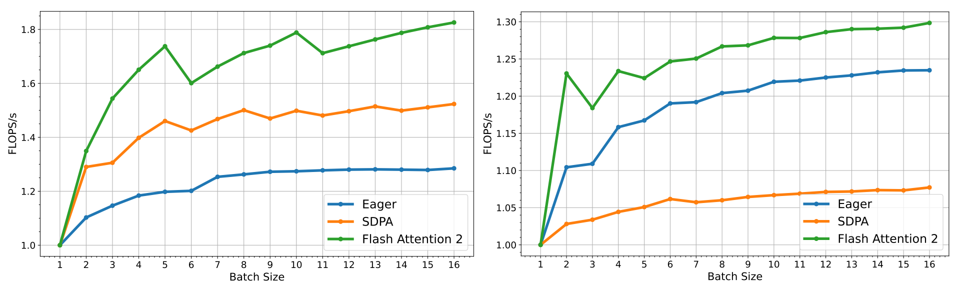


(a) a100 (b) h100

Figure 4: Cutlass Group GEMM scales similarly to batch size 1 Linear layer's matrix multiplication, starting from group size 4.

4.2 Attention layer efficiency

Our method does not modify attention layer at all. Instead, attention just performs batched operation with batch size equal to number of layers. This increase its performance to implementation FLOPS peak. We show relative FLOPS speedups in Figure 5.



(a) a100 (b) h100

Figure 5: Diagonal batching increase attention performance by treating groups as batches—similar to increasing the model's overall batch size.

4.3 Inference scaling

The performance increase for individual operations directly translates into overall model speedup. We evaluate this effect on Llama ARMT models of varying sizes—160M (Table 7), 1B (Table 1), 3B (Table 5), and 8B (Table 6).

Across all model sizes and batch configurations, our implementation consistently achieves substantial speedups over the default ARMT implementation. Gains are particularly pronounced for smaller segment sizes. This is because, with larger matrix multiplications, hardware utilization is already near peak FLOPS, leaving less room for group scaling.

A key implication of these results is that researchers can prioritize quality-driven choices for segment size without being overly constrained by performance. Diagonal batching decouples performance from segment size, allowing better flexibility in architectural decisions.

4.4 Diagonal batching vs mini-batching

We evaluate the effectiveness of diagonal batching compared to standard mini-batching by measuring compute time per segment under identical hardware and model configurations. As shown in Figure 6, diagonal batching achieves compute scaling per segment that closely matches micro-batching across almost all tested scenarios.

Method	Sequence Length					
	4096	**8192**	**16384**	**32768**	**65536**	**131072**
Llama-3.2-1B	0.024	0.026	0.376	0.926	2.460	8.160
Configuration: (512, 128)						
LLama-3.2-1B-ARMT	0.147	0.574	1.15	2.29	4.52	8.98
Diagonal Batching: LLama-3.2-1B-ARMT	0.283 x0.52	0.248 x2.32	0.454 x2.53	0.861 x2.66	1.67 x2.71	3.3 x2.72
Configuration: (1024, 128)						
LLama-3.2-1B-ARMT	0.149	0.291	0.578	1.15	2.3	4.48
Diagonal Batching: LLama-3.2-1B-ARMT	0.119 x1.25	0.196 x1.49	0.351 x1.65	0.656 x1.75	1.27 x1.81	2.48 x1.81
Configuration: (2048, 128)						
LLama-3.2-1B-ARMT	0.094	0.177	0.344	0.679	1.35	2.68
Diagonal Batching: LLama-3.2-1B-ARMT	0.108 x0.87	0.176 x1.01	0.304 x1.13	0.571 x1.19	1.11 x1.22	2.18 x1.23
Configuration: (4096, 128)						
LLama-3.2-1B-ARMT	0.082	0.155	0.301	0.594	1.18	2.35
Diagonal Batching: LLama-3.2-1B-ARMT	0.102 x0.80	0.172 x0.90	0.295 x1.02	0.553 x1.07	1.07 x1.10	2.1 x1.12

Table 1: Diagonal Batching allows to speed-up the execution for longer sequences — from 1.1× to 2.7× compared to base ARMT at 131072 sequence length. Execution time comparison (in seconds) and relative speedups across different sequence lengths compared to LLama-3.2-1B-ARMT. Configuration format: (segment_size, memory_tokens). Measured on Nvidia A100 GPU.

To provide an upper bound on achievable performance, we also report the Ideal Even Load case, than all segments computations computed with full grouped layer with maximum achievable FLOPS. One can see this even load setup is much better, mostly matching or overcoming the biggest batch sizes. The gap between them is our current implementation inefficiency.

Notably, diagonal batching delivers substantial performance improvements for larger models (starting from 1B parameters), particularly when segment sizes are moderate. For these configurations, diagonal batching matches large batch sizes.

These findings suggest that diagonal batching effectively captures the utilization benefits of large-batch inference—through parallelized scheduling rather than increased memory allocation.

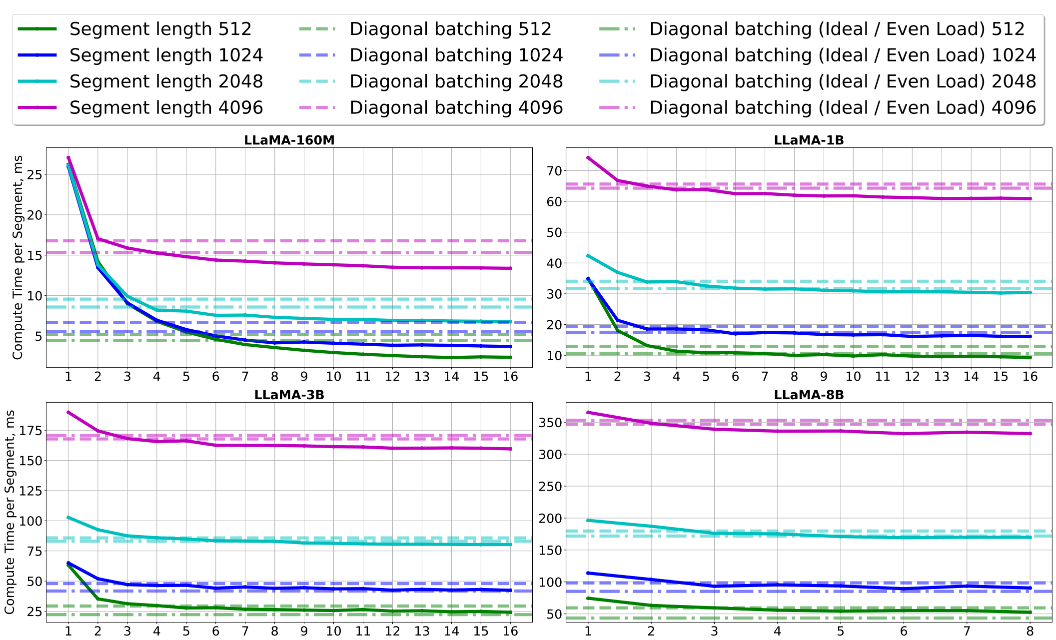


Figure 6: Ideal batch-size scaling vs grouped batching on Nvidia A100 for Llama models, time per segment in batch (group)

4.5 Error accumulation

We conducted an empirical investigation on error accumulation during inference stage with Diagonal Batching. Our experiments show that the overall error is less than 2% for all sequences shorter than 32,768 tokens. This is comparable to other efficient layers implementations used in production. For example, we observed FlashAttention2 [5] gives 1-2% relative logits error compared to other attention implementations on same random input sequences.

The detailed error values for each segment are presented in Table 2. The error is calculated as the ratio of the Frobenius norm of difference between logits of base ARMT implementation and logits of ARMT with Diagonal Batching to the norm of logits of base ARMT. However, we the effect of error accumulation on downstream tasks is negligible. To prove this, we evaluated the trained ARMT model both in original implementation and with Diagonal Batching; the results are presented in Table 3 in Appendix A. These results show that both implementations achieve the same results on the BABILong benchmark [17], while Table 4 in Appendix A shows that diagonal batching can increase the relative speed by up to 3.2x for 64k-length token sequences.

Number of segments	1	2	4	8	16	32
Diagonal Batching, Error, %	0.00	1.10	1.49	1.75	1.89	1.87
FlashAttention2 [5] vs torch SDPA, Error, %	1.25	1.15	1.17	1.22	1.36	1.45

Table 2: During inference with diagonal batching, error accumulates but does not exceed 2%, which is comparable to the change of attention implementation (FlashAttention vs SDPA). The results for ARMT with Llama-3.2-1B-Instruct are shown with a segment size of 1024 tokens.

5 Conclusion

Long-context inference with transformer models still suffers from quadratic compute and linear memory growth. Several linear complexity architectures, such as Mamba, RWKV, and Recurrent Memory Transformers (RMTs), aim to address this. RMTs, in particular, offer the advantage of minimal architectural changes, ensuring compatibility with existing models and algorithms.

This paper demonstrated that the principal bottleneck in both RMTs and their layer-memory variants (PRMTs) is not algorithmic complexity but scheduling: recurrent dependencies force fine-grained synchronization, that underutilizes modern accelerators. We introduced *Diagonal Batching*, a simple but powerful scheduling scheme that reorganizes the layer–segment computation grid into concurrency-friendly diagonals, thereby enabling up to N_Layers operations per kernel without altering exact recurrence. Our experiments demonstrate that a Llama-1B ARMT equipped with diagonal batching achieves a 3.3x latency decrease over the vanilla Llama-1B and a 1.8x speedup over a sequential RMT implementation on a 131,072 token context task, all while maintaining high exactness of resulting logits (with only a 1% relative error).

Considering these advantages, Diagonal Batching turns theoretically appealing compute scaling of PRMTs into a practical solution for exact linear-time inference on extremely long contexts. By eliminating the major performance barrier, it positions memory-augmented recurrent Transformers as a competitive and scalable foundation for next-generation LLM applications that require efficient long-range input processing.

Limitations

Despite its advantages, Diagonal Batching has several practical limitations. First, it is not directly compatible with the Recurrent Memory Transformers (RMTs) due to intra-layer recurrence. However, a more promising approach is to focus on Parallel RMTs, which has already been shown in previous works to be more effective [28]. Second, our current implementation assumes a uniform layer configuration. When models employ heterogeneous layers or varied hidden sizes, applying the technique requires more intricate grouping logic and manual engineering. Finally, the achievable speedup increases with the number of layers. Therefore, shallower models or models with very few layers will only see modest performance gains.

References

[1] Joshua Ainslie, James Lee-Thorp, Michiel de Jong, Yury Zemlyanskiy, Federico Lebron, and Sumit Sanghai. Gqa: Training generalized multi-query transformer models from multi-head checkpoints. In *Proceedings of the 2023 Conference on Empirical Methods in Natural Language Processing*, pages 4895–4901, 2023.

[2] Aydar Bulatov, Yuri Kuratov, Yermek Kapushev, and Mikhail Burtsev. Beyond attention: Breaking the limits of transformer context length with recurrent memory. *Proceedings of the AAAI Conference on Artificial Intelligence*, 38(16):17700–17708, Mar. 2024.

[3] Aydar Bulatov, Yury Kuratov, and Mikhail Burtsev. Recurrent memory transformer. *Advances in Neural Information Processing Systems*, 35:11079–11091, 2022.

[4] Zihang Dai, Zhilin Yang, Yiming Yang, Jaime Carbonell, Quoc Le, and Ruslan Salakhutdinov. Transformer-XL: Attentive language models beyond a fixed-length context. In *Proceedings of the 57th Annual Meeting of the Association for Computational Linguistics*, pages 2978–2988, Florence, Italy, July 2019. Association for Computational Linguistics.

[5] Tri Dao. FlashAttention-2: Faster attention with better parallelism and work partitioning. In *International Conference on Learning Representations (ICLR)*, 2024.

[6] Tri Dao, Daniel Y. Fu, Stefano Ermon, Atri Rudra, and Christopher Ré. FlashAttention: Fast and memory-efficient exact attention with IO-awareness. In *Advances in Neural Information Processing Systems (NeurIPS)*, 2022.

[7] Tri Dao and Albert Gu. Transformers are ssms: Generalized models and efficient algorithms through structured state space duality. In *International Conference on Machine Learning*, pages 10041–10071. PMLR, 2024.

[8] Jacob Devlin, Ming-Wei Chang, Kenton Lee, and Kristina Toutanova. BERT: Pre-training of Deep Bidirectional Transformers for Language Understanding. In *Proceedings of the 2019 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies, Volume 1 (Long and Short Papers)*, pages 4171–4186, 2019.

[9] Abhimanyu Dubey, Abhinav Jauhri, Abhinav Pandey, Abhishek Kadian, Ahmad Al-Dahle, Aiesha Letman, Akhil Mathur, Alan Schelten, Amy Yang, Angela Fan, et al. The llama 3 herd of models. *arXiv preprint arXiv:2407.21783*, 2024.

[10] Elias Frantar, Saleh Ashkboos, Torsten Hoefler, and Dan Alistarh. GPTQ: Accurate post-training compression for generative pretrained transformers. *arXiv preprint arXiv:2210.17323*, 2022.

[11] Aaron Grattafiori, Abhimanyu Dubey, Abhinav Jauhri, Abhinav Pandey, Abhishek Kadian, Ahmad Al-Dahle, Aiesha Letman, Akhil Mathur, Alan Schelten, Alex Vaughan, et al. The llama 3 herd of models. *arXiv preprint arXiv:2407.21783*, 2024.

[12] Albert Gu and Tri Dao. Mamba: Linear-time sequence modeling with selective state spaces. *arXiv preprint arXiv:2312.00752*, 2023.

[13] Albert Gu, Karan Goel, and Christopher Re. Efficiently modeling long sequences with structured state spaces. In *International Conference on Learning Representations*, 2021.

[14] DeLesley Hutchins, Imanol Schlag, Yuhuai Wu, Ethan Dyer, and Behnam Neyshabur. Block-recurrent transformers. In Alice H. Oh, Alekh Agarwal, Danielle Belgrave, and Kyunghyun Cho, editors, *Advances in Neural Information Processing Systems*, 2022.

[15] Sam Ade Jacobs, Masahiro Tanaka, Chengming Zhang, Minjia Zhang, Shuaiwen Leon Song, Samyam Rajbhandari, and Yuxiong He. Deepspeed ulysses: System optimizations for enabling training of extreme long sequence transformer models. *arXiv preprint arXiv:2309.14509*, 2023.

[16] Samy Jelassi, David Brandfonbrener, Sham M Kakade, and Eran Malach. Repeat after me: Transformers are better than state space models at copying. In *International Conference on Machine Learning*, pages 21502–21521. PMLR, 2024.

[17] Yuri Kuratov, Aydar Bulatov, Petr Anokhin, Ivan Rodkin, Dmitry Sorokin, Artyom Sorokin, and Mikhail Burtsev. Babilong: Testing the limits of llms with long context reasoning-in-a-haystack. In A. Globerson, L. Mackey, D. Belgrave, A. Fan, U. Paquet, J. Tomczak, and C. Zhang, editors, *Advances in Neural Information Processing Systems*, volume 37, pages 106519–106554. Curran Associates, Inc., 2024.

[18] Benjamin Lefaudeux, Francisco Massa, Diana Liskovich, Wenhan Xiong, Vittorio Caggiano, Sean Naren, Min Xu, Jieru Hu, Marta Tintore, Susan Zhang, Patrick Labatut, Daniel Haziza, Luca Wehrstedt, Jeremy Reizenstein, and Grigory Sizov. xformers: A modular and hackable transformer modelling library. `https://github.com/facebookresearch/xformers`, 2022.

[19] Ji Lin, Jiaming Tang, Haotian Tang, Shang Yang, Wei-Ming Chen, Wei-Chen Wang, Guangxuan Xiao, Xingyu Dang, Chuang Gan, and Song Han. Awq: Activation-aware weight quantization for on-device llm compression and acceleration. *Proceedings of Machine Learning and Systems*, 6:87–100, 2024.

[20] Aixin Liu, Bei Feng, Bin Wang, Bingxuan Wang, Bo Liu, Chenggang Zhao, Chengqi Dengr, Chong Ruan, Damai Dai, Daya Guo, et al. Deepseek-v2: A strong, economical, and efficient mixture-of-experts language model. *arXiv preprint arXiv:2405.04434*, 2024.

[21] Hao Liu, Matei Zaharia, and Pieter Abbeel. Ringattention with blockwise transformers for near-infinite context. In *The Twelfth International Conference on Learning Representations*, 2024.

[22] William Merrill, Jackson Petty, and Ashish Sabharwal. The illusion of state in state-space models. In *International Conference on Machine Learning*, pages 35492–35506. PMLR, 2024.

[23] OpenAI. Gpt-4 technical report, 2023.

[24] Bo Peng, Eric Alcaide, Quentin Anthony, Alon Albalak, Samuel Arcadinho, Stella Biderman, Huanqi Cao, Xin Cheng, Michael Chung, Leon Derczynski, Xingjian Du, Matteo Grella, Kranthi Gv, Xuzheng He, Haowen Hou, Przemyslaw Kazienko, Jan Kocon, Jiaming Kong, Bartłomiej Koptyra, Hayden Lau, Jiaju Lin, Krishna Sri Ipsit Mantri, Ferdinand Mom, Atsushi Saito, Guangyu Song, Xiangru Tang, Johan Wind, Stanisław Woźniak, Zhenyuan Zhang, Qinghua Zhou, Jian Zhu, and Rui-Jie Zhu. RWKV: Reinventing RNNs for the transformer era. In Houda Bouamor, Juan Pino, and Kalika Bali, editors, *Findings of the Association for Computational Linguistics: EMNLP 2023*, pages 14048–14077, Singapore, December 2023. Association for Computational Linguistics.

[25] Alec Radford, Jeff Wu, Rewon Child, David Luan, Dario Amodei, and Ilya Sutskever. Language models are unsupervised multitask learners. 2019.

[26] Jack W. Rae, Anna Potapenko, Siddhant M. Jayakumar, Chloe Hillier, and Timothy P. Lillicrap. Compressive transformers for long-range sequence modelling. In *International Conference on Learning Representations*, 2020.

[27] Machel Reid, Nikolay Savinov, Denis Teplyashin, Dmitry Lepikhin, Timothy Lillicrap, Jean-baptiste Alayrac, Radu Soricut, Angeliki Lazaridou, Orhan Firat, Julian Schrittwieser, et al. Gemini 1.5: Unlocking multimodal understanding across millions of tokens of context. *arXiv preprint arXiv:2403.05530*, 2024.

[28] Ivan Rodkin, Yuri Kuratov, Aydar Bulatov, and Mikhail Burtsev. Associative recurrent memory transformer. *CoRR*, 2024.

[29] Imanol Schlag, Kazuki Irie, and Jürgen Schmidhuber. Linear transformers are secretly fast weight programmers, 2021.

[30] Noam Shazeer. Fast transformer decoding: One write-head is all you need. *arXiv preprint arXiv:1911.02150*, 2019.

[31] Lena Strobl, William Merrill, Gail Weiss, David Chiang, and Dana Angluin. What formal languages can transformers express? a survey. *Transactions of the Association for Computational Linguistics*, 12, 2024.

[32] Sainbayar Sukhbaatar, Arthur Szlam, Jason Weston, and Rob Fergus. End-to-end memory networks, 2015.

[33] Yutao Sun, Li Dong, Shaohan Huang, Shuming Ma, Yuqing Xia, Jilong Xue, Jianyong Wang, and Furu Wei. Retentive network: A successor to transformer for large language models. *arXiv preprint arXiv:2307.08621*, 2023.

[34] Ashish Vaswani, Noam Shazeer, Niki Parmar, Jakob Uszkoreit, Llion Jones, Aidan N Gomez, Łukasz Kaiser, and Illia Polosukhin. Attention is All you Need. In *Advances in neural information processing systems*, pages 5998–6008, 2017.

[35] Jason Weston, Sumit Chopra, and Antoine Bordes. Memory networks. In Yoshua Bengio and Yann LeCun, editors, *3rd International Conference on Learning Representations, ICLR 2015, San Diego, CA, USA, May 7-9, 2015, Conference Track Proceedings*, 2015.

[36] Samuel Williams, Andrew Waterman, and David Patterson. Roofline: an insightful visual performance model for multicore architectures. *Communications of the ACM*, 52(4):65–76, 2009.

[37] Heming Xia, Tao Ge, Peiyi Wang, Si-Qing Chen, Furu Wei, and Zhifang Sui. Speculative decoding: Exploiting speculative execution for accelerating seq2seq generation. In Houda Bouamor, Juan Pino, and Kalika Bali, editors, *Findings of the Association for Computational Linguistics: EMNLP 2023*, pages 3909–3925, Singapore, December 2023. Association for Computational Linguistics.

[38] Songlin Yang, Bailin Wang, Yu Zhang, Yikang Shen, and Yoon Kim. Parallelizing linear transformers with the delta rule over sequence length. In *The Thirty-eighth Annual Conference on Neural Information Processing Systems*.

Task	Length, tokens	LLama-3.2-1B ARMT	LLama-3.2-1B ARMT, Diagonal Batching
QA1	0K	100	100
	1K	100	100
	2K	100	100
	4K	100	100
	8K	100	100
	16K	100	100
	32K	100	100
	64K	70	69
QA2	0K	100	100
	1K	100	100
	2K	100	100
	4K	100	100
	8K	99	100
	16K	98	98
	32K	94	94
	64K	47	46

Table 3: Diagonal Batching maintains the same scores as the original ARMT inference method on the BABILong benchmark. Scores of the models were evaluated on the first two tasks: QA1 and QA2.

Task	Length, tokens	LLama-3.2-1B, ARMT	LLama-3.2-1B, ARMT, Diagonal Batching	Speed-up ($\times$ times)
QA1	2K	13.43	15.06	0.89
	4K	22.45	17.99	1.25
	8K	41.41	22.49	1.84
	16K	79.16	33.12	2.39
	32K	153.68	54.20	2.84
	64K	302.15	94.36	3.20
QA2	2K	13.08	14.93	0.88
	4K	22.66	18.21	1.24
	8K	41.66	22.70	1.84
	16K	79.80	33.38	2.39
	32K	153.82	53.46	2.88
	64K	303.40	94.69	3.20

Table 4: Diagonal Batching significantly speeds up ARMT inference on longer inputs. Inference time (in seconds) and relative speed-up of the models are given on the BABILong dataset, first two tasks.

A Evaluating Models with Diagonal Batching

Although diagonal Batching significantly speeds up the inference, it also introduces some numerical drifts due to the optimized execution procedure. To estimate the effect of these drifts on practical tasks, we evaluated the ARMT model on BABILong benchmark [17] with and without diagonal Batching. The ARMT model was trained on the BABILong dataset with curriculum learning on length up to 8192 tokens, similar to the approach described in [17]. After, we evaluated this model with and without diagonal batching on QA1 and QA2 tasks from BABILong. Note that we did not change the weights of the model in this experiment; we simply applied the proposed Diagonal Batching grouping method.

The evaluation results are presented in Table 3. As one can see, despite the numerical drifts during forward pass, the generation results remain almost unchanged up to the 65536 input length. These results show that diagonal batching preserves the quality of the generation of trained ARMT model and can be used as drop-in replacement to speed-up the inference.

We also compared the inference time of these two approaches on the same benchmark. In this experiment, we measure not the forward pass time, but the generation time on the BABILong. Table 4 shows that the diagonal batching approach significantly speeds up the generation, up to 3 times on the input length of 65536 tokens. During both of these experiments, we used the following ARMT configuration - the size of the segment was set to 1024 tokens, the number of memory tokens was set to 16 and the associative memory hidden size is 64.

Finally, we implemented backward pass for diagonal batching to support training. Aligning the training and inference code eliminates a discrepancy that is likely the source of logit-level floating-point drift.

B Additional measurements

To clearly illustrate the speedup provided by the developed diagonal batching algorithm, we present relative improvements across various configurations and sequence lengths. Results for speedup against original ARMT implementation is shown in Table 9 and against underlying Llama model in Table 8. These measurements provide additional insights into how our method scales and compares to the baseline implementations.

We also present results for different size models of Llama-3 family [11]: LLaMA-160M (Table 7), 1B (Table 1), 3B (Table 5), and 8B (Table 6) models.

Method	**Sequence Length**					
	4096	**8192**	**16384**	**32768**	**65536**	**131072**
Llama-3.2-3B	0.168	0.344	0.769	1.95	5.59	18.2
Configuration: (1024, 128)						
LLama-3.2-3B-ARMT	0.272	0.537	1.05	2.02	4.09	8.23
Diagonal Batching: LLama-3.1-3B-ARMT	0.274 x0.99	0.454 x1.18	0.833 x1.26	1.58 x1.28	3.1 x1.32	6.14 x1.34
Configuration: (4096, 128)						
LLama-3.2-3B-ARMT	0.203	0.39	0.765	1.52	3.01	6.01
Diagonal Batching: LLama-3.2-3B-ARMT	0.239 x0.85	0.411 x0.95	0.739 x1.04	1.4 x1.09	2.72 x1.11	5.37 x1.12

Table 5: Diagonal batching speed-ups the execution - from 1.1 to 1.3 times comparing to base ARMT for 131072 sequence length. Execution time comparison (in seconds) and relative speedups across different sequence lengths compared to LLama-3.2-3B-ARMT. Configuration in format (segment_size, memory_tokens). Nvidia A100 GPU.

Method	**Sequence Length**					
	4096	**8192**	**16384**	**32768**	**65536**	**131072**
Llama-3.1-8B	0.332	0.682	1.48	3.61	9.82	30.4
Configuration: (1024, 128)						
LLama-3.1-8B-ARMT	0.497	0.936	1.82	3.63	7.22	14.4
Diagonal Batching: LLama-3.1-8B-ARMT	0.478 x1.04	0.86 x1.09	1.64 x1.11	3.2 x1.13	6.34 x1.14	12.6 x1.14
Configuration: (4096, 128)						
LLama-3.1-8B-ARMT	0.384	0.754	1.48	2.95	5.86	11.7
Diagonal Batching: LLama-3.1-8B-ARMT	0.432 x0.89	0.781 x0.97	1.46 x1.01	2.83 x1.04	5.6 x1.05	11.1 x1.05

Table 6: Diagonal batching speed-ups the execution - from 1.05 to 1.14 times comparing to base ARMT for 131072 sequence length. Execution time comparison (in seconds) and relative speedups across different sequence lengths compared to LLama-3.2-8B-ARMT. Configuration in format (segment_size, memory_tokens). Nvidia A100 GPU.

Method	Sequence Length					
	4096	8192	16384	32768	65536	131072
Llama-160M	0.017	0.033	0.075	0.196	0.594	2.03
Configuration: (1024, 128)						
LLama-160M-ARMT	0.105	0.211	0.422	0.877	1.72	3.37
Diagonal Batching: LLama-160M-ARMT	0.061 x1.72	0.087 x2.43	0.138 x3.06	0.243 x3.61	0.451 x3.81	0.855 x3.94
Configuration: (4096, 128)						
LLama-160M-ARMT	0.031	0.057	0.111	0.216	0.432	0.855
Diagonal Batching: LLama-160M-ARMT	0.046 x0.67	0.062 x0.92	0.094 x1.18	0.156 x1.38	0.284 x1.52	0.537 x1.59

Table 7: Diagonal batching speed-ups the execution - from 1.6 to 3.9 times comparing to base ARMT for 131072 sequence length. Execution time comparison (in seconds) and relative speedups across different sequence lengths compared to LLama-160M-ARMT. Configuration in format (segment_size, memory_tokens). Nvidia A100 GPU.

Method	Sequence Length					
	4096	8192	16384	32768	65536	131072
LLama-3.2-1B, configuration: (512, 128)	0.085	0.105	0.828	1.075	1.473	2.473
LLama-3.2-1B, configuration: (1024, 128)	0.202	0.133	1.071	1.412	1.937	3.290
LLama-3.2-1B, configuration: (2048, 128)	0.222	0.148	1.237	1.622	2.216	3.743
LLama-3.2-1B, configuration: (4096, 128)	0.235	0.151	1.275	1.675	2.299	3.886

Table 8: Diagonal batching ARMT implementation allows to speedup the execution for longer sequences due to linear complexity - from 2.4 times to 3.8 times with respect to LLama-3.2-1B for 131072 sequence length. Table shows Diagonal Batching executor speedup against original LLama-3.2-1B for different methods across sequence lengths. Configuration in format (segment_size, memory_tokens). Measured on Nvidia A100 GPU.

Method	Sequence Length					
	4096	8192	16384	32768	65536	131072
LLama-3.2-1B, configuration: (512, 128)	0.519	2.315	2.533	2.660	2.707	2.721
LLama-3.2-1B, configuration: (1024, 128)	1.252	1.485	1.647	1.753	1.811	1.806
LLama-3.2-1B, configuration: (2048, 128)	0.870	1.006	1.132	1.189	1.216	1.229
LLama-3.2-1B, configuration: (4096, 128)	0.804	0.901	1.020	1.074	1.103	1.119

Table 9: Diagonal batching allows to speedup the execution for longer sequences - from 1.1 times to 2.7 times with respect to base ARMT for 131072 sequence length. In cases when diagonal batching is slower, we can fall back to the original inference algorithm at runtime. Table shows Diagonal Batching executor speedup against original ARMT inplementation for different methods across sequence lengths. Configuration in format (segment_size, memory_tokens). Measured on Nvidia A100 GPU.