Algorithmics Tutorial Sheet 3

Problem Solving Questions

Name: Palak Jain
Soton email address: ppj1u18@soton.ac.uk
ID: 30012627

1. Let us take any 2000 consecutive numbers in the range n<100000. Because the hash values are derived by doing $(2d_1 + 3d_1 + 5d_3 + 7d_4 + 11d_5)\%47$, there are 47 possible hash values that we can have, as there are 47 possible remainders (0,1,2,....,46).

Now, let's assume, the statement is wrong. Then, we cannot find 43 or more numbers among 2000 consecutive numbers, whose hash value equals to x where 0<= x <= 46. Since all these numbers are consecutive, we can calculate 2000/47 to find the minimum of numbers which have the same hash value. The result of this division is 42.55. Thus, this gives rise to a contradiction, as there are in fact at least 43 numbers which have the same hash value. Therefore in this case, the hash table therefore has 47 hash values, and all the numbers with the same hash value will be linked through a linked list. there will effectively be 47 linked lists, each of at least size 43.

2. The compression algorithm can only take any binary sequence of characters, however, Morty's diary was written in the sequence of three characters. As a result, we need to to modify Morty's text by rewriting it as a sequence of binary numbers. Now, Morty's text is written using 3 different characters; triangle, circle, and square. We want to assign binary values to each of the characters.

If we assign the value 0 to the triangle, and the value 1 to the circle, this compels us to assign a 2-bit binary number to the square. Let's say the square is assigned 01. Now, when reading '01' in the message in binary sequence, it will be difficult to interpret if the message is implying a triangle and circle or just a square. However, If we only assign 2-bit binary numbers to all the 3 characters, let's say, the triangle is assigned the value 00, the circle, 01, and the square the value 11, Morty's message can be translated to a binary sequence and the message can be interpreted much more easily.

It takes one m operation to translate each shape character into a 2-bit binary number. Now, n = 2m, as each figure must be converted to a 2-bit binary number and the message is effectively twice as long now. As a result, the new time complexity is $O((2m)^5 + 5(2m)^3 + 1)$. If we look at the asymptotic leading functional behaviour, the time complexity is $O(m^5)$, and earlier it was $O(n^5)$. We can see that there isn't a significant difference between the time complexities. Therefore it is reasonable to conclude that this is a relatively efficient method of binary conversion of Morty's initial message.