

Consider an array where the elements are with incremental natural numbers starting with 1 at the top right corner and following a diagonal pattern:

$$a = \begin{bmatrix} 1 & 2 & 4 & 7 & 11 & 16 & 22 & 29 \\ 3 & 5 & 8 & 12 & 17 & 23 & 30 & 38 \\ 6 & 9 & 13 & 18 & 24 & 31 & 39 & 48 \\ 10 & 14 & 19 & 25 & 32 & 40 & 49 & 59 \\ 15 & 20 & 26 & 33 & 41 & 50 & 60 & 71 \\ 21 & 27 & 34 & 42 & 51 & 61 & 72 & 84 \\ 28 & 35 & 43 & 52 & 62 & 73 & 85 & 98 \end{bmatrix}$$

What follows is the procedure to obtain a formula that gives the value for a_{ij} without having to find it iteratively.

Let b be the first row of a :

$$b = [1 \quad 2 \quad 4 \quad 7 \quad 11 \quad 16 \quad \dots]$$

Each number has a relationship with the previous one

$$\begin{aligned} b[0] &= 1 + 0 = 1 \\ b[1] &= 1 + 1 = 2 \\ b[2] &= 2 + 2 = 4 \\ b[3] &= 4 + 3 = 7 \\ b[4] &= 7 + 4 = 11 \\ b[5] &= 11 + 5 = 16 \\ &\dots \\ b[n] &= b[n-1] + n \end{aligned}$$

Extending $b[n-1]$ on each line shows a pattern:

$$\begin{aligned}
b[0] &= 1 \\
b[1] &= b[0] + 1 = 0(n-1) + 1 + n = 1n + 1 \\
b[2] &= b[1] + 2 = 1(n-1) + 1 + n = 2n + 0 \\
b[3] &= b[2] + 3 = 2(n-1) + 0 + n = 3n - 2 \\
b[4] &= b[3] + 4 = 3(n-1) - 2 + n = 4n - 5 \\
b[5] &= b[4] + 5 = 4(n-1) - 5 + n = 5n - 9 \\
b[6] &= b[5] + 6 = 5(n-1) - 9 + n = 6n - 14 \\
&\dots \\
b[n] &= n^2 - c[n]
\end{aligned} \tag{1}$$

where

$$c = [-1 \quad -1 \quad 0 \quad 2 \quad 5 \quad 9 \quad 14 \quad \dots]$$

$$\begin{aligned}
c[0] &= -1 \\
c[1] &= -1 = 0 - 1 \\
c[2] &= 0 = 1 - 1 \\
c[3] &= 2 = 2 + 0 \\
c[4] &= 5 = 3 + 2 \\
c[5] &= 9 = 4 + 5 \\
c[6] &= 14 = 5 + 9 \\
&\dots \\
c[n] &= (n-1) + c[n-1]
\end{aligned}$$

Let's develop one, say $c[6]$, to unveil the pattern:

$$\begin{aligned}
c[6] &= (6-1) + c[5] \\
&= (6-1) + (5-1) + c[4] \\
&= (6-1) + (5-1) + (4-1) + c[3] \\
&= (6-1) + (5-1) + (4-1) + (3-1) + c[2] \\
&= (6-1) + (5-1) + (4-1) + (3-1) + (2-1) + c[1] \\
&= (6-1) + (5-1) + (4-1) + (3-1) + (2-1) + (1-1) + c[0]
\end{aligned}$$

that looks like:

$$\begin{aligned}
c[n] &= \sum_{i=1}^n (i-1) + c[0] = \sum_{i=1}^n i - \sum_{i=1}^n 1 + c[0] \\
&= \frac{n(n+1)}{2} - n - 1 = \frac{n^2 + n - 2n - 2}{2} \\
&= \frac{n^2 - n - 2}{2}
\end{aligned} \tag{2}$$

And from (1) and (2) we have:

$$\begin{aligned}
b[n] &= n^2 - \frac{n^2 - n - 2}{2} = \frac{2n^2 - n^2 + n + 2}{2} \\
&= \frac{n^2 + n + 2}{2}
\end{aligned} \tag{3}$$

That gives us the formula to find out the elements of the first row. To apply it to the rest of the rows, in (3) $n = ij$ and we offset the elements with the row i :

$$a_{ij} = i + \frac{(ij)^2 + (ij) + 2}{2}$$