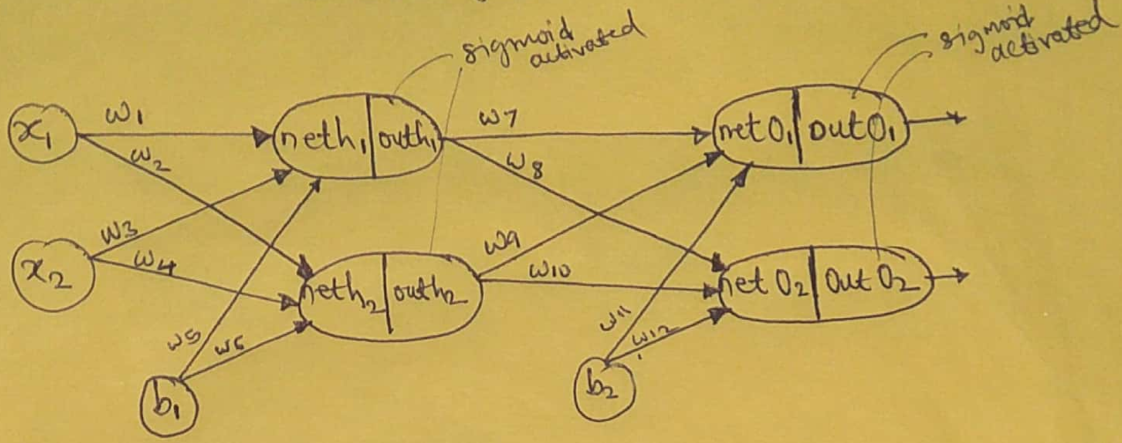
Backpropagation



Target Values

$T_1 = 0.1$

$T_2 = 0.99$

Error (Loss) (MSE)

$$E_{total} = \frac{1}{2} \sum_{i=1}^{2} (T_i - OutO_i)^2$$

$$E_{total} = \frac{1}{2}(T_1 - OutO_1)^2 + \frac{1}{2}(T_2 - OutO_2)^2$$

$$\frac{\partial E_{total}}{\partial w_7} = \frac{\partial E_{total}}{\partial outO_1} * \frac{\partial outO_1}{\partial netO_1} * \frac{\partial netO_1}{\partial w_7} \quad — ①$$

$$OutO_1 = \frac{1}{1 - \exp\{-netO_1\}}$$

$$\boxed{\begin{aligned} f(x) &= \frac{1}{1-e^{-x}} \\ f'(x) &= f(x)(1-f(x)) \end{aligned}}$$

$$\boxed{\frac{\partial outO_1}{\partial netO_1} = (outO_1)(1 - OutO_1)} \quad — ②$$

$b_2 = 1$

$$netO_1 = w7 * outh_1 + w9 * outh_2 + 1 * w_{11}$$

$$\boxed{\frac{\partial netO_1}{\partial w_7} = outh_1} \quad — ③$$

$$\boxed{\frac{\partial E_{total}}{\partial outO_1} = \frac{-1}{2} \cdot 2\,(T_1 - OutO_1)^{2-1} = -(T_1 - OutO_1)} \quad — ④$$

Substituting 2, 3, 4 in ①

$$\frac{\partial E_{total}}{\partial w_7} = -(T_1 - OutO_1) * [(OutO_1)(1 - OutO_1)] * outh_1$$

... which can be solved easily.

Update in w_7 :

$$w_7 = w_7 - \alpha * \frac{\partial E_{total}}{\partial w_7} \qquad (\alpha = \text{learning rate})$$

Similarly we find updates for $w_8, w_9, w_{10}, w_{11}, w_{12}$

$$\frac{\partial E_{total}}{\partial w_{11}} = \underbrace{\frac{\partial E_{total}}{\partial out O_1} * \frac{\partial out O_1}{\partial net O_1}}_{\text{known from previous calculation}} * \frac{\partial net O_1}{\partial w_{11}}$$

$$\frac{\partial net O_1}{\partial w_{11}} = 1$$

$$\therefore \quad w_{11} = w_{11} - \alpha * \frac{\partial E_{total}}{\partial w_{11}}$$

$\|^{ly}$ for w_{12}

Updating weights $w_1, w_2, w_3, w_4, w_5, w_6$.

$$\frac{\partial E_{total}}{\partial w_1} = \frac{\partial E_{total}}{\partial out h_1} * \frac{\partial out h_1}{\partial net h_1} * \frac{\partial net h_1}{\partial w_1} \qquad \text{— (5)}$$

$$\frac{\partial E_{total}}{\partial out h_1} = \frac{\partial E_{01}}{\partial out h_1} + \frac{\partial E_{02}}{\partial out h_1}$$

$$E_{01} = \frac{1}{2}(T_1 - out O1)^2$$

$$out O1 = \frac{1}{1+e^{-net O1}}$$

$$net O1 = w_7 * out h_1 + w_9 * out h_2 + w_{11} * b_2$$

$$\frac{\partial net O1}{\partial out h_1} = w_7$$

$$\frac{\partial E_{01}}{\partial out h_1} = \underbrace{\frac{\partial E_{01}}{\partial out O1}}_{-(T_1 - out O_1)} * \underbrace{\frac{\partial out O_1}{\partial net O_1}}_{\checkmark \text{ from } (2)} * \underbrace{\frac{\partial net O1}{\partial out h_1}}_{w_7}$$

Similarly, $$\frac{\partial E_{02}}{\partial out h_1} = \frac{\partial E_{02}}{\partial out O_2} * \frac{\partial out O_2}{\partial net O_2} * \frac{\partial net O_2}{\partial out h_1}$$

$$\frac{\partial E_{02}}{\partial out h_1}$$

$$\frac{\partial EO2}{\partial outO_2} = -(T_2 - outO_2)$$

$$EO_2 = \frac{1}{2}(T_2 - outO_2)^2$$

$$outO_2 = \frac{1}{1+e^{-netO_2}}$$

$$\frac{\partial outO_2}{\partial netO_2} = (outO_2)(1 - outO_2)$$

$$netO_2 = w8 * outh_1 + w10 * outh_2 + w12 * b_2$$

$$\frac{\partial netO_2}{\partial outh_1} = w8$$

Thus, we get $\frac{\partial E_{total}}{\partial outh_1}$

Now, $outh_1 = \frac{1}{1+e^{-neth_1}}$

$$\frac{\partial outh_1}{\partial neth_1} = (outh_1)(1 - outh_1)$$

$$neth_1 = w_1 * x_1 + w_3 * x_2 + w_5 * b_1$$

$$\frac{\partial neth_1}{\partial w_1} = x_1$$

Thus we get $\frac{\partial E_{total}}{\partial w_1}$.

$$\therefore w_1 = w_1 - \alpha * \frac{\partial E_{total}}{\partial w_1}$$

Similarly, we find w_2, w_3, w_4, w_5, w_6.