Department of Physics, Indian Institute of Technology, Kanpur

PHY-224A: OPTICS

Lecture -5: Reflection and refraction of light

Date: 17 Aug. 2016, Time :1700 hours Instructor: S.A. Ramakrishna

In the previous class, we derived the complex Fresnel coefficients for the amplitudes of the S- and P-polarized light as

$$t_s = \frac{E_t}{E_i} = \frac{2k_{iz}/\mu_1}{k_{iz}/\mu_1 + k_{tz}/\mu_2}, \tag{1}$$
$$r_s = \frac{E_r}{E_i} = \frac{k_{iz}/\mu_1 - k_{tz}/\mu_2}{k_{iz}/\mu_1 + k_{tz}/\mu_2}, \tag{2}$$
$$t_p = \frac{E_t}{E_i} = \frac{Z_2}{Z_1}\frac{2k_{iz}/\varepsilon_1}{k_{iz}/\varepsilon_1 + k_{tz}/\varepsilon_2}, \tag{3}$$
$$r_s = \frac{E_r}{E_i} = \frac{k_{iz}/\varepsilon_1 - k_{tz}/\varepsilon_2}{k_{iz}/\varepsilon_1 + k_{tz}/\varepsilon_2}. \tag{4}$$

From these, we can calculate the reflected and transmitted intensities. The light intensity is given by

$$|\vec{S}| = \frac{1}{2}\sqrt{\frac{\varepsilon_0\varepsilon}{\mu_0}}E_0^2 = \frac{1}{2}c\sqrt{\varepsilon}\varepsilon_0 E_0^2. \tag{5}$$

Hence the amount of energy incident per unit time unit area of the interface (flux of light) is given by the component of the Poynting vector normal to the interface:

$$J_i = \vec{S}_i \cdot \hat{z} = S_i \cos\theta_i = \frac{1}{2}c\sqrt{\varepsilon_1}\varepsilon_0 E_i^2 \cos\theta_i. \tag{6}$$

Similarly, the energies of the reflected and the transmitted waves leaving a unit area of the interface per unit time is are

$$J_r = S_r \cos\theta_r = \frac{1}{2}c\sqrt{\varepsilon_1}\varepsilon_0 E_r^2 \cos\theta_r, \tag{7}$$
$$J_t = S_t \cos\theta_t = \frac{1}{2}c\sqrt{\varepsilon_2}\varepsilon_0 E_t^2 \cos\theta_t. \tag{8}$$

The ratios of the energies

$$R_s = \frac{J_r}{J_i} = \frac{E_r^2}{E_i^2} = r_s^2, \tag{9}$$
$$T_s = \frac{J_t}{J_i} = \frac{n_2\cos\theta_t E_t^2}{n_1\cos\theta_i E_i^2} = \frac{n_2\cos\theta_t}{n_1\cos\theta_i}t_s^2, \tag{10}$$

are the reflectivity and transmittivity of the interface for the S-polarized light. Analogous expressions can be written for the P-polarized light. The student should verify at this point that $R + T = 1$ for each of the the S- and P- polarizations (note what are the conditions for this) - this is required by energy conservation and the formulae are consistent with it.

We can re-write the expressions that we derived in the previous lecture in terms of the angles of incidence, reflection and transmission when we want to explicitly understand the behaviour of these quantities in terms of angles. Further when we consider primarily optical frequencies, there is almost no magnetic activity and $\mu \simeq 1$ for the two media, and $n = sqrt\varepsilon$. Then the expressions for the Fresnel coefficients look like:

$$t_s = \frac{2n_1 \cos\theta_i}{n_1 \cos\theta_i + n_2 \cos\theta_t} = \frac{2 \sin\theta_t \cos\theta_i}{\sin(\theta_i + \theta_t)} \tag{11}$$

$$r_s = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} = \frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}, \tag{12}$$

$$t_p = \frac{2n_1 \cos\theta_i}{n_2 \cos\theta_i + n_1 \cos\theta_t} = \frac{2 \sin\theta_t \cos\theta_i}{\sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t)}, \tag{13}$$

$$r_p = \frac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_2 \cos\theta_i + n_1 \cos\theta_t} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}. \tag{14}$$

Note the following observations and the consequences of these coefficients:

- These expressions are more convenient to use if one has real refractive indices (n) and angles lesser than the critical angle for total internal reflection (real angles).

- Then all the Fresnel coefficients are real and the phases are either 0 or π – this is the additional phase due to the scattering from the interface.

- t_p and t_s have the same phase as the incident wave

- r_s has the same sign or phase over all the angles (whether you have an overall phase of zero or π will depend on the direction that you choose for the electric fi

  eld of the reflected wave during the derivation.

- **Brewster's angle**: r_p goes through a zero with increasing angle and changes sign across that point. The angle at which $r_p = 0$ is called the *Brewster's angle.* This point is given essentially by the divergence of the denominator $\tan(\theta_i + \theta_t) \to \infty$ or $(\theta_i + \theta_t) = \pi/2$. This yields for the Brewster's angle, $\sin\theta_t = \sin(\pi - \theta_i) = \cos\theta_i$. Using the Ibn Sahl's law for refraction, we also obtain $\sin\theta_t = n_1 \sin\theta_i / n_2$. Putting them together, we obtain

  $$\tan\theta_i = \frac{n_2}{n_1}, \tag{15}$$

  for the Brewster's angle.

  Note that it is not possible for r_s to have such a zero.

The physical reason for the zero in r_p can be envisaged as follows. The incident wave polarizes the molecules of the medium and sets up a polarization in the medium. These oscillating dipoles in turn radiate and give rise to the reflected and transmitted radiation. At the Brewster angle, note that the dipoles point along the direction of the reflected wave. As a dipole radiates nothing along its axis, there cannot be any radiation along the direction of the reflected wave. As a consequence, if you have unpolarized light containing light of both S- and P- polarizations incident as with sunlight, for example, you will see polarized light (s-polarized) when you detect the reflection at the Brewster's angle an interface. This can be used, for example, to determine the axis of a polarizer as was done by you in the laboratory. This effect was used by the noble laurette C.V. Raman to good effect in proving that light scattered by the sea water is blue. Lord Rayleigh had declared that the blue colour of the see was just due to the reflection of the blue sky. Using this Brewster's effect, Prof. Raman could exclude the reflection of the sky and measure only the light coming from the ocean. The interested student is asked to understand how exactly could it be done.

- Note that r_p and r_s are unequal in magnitude in general except for $\theta = 0,\ \pi$ or for angles greater than the critical angle.

- **Critical angle for total internal reflection**: If the wave is incident from a medium of larger index,$k_1 > k_2$ in magnitude. Then there is a critical incident angle θ_c, at which the transmitted wave is along the interface $k_{2z} = 0$. This happens when $k_x = n_1\omega/c \sin\theta_c = n_2\omega/c$. Thus we obtain the critical angle to be

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_2}\right). \tag{16}$$

For larger angles of incidence, k_{2z} In other words, the becomes purely imaginary. In other words, the transmitted wave, $\exp[i(k_x x + k_{2z}z - \omega t)] = \exp[i(k_x x - \omega t)]\exp(-k_{2z}z)$, essentially is exponentially decaying in nature normal to the interface and propagating along the interface. Such waves are called evanescent waves. They carry no energy away from the interface (Prove in the assignment that the normal component of the Poynting vector is zero). Hence all the incident intensity goes into the reflected intensity (reflectance magnitude should be unity). To understand this, note that for $k_x > n_2\omega/c$, the z-component of the wave vector becomes an imaginary quantity, $k_{tz} = i\sqrt{k_x^2 - n_1^2\omega^2/c^2} = i\kappa_{tz}$. The Fresnel coefficients take the form

$$r_p = \frac{k_{iz}/\varepsilon_1 - i\kappa_{tz}/\varepsilon_2}{k_{iz}/\varepsilon_1 + i\kappa_{tz}/\varepsilon_2} = \frac{\xi_p}{\xi_p^*} = e^{i\phi_p}, \tag{17}$$

$$r_s = \frac{\xi_s}{\xi_s^*} = e^{i\phi_p}, \tag{18}$$

which are unimodular. But note that the phase shifts upon reflectiona re different for the S- and P-polarized light.