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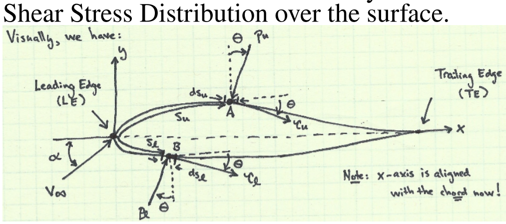
Concepts

Variables

Pressure - Normal force per unit area exerted on a surface due to the time rate of change of momentum of the gas particles impacing said surface.
$P = \lim_{dA \to 0} \frac{dF_n}{dA}$
Density - Mass per unit volume.
$\rho = \lim_{dV \to 0} \frac{dm}{dV}$
Temperature - Change in kinetic energy due to random molecular motion.
$KE = \frac{3}{2}kT$
Flow Velocity - Velocity due to organized motion.
$\vec{v} = \frac{d}{dt}\vec{x}$

Forces and Moments

Causes of forces and moments on an aerodynamic body:
Pressure Distribution over the body surface.
Shear Stress Distribution over the surface.



Lift and Drag per Unit Span:
$L' = N' \cos\alpha - A' \sin\alpha$
$D' = N' \sin\alpha + A' \cos\alpha$
Normal Force per Unit Span:
$N' = \int_{LE}^{TE} (P_u \cos\theta + \tau_u \sin\theta) ds_u + \int_{LE}^{TE} (P_l \cos\theta - \tau_l \sin theta) ds_l$
Axial Force per Unit Span:
$A' = \int_{LE}^{TE} (-P_u \sin\theta + \tau_u \cos\theta) ds_u + \int_{LE}^{TE} (P_l \sin\theta + \tau_l \cos\theta) ds_l$
Moment per Unit Span:
$M'_* = \int_{LE}^{TE} \left[(P_u \cos\theta + \tau_u \sin\theta)(x - x^*) - (P_u \sin\theta - \tau_u \cos\theta)(y - y*) \right] ds_u$
$+ \int_{LE}^{TE} \left[(-P_l \cos\theta + \tau_u \sin\theta)(x - x^*) + (P_l \sin\theta + \tau_l \cos\theta)(y - y*) \right] ds_l$
When $x^* = y^* = 0$ we obtain the moment about the leading edge.
Center of Pressure:
$x_{cp} = -\frac{M'_{LE}}{N'}$

Force and Moment Coefficients

Dynamic Pressure: $q = \frac{1}{2}\rho_\infty v_\infty^2$ **Reference Area**: S
Reference Length: L or c
Lift Coefficient: $c_L = \frac{L}{q_\infty S}$
Drag Coefficient: $c_D = \frac{D}{q_\infty S}$
Moment Coefficient: $c_M = \frac{M}{q_\infty S L}$

Buckingham Pi Theorem

If we have a physically meaningful equation such as:
$f(P_1, P_2, ..., P_N) = 0$
where P_i are physical variables in terms of K independent physical units (mass, length, time, temperature) then it can be restated as:
$F(\Pi_1, \Pi_2, ..., \Pi_{N-K}) = 0$ or
$\Pi_{N-K} = G(\Pi_1, \Pi_2, ..., \Pi_{N-K-1})$
where Π_i are dimensionaless variables known as Pi Groups.

Calculus Review

Vector Calculus

Magnitude: $|\vec{A}| = \sqrt{A_1^2 + A_2^2 + a_3^2}$
Dot Product: $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z = |\vec{A}||\vec{B}| \cos\theta$
Cross Product:$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\hat{i} + (A + z B_x - a_x B_z)\hat{j} + (A_x B_y - A_y B_x)\hat{k}$
$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin\theta$
Gradient of Scalar Field: $\vec{\nabla} p = \frac{\partial p}{\partial x}\hat{i} + \frac{\partial p}{\partial y}\hat{j} + \frac{\partial p}{\partial z}\hat{k}$ vector field
Directional Derivative: $\frac{dp}{ds} = \vec{\nabla} p \cdot \vec{s}$-> unit vector in $\vec{s}$ direction.
Divergence of a Vector Field: $\text{div}\vec{V} = \vec{\nabla} \cdot \vec{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$ scalar field
Curl of a Vector Field: $\text{curl}\vec{V} = \vec{\nabla} \times \vec{V} = (\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z})\hat{i} + (\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x})\hat{j} + (\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y})\hat{k}$

Integrals

Stokes' Theorem: $\oint_c \vec{A} \cdot \vec{ds} = \iint_S (\vec{\nabla} \times \vec{A}) \cdot \vec{dS}$
Divergence Theorem: $\oiint_S \vec{A} \cdot \vec{dS} = \iiint_{\mathscr{V}} (\vec{\nabla} \cdot \vec{A} d\mathscr{V})$
Gradient Theorem: $\oiint_S p\vec{dS} = \iiint_{\mathscr{V}} \vec{\nabla} p d\mathscr{V}$