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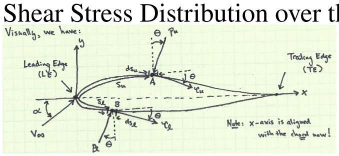
Concepts

Variables

Pressure - Normal force per unit area exerted on a surface due to the time rate of change of momentum of the gas particles impacing said surface.
$P = \lim_{dA\to 0} \frac{dF_n}{dA}$
Density - Mass per unit volume.
$\rho = \lim_{dV\to 0} \frac{dm}{dV}$
Temperature - Change in kinetic energy due to random molecular motion.
$KE = \frac{3}{2}kT$
Flow Velocity - Velocity due to organized motion.
$\vec{v} = \frac{d}{dt}\vec{x}$

Forces and Moments

Causes of forces and moments on an aerodynamic body:
Pressure Distribution over the body surface.
Shear Stress Distribution over the surface.



Lift and Drag per Unit Span:
$L' = N'\cos\alpha - A'\sin\alpha$
$D' = N'\sin\alpha + A'\cos\alpha$
Normal Force per Unit Span:
$N' = \int_{LE}^{TE}(P_u\cos\theta + \tau_u\sin\theta)ds_u + \int_{LE}^{TE}(P_l\cos\theta - \tau_l\sin theta)ds_l$
Axial Force per Unit Span:
$A' = \int_{LE}^{TE}(-P_u\sin\theta + \tau_u\cos\theta)ds_u + \int_{LE}^{TE}(P_l\sin\theta + \tau_l\cos\theta)ds_l$
Moment per Unit Span:
$M'_* = \int_{LE}^{TE}\left[(P_u\cos\theta + \tau_u\sin\theta)(x-x^*) - (P_u\sin\theta - \tau_u\cos\theta)(y-y*)\right]ds_u$
$+\int_{LE}^{TE}\left[(-P_l\cos\theta + \tau_u\sin\theta)(x-x^*) + (P_l\sin\theta + \tau_l\cos\theta)(y-y*)\right]ds_l$
When $x^* = y^* = 0$ we obtain the moment about the leading edge.
Center of Pressure:
$x_{cp} = -\frac{M'_{LE}}{N'}$

Force and Moment Coefficients

Dynamic Pressure: $q = \frac{1}{2}\rho_\infty v_\infty^2$ **Reference Area**: S
Reference Length: L or c
Lift Coefficient: $c_L = \frac{L}{q_\infty S}$
Drag Coefficient: $c_D = \frac{D}{q_\infty S}$
Moment Coefficient: $c_M = \frac{M}{q_\infty SL}$

Buckingham Pi Theorem

If we have a physically meaningful equation such as:
$f(P_1, P_2, ..., P_N) = 0$
where P_i are physical variables in terms of K independent physical units (mass, length, time, temperature) then it can be restated as:
$F(\Pi_1, \Pi_2, ..., \Pi_{N-K}) = 0$ or
$\Pi_{N-K} = G(\Pi_1, \Pi_2, ..., \Pi_{N-K-1})$
where Π_i are dimensionless variables known as Pi Groups.

Calculus Review

Vector Calculus

Magnitude: $|\vec{A}| = \sqrt{A_1^2 + A_2^2 + a_3^2}$
Dot Product: $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z = |\vec{A}||\vec{B}|\cos\theta$
Cross Product: $\vec{A}\times\vec{B} = (A_yB_z - A_zB_y)\hat{i} + (A+zB_x - a_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$
$|\vec{A}\times\vec{B}| = |\vec{A}||\vec{B}|\sin\theta$
Gradient of Scalar Field: $\nabla p = \frac{\partial p}{\partial x}\hat{i} + \frac{\partial p}{\partial y}\hat{j} + \frac{\partial p}{\partial z}\hat{k}$ vector field
Directional Derivative: $\frac{dp}{ds} = \vec{\nabla}p\cdot\vec{s}$-> unit vector in $\vec{s}$ direction.
Divergence of a Vector Field: $\text{div}\vec{V} = \vec{\nabla}\cdot\vec{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$ scalar field
Curl of a Vector Field: $\text{curl}\vec{V} = \vec{\nabla}\times\vec{V} = (\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z})\hat{i} + (\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x})\hat{j} + (\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y})\hat{k}$

Integrals

Stokes' Theorem: $\oint_c \vec{A}\cdot d\vec{s} = \iint_S(\vec{\nabla}\times\vec{A})\cdot d\vec{S}$
Divergence Theorem: $\oiint_S \vec{A}\cdot d\vec{S} = \iiint_{\mathcal{V}}(\vec{\nabla}\cdot\vec{A}d\mathcal{V})$
Gradient Theorem: $\oiint_S pd\vec{S} = \iiint_{\mathcal{V}}\vec{\nabla}pd\mathcal{V}$

Basic Flow Equations

Fixed control volumes for basic flow equations.

Conservation of Mass

Net mass flow out of $\mathcal{V}$ = Time rate of decrease of mass inside $\mathcal{V}$.
Integral Continuity Equation:
$\frac{\partial}{\partial t}\iiint_{\mathcal{V}}\rho d\mathcal{V} + \oiint_S \rho\vec{V}\cdot d\vec{S} = 0$
Differential Continuity Equation:
$\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{V}) = 0$

Conservation of Momentum

Net force applied to $\mathcal{V}$ = Time rate of change of momentum of fluid in $\mathcal{V}$
$\frac{\partial}{\partial t}\iiint_{\mathcal{V}}\rho\vec{V}d\mathcal{V} + \oiint_S(\rho\vec{V}\cdot d\vec{S})\vec{V} = -\oiint_S pd\vec{S} + \iiint_{\mathcal{V}}\rho\vec{f}d\mathcal{V} + \vec{F}_{viscous}$

1. Body Forces (e.g. gravity) which act on the fluid inside $\mathcal{V}$
   $\iiint_{\mathcal{V}}\rho\vec{f}d\mathcal{V}$
2. Pressure and shear surface forces acting on S
   $-\oiint_S pd\vec{S} + \vec{F}_{viscous}$
3. Net flow of momentume out of volume across S
   $\oiint_S(\rho\vec{V}\cdot d\vec{S})\vec{V}$
4. Time rate of change of momentum due to unsteadiness in volume
   $\frac{\partial}{\partial t}\iiint_{\mathcal{V}}\rho\vec{V}d\mathcal{V}$

Navier-Stokes Equations:
$\frac{\partial}{\partial t}(\rho u) + \vec{\nabla}\cdot(\rho u\vec{V}) = -\frac{\partial p}{\partial x} + \rho f_x + (\mathcal{F}_x)_{viscous}$
$\frac{\partial}{\partial t}(\rho v) + \vec{\nabla}\cdot(\rho v\vec{V}) = -\frac{\partial p}{\partial y} + \rho f_y + (\mathcal{F}_y)_{viscous}$

Conservation of Energy

$$\underbrace{\frac{\partial}{\partial t}\iiint_{\mathcal{V}}\rho\left(e+\frac{V^2}{2}\right)d\mathcal{V}}_{\text{Net Time Rate of Change of Total Energy Due to Unsteadiness in }\mathcal{V}} + \underbrace{\oiint_S \rho\left(e+\frac{V^2}{2}\right)\vec{V}\cdot d\vec{S}}_{\text{Net Rate of Flow of Total Energy Out of }\mathcal{V}} =$$
$$\underbrace{\iiint_{\mathcal{V}}\dot{q}\rho d\mathcal{V}}_{\text{Rate of Volumetric Heating}} + \underbrace{\dot{Q}_{viscous}}_{\text{Heat Addition Due to Viscosity}}$$
$$\underbrace{-\oiint_S p\vec{V}\cdot d\vec{S}}_{\text{Rate of Pressure Work}} + \underbrace{\iiint_{\mathcal{V}}\rho\vec{f}\cdot\vec{V}d\mathcal{V}}_{\text{Rate of Work Done by Body Forces}}$$
$$+\underbrace{\dot{W}_{viscous}}_{\text{Rate of Viscous Work}}$$

$$\frac{\partial}{\partial t}\left(\rho\left(e+\frac{V^2}{2}\right)\right) + \vec{\nabla}\cdot\left(\rho\left(e+\frac{V^2}{2}\right)\right) = \rho\dot{q} + \dot{Q}'_{viscous} - \vec{\nabla}\cdot(p\vec{V}) + \rho\vec{f}\cdot\vec{V} + \dot{W}'_{viscous}$$

Kinematics

$\vec{\nabla}\cdot\vec{V}$: time rate of change of volume of a fluid element per unit volume $\rightarrow$ **Dilation**
Stream Line: Curve everywhere tangent to the velocity.
$\frac{v}{u} = \frac{dy}{dx}$

Path Line: Trajectory of a particle released from a point in time.
Streak Line: Line connecting all particles that have passed through a given point.
For a steady flow the three lines coincide.

Substantial Derivative

Time rate of change of density of a given material fluid element as it moves through space and time.
$\frac{D\rho}{Dt} = \frac{\partial\rho}{\partial t} + \vec{V} \cdot \vec{\nabla}\rho$
Continuity Equation: $\frac{D\rho}{Dt} = -\rho\vec{\nabla} \cdot \vec{V}$
Material Acceleration: $(\frac{Du}{Dt}; \frac{Dv}{Dt}; \frac{Dz}{Dt})$
Navier-Stokes:
$\rho\frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \rho f_x + (\mathcal{F}_x)_{viscous}$ $\rightarrow$ applicable to y and z as well.

Vorticity and Strain

$\vec{\omega} = \frac{1}{2}\vec{\nabla} \times \vec{V}$ $\rightarrow$ angular velocity
$\vec{\xi} = \vec{\nabla} \times \vec{V}$ $\rightarrow$ Vorticity
$\epsilon_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$ $\rightarrow$ Rate of Strain in xy plane.
$\Gamma = -\oint_C \vec{V} \cdot \vec{ds}$ $\rightarrow$ Circulation
Relation between circulation and vorticity: $\vec{xi} \cdot \vec{n} = \frac{d\Gamma}{dS}$

Stream Functions and Velocity Potentials

Stream Function: For every incompressible, two-dimensional flow, there exists a scalar stream function such that: $u = \frac{\partial\psi}{\partial y}$, $v = -\frac{\partial\psi}{\partial x}$ DON'T FORGET +C
Velocity Potential: For every irrotational flow, there exists a scalar velocity potential such that: $\vec{V} = \vec{\nabla}\phi/rightarrow\frac{\partial\phi}{\partial x} = y, \frac{\partial\phi}{\partial y} = x$
Euler's Equation: $\frac{1}{2}d(V^2) = -\frac{1}{\rho}dp$
If flow is barotropic, $\rho = \rho(p)$ is a function of pressue only and can integrate the equation. Occurs in incompressible flow, isothermal flow, or isentropic (frictionless) flow.
Bernoulli's Equation: $p + \frac{1}{2}\rho V^2 =$ constant - along a streamline for steady, inviscid, incompressible flow with no body forces.

Flow in a Duct

$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$ $\rightarrow$ Steady, inviscid flow in a duct with no body forces. For incompressible flow remove rho.