Indian Institute of Technology, Kanpur
Department of Physics

PHY-224A: OPTICS
Lecture -14: Interference of Light

Date: 22 Sept. 2014, **Time :1700 hours** **Instructor: S.A. Ramakrishna**

1 Interference of two optical fields

The basic rule of electromangetic fields is the fields add and the corresponding intensities do not. The intensity at a point corresponding to a propagating electromagnetic wave is given by

$$I = \varepsilon v_{\text{ph}} \langle E^2 \rangle = c\sqrt{\frac{\varepsilon}{\mu}} \langle E^2 \rangle$$

where the $\langle \quad \rangle$ indicates time averaging over several time periods for the electric field of the radiation. For a vectorial wave (which em waves are), and using a phasor notation where the time-harmonic fields that we talk of are given by real parts of the complex phasors $\vec{E}(\vec{r}, t) = \text{Re}[\vec{E}(\vec{r}) \exp(-i\omega t)]$, we have then

$$\langle \vec{E}^2 \rangle = \langle \frac{1}{4} [\vec{E}^2 e^{-2i\omega t} + \vec{E}^{*2} e^{2i\omega t} + 2\vec{E} \cdot \vec{E}^*] \rangle.$$

Hence, we have

$$\langle \vec{E}^2 \rangle = \frac{1}{2} \vec{E} \cdot \vec{E}^* = \frac{1}{2} [E_x^2 + E_y^2 + E_z^2].$$

Now suppose two monochromatic waves with the same frequency, $\vec{E}_1$ and $\vec{E}_2$, are superposed at the same point in space, P, then

$$\vec{E} = \vec{E}_1 + \vec{E}_2, \tag{1}$$

$$\vec{E}^2 = \vec{E}_1^2 + \vec{E}_2^2 + 2\vec{E}_1 \cdot \vec{E}_2.$$

The intensity is given by

$$\begin{aligned} I \sim \langle \vec{E}^2 \rangle &= \langle \vec{E}_1^2 \rangle + \langle \vec{E}_2^2 \rangle + 2\langle \vec{E}_1 \cdot \vec{E}_2 \rangle \\ &= I_1 + I_2 + J_{12}, \end{aligned} \tag{2}$$

where

$$J_{12} \sim 2\langle \vec{E}_1 \cdot \vec{E}_2 \rangle = \frac{1}{2}(\vec{E}_1 \cdot \vec{E}_2^* + \vec{E}_1^* \cdot \vec{E}_2) \quad = (E_{x1}E_{x2}\cos\delta_x + E_{y1}E_{y2}\cos\delta_y + E_{z1}E_{z2}\cos\delta_z),$$

where $\delta_{x,y,z}$ are the phase difference between the respective components of the two waves (Example, $\delta_x = \phi_{x2} - \phi_{x1}$). We will assume that this phase difference is maintained over the timescales relevant for our measurements.

Let us analyse the simplest cases. If $\delta_x = \delta_y = \delta_z = \delta$ (say), and for transverse waves polarized along the x-axis (say), we have $J_{12} \sim E_{x1}E_{x2}\cos\delta$, $I_1 \sim 1/2\ E_{x1}^2$ and $I_2 \sim 1/2 E_{x2}^2$. Hence,

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta \tag{3}$$

The intensities due to the two waves do not add. There are interference terms which make the intensity at P very different from the sum of the intensities. The maximum possible intensity is $I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2$ for $\delta = 2m\pi$ and the minimum possible intensity is $I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2$ for $\delta = 2(m+1)\pi$, where m is an integer. Interference is a property unique to waves.

2 Interference of two beams: Wavefront division

We will discuss now the classic Youngs double slit interference. Consider a wave-front of a plane wave that is incident on two slits. We will assume perfect coherence of the wave over the wave front here. Coherence here refers to the idea that a definite relationship of the phase between any two points on the wavefront exists, and does not change over the timescales relevant for the experiment. Now consider a perfectly opaque screen containing two long slits placed apart by a distance of $2d$. At this stage, it will suffice for us to declare that the size of the slits are very small and that they almost act as point sources – the size of the slits are assumed small with respect to the wavelength. Now let us examine what intensity of light will be measured at a point P, on a screen that is placed beyond the two slits. The plane of the screen is taken to be the x-y plane.

Now the path lengths between the slits and the point, P, can be written as

$$s_1^2 = (x-d)^2 + y^2 + z_0^2, \qquad s_2^2 = (x+d)^2 + y^2 + z_0^2,$$

and

$$s_2^2 - s_1^2 = 2x(2d) \Rightarrow s_2 - s_1 = \frac{2x\ (2d)}{s_2 + s_1} \simeq \frac{2x\ (2d)}{2z_0},$$

where it is assumed that $z_0 \gg 2d$: i.e, the screen is sufficient far from the slits as well as the distances on the screen (x, y) are small compared to z_0. Thus, we obtain the difference of the two paths as

$$\Delta s = \frac{x\ (2d)}{z_0} \tag{4}$$

and the corresponding optical phase difference is

$$\Delta\phi = \frac{nkx\ (2d)}{z_0} = \frac{2\pi n x\ (2d)}{\lambda z_0} \tag{5}$$

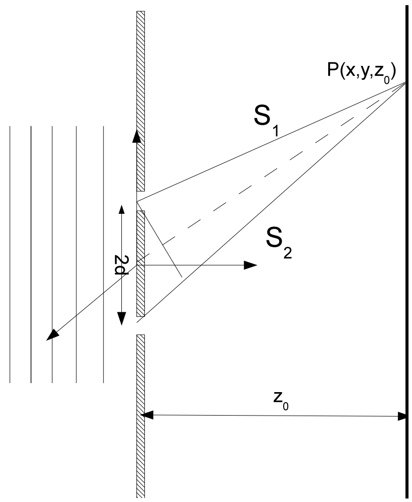

Figure 1: A schematic diagram of the Youngs's double slit experiment and the associated path lengths.

At the point P, the field will be a maximum or minimum depending on the position x. It will be a maximum if

$$\frac{2\pi nx\ (2d)}{\lambda z_0} = 0, 2\pi,\ 4\pi, \cdots$$

$$\text{or} \qquad x = m\frac{\lambda z_0}{n\ (2d)} \tag{6}$$

Similarly, it will be a minimum if

$$x = (m + 1/2)\frac{\lambda z_0}{n\ (2d)}$$

These intensity maxima and minima are called interference fringes. Note that there will be no dependence of the intensity on the y-coordinates. We call this interference by wave-front division because we take light from two different places on a wavefront and interfere them.

Now we will assume the two slits are of identical nature, transmit the same amounts of the incident fields with the same phase - effectively they both act as point sources with the same phase. The assumption of a plane wave incident is useful for this purpose. In that case, the electric field contributions due to slit-1 and slit-2 at the point P on the screen beyond the slits will be of equal amplitude and will only have a phase difference that depends on the path difference. We will assume that the electric fields involved are polarized only along the x-direction – justified due to the consideration $z_0 \gg 2d$. In that case, the intensity maxima will be 4 times the intensity due to one slit and the intensity at the minima will be zero.

A serious point is that interference does not violate any conservation law, such as for energy. Interference only redistributes the energy in the optical fields and does nothing more. One can prove this (problem -1 of assignment) by integrating the energy available in the interference pattern over the entire screen and checking if this is the sum of the energies contributed individually by the two slits.

A second important point concerns the phase of the wave emitted by the two slits, which we assumed to be equal in the above arguments. What if one of the slit (say, slit-1) transmits the wave with an extra phase shift of ϕ_1 compared to the other. You are invited to prove that it will merely lead to a spatial shift of all the fringes along the x-axis (problem-2 of assignment) . This same effect would be seen if a plane wavewere to be incident on the two slits at a small oblique angle θ with respect to the normal of the screen with the slits (problem -3 for assignment). I do not suggest a very large angle as that would complicate the scattering from the two slits which would confuse you at the moment.

A third question that arises is that what would happen if the sources were point sources instead of slits? How would the interference pattern then appear – would it continue to be straight lines as in the earlier case? Note the y-invariance. Try to work out the pattern by similar arguments as presented here (problem 4 of the assignment).

In much of the discussions from now onwards, we will assume that the interfering electromagnetic waves have a simple spatial structure and the same polarization. Then we can work with scalars for the fields which considerably simplifies the notation.

Read about alternative ways of accomplishing two-beam interferences by wavefront division such as a Fresnel's mirror, Lloyd's mirror and the fresnel's bi-prism (problem -5 of the assignment).