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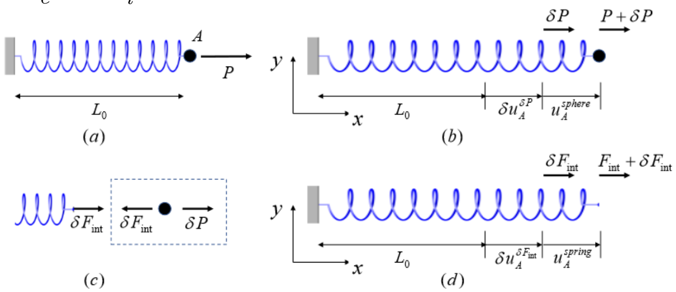
Virtual Force

$\delta W_e^* + \delta W_i^* = 0$



$\delta W_e^* = u_A^{sphere}\delta P \rightarrow \delta F_{int} = \delta P$

$\delta W_e^* = -u_A^{spring}\delta F_{int}$

$\left(u_A^{sphere} - u_A^{spring}\right)\delta P = 0$

This shows that the sum of the external and internal virtual work due to an external virtual force (or moment) vanishes for structure in static equilibrium, if the displacements and deformations are compatible.

Virtual Force Method for Computing Deflections

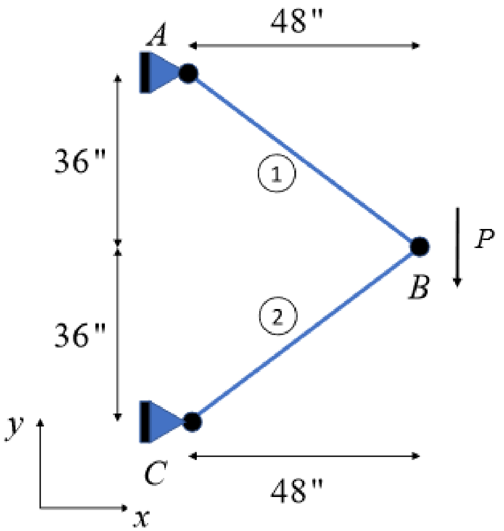
$\delta W_e^* = u\delta P$ or $\delta W_E^* = \theta\delta M$

Where u is the real displacement of the point at which the virtual force is applied. The internal virtual work can be written for a multi-component member as follows:

$\delta W_{ie}^* = \sum_{N_m} \delta F_{int}\Delta$

Replace δP with 1, $\bar{1}u = \sum_{N} \bar{f}_{int}\Delta$

Truss Example



$\delta W_{ie,bar}^* = \int_L \varepsilon\bar{\sigma}Adx \rightarrow \delta W_{ie,bar}^* = \int_L \frac{\sigma}{E}\bar{\sigma}Adx$

For the case that the real and virtual stresses are constant in the bar, the above expression can be expressed in terms of the real internal force, N, and the virtual force, $\bar{n}$, as follows:

$\delta W_{ie,bar}^* = \frac{N\bar{n}L}{EA}$

For a truss composed of multiple bars:

$\delta W_{ie,truss}^* = \sum_{i=1}^{N_b} \frac{N_i\bar{n}_iL_i}{E_iA_i}$

Using the balance of external and internal virtual work, i.e. $W_e^* = \delta W_{ie,truss}^*$, and applying a dummy load in a particular direction, the displacement, d, at the chosen joint in this direction can be computed by:

$\bar{1}d = \sum_{i=1}^{N_b} \frac{N_i\bar{n}_iL_i}{E_iA_i}$

Applying to example above:

Step 1: Compute internal forces, N_i, in the bars due to real load P. Truss must be statically determinate.

$N_1 = \frac{5}{6}P$ and $N_2 = -\frac{5}{6}P$

Step 2: Compute internal forces, $\bar{n}_i$, in the bars due to the dummy loads $\bar{1}$. First, we apply a dummy load in the horizontal direction to compute the horizontal displacement.

$\bar{n}_1^u = \frac{5}{8}$ and $\bar{n}_2^u = \frac{5}{8}$

Do the same for a vertical dummy load at joint B.

$\bar{n}_1^v = -\frac{5}{6}$ and $\bar{n}_2^v = \frac{5}{6}$

Step 3: To evaluate the internal work, summarize in a table:

bar	N_i	$\bar{n}_i^u$	$\bar{n}_i^v$	A_i	L_i	E_i
1	$\frac{5}{6}P$	$\frac{5}{8}$	$-\frac{5}{6}$	0.15	60.0	$3\cdot 10^6$
2	$-\frac{5}{6}P$	$\frac{5}{8}$	$\frac{5}{6}$	0.20	60.0	$3\cdot 10^6$

Step 4: For each dummy load, evaluate the balance of external and internal forces. For the displacement in horizontal:

$\bar{1}u = \sum_{i=1}^{2} \frac{N_i\bar{n}_i^uL_i}{E_iA_i} = 8.33\cdot 10^{-3}in$

Vertical:

$\bar{1}v = \sum_{i=1}^{2} \frac{N_i\bar{n}_i^vL_i}{E_iA_i} = -44.4\cdot 10^{-3}in$

Thermal Loading Assuming that the material properties are constant in the bar and expressing the virtual stress in terms of the internal force, $\bar{n}$, we obtain:

$\delta W_{ie,bar}^{*,thermal} = \alpha\Delta T\bar{n}L$

Defining an internal force due to differential heating/cooling as:

$N^{thermal} = EA\alpha\Delta T$

we can write the internal virtual work by replacing the internal force due to mechanical loading, N, with one for thermal loading, $N^{thermal}$.

$\delta W_{ie,bar}^* = \frac{N^{thermal}\bar{n}L}{EA}$

From here follow same procedure as previous.

Beam Example