Indian Institute of Technology, Kanpur
Department of Physics

PHY-224A: OPTICS

Lecture -16: Interference of light - 3

Date: 22 Sept. 2014, **Time :1700 hours** **Instructor: S.A. Ramakrishna**

1 Interference of two beams - continued

1.1 Interference fringes with thin films

A thin film of a transparent material can give rise to non-localised fringes as shown above. However, when the point P literally lies on the film itself, a special case manifests. Note that the fields at the point P may be imaged by a lens onto another point P' where they can be measured. Under such conditions, note that for all source points of a source of reasonable extension, the thickness t is the same for all the rays. If the spread in angles being considered is small, then the spread in the phase differences due to the various different paths can be substantially lesser than 2π. Then one may image fringes at P' that are *localised at P*. The maximum of intensity occurs at P', if

$$2nt\langle\cos\phi\rangle \pm \lambda/2 = m\lambda \qquad \text{where} \quad m = 0, 1, 2, \cdots, \tag{1}$$

where $\langle\cos\phi\rangle$ represents the average of the cosine of ϕ that are possible with various points on the source. Hence the fringes in this case are the loci of points in the film at which the optical thickness is constant and they are called *fringes of equal thickness*. One usually sees these fringes in the form of multicolour from a thin oil film that coats a wet road on rainy days, from a soap bubble and a variety of other situations. If a film is of constant thickness, then it will be uniformly intense.

This is used to test the figure of a an optical surface by placing the surface on a plane reference surface. The non-uniformities on the test surface will cause air films to be formed in-between which will give rise to fringes in reflection. If a wedge is formed in between these surfaces, at normal incidence of the light, the maxima will occur when $2t + \lambda/2 = m\lambda$ which implies that the thicknesses difference corresponding to successive bright fringes will be $t_{m+1} - t_m = \lambda/2$. If a wedge angle of $\alpha = t_m/x_m$ is present, the lateral distance between two fringes will be $\Delta x = \lambda/(2\alpha)$. Similarly, if one considers a wedge formed of a glass slab of refractive index, then the lateral distance between two bright fringes will be $\Delta x = \lambda/(2n\alpha)$. These are also called *wedge fringes*.

The case of the famous *Newtons rings* obtained by placing a convex lens on a glass plate. You are required to work out the radii of the fringes of equal thickness in this case and their dependence on the order (problem for the assignment). If the lens and the plate are separated out slowly, the fringes move inwards and collapse at the centre (Explain why?) with one fringe collapsing for each $\lambda/2$ increase of the separation. Hence this can be sensitively used to measure the wavelength of light.

1.2 The Michelson Interferometer

The Michelson interferometer is a two-beam interference device that works by amplitude division. Consider the Fig. 1 showing the configuration of a Michelson interferometer consisting of a 50-50 beam-splitter (BS) and two mirrors on the two arms of the interferometer, typically at 90^o. One of the two mirrors is fixed (M_1) and the other is movable along the arm (M_2). First, consider a finely collimated beam (as from a laser) that is incident on the beamsplitter. The two beams separated by the beam splitter are superposed on a screen on the other side of the mirrors. At the screen, the two wavefronts interfere and give rise to an interference pattern. The two beams suffer from different phase shifts depending on the optical pathlengths in the two arms. Depending on the difference of these pathlengths and corresponding phase shifts, we have the conditions of constructive or destructive interference on the screen. If the distance from the BS to M_2 were L_2 and the distance from the BS to M_1 were L_1, then constructive interference happens when

$$2k\Delta L = 2\frac{2\pi}{\lambda}(L_2 - L_1) = 2\pi m, \qquad \Rightarrow \Delta L = m\frac{\lambda}{2}$$

For change in the position of M_1 by a quarter wavelength, one drives the system from constructive to destructive interference. A photodetector placed along the beam in that arm would detect the maxima and minima as a function of the position of mirror M_2.

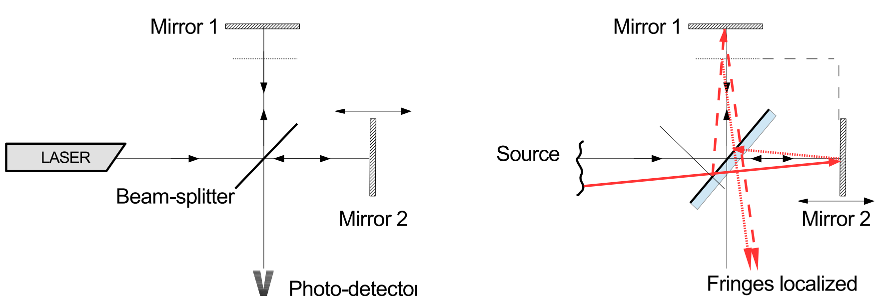

Figure 1: Left: A schematic diagram of the Michelson interferometer and the paths associated with a fine collimated beam. Right: Schematic diagram shows how the oblique rays from an extended / diverging light source form fringes localised at infinity. The projected location of the mirror M2 along the path of Mirror M1 allows better understanding of the system.

Traditionally, one considers a diverging light source as shown in the right panel of Fig. 1. This generally creates a circular fringe pattern of dark and bright rings, and how does one understand the fringe pattern. One way to understand the interference is to consider the image of one mirror (M_2') projected into the arm of the fixed one (M_1). When the path lengths are equal, thenM_1 and the image of M_2 superpose (and if perfectly aligned) and there would be perfect constructive interference from the two reflections as the path difference is zero for any angle of incidence. This leads to a broad field of brightness. As M_1 moves away outwards, a path difference develops between the reflections from M! and M2 and both the reflected rays are parallel to each other. This results in a circular fringe (due to symmetry about the incident direction) localised at infinity.

Actually, a small misalignment of the two mirrors would be present and at approximately zero pathlength difference, instead of a broad field, one would see straight fringes corresponding to the wedge fringes that result from equal distances between the mirror M_1 and the projected mirror M_2'. Also note the multiple (three) passes through the beamsplitter substrate that light through one arm (L_2) in this case makes compared to one pass in the other arm. This creates the requirement of a compensating transparent plate of equal thickness in the L_1 arm.

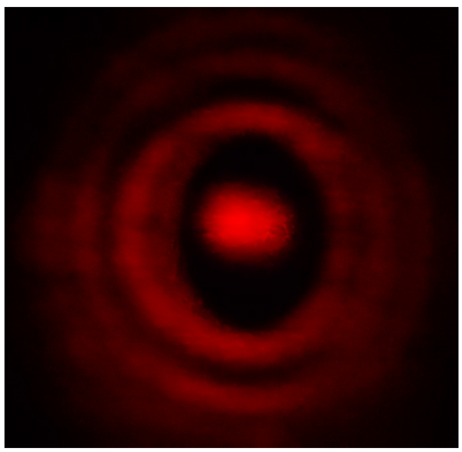

Figure 2: The circular fringes that become visible with a Michelson interferometer. The wavelength used for the photograph is 632.8 nm

The Michelson interferometer is a very useful instrument where the fringed shift can be used to resolve very small motions and vibrations. associated with mirrors. While even 1/20th of a fringe shift can be easily resolved, special techniques have been developed for detecting even several hundredths of a fringed shift. This corresponds to resolution of path length differences of lesser than $\lambda/100$, which are of the order of few nanometers with visible light. This is used in finely controlling the motion of very fine piezostages over long distances in modern equipment, for example, in electron beam lithography.