Department of Physics, Indian Institute of Technology, Kanpur

PHY-224A: OPTICS

Lecture -4: Reflection and refraction of light

Date: 09 Aug. 2016, **Time :1700 hours** **Instructor: S.A. Ramakrishna**

The reflection and refraction of light has been understand by man for a long time. Since school, you have learnt that light reflects off the surface of water or glass and so on. You have learnt that light bends or changes its direction as it moves from one medium to another. The directions of the incident light and the refracted light are dictated by a so-called *Snel's law*:

$$\theta_r = \theta_i, \qquad n_2 \sin\theta_t = n_1 \sin\theta_i, \tag{1}$$

where θ_i, θ_r and θ_t are the angles (as in Fig. 1 a) the incident, reflected and refracted light make with the normal to the interface between the two media with refractive indices n_1 and n_2 respectively. Of course, the French would rather like to call this the *Descartes law*. But the knowledge of this is much more ancient. There is documentary evidence that it was known as early as 984 AD, when Ibn Sahl, who lived in Baghdad wrote a treatise *Burning Instruments*, where he clearly stated a law of refraction for light passing across a plane interface from a material medium into air. This law, which is completely identical to what we now call the Snell laws of refraction, defined the refractive index n of a medium in terms of the incident and refracted rays as shown in Fig. 1 b. But I would speculate that the knowledge of refractive properties is bound to be even earlier, maybe even a couple of thousand years. There was an ancient diamond and gem making industry in India that was famous all over the world. It would have been next to impossible to decide the cuts for a well shaped gem without a proper knowledge of light refraction. I doubt if that would have merely been based on some experience with no codification. After all very valuable materials like diamonds were involved. But giving precendence to documentary evidence, I would rather call this the *Ibn-Sahl law*.

In any case, this was an empirical law based on experimental observations. Now that we know that light is an electromagnetic wave, we will now deduce and understand this law from electromagnetic considerations. We will also further deduce from a theoretical perspective the amounts of light that are reflected or transmitted across the interface.

1 Fresnel coefficients

I will now outline the derivation of the Fresnel coefficients for the reflected and refracted light across an interface between two media with dielectric permittivities ε_1 and ε_2 and magnetic permeabilities μ_1 and μ_2. Consider Fig. 1a for the geometry involved. Let us consider a plane wave that is incident from medium -1 on the interface with medium -2. First of all, we note that there is invariance in the x-y plane (nothing changes along X and Y). The incident wave direction breaks the rotational symmetry. The plane defined by the incident wave-vector $\vec{k}_1$ and the normal to the interface $\hat{n}$ is called the plane of incidence. We will take this to be the x-z plane. The breaking of rotational symmetry also makes two different cases of linear polarization different. Consider the polarization when the electric field of the incident wave lies in the plane of the incidence (P-polarization or π polarization) and when the magnetic field of the incident wave liesin the plane of polarizaton (S or σ polarization). Due to the fact that the magnetic field is perpendicular to the plane of incidence for P-polarized light, it is also called Transverse Magnetic (TM) polarization. Similarly, the case of S-polarization is also called Transverse Electric (TE) polarization. The boundary conditions for the waves turns out to be different in these two cases as can be readily seen. In case of P-polarized light, there are electric field components both parallel and perpendicular to the interface while for the S-polarized case, the light is only parallel to the interface. The case of S-polarization is considered here by me, while the calculation for the P-polarized case is left to you as an exercise.

Representing electric fields of the incident, reflected and transmitted S-polarized plane waves as

$$\vec{E}_{\text{inc}} = E_0 \exp[i(k_{ix}x + k_{iz}z - \omega_i t)]\hat{y}, \tag{2}$$

$$\vec{E}_{\text{ref}} = E_r \exp[i(k_{rx}x + k_{rz}z - \omega_r t)]\hat{y}, \tag{3}$$

$$\vec{E}_{\text{trans}} = E_t \exp[i(k_{tx}x + k_{tz}z - \omega_t t)]\hat{y}, \tag{4}$$

One might of course ask why should one assume both reflecte and transmitted fields. I urge the student to try to do this calculation using only one reflected or transmitted field. You would find it impossible to satisfy the continuity conditions on Maxwell's equations across the interface. we note that the incident, reflected and transmitted wave-vectors lie in the plane of incidence. If you assume otherwise that the reflected and transmitted wave-vectors have out-of-plane components, those components would have no source and those components turn out to be zero (exercise left to the student). The continuity conditions on Maxwell's equations require us to equate these components, all of which are parallel to the interface, on both sides of the interface as

$$\vec{E}_{\text{inc}} + \vec{E}_{\text{ref}} = \vec{E}_{\text{trans}}$$

If these conditions are going to be satisfied at all times, the frequencies of all the terms would have to be equal: $\omega_i = \omega_r = \omega_t = \omega$ (say). So the frequency cannot change upon simple reflection or refractive processes. Similarly, noting that this condition would have be satisfied at all points along the interface (along x), we note that $k_{ix} = k_{rx} = k_{tx} = k_x$.

We note that the wavevector components for the three fields would need to satisfy the dispersion

$$k_x^2 + +k_{iz}^2 = \varepsilon_1\mu_1\frac{\omega^2}{c^2},$$
$$k_x^2 + +k_{rz}^2 = \varepsilon_1\mu_1\frac{\omega^2}{c^2},$$
$$k_x^2 + +k_{tz}^2 = \varepsilon_2\mu_2\frac{\omega^2}{c^2}, \tag{5}$$

Using these, and the fact that k_x is the component of the wave-vectors parallel to the interface, we can write

$$k_x = \sqrt{\varepsilon_1\mu_1}\,\frac{\omega}{c}\sin\theta_i = \sqrt{\varepsilon_1\mu_1}\,\frac{\omega}{c}\sin\theta_r = \sqrt{\varepsilon_2\mu_2}\,\frac{\omega}{c}\sin\theta_t.$$

These yield Ibn-Sahl laws of refraction and reflection given by Equation (1). Further, we see that $k_{rz} = -k_{iz}$ in order to represent a wave that is flowing away from the interface in the backward direction. Thus, we see that the laws of reflection and refraction are the result of a phase matching condition and have a justification now from fundamental principles.

Now we will proceed to calculate the reflection and transmission coefficient from the Maxwell equations. Let us take the interface to be the $z = 0$ plane for simplicity (it actually does not matter if the plane were shifted in which case only an overall kinematic phase would occur for the reflected and transmitted waves). Then, equating the tangential electric fields across the interface,

$$E_0 + E_r = E_t. \tag{6}$$

We use the Maxwell equation for the plane harmonic wave $\vec{k}\times\vec{E} = \omega\mu_0\mu\vec{H}$ to obtain the associated magnetic field components as

$$H_x = -\frac{k_z E_y}{\mu_0\mu\omega}, \qquad H_z = \frac{k_x E_y}{\mu_0\mu\omega}. \tag{7}$$

Continuity of the tangential ($x-$) component of the $\vec{H}$ fields across the interface implies

$$\frac{k_{iz}E_i}{\mu_0\mu_1\omega} + \frac{k_{rz}E_r}{\mu_0\mu_1\omega} = \frac{k_{tz}E_t}{\mu_0\mu_2\omega}. \tag{8}$$

Noting $k_{rz} = -k_{iz}$, and eleiminating E_r from the above equations, we get the transmission coefficient as

$$T = \frac{E_t}{E_i} = \frac{2k_{iz}/\mu_1}{k_{iz}/\mu_1 + k_{tz}/\mu_2}. \tag{9}$$

Similarly, eliminating E_t, we obtain the reflection coefficient as

$$R = \frac{E_r}{E_i} = \frac{k_{iz}/\mu_1 - k_{tz}/\mu_2}{k_{iz}/\mu_1 + k_{tz}/\mu_2}. \tag{10}$$

A similiar analysis can be made for the P-polarized light. The results are very similar: one only needs to replace the electric fields by the H field components and the μ_i are replaced with the corresponding ε_i everywhere in the above two expressions.