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ASEN 3128
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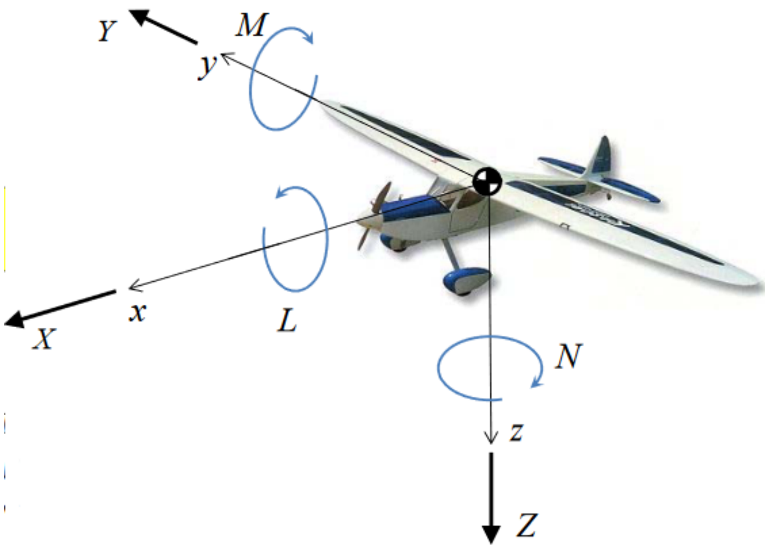
Nomenclature



$\mathbf{V}_B^E$ = velocity in intertial frame written in body coordinate system
$\mathbf{F}_B^{aero} = X\mathbf{e}_x + Y\mathbf{e}_y + Z\mathbf{e}_z = [X; Y; Z]$
$\mathbf{M}_B^{aero} = \mathbf{G}_B^{aero} = L\mathbf{e}_x + M\mathbf{e}_y + N\mathbf{e}_z = [L; M; N]$
$\mathbf{V}_B^E = u^E\mathbf{e}_x + v^E\mathbf{e}_y + w^E\mathbf{e}_z = [u^E; v^E; w^E]$
$V_g = |\mathbf{V}_B^E| = \mathbf{V}_E^E = \sqrt{(u^E)^2 + (v^E)^2 + (w^E)^2}$
$\omega_B^E = p^E\mathbf{e}_x + q^E\mathbf{e}_y + r^E\mathbf{e}_z = [p; q; r]$

Four Control Surfaces

Rudder: $+\delta_r$ = towards -y = negative moment & positive force
Elevator: $+\delta_e$ = down = negative moment & negative force
Aileron: $+\delta_a$ = right (+y) down = negative moment
Throttle: $+\delta_t$ = no moment, positive force.

Wind

Background Wind: $\mathbf{V}^E = \mathbf{V} + \mathbf{W}$ **Wind Angles**:
$V = |\mathbf{V}_B|$
$a = \arctan\frac{w}{u}$, $\beta = \arcsin\frac{v}{V}$
$u = V\cos\beta\cos\alpha$, $v = V\sin\beta$, $w = V\cos\beta\sin\alpha$
α = angle of attack, β = sideslip angle

Euler Angles

$R_E^B(\phi,\theta,\psi) = R_{v2}^B(\phi)R_{v1}^{v2}(\theta)R_E^{v1}(\psi)$
Body to Inertial Frame Transformation:
$\mathbf{p}_B = R_E^B\mathbf{p}_E \rightarrow \mathbf{p}_E = R_B^E\mathbf{p}_B$

$$R_B^E = (R_E^B)^T = \begin{pmatrix} c_\theta c_\psi & s_\phi s_\theta c_\psi - c_\phi s_\psi & c_\phi s_\theta c_\psi + s_\phi s_\psi \\ c_\theta c_\psi & s_\phi s_\theta c_\psi + c_\phi s_\psi & c_\phi s_\theta c_\psi - s_\phi s_\psi \\ -s_\theta & s_\phi c_\theta & c_\phi c_\theta \end{pmatrix}$$

Stability Frame: $\mathbf{p}_s = R_B^s\mathbf{p}_B$

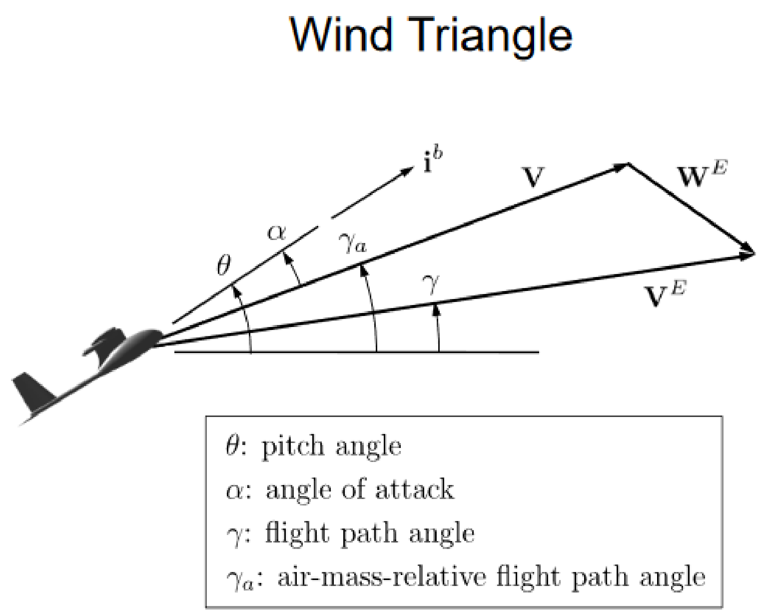
$$R_B^s(\alpha) = \begin{pmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{pmatrix}$$

Wind Frame: $\mathbf{p}_w = R_s^w(\alpha)\mathbf{p}_s$

$$R_B^w(\alpha,\beta) = R_s^w(\beta)R_B^s(\alpha) = \begin{pmatrix} \cos\beta\cos\alpha & \sin\beta & \cos\beta\sin\alpha \\ -\sin\beta\cos\alpha & \cos\beta & -\sin\beta\sin\alpha \\ -\sin\alpha & 0 & \cos\alpha \end{pmatrix}$$

$$R_w^B(\alpha,\beta) = (R_B^w)^T(\alpha,\beta)$$

Wind Triangle



$$\gamma_a = \theta - \alpha$$

Kinematics and Dynamics

Name	Description
x_E	Intertial x (North) position
y_E	Interial y (East) position
z_E	Inertial z (Down) position
ϕ	Roll Angle
θ	Pitch Angle
ψ	Yaw Angle
u^E	Inertial Velocity along $\hat{i}_B$
v^E	Inertial Velocity along $\hat{j}_B$
w^E	Inertial Velocity along $\hat{k}_B$
p	Angular Velocity along $\hat{i}_B$ (Roll rate)
q	Angular Velocity along $\hat{j}_B$ (Pitch rate)
r	Angular Velocity along $\hat{k}_B$ (Yaw rate)

Kinematics

$\frac{d\vec{r}_E}{dt} = \vec{V}_E^E = R_B^E \cdot \vec{V}_B^E$

$\omega_B = R(\phi)_{v2}^B R(\theta)_{v1}^{v2}(0;0;\dot{\psi}) + R(\phi)_{v2}^B(0;\dot{\theta};0) + (\dot{\phi};0;0)$ **Gravity Force in Inertial Frame**

$\mathbf{f}_E^g = (0;0;mg)$

Gravity Force in Body Frame

$\mathbf{f}_B^g = \mathbf{R}_E^B(0;0;mg) = \begin{pmatrix} -mg\sin\theta \\ mg\cos\theta\sin\phi \\ mg\cos\theta\cos\phi \end{pmatrix}$

Aerodynamic Force in Body Frame

$\mathbf{f}_B^a = (X;Y;Z)$

Aerodynamic Moment in Body Frame

$\mathbf{G}_B = \mathbf{G}_B^a = (L;M;N)$

Equations of Motion

$$\begin{pmatrix} \dot{x}_E \\ \dot{y}_E \\ \dot{z}_E \end{pmatrix} = \begin{pmatrix} c_\theta c_\psi & s_\phi s_\theta c_\psi - c_\phi s_\psi & c_\phi s_\theta c_\psi + s_\phi s_\psi \\ c_\theta c_\psi & s_\phi s_\theta c_\psi + c_\phi s_\psi & c_\phi s_\theta c_\psi - s_\phi s_\psi \\ -s_\theta & s_\phi c_\theta & c_\phi c_\theta \end{pmatrix} \begin{pmatrix} u^E \\ v^E \\ w^E \end{pmatrix}$$

$$\begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{pmatrix} \begin{pmatrix} p \\ q \\ r \end{pmatrix}$$

$$\begin{pmatrix} \dot{u}^E \\ \dot{v}^E \\ \dot{w}^E \end{pmatrix} = \begin{pmatrix} rv^E - qw^E \\ pw^E - ru^E \\ qu^E - pv^E \end{pmatrix} + g\begin{pmatrix} -\sin\theta \\ \cos\theta\sin\phi \\ \cos\theta\cos\phi \end{pmatrix} + \frac{1}{m}\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$\begin{pmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{pmatrix} = \begin{pmatrix} \Gamma_1 pq - \Gamma_2 qr \\ \Gamma_5 pr - \Gamma_6(p^2 - r^2) \\ \Gamma_7 pq - \Gamma_1 qr \end{pmatrix} + \begin{pmatrix} \Gamma_3 L + \Gamma_4 N \\ \frac{1}{I_y} M \\ \Gamma_4 L + \Gamma_8 N \end{pmatrix}$$

Quadrotor

Forces and moments = gravity, aerodynamics, and thrust (control)

$\mathbf{f} = \mathbf{f}^{grav} + \mathbf{f}^{aero} + \mathbf{f}^{cntl}$

$\mathbf{m} = \mathbf{m}^{aero} + \mathbf{m}^{cntl}$

$mathbff f_B^{cntl} = (0;0;Z_c)$ and $mathbfm_B^{cntl} = (L_c;M_c;N_c)$