

2nd Law

The Second Law of Thermodynamics: Processes occur in a certain direction and energy has quality as well as quantity.

Thermal Energy Reservoirs: A hypothetical body with a relatively large thermal energy capacity (mass \times specific heat) that can supply or absorb finite amounts of heat without undergoing any change in temperature.

Heat Engines

- Devices that convert heat to work
- Receive heat from high-temp source
- Convert part of this heat to work
- Reject remaining waste heat to low-temperature sink - Kelvin-Planck Statement
- Operate on a cycle
- MUST waste some energy by transferring to low-temperature reservoir in order to complete cycle

Notation:

- $Q_{in} = Q_H$ = amount of heat supplied from a high-temp source
- $Q_{out} = Q_L$ = amount of heat rejected to a low temperature sink
- W_{out} = amount of work delivered out of system by working fluid
- W_{in} = amount of work input to system

$$W_{net,out} = W_{out} - W_{in} \text{ (kJ)}$$

$$W_{net,out} = Q_{in} - Q_{out} \text{ (kJ)}$$

Thermal Efficiency

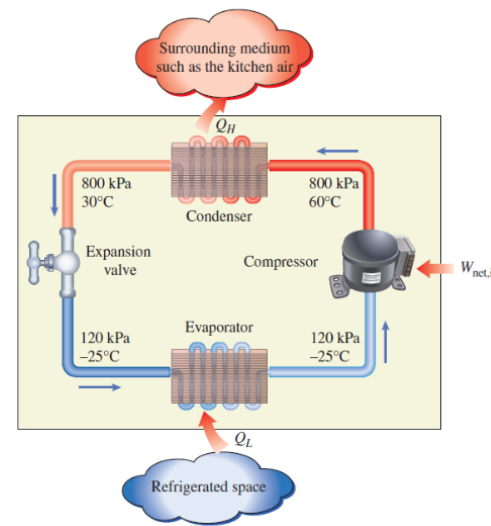
$$\eta = \frac{W_{net,out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{Q_L}{Q_H}$$

Refrigerators and Heat Pumps

Coefficient of Performance: efficiency of a refrigerator.

$$COP_R = \frac{\text{Desired Output}}{\text{Required Input}} = \frac{Q_L}{W_{net,in}} = \frac{Q_L}{Q_H - Q_L} = \frac{1}{Q_H/Q_L - 1}$$

Clausius Statement: It is impossible to construct a device that operates in a cycle and produces no effect other than the transfer of heat from a lower-temperature body to a higher-temperature body.



Reversible and Irreversible Processes

Reversible Process: A process that can be reversed without leaving any trace on the surroundings - theoretical to find limits.

Irreversible Process: A process that is not reversible.

Irreversibilities:

- Friction
- Unrestrained expansion
- Mixing of two fluids
- Heat transfer across a finite temperature difference
- Electric resistance
- Inelastic deformation of solids
- Chemical reactions

The Carnot Cycle

The Carnot Cycle is composed of four reversible processes - two isothermal and two adiabatic - and it can be executed either in a closed or steady-flow system.

- Reversible Isothermal Expansion - process 1-2, $T_H = \text{constant}$
- Reversible Adiabatic Expansion - process 2-3, $T_H \rightarrow T_L$
- Reversible Isothermal Compression - process 3-4, $T_L = \text{constant}$
- Reversible Adiabatic Compression - process 4-1, $T_L \rightarrow T_H$

The Carnot Cycle is completely reversible - in which case it becomes the Carnot refrigeration cycle.

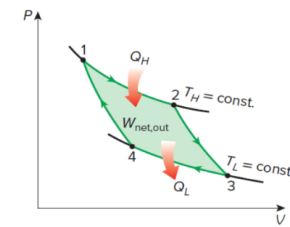


FIGURE 7-35
 P-V diagram of the Carnot cycle.

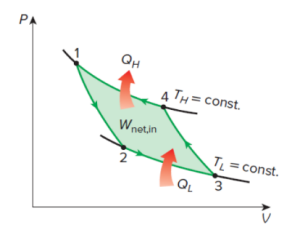


FIGURE 7-36
 P-V diagram of the reversed Carnot cycle.

The Carnot Principles: The efficiency of an irreversible heat engine is always less than the efficiency of a reversible one operating between the same two reservoirs and the efficiencies of all reversible heat engines operating between the same two reservoirs are the same.

The Thermodynamic Temperature Scale: A temperature scale that is independent of the properties of the substances that are used to measure temperature.

$$T_H = T_L \frac{Q_H}{Q_L}$$

The Carnot Heat Engine

Any heat engine: $\eta_{th} = 1 - \frac{Q_L}{Q_H}$

Carnot heat engine: $\eta_{th,rev} = 1 - \frac{T_L}{T_H}$

$$\eta_{th} \begin{cases} < \eta_{th,rev} & \text{irreversible heat engine} \\ = \eta_{th,rev} & \text{reversible heat engine} \\ > \eta_{th,rev} & \text{impossible heat engine} \end{cases}$$

Amount of heat rejected per cycle: $Q_{L,rev} = \frac{T_L}{T_H} Q_{H,rev}$

Quality of Energy: The higher the temperature of the thermal energy, the higher its quality. Directly relates to face that you can use temperature to measure efficiency in $\eta_{th,rev}$.

The Carnot Refrigerator and Heat Pump

Any refrigerator or heat pump:

$$COP_R = \frac{1}{Q_H/Q_L - 1} \text{ and } COP_{HP} = \frac{1}{1 - Q_L/Q_H}$$

Carnot refrigerator or heat pump:

$$COP_{R,rev} = \frac{1}{T_H/T_L - 1} \text{ and } COP_{HP,rev} = \frac{1}{1 - T_L/T_H}$$

$$COP_R \begin{cases} < COP_{R,rev} & \text{irreversible refrigerator} \\ = COP_{R,rev} & \text{reversible refrigerator} \\ > COP_{R,rev} & \text{impossible refrigerator} \end{cases}$$

Entropy

- Processes can only occur in a certain direction and that direction must comply with the increase of entropy principle: $S_{\text{gen}} \geq 0$.
- Entropy is a nonconserved property, and there is no such thing as the conservation of entropy principle. Entropy is conserved during the idealized reversible processes only and increases during all actual processes.
- Entropy generation is the measure of the magnitudes of the irreversibilities during the performance of engineering systems. It is also used to establish criteria for the performance of engineering devices.

Clausius Inequality: $\oint \frac{\delta Q}{T} \leq 0$

$$\delta W_C = \delta Q_R - dE_C$$

Reversible cyclic device:

$$\frac{\delta Q_R}{T_R} = \frac{\delta Q}{T} \rightarrow \delta W_C = T_R \frac{\delta Q}{T} - dE_C$$

For a system undergoing a cycle: $W_C = T_R \oint \frac{\delta Q}{T}$

$$\oint \left(\frac{\delta Q}{T} \right)_{\text{int,rev}} = 0 \text{ and } dS = \left(\frac{\delta Q}{T} \right)_{\text{int,rev}} \text{ (kJ/K)}$$

Entropy: $\Delta S = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T} \right)_{\text{int,rev}}$

Special Case:

Internally Reversible Isothermal Heat Transfer Process $\Delta S = \frac{Q}{T_0}$ (kJ/K) where T_0 is the constant temperature of the system and Q is the heat transfer for the internally reversible process.

Generated Entropy:

$$S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{sys}} + \Delta S_{\text{surr}} \geq 0 \rightarrow \Delta S_{\text{sys}} = S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + S_{\text{gen}}$$

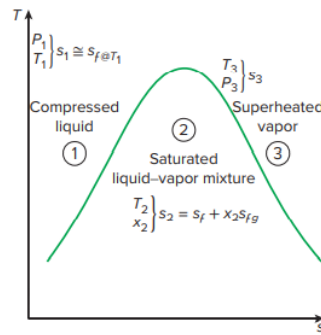
The Increase of Entropy Principle:

$$S_{\text{gen}} \begin{cases} < 0 \text{ impossible process} \\ = 0 \text{ reversible process} \\ > 0 \text{ irreversible process} \end{cases}$$

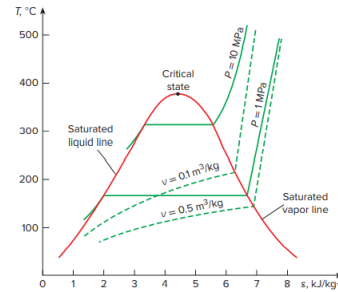
Entropy Change of Pure Substances

Once the state of the system is fixed the value of entropy is also fixed.

$$\Delta S = m \Delta s = m(s_2 - s_1) \text{ (kJ/K)}$$



T-s Diagram for water ↓



Isentropic Processes

$$\Delta S = 0 \text{ or } S_2 = S_1 \text{ (kJ/kgK)}$$

Property Diagrams involving Entropy

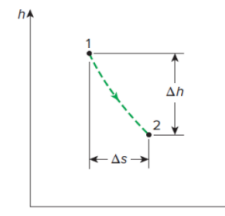
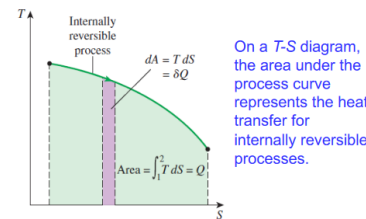


FIGURE 8-18
For adiabatic steady-flow devices, the vertical distance Δh on an h - s diagram is a measure of work, and the horizontal distance Δs is a measure of irreversibilities.

Mollier diagram: The h - s diagram

$$\delta Q_{\text{int rev}} = T ds \quad Q_{\text{int rev}} = \int_1^2 T ds$$

$$\delta q_{\text{int rev}} = T ds \quad q_{\text{int rev}} = \int_1^2 T ds$$

$$Q_{\text{int rev}} = T_0 \Delta S \quad q_{\text{int rev}} = T_0 \Delta s$$

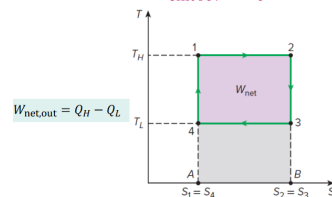


FIGURE 8-19 The T-s diagram of a Carnot cycle

What is Entropy?

Boltzmann Relation: $S = k \ln W$

$$k = 1.3806 \times 10^{-23} \text{ J/K}$$

Gibb's Formulation: $S = -k \sum P_i \log P_i$

Pure Crystal: $T = 0 \text{ K}$ then Entropy = 0

The entropy of a pure crystalline substance at absolute zero temperature is zero (Third Law of Thermodynamics)

The Tds Relations

The First Tds or Gibbs Equation:

$$\delta Q_{\text{int,rev}} - \delta W_{\text{int,rev,out}} = dU$$

$$\delta Q_{\text{int,rev}} = TdS \text{ and } \delta W_{\text{int,rev,out}} = PdV$$

$$TdS = dU + PdV \text{ (kJ)}$$

$$TdS = du + Pdv$$

The Second Tds Equation:

$$h = u + Pv$$

$$dh = du + Pdv + vdP \text{ and } Tds = du + Pdv$$

$$TdS = dh - vdP$$

Finally:

$$ds = \frac{du}{T} + \frac{Pdv}{T} = \frac{dh}{T} - \frac{vdP}{T}$$

Entropy Change of Liquids and Solids

Liquids and solids can be approximated as incompressible substances $\rightarrow dv = 0$.

$$ds = \frac{du}{T} = \frac{cdT}{T} \rightarrow c_P = c_v = c \text{ and } du = cdT$$

$$S_2 - S_1 = \int_1^2 d(T) \frac{dT}{T} = c_{\text{avg}} \ln \frac{T_2}{T_1} \text{ (kJ/kgK)}$$

Isentropic: $s_2 - s_1 = c_{\text{avg}} \ln \frac{T_2}{T_1} = 0 \rightarrow T_2 = T_1$

Entropy Change of Ideal Gases

$$Pv = RT$$

$$du = c_v dT$$

$$dh = c_p dT$$

$$ds = c_v \frac{dT}{T} + R \frac{dv}{v} \rightarrow s_2 - s_1 = \int_1^2 c_v(T) \frac{dT}{T} + R \ln \frac{v_2}{v_1}$$

$$s_2 - s_1 = \int_1^2 c_p(T) \frac{dT}{T - R \ln \frac{P_2}{P_1}}$$

Constant Specific Heats (Approximate Analysis)

$$s_2 - s_1 = c_{v, \text{avg}} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

$$s_2 - s_1 = c_{p, \text{avg}} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

Variable Specific Heats (Exact Analysis)

$$s_2 - s_1 = \int_1^2 c_v(T) \frac{dT}{T} + R \ln \frac{v_2}{v_1}$$

Choose absolute zero as reference temperature and define a function s° as:

$$s^\circ = \int_0^T c_p(T) \frac{dT}{T}$$

$$s_2^\circ - s_1^\circ = \int_1^2 c_p(T) \frac{dT}{T}$$

On a unit mass basis:

$$s_2 - s_1 = s_2^\circ - s_1^\circ - R \ln \frac{P_2}{P_1} \quad (\text{kJ/kgK})$$

Isentropic Processes of Ideal Gases

Constant Specific Heats

$$\left(\frac{T_2}{T_1} \right)_{s=\text{const}} = \left(\frac{V_1}{V_2} \right)^{k-1}$$

$$\left(\frac{T_2}{T_1} \right)_{s=\text{const}} = \left(\frac{P_2}{P_1} \right)^{(k-1)/k}$$

$$\left(\frac{V_1}{V_2} \right)^{k-1} = \left(\frac{P_2}{P_1} \right)^{(k-1)/k}$$

Variable Specific Heats

Relative Pressure, $P_r = \exp(s^\circ/R)$

Relative Specific Volume, T/P_r

$$\left(\frac{P_2}{P_1} \right)_{s=\text{const}} = \frac{P_{r2}}{P_{r1}}$$

$$\left(\frac{v_2}{v_1} \right)_{s=\text{const}} = \frac{v_{r2}}{v_{r1}}$$

Reverse Steady-Flow Work

$$w_{\text{rev}} = - \int_1^2 v dP - \Delta ke - \Delta pe \rightarrow w_{\text{rev}} = -v(P_2 - P_1) - \Delta ke - \Delta pe$$

When KE and PE are negligible: $w_{\text{rev}} = - \int_1^2 v dP$

For steady flow of a liquid through a device that involves no work, like a pipe, the work term is zero:

$$v(P_2 - P_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) = 0$$

Isentropic Efficiencies of Steady-Flow Devices

Turbine: $\eta_T = \frac{w_a}{w_s}$ where w_a is actual turbine work and w_s is isentropic turbine work.

$$\eta_T = \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

Compressors and Pumps:

$\eta_C = \frac{w_s}{w_a} = \frac{h_{2s} - h_1}{h_{2a} - h_1}$ when kinetic and potential energies are negligible.

$$\eta_P = \frac{w_s}{w_a} = \frac{v(P_2 - P_1)}{h_{2a} - h_1}$$

Nozzles: $\eta_N = \frac{V_{2a}^2}{V_{2s}^2} = \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$ where V is velocity and

$$h_1 = h_{2a} + \frac{V_{2a}^2}{2}$$

Entropy Balance

Total entropy entering - total entropy leaving + total entropy generated = change in total entropy of the system.

$$S_{\text{in}} - S_{\text{out}} + S_{\text{gen}} = \Delta S_{\text{system}}$$

$$\Delta S_{\text{system}} = S_{\text{final}} - S_{\text{initial}} = S_2 - S_1$$

Refer to Lecture 7 Page 21 for more information.

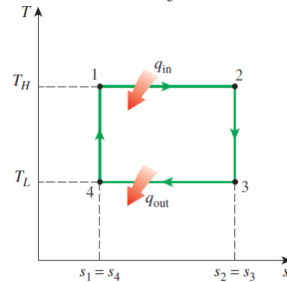
Power Cycles

Ideal Cycle: A cycle that resembles the actual cycle closely but is made up of totally internally reversible processes. These cycles are not necessarily externally reversible.

Carnot Cycle

The Carnot Cycle is composed of 4 totally reversible processes: isothermal heat addition, isentropic expansion, isothermal heat rejection, and isentropic compression.

For both ideal and actual cycles the thermal efficiency increases with an increase in the average temperature at which the heat is supplied to the system or with a decrease in the average temperature at which the heat is rejected from the system.



Derivation of the Efficiency of the Carnot Cycle

$$q_{\text{in}} = T_H(s_2 - s_1) \quad q_{\text{out}} = T_L(s_3 - s_4) = T_L(s_2 - s_1) \quad s_2 = s_3 \text{ and } s_4 = s_1$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}}$$

Air-Standard Assumptions

- The working fluid is air which continuously circulates in a closed loop and always behaves as an ideal gas.
- All the processes that make up the cycle are internally reversible.
- The combustion process is replaced by a heat-addition process from an external source.

- The exhaust process is replaced by a heat-rejection process that restores the working fluid to its initial state.

Cold Air Standard Assumptions: When the working fluid is considered to be air with constant specific heats at room temperature (25°C)

Air Standard Cycle: A cycle for which the air-standard assumptions are applicable.

Reciprocating Engines

$$\text{Compression Ratio: } r = \frac{V_{\text{max}}}{V_{\text{min}}} = \frac{V_{\text{BDC}}}{V_{\text{TDC}}}$$

$$W_{\text{net}} = \text{MEP} \times \text{Piston Area} \times \text{Stroke} = \text{MEP} \times \text{Displacement Volume}$$

Mean Effective Pressure (MEP):

$$\text{MEP} = \frac{W_{\text{net}}}{V_{\text{max}} - V_{\text{min}}} = \frac{w_{\text{net}}}{v_{\text{max}} - v_{\text{min}}}$$

The engine with a larger MEP delivers more work per cycle and thus performs better.

The Otto Cycle

The ideal cycle for spark-ignition engines.

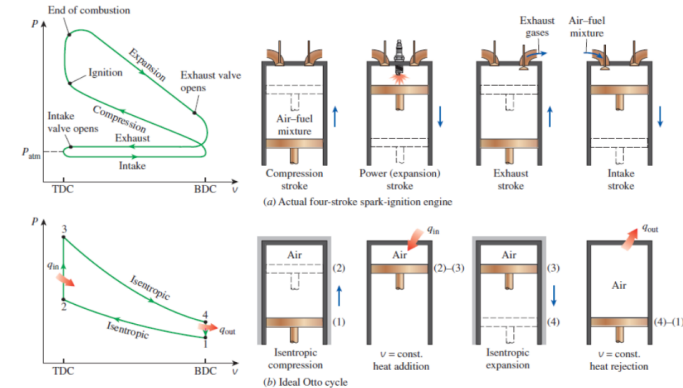


FIGURE 9-12

Actual and ideal cycles in spark-ignition engines and their P-v diagrams.

Four-stroke cycle
1 cycle=4 stroke=2 revolution
Two-stroke cycle
1 cycle=2 stroke=1 revolution

- 1-2 Isentropic compression
- 2-3 Constant-volume heat addition
- 3-4 Isentropic expansion
- 4-1 Constant-volume heat rejection

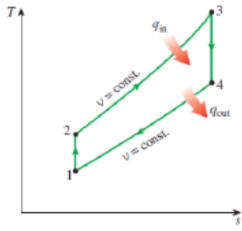
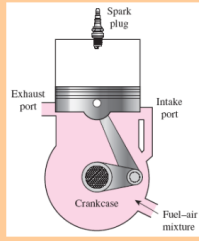


FIGURE 9-15
T-s diagram of the ideal Otto

The two-stroke engines are generally less efficient than their four-stroke counterparts but they are relatively simple and inexpensive, and they have high power-to-weight and power-to-volume ratios.



Schematic of a two-stroke reciprocating engine.

For air $k=1.4$ and the thermal efficiency of the Otto Cycle increases with the specific heat ratio, k , of the working fluid.

$$(q_{in} - q_{out}) + (w_{in} - w_{out}) = u_{exit} - u_{inlet}$$

$$q_{in} = u_3 - u_2 = c_v(T_3 - T_2)$$

$$q_{out} = u_4 - u_1 = c_v(T_4 - T_1)$$

$$\eta_{th,Otto} = \frac{w_{net}}{q_{in} - q_{out}} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)}$$

$$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{k-1} = \left(\frac{v_3}{v_4}\right)^{k-1} = \frac{T_4}{T_3}$$

$$r = \frac{V_{max}}{V_{min}} = \frac{V_1}{V_2} = \frac{v_1}{v_2}$$

Cold-Air Standard Assumption: $\eta_{th,Otto} = 1 - \frac{1}{r^{k-1}}$

Diesel Cycle

The ideal cycle for the compression-ignition engines.

In diesel engines, only air is compressed during the compression stroke, eliminating the possibility of autoignition (engine knock). Therefore, diesel engines can be designed to operate at much higher compression ratios than SI engines, typically between 12 and 24.

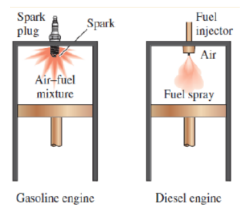


FIGURE 9-20 In diesel engines, the spark plug is replaced by a fuel injector, and only air is compressed during the compression process.

- 1-2 isentropic compression
- 2-3 constant-pressure heat addition
- 3-4 isentropic expansion
- 4-1 constant-volume heat rejection.

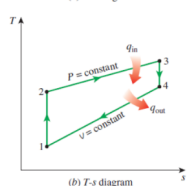
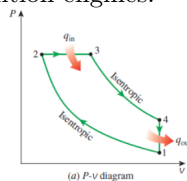


FIGURE 9-21 T-s and P-v diagrams for the ideal Diesel cycle.

$$q_{in} - w_{b,out} = u_3 - u_2 \rightarrow q_{in} = P_2(v_3 - v_2) + (u_3 - u_2) = h_3 - h_2 = c_p(T_3 - T_2)$$

$$-q_{out} = u_1 - u_4 \rightarrow q_{out} = u_4 - u_1 = c_v(T_4 - T_1)$$

$$\eta_{th,Diesel} = \frac{W_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{T_4 - T_1}{k(T_3 - T_2)} = 1 -$$

$$\frac{T_1(T_4/T_1 - 1)}{kT_2(T_3/T_2 - 1)}$$

$$r_c = \frac{V_3}{V_2} = \frac{v_3}{v_2}$$

$\eta_{th,Diesel} = 1 - \frac{1}{r^{k-1}} \left[\frac{r_c^k - 1}{k(r_c - 1)} \right]$ The thermal efficiency of the Diesel Cycle decreases as the cutoff ratio increases and increases as the compression ratio increases.

Brayton Cycle

The ideal cycle for gas-turbine engines.

The combustion process is replaced by a constant-pressure heat-addition process from an external source, and the exhaust process is replaced by a constant-pressure heat-rejection process to the ambient air.

- 1-2 Isentropic compression (in a compressor)
- 2-3 Constant-pressure heat addition
- 3-4 Isentropic expansion (in a turbine)
- 4-1 Constant-pressure heat rejection

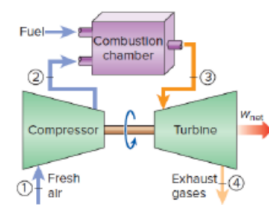


FIGURE 9-25 An open-cycle gas-turbine engine.

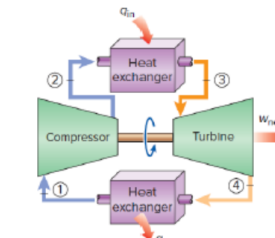
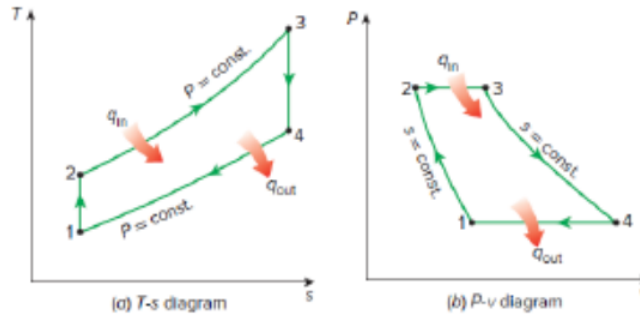


FIGURE 9-26 A closed-cycle gas-turbine engine.



The thermal efficiency of the ideal Brayton Cycle increases as the pressure ratio increases.

$$(q_{in} - q_{out}) + (w_{in} - w_{out}) = h_{exit} - h_{inlet}$$

$$q_{in} = h_3 - h_2 = c_p(T_3 - T_2)$$

$$q_{out} = h_4 - h_1 = c_p(T_4 - T_1)$$

$$\eta_{th,Brayton} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{c_p(T_4 - T_1)}{c_p(T_3 - T_2)} = 1 - \frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} = \left(\frac{P_3}{P_4}\right)^{(k-1)/k} = \frac{T_3}{T_4}$$

$$r_p = \frac{P_2}{P_1} \text{ Pressure Ratio}$$

Only for ideal cycle and cold-air standard assumptions:

$$\eta_{th,Brayton} = 1 - \frac{1}{r_p^{(k-1)/k}}$$

Development of Gas Turbines

- Increasing the turbine inlet (or firing) temperatures
- Increasing the efficiencies of turbomachinery components (turbines, compressors)
- Adding modification to the basic cycle (intercooling, regeneration or recuperation, and reheating)

Deviation of Actual Gas-Turbine Cycles from Idealized Ones:

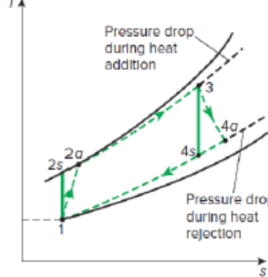


FIGURE 9-32

The deviation of an actual gas-turbine cycle from the ideal Brayton cycle as a result of irreversibilities.

Irreversibilities in turbine and compressors, pressure drops, heat losses.

Isentropic efficiencies of the compressor and turbine:

$$\eta_C = \frac{w_s}{w_a} = \frac{h_{2s} - h_1}{h_{2a} - h_1} \text{ and } \eta_T = \frac{w_a}{w_s} = \frac{h_3 - h_{4a}}{h_3 - h_{4s}}$$

Brayton Cycle with Regeneration

In gas-turbine engines, the temperature of the exhaust gas leaving the turbine is often considerably higher than the temperature of the air leaving the compressor.

Therefore, the high-pressure air leaving the compressor can be heated by the hot exhaust gases in a counter-flow heat exchanger (a *regenerator* or a *recuperator*).

The thermal efficiency of the Brayton cycle increases as a result of regeneration since less fuel is used for the same work output.

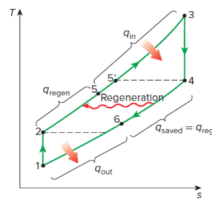
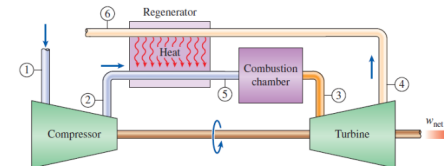


FIGURE 9-35
T-s diagram of a Brayton cycle with regeneration.



A gas-turbine engine with regenerator.

General Knowledge

Isentropic: Constant entropy

Isothermal: Constant temperature

Adiabatic: No Q

Adiabatic and Reversible: Isentropic