

## Two-Body Problem

Newton's Law:  $\Sigma \vec{F} = \frac{d(m\vec{v})}{dt} = m\vec{a}$

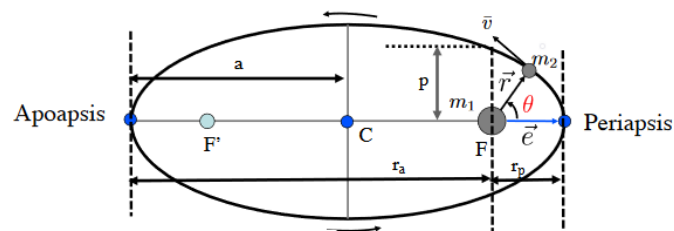
Universal Law of Gravitation:  $\vec{F}_g = -\frac{Gm_1m_2}{r^2} \frac{\vec{r}}{|\vec{r}|}$

Apply Newton's Laws to a two-body problem with the assumptions:

1. Only system force: Gravity  $\rightarrow$  acts along the line joining the centers of the bodies.
2. Mass of each body is constant.
3. Treat each body as a spherically symmetrical point mass with uniform density.

Orbital Properties:

- a = semimajor axis
- b = semiminor axis
- p = semiperimeter
- $r_a/r_p$  = radii of apoapsis/periapsis
- $\vec{e}$  = eccentricity



Useful Equations:

$$a = \frac{1}{2}(r_a + r_p)$$

$$p = \frac{b^2}{a} = a(1 - e^2) = \frac{h^2}{\mu}$$

$$r_a = \frac{p}{1 - e} = a(1 + e)$$

$$r_p = \frac{p}{1 + e} = a(1 - e)$$

$$e = \frac{r_a - r_p}{r_a + r_p} = \frac{c}{a} = \frac{\sqrt{a^2 - b^2}}{a} = \sqrt{1 + \frac{2h^2\varepsilon}{\mu^2}}$$

$$b = a\sqrt{1 - e^2}$$

$$\text{Angular Momentum: } \vec{h} = \vec{r} \times \vec{v} = \sqrt{\mu a(1 - e^2)}$$

$$\text{Eccentricity Vector: } \vec{e} = \frac{\vec{v} \times \vec{h}}{\mu} - \frac{\vec{r}}{r}$$

$$\text{Specific Energy: } \varepsilon = \frac{v^2}{2} - \frac{\mu}{r} = \frac{\mu^2(e^2 - 1)}{2h^2}$$

- $\varepsilon < 0$  Motion of Body 2 is bounded wrt Body 1
- $\varepsilon \geq 0$  Motion of Body 2 is unbounded wrt Body 1

$$\text{Conic Equation: } r = \frac{h^2/\mu}{1 + e \cos \theta} = \frac{p}{1 + e \cos \theta}$$

$$\text{Vis-Viva Equation: } v = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}}$$

True Anomaly: