Contents

Preface **v**

Preface to the second edition **viii**

Notations **ix**

1 Preliminaries **1**
1.1 Why Quantum Field Theory 1
1.2 Creation and annihilation operators 3
1.3 Special relativity 5
1.4 Space and time in relativistic quantum theory 8
1.5 Natural units 9

2 Classical Field Theory **12**
2.1 A quick review of particle mechanics 12
2.1.1 Action principle and Euler-Lagrange equations . . . 12
2.1.2 Hamiltonian formalism and Poisson brackets 14
2.2 Euler-Lagrange equations in field theory 15
2.2.1 Action functional and Lagrangian 15
2.2.2 Euler-Lagrange equations 17
2.3 Hamiltonian formalism 19
2.4 Noether's theorem 21

3 Quantization of scalar fields **28**
3.1 Equation of motion 28
3.2 The field and its canonical quantization 29
3.3 Fourier decomposition of the field 30
3.4 Ground state of the Hamiltonian and normal ordering . . . 34
3.5 Fock space 36
3.6 Complex scalar field 37
3.6.1 Creation and annihilation operators 37
3.6.2 Particles and antiparticles 39
3.6.3 Ground state and Hamiltonian 40
3.7 Propagator 41

4 Quantization of Dirac fields **47**
4.1 Dirac Hamiltonian . . . 47
4.2 Dirac equation . . . 51
4.3 Plane wave solutions of Dirac equation . . . 54
4.3.1 Positive and negative energy spinors . . . 54
4.3.2 Explicit solutions in Dirac-Pauli representation . . . 56
4.4 Projection operators . . . 59
4.4.1 Projection operators for positive and negative energy states . . . 59
4.4.2 Helicity projection operators . . . 60
4.4.3 Chirality projection operators . . . 61
4.4.4 Spin projection operators . . . 62
4.5 Lagrangian for a Dirac field . . . 63
4.6 Fourier decomposition of the field . . . 65
4.7 Propagator . . . 69

5 The S-matrix expansion **72**
5.1 Examples of interactions . . . 73
5.2 Evolution operator . . . 75
5.3 S-matrix . . . 80
5.4 Wick's theorem . . . 82

6 From Wick expansion to Feynman diagrams **87**
6.1 Yukawa interaction : decay of a scalar . . . 87
6.2 Normalized states . . . 94
6.3 Sample calculation of a matrix element . . . 97
6.4 Another example: fermion scattering . . . 101
6.5 Feynman amplitude . . . 105
6.6 Feynman rules . . . 106
6.7 Virtual particles . . . 110
6.8 Amplitudes which are not S-matrix elements . . . 112

7 Cross sections and decay rates **115**
7.1 Decay rate . . . 115
7.2 Examples of decay rate calculation . . . 117
7.2.1 Decay of a scalar into a fermion-antifermion pair . . . 117
7.2.2 Muon decay with 4-fermion interaction . . . 122
7.3 Scattering cross section . . . 130
7.4 Generalities of 2-to-2 scattering . . . 133
7.4.1 CM frame . . . 135
7.4.2 Lab frame . . . 137
7.5 Inelastic scattering with 4-fermion interaction . . . 140
7.5.1 Cross-section in CM frame . . . 142
7.5.2 Cross-section in Lab frame . . . 143
7.6 Mandelstam variables . . . 144

8 Quantization of the electromagnetic field **146**
8.1 Classical theory of electromagnetic fields 146
8.2 Problems with quantization 149
8.3 Modifying the classical Lagrangian 150
8.4 Propagator 153
8.5 Fourier decomposition of the field 156
8.6 Physical states 158
8.7 Another look at the propagator 162
8.8 Feynman rules for photons 164

9 Quantum electrodynamics **166**
9.1 Local gauge invariance 166
9.2 Interaction Hamiltonian 170
9.3 Lowest order processes 172
9.4 Electron-electron scattering 174
9.5 Electron-positron scattering 180
9.6 $e^-e^+ \to \mu^-\mu^+$ 182
9.7 Consequence of gauge invariance 184
9.8 Compton scattering 185
9.9 Scattering by an external field 194
9.10 Bremsstrahlung 197

10 P, T, C and their combinations **200**
10.1 Motivations from classical physics 200
10.2 Parity 201
10.2.1 Free scalar fields 201
10.2.2 Free Dirac field 202
10.2.3 Free photon field 204
10.2.4 Interacting fields 205
10.3 Charge conjugation 207
10.3.1 Free fields 207
10.3.2 Interactions 211
10.4 Time reversal 212
10.4.1 Antilinearity 212
10.4.2 Free fields 213
10.4.3 Interactions 216
10.5 CP 217
10.6 CPT 218

11 Electromagnetic form factors **222**
11.1 General electromagnetic vertex 222
11.2 Physical interpretation of form factors 224
11.2.1 Charge form factor F_1 224
11.2.2 Anomalous magnetic moment F_2 228
11.2.3 Electric dipole moment $\widetilde{F}_2$ 228

11.2.4 Anapole moment $\widetilde{F}_3$ 229
11.3 Anomalous magnetic moment of the electron 230
11.4 Charge form factor 239
11.5 Electron-proton scattering 241

12 Renormalization **245**
12.1 Degree of divergence of a diagram 245
12.1.1 Superficial degree of divergence 245
12.1.2 Superficial vs. real degree of divergence 248
12.2 Specific examples in QED 250
12.3 Outline of the program 252
12.4 Ward-Takahashi identity 253
12.5 General forms for divergent amplitudes 256
12.5.1 Fermion self-energy 256
12.5.2 Vacuum polarization 257
12.5.3 Vertex function 259
12.6 Regularization of self-energy diagrams 260
12.6.1 Vacuum polarization diagram 260
12.6.2 Fermion self-energy diagram 265
12.7 Counterterms 267
12.7.1 Vacuum polarization diagram 267
12.7.2 Fermion self-energy diagram 270
12.7.3 Vertex function 272
12.8 Full Lagrangian 273
12.9 Observable effects of renormalization 275
12.9.1 Modification of Coulomb interaction 275
12.9.2 Running coupling constant 276
12.9.3 Cancellation of infra-red divergences 279

13 Symmetries and symmetry breaking **283**
13.1 Classification of symmetries 283
13.2 Groups and symmetries 284
13.2.1 Symmetry group 284
13.2.2 Examples of continuous symmetry groups 286
13.2.3 Generators of continuous groups 289
13.2.4 Representations 292
13.3 Approximate symmetries 293
13.4 Spontaneous breaking of symmetries 294
13.4.1 Discrete symmetry 295
13.4.2 U(1) symmetry 299
13.4.3 Non-Abelian symmetry 300
13.5 Goldstone's theorem 301
13.5.1 Appearance of massless states 301
13.5.2 Examples of Nambu-Goldstone bosons 303

13.5.3 Interaction of Goldstone bosons 305
13.6 Higgs mechanism 308

14 Yang-Mills theory of non-Abelian gauge fields **312**
14.1 Gauge fields of non-Abelian symmetry 312
14.2 Pure gauge Lagrangian 315
14.3 Interactions of non-Abelian gauge fields 318
14.3.1 Gauge interactions of other particles 318
14.3.2 Self-interactions of gauge bosons 319
14.4 Equations of motion and conserved currents 321
14.5 Quantization of non-Abelian gauge fields 322
14.6 Quantum Chromodynamics 323

15 Standard electroweak theory **326**
15.1 Gauge group 326
15.1.1 Choice of gauge group 326
15.1.2 Pure gauge Lagrangian 328
15.2 Spontaneous symmetry breaking 330
15.2.1 Introducing the Higgs boson multiplet 330
15.2.2 Gauge boson masses 332
15.2.3 Scalar modes 333
15.3 Fermions in the theory 335
15.3.1 Gauge interactions 335
15.3.2 Electron mass 338
15.3.3 Yukawa couplings 339
15.3.4 Other fermions in the model 340
15.4 Gauge boson decay 342
15.5 Scattering processes 346
15.5.1 Forward-backward asymmetry 346
15.5.2 Low energy weak interactions 349
15.5.3 High energy scattering 351
15.6 Propagator for unstable particles 355
15.7 Global symmetries of the model 356

A Useful formulas **359**
A.1 Representation of γ-matrices 359
A.2 Traces of γ-matrices 361
A.3 The antisymmetric tensor 364
A.4 Useful integration formulas 365
A.4.1 Angular integrations in N-dimensional space 365
A.4.2 Momentum integration in loops 367

B Answers to selected exercises **369**

Index **373**