HYDROCYCLONE CALCULATIONS GRAPH

Tyshkevich Dmitry

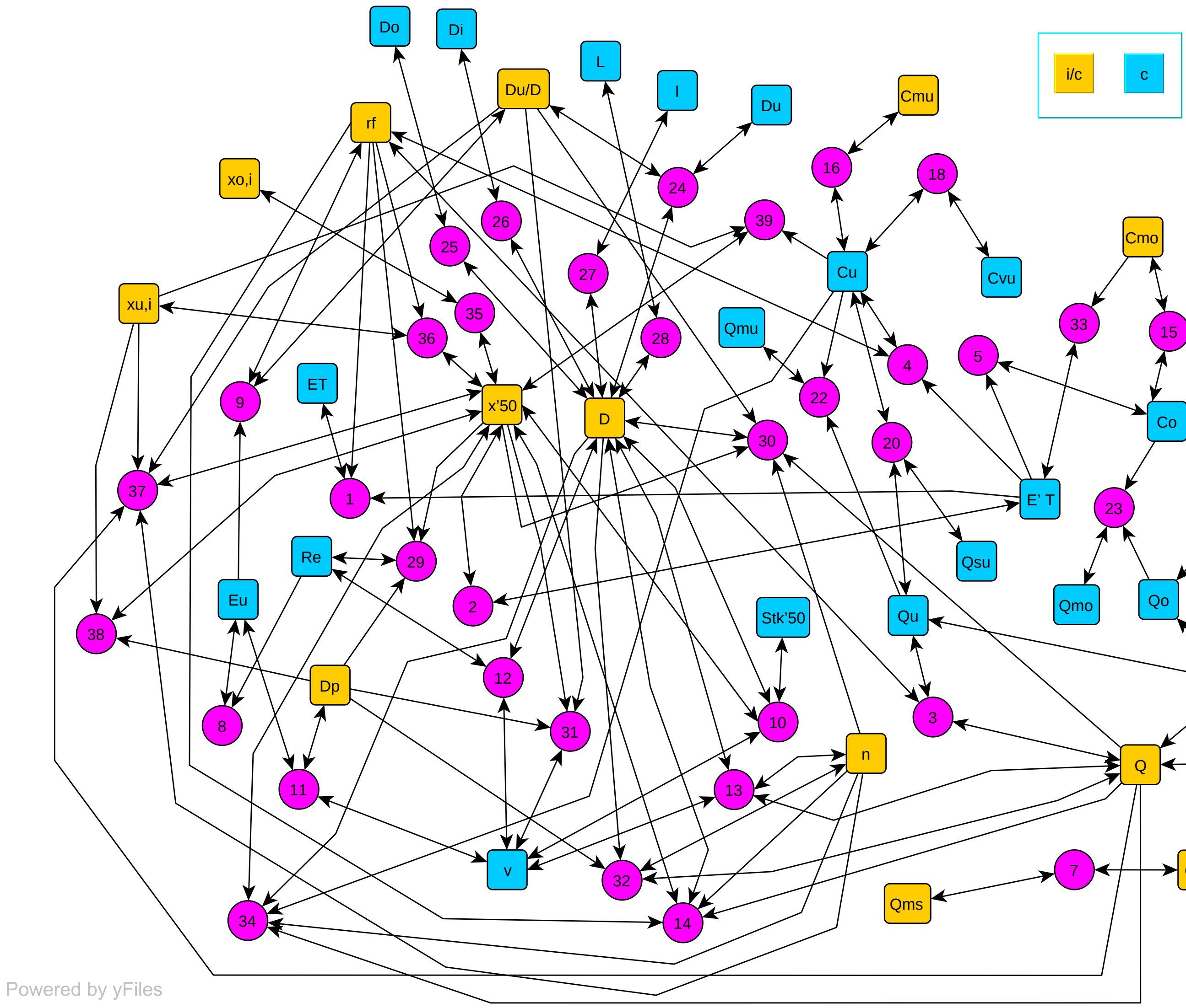

FIGURE 1. Calculations Graph

1. Commentary to the Graph

Box–shaped verteces correspond to the **Parameters**. Yellow–colored verteces correspond to **input/calculated** Parameters. Blue–colored verteces correspond to only **calculated** Parameters.

Circle-shaped verteces correspond to the **Equations** by wich Paramereters are calculated in the Project. Inside the circle there is a number corresponding to the equation reference number (see the section 3 below).

The absence of an arrow at a Parameter vertex means that this Parameter enters only into the right–hand side of an equation which sends an (unarrowed) arc to this parameter and only partcipates in calculation of some left–hand side Parameter, which is to be calculated. The presence of an an arrow at a Parameter vertex means that this Parameter enters into the left–hand side (and is to be calculated) of a corresponding equation which sends an (arrowed) arc to this parameter.

2. Commentary to the Equations

The expression "all terms are derivable" at a *sum* or *product equation* means that each Parameter of the equation can enter into the left–hand side of this general equation (i.e. is derived from the other Parameters and is to be calculated).

By the blue color the constant (regarding Calculation Graph in Figure 1) Parameters (i.e. the Material, Machine Geometry Parameters and constant numbers) and values calculated by using these constant Parameters are highlighted.

By the green color auxiliary values, parameters, and functions are highlighted.

By the red color the new exclusive power-and-fast-implemented functions expL, eeI, ecel (see the Sections 4, 5) are highlighted.

3. Equations

$$E_T = (1 - r_f)E'_T + r_f \tag{1}$$

$$\begin{cases} E'_T = 0.5\left(1 + \operatorname{erf}\left(\dfrac{\ln(x_g) - \ln(x'_{50})}{\sqrt{2}\sqrt{\ln^2(\sigma_g) + \ln^2(\sigma_s)}}\right)\right) \\ x'_{50} = \dfrac{x_g}{\exp\left(\operatorname{erf}^{-1}\left(2E'_T - 1\right)\sqrt{2}\sqrt{\ln^2(\sigma_g) + \ln^2(\sigma_s)}\right)} \end{cases} \tag{2}$$

$$r_f Q = Q_u \quad (\textit{product equation}\text{: all terms are derivable }) \tag{3}$$

$$\begin{cases} c_u = c\left(1 + \dfrac{1 - r_f}{r_f} E'_T\right) \\ r_f = \dfrac{1}{1 + \dfrac{\dfrac{c_u}{c} - 1}{E'_T}} \end{cases} \tag{4}$$

$$c_o = c(1 - E'_T) \tag{5}$$

$$Q_m = \rho_{sus} Q \quad (product\ equation\text{: all terms are derivable }) \tag{6}$$

$$Q_{ms} = c_m \rho_{sus} Q \quad (product\ equation\text{: all terms are derivable }) \tag{7}$$

$$Eu = \beta_1 Re^{\beta_2} \exp(-\beta_3 c_v) \tag{8}$$

$$\begin{cases} r_f = \gamma_1 (D_u/D)^{\gamma_2} Eu^{-\gamma_3} \\ D_u/D = \left(\dfrac{r_f Eu^{\gamma_3}}{\gamma_1}\right)^{\dfrac{1}{\gamma_2}} \end{cases} \tag{9}$$

$$18\eta \cdot D \cdot Stk'_{50} = {x'_{50}}^2 \cdot (\rho_s - \rho) \cdot v \quad (product\ equation\text{: all terms are derivable }) \tag{10}$$

$$\rho \cdot Eu \cdot v^2 = 2\Delta p \quad (product\ equation\text{: all terms are derivable }) \tag{11}$$

$$\eta \cdot Re = \rho \cdot D \cdot v \quad (product\ equation\text{: all terms are derivable }) \tag{12}$$

$$\pi D^2 \cdot n \cdot v = 4Q \quad (product\ equation\text{: all terms are derivable }) \tag{13}$$

$$\begin{cases} D = \left(\dfrac{2}{9\pi} \dfrac{{x'_{50}}^2 (\rho_s - \rho) \beta_1 Q \left(\dfrac{4\rho Q}{\pi \eta n}\right)^{\beta_2} \exp(-(\alpha_3 + \beta_3) c_v)}{\eta \alpha_1 n \ln^{\alpha_2}\left(\dfrac{1}{r_f}\right)}\right)^{\dfrac{1}{3 + \beta_2}} \\ x'_{50} = \sqrt{\dfrac{9\pi}{2} \dfrac{\eta \alpha_1 n \ln^{\alpha_2}\left(\dfrac{1}{r_f}\right) D^{3+\beta_2}}{(\rho_s - \rho) \beta_1 Q \left(\dfrac{4\rho Q}{\pi \eta n}\right)^{\beta_2} \exp(-(\alpha_3 + \beta_3) c_v)}} \end{cases} \tag{14}$$

$$\begin{cases} c_{mo} = \dfrac{c_o}{\rho\left(1 + \dfrac{c_o}{\rho_s}\left(\dfrac{\rho_s}{\rho} - 1\right)\right)} \\[2ex] c_o = \dfrac{1}{\dfrac{1}{\rho c_{mo}} - \dfrac{1}{\rho_s}\left(\dfrac{\rho_s}{\rho} - 1\right)} \end{cases} \tag{15}$$

$$\begin{cases} c_{mu} = \dfrac{c_u}{\rho\left(1 + \dfrac{c_u}{\rho_s}\left(\dfrac{\rho_s}{\rho} - 1\right)\right)} \\[2ex] c_u = \dfrac{1}{\dfrac{1}{\rho c_{mu}} - \dfrac{1}{\rho_s}\left(\dfrac{\rho_s}{\rho} - 1\right)} \end{cases} \tag{16}$$

$$\rho_s \cdot c_{vo} = c_o \quad (\textit{product equation}\text{: all terms are derivable }) \tag{17}$$

$$\rho_s \cdot c_{vu} = c_u \quad (\textit{product equation}\text{: all terms are derivable }) \tag{18}$$

$$Q = Q_o + Q_u \quad (\textit{sum equation}\text{: all terms are derivable }) \tag{19}$$

$$Q_{su} = Q_u c_u \quad (\textit{product equation}\text{: all terms are derivable }) \tag{20}$$

$$Q_{so} = Q_o c_o \quad (\textit{product equation}\text{: all terms are derivable }) \tag{21}$$

$$Q_{mu} = Q_u\left(c_u + (\rho_s - c_u)\frac{\rho}{\rho_s}\right) \tag{22}$$

$$Q_{mo} = Q_o\left(c_o + (\rho_s - c_o)\frac{\rho}{\rho_s}\right) \tag{23}$$

$$D_u = Du/D \cdot D \quad (\textit{product equation}\text{: all terms are derivable }) \tag{24}$$

$$D_o = Do/D \cdot D \quad (\textit{product equation}\text{: all terms are derivable }) \tag{25}$$

$$D_i = Di/D \cdot D \quad (\textit{product equation}\text{: all terms are derivable }) \tag{26}$$

$$l = l/D \cdot D \quad (\textit{product equation}\text{: all terms are derivable }) \tag{27}$$

$$L = L/D \cdot D \quad (\textit{product equation}\text{: all terms are derivable }) \tag{28}$$

$$Re = \frac{{x'_{50}}^2(\rho_s - \rho)\Delta p}{9\eta^2\alpha_1 \exp(\alpha_3 c_v) \ln^{\alpha_2}\left(\frac{1}{r_f}\right)} \tag{29}$$

$$\begin{cases}
\textit{We have to solve the transcendental equation:} \\[1em]
?D: \quad \frac{2}{9\pi}\frac{{x'_{50}}^2(\rho_s-\rho)Q}{\eta n D^3}\beta_1\left(\frac{4\rho Q}{\pi\eta n D}\right)^{\beta_2}\exp(-\beta_3 c_v) = \\[1em]
\alpha_1\left(-\ln(\gamma_1) - \gamma_2\ln(Du/D) + \gamma_3\ln\left(\beta_1\left(\frac{4\rho Q}{\pi\eta n D}\right)^{\beta_2}\exp(-\beta_3 c_v)\right)\right)^{\alpha_2}\exp(\alpha_3 c_v) \\[1em]
\textit{The solution can be given by the formula:} \\[1em]
D = \left(\exp\left(\mathcal{E}\cdot\mathcal{F}\right)\cdot\exp\left(\mathrm{expL}_{+}^{\infty}\left(\mathcal{F}\cdot\mathcal{G}\cdot(\mathcal{A}\cdot\mathcal{B})^{\frac{1}{\alpha_2}}\right)\right)\right)^{\dfrac{-\alpha_2}{3+\beta_2}}, \quad \textit{where} \\[1em]
\mathcal{A} = \beta_1\left(\frac{4\rho Q}{\pi\eta n}\right)^{\beta_2}\exp(-\beta_3 c_v) \\[1em]
\mathcal{B} = \frac{2}{9\pi}\frac{{x'_{50}}^2(\rho_s-\rho)Q\exp(-\alpha_3 c_v)}{\eta\alpha_1 n} \\[1em]
\mathcal{E} = -\ln(\gamma_1) - \gamma_2\ln(Du/D) + \gamma_3\ln(\mathcal{A}) \\[1em]
\mathcal{F} = \frac{3+\beta_2}{\alpha_2\beta_2\gamma_3} \\[1em]
\mathcal{G} = \exp(-\mathcal{E}\cdot\mathcal{F})
\end{cases} \tag{30}$$

$$\begin{cases} \textit{We have to solve the transcendental equation:} \\ ?v: \quad \dfrac{{x'_{50}}^2(\rho_s-\rho)\Delta p}{9\eta^2\left(\dfrac{2\Delta p}{\rho\beta_1\exp(-\beta_3 c_v)v^2}\right)^{\frac{1}{\beta_2}}} = \\ \alpha_1\left(-\ln(\gamma_1)-\gamma_2\ln(Du/D)+\gamma_3\ln\left(\dfrac{2\Delta p}{\rho v^2}\right)\right)^{\alpha_2}\exp(\alpha_3 c_v) \\ \textit{The solution can be given by the formula:} \\ v = \left(\mathcal{E}\cdot\exp\left(\mathrm{expL\_}\left(-\mathcal{B}^{\frac{1}{\alpha_2}}\cdot\mathcal{D}\cdot\mathcal{E}\right)\right)\right)^{\frac{\alpha_2\beta_2}{2}}, \quad \textit{where} \\ \mathcal{A} = \dfrac{2\Delta p}{\rho} \\ \mathcal{B} = \dfrac{{x'_{50}}^2(\rho_s-\rho)\Delta p}{9\eta^2\left(\dfrac{\mathcal{A}}{\beta_1\exp(-\beta_3 c_v)}\right)^{\frac{1}{\beta_2}}\alpha_1\exp(\alpha_3 c_v)} \\ \mathcal{C} = \alpha_1\big(-\ln(\gamma_1)-\gamma_2\ln(Du/D)+\gamma_3\ln(\mathcal{A})\big) \\ \mathcal{D} = \dfrac{1}{\alpha_2\beta_2\gamma_3} \\ \mathcal{E} = \exp(\mathcal{C}\cdot\mathcal{D}) \end{cases} \tag{31}$$

$$\begin{cases} Q = \left(\dfrac{\pi^2\Delta p\, D^4}{8\rho\beta_1\exp(-\beta_3 c_v)\left(\dfrac{4\rho}{\pi\eta D}\right)^{\beta_2}}\right)^{\frac{1}{2+\beta_2}}\cdot n \\ n = \left(\dfrac{8\rho\beta_1\exp(-\beta_3 c_v)\left(\dfrac{4\rho}{\pi\eta D}\right)^{\beta_2}}{\pi^2\Delta p\, D^4}\right)^{\frac{1}{2+\beta_2}}\cdot Q \end{cases} \tag{32}$$

$$E'_T = 1 - \frac{\rho\rho_s c_{mo}}{c\big(c_{mo}(\rho-\rho_s)+\rho_s\big)} \tag{33}$$

$$\left\{\begin{array}{l}
\textit{We have to solve the transcendental equation:} \\[2ex]
?x'_{50}: \quad x'_{50} = \left(\dfrac{9\pi\eta\alpha_1 n\left(\ln\left(1+\dfrac{\frac{c_u}{c}-1}{0.5\left(1+\operatorname{erf}\left(\frac{\ln(x_g)-\ln(x'_{50})}{\sqrt{2}\sqrt{\ln^2(\sigma_g)+\ln^2(\sigma_s)}}\right)\right)}\right)\right)^{\alpha_2}}{2(\rho_s-\rho)\beta_1 Q\left(\dfrac{4\rho Q}{\pi\eta n}\right)^{\beta_2}\exp\big(-(\alpha_3+\beta_3)c_v\big)}\right)^{\frac{1}{2}} \cdot D^{\frac{3+\beta_2}{2}} \\[2ex]
\textit{Let's denote} \\[2ex]
\mathcal{A} = \left(\dfrac{9\pi\eta\alpha_1\exp\big((\alpha_3+\beta_3)c_v\big)n}{2(\rho_s-\rho)\beta_1 Q\left(\dfrac{4\rho Q}{\pi\eta n}\right)^{\beta_2}}\right)^{\frac{1}{2}} \cdot D^{\frac{3+\beta_2}{2}} \\[2ex]
\mathcal{B} = 2\left(\dfrac{c_u}{c}-1\right) \qquad\qquad \mathcal{E} = \sqrt{2}\sqrt{\ln^2(\sigma_g)+\ln^2(\sigma_s)} \\[2ex]
a_1 = \dfrac{\ln\left(\frac{xg}{\mathcal{A}}\right)}{\mathcal{E}} \qquad\qquad a_2 = \dfrac{\alpha_2}{2\mathcal{E}} \qquad\qquad a_3 = \mathcal{B} \\[2ex]
\textit{Then the above equation is equivalent to the transcendental equation to be solved:} \\[2ex]
?x: \quad x = a_1 - a_2\ln\left(\ln\left(1+\dfrac{a_3}{1+\operatorname{erf}(x)}\right)\right) \\[2ex]
\textit{by using the relations:} \\[2ex]
x'_{50} = x_g\cdot\exp(-x\cdot\mathcal{E}), \qquad\qquad x = \dfrac{\ln(x_g)-\ln(x'_{50})}{\mathcal{E}}
\end{array}\right. \tag{34}$$

$$\begin{cases} \textit{We solve the transcendental equation:} \\ \mathrm{eceI}(b, z'_{50}, z_{oi}) = 2i \cdot \mathrm{erfc}(a \cdot z'_{50}) \\ \textit{with respect to the } z'_{50} \textit{ (to find } x'_{50} \textit{) or } z_{oi} \textit{ (to find } x_{oi} \textit{) where} \\ a = \dfrac{\ln(\sigma_s)}{\sqrt{\ln^2(\sigma_g) + \ln^2(\sigma_s)}} \qquad b = \dfrac{\ln(\sigma_g)}{\ln(\sigma_s)} \\ z'_{50} = \dfrac{\ln(x_g) - \ln(x'_{50})}{\sqrt{2}\ln(\sigma_s)} \qquad z_{oi} = \dfrac{\ln(x_{oi}) - \ln(x_g)}{\sqrt{2}\ln(\sigma_g)} \\ \textit{Getting calculated } z'_{50} \textit{ or } z_{oi} \textit{ we then can calculate } x'_{50} \textit{ or } x_{oi} \textit{ by the relations:} \\ x'_{50} = x_g \cdot \exp(-z'_{50} \cdot \sqrt{2}\ln(\sigma_s)) \qquad x_{oi} = x_g \cdot \exp(z_{oi} \cdot \sqrt{2}\ln(\sigma_g)) \end{cases} \tag{35}$$

$$\begin{cases} \textit{We solve the transcendental equation:} \\ 1 + \mathrm{erf}(z_{ui}) + \widehat{r_f} \cdot \mathrm{eeI}(b, z'_{50}, z_{ui}) = 2i \cdot \left(1 + \widehat{r_f} \cdot \mathrm{erf}(a \cdot z'_{50})\right) \\ \textit{with respect to the } z'_{50} \textit{ (to find } x'_{50} \textit{) or } z_{ui} \textit{ (to find } x_{ui} \textit{) where} \\ a = \dfrac{\ln(\sigma_s)}{\sqrt{\ln^2(\sigma_g) + \ln^2(\sigma_s)}} \qquad b = \dfrac{\ln(\sigma_g)}{\ln(\sigma_s)} \qquad \widehat{r_f} = \dfrac{1 - r_f}{1 + r_f} \\ z'_{50} = \dfrac{\ln(x_g) - \ln(x'_{50})}{\sqrt{2}\ln(\sigma_s)} \qquad z_{ui} = \dfrac{\ln(x_{ui}) - \ln(x_g)}{\sqrt{2}\ln(\sigma_g)} \\ \textit{Getting calculated } z'_{50} \textit{ or } z_{ui} \textit{ we then can calculate } x'_{50} \textit{ or } x_{ui} \textit{ by the relations:} \\ x'_{50} = x_g \cdot \exp(-z'_{50} \cdot \sqrt{2}\ln(\sigma_s)) \qquad x_{ui} = x_g \cdot \exp(z_{ui} \cdot \sqrt{2}\ln(\sigma_g)) \end{cases} \tag{36}$$

$$
\left\{
\begin{array}{l}
\textit{We solve the transcendental equation:} \\[2ex]
?z'_{50}: \quad 1+\mathrm{erf}(z_{ui})+\widehat{R_f}(z'_{50})\cdot \mathrm{eeI}(b,z'_{50},z_{ui}) = 2i\cdot\big(1+\widehat{R_f}(z'_{50})\cdot\mathrm{erf}(a\cdot z'_{50})\big), \\[2ex]
\ \textit{where} \\[2ex]
a=\dfrac{\ln(\sigma_s)}{\sqrt{\ln^2(\sigma_g)+\ln^2(\sigma_s)}} \qquad\qquad b=\dfrac{\ln(\sigma_g)}{\ln(\sigma_s)} \qquad\qquad z_{ui}=\dfrac{\ln(x_{ui})-\ln(x_g)}{\sqrt{2}\ln(\sigma_g)} \\[2ex]
\widehat{R_f}(\zeta)=\dfrac{1-R_f(\zeta)}{1+R_f(\zeta)} \qquad\qquad R_f(\zeta)=\mathrm{expL}_{+}^{\infty}\left(\dfrac{3+\beta_2}{\alpha_2\beta_2\gamma_3}\cdot\mathfrak{a}(\zeta)^{-\frac{3+\beta_2}{\alpha_2\beta_2}}\right) \\[2ex]
\mathfrak{a}(\zeta)=\mathcal{A}\cdot\big(x_g\cdot\exp(-\zeta\cdot\sqrt{2}\ln(\sigma_s))\big)^{-\frac{2\beta_2}{3+\beta_2}} \\[2ex]
\mathcal{A}=(\gamma_1(D_u/D)^{\gamma_2})^{-\frac{1}{\gamma_3}}\,\beta_1\left(\mathcal{B}\cdot\left(\left(\frac{\pi}{12}\right)^2\frac{\beta_1}{2\alpha_1}\left(\frac{\rho_s}{\rho}-1\right)\mathrm{e}^{-(\alpha_1+\beta_3)c_v}\cdot\mathcal{B}^{\beta_2+1}\right)^{-\frac{1}{3+\beta_2}}\right)^{\beta_2}\mathrm{e}^{-\beta_3 c_v} \\[2ex]
\mathcal{B}=\dfrac{\rho Q}{\eta n} \\[2ex]
\ \textit{Getting calculated } z'_{50} \textit{ we then calculate } x'_{50} \textit{ by the relation:} \\[2ex]
x'_{50}=x_g\cdot\exp(-z'_{50}\cdot\sqrt{2}\ln(\sigma_s))
\end{array}
\right.
\tag{37}
$$

$$
\begin{cases}
\textit{We solve the transcendental equation:} \\
?z'_{50}: \quad 1+\operatorname{erf}(z_{ui})+\widehat{R_f}(z'_{50})\cdot \mathrm{eeI}(b,z'_{50},z_{ui}) = 2i\cdot\left(1+\widehat{R_f}(z'_{50})\cdot\operatorname{erf}(a\cdot z'_{50})\right), \\
\textit{where} \\
a=\dfrac{\ln(\sigma_s)}{\sqrt{\ln^2(\sigma_g)+\ln^2(\sigma_s)}} \qquad b=\dfrac{\ln(\sigma_g)}{\ln(\sigma_s)} \qquad z_{ui}=\dfrac{\ln(x_{ui})-\ln(x_g)}{\sqrt{2}\ln(\sigma_g)} \\
\widehat{R_f}(\zeta)=\dfrac{1-R_f(\zeta)}{1+R_f(\zeta)} \qquad R_f(\zeta)=\mathrm{expL}_{-}\left(\dfrac{-1}{\alpha_2\beta_2\gamma_3}\cdot\mathfrak{a}(\zeta)^{\frac{1}{\alpha_2\beta_2}}\right) \\
\mathfrak{a}(\zeta)=\mathcal{A}\cdot\left(x_g\cdot\exp(-\zeta\cdot\sqrt{2}\ln(\sigma_s))\right)^{2\beta_2} \\
\mathcal{A}=(\gamma_1(D_u/D)^{\gamma_2})^{-\frac{1}{\gamma_3}}\,\beta_1\left(\dfrac{(\rho_s-\rho)\mathrm{e}^{-\alpha_3 c_v}\,\Delta p}{9\eta^2\alpha_1}\right)^{\beta_2}\mathrm{e}^{-\beta_3 c_v} \\
\textit{Getting calculated } z'_{50} \textit{ we then calculate } x'_{50} \textit{ by the relation:} \\
x'_{50}=x_g\cdot\exp(-z'_{50}\cdot\sqrt{2}\ln(\sigma_s))
\end{cases} \tag{38}
$$

$$
\begin{cases}
\textit{We solve the transcendental equation:} \\
?z'_{50}: \quad 1+\operatorname{erf}(z_{ui})+\widehat{R_f}(z'_{50})\cdot \mathrm{eeI}(b,z'_{50},z_{ui}) = 2i\cdot\left(1+\widehat{R_f}(z'_{50})\cdot\operatorname{erf}(a\cdot z'_{50})\right), \\
\textit{where} \\
a=\dfrac{\ln(\sigma_s)}{\sqrt{\ln^2(\sigma_g)+\ln^2(\sigma_s)}} \qquad b=\dfrac{\ln(\sigma_g)}{\ln(\sigma_s)} \qquad z_{ui}=\dfrac{\ln(x_{ui})-\ln(x_g)}{\sqrt{2}\ln(\sigma_g)} \\
\widehat{R_f}(\zeta)=\dfrac{1-R_f(\zeta)}{1+R_f(\zeta)} \qquad R_f(\zeta)=\dfrac{1}{1+\dfrac{2\left(\dfrac{c_u}{c}-1\right)}{1+\operatorname{erf}(a\cdot\zeta)}} \\
\textit{Getting calculated } z'_{50} \textit{ we then calculate } x'_{50} \textit{ by the relation:} \\
x'_{50}=x_g\cdot\exp(-z'_{50}\cdot\sqrt{2}\ln(\sigma_s))
\end{cases} \tag{39}
$$

4. The functions expL_+^∞, expL_+^0, expL_-

The transcendental equation

$$z = x\exp(z) \tag{40}$$

with respect to z given x has a *unique* solution if $x < 0$ and *exactly two* solutions if

$$0 < x < \exp(-1) \approx 0.3678794411714423215955524$$

If $x = \exp(-1)$ then (40) has a unique solution $z = 1$; if $x > \exp(-1)$ then (40) *has no* solution.

So for the domain $(0, \exp(-1)]$ we have two branches of $z(x)$: the first one, expL_+^∞, for which

$$\text{if } x \to 0 \quad \text{then } z \to \infty$$

and the second one, expL_+^0, for which

$$\text{if } x \to 0 \quad \text{then } z \to 0.$$

By expL_- we denote the function $0 > x \mapsto z(x)$.

5. The functions eeI, eceI

We introduce the function

$$\mathrm{eeI}(a,b,x) = \frac{2}{\sqrt{\pi}} \int\limits_{-\infty}^{x} \mathrm{erf}(at+b)\exp(-t^2)dt \tag{41}$$

This function satisfies the following notable equalities

$$\mathrm{eeI}(a,b,x) = \begin{cases} \mathrm{erf}(ax+b)\,\mathrm{erf}(x) - 1 - \mathrm{eeI}\left(\frac{1}{a}, -\frac{b}{a}, ax+b\right), & a > 0 \\ \mathrm{erf}(b)\big(1+\mathrm{erf}(x)\big), & a = 0 \\ \mathrm{erf}(ax+b)\,\mathrm{erf}(x) + 1 + \mathrm{eeI}\left(-\frac{1}{a}, -\frac{b}{a}, -ax-b\right), & a < 0 \end{cases}$$

$$\mathrm{eeI}(-a,-b,x) = -\,\mathrm{eeI}(a,b,x)$$

Some plots regarding eeI are shown in Figures 5, 6.

Also we introduce the function

$$\mathrm{eceI}(a,b,x) = \frac{2}{\sqrt{\pi}} \int\limits_{-\infty}^{x} \mathrm{erfc}(at+b)\exp(-t^2)dt = 1 + \mathrm{erf}(x) + \mathrm{eeI}(a,b,x) \tag{42}$$

6. The Feed Functions and their connection with eeI, eceI

The **Feed Functions** are defined by the following formulae (by the red color the main argument is highlighted; by the blue color the constant Parameters correspondigly to the equations in the Section 3 are highlighted; by the green color auxiliary parameters are highlighted):

$$F_o(r_f, x'_{50}, x) = \frac{1}{1 - E_T(r_f, x'_{50})} \int\limits_0^x \big(1 - G(r_f, x'_{50}, t)\big)\dot{F}(t)dt$$

$$F_u(r_f, x'_{50}, x) = \frac{1}{E_T(r_f, x'_{50})} \int\limits_0^x G(r_f, x'_{50}, t)\dot{F}(t)dt$$

($\dot{F}(t)$ is a derivative of $F(t)$, $\frac{dF}{dt}$) where

$$E_T(r_f, x'_{50}) = (1 - r_f)E'_T(x'_{50}) + r_f;$$

$$E'_T(x'_{50}) = 0.5\left(1 + \operatorname{erf}\left(\frac{\ln(x_g) - \ln(x'_{50})}{\sqrt{2}\sqrt{\ln^2(\sigma_g) + \ln^2(\sigma_s)}}\right)\right);$$

$$F(x) = 0.5\left(1 + \operatorname{erf}\left(\frac{\ln(x) - \ln(x_g)}{\sqrt{2}\ln(\sigma_g)}\right)\right);$$

$$G(r_f, x'_{50}, x) = (1 - r_f)G'(x'_{50}, x) + r_f;$$

$$G'(x'_{50}, x) = 0.5\left(1 + \operatorname{erf}\left(\frac{\ln(x) - \ln(x'_{50})}{\sqrt{2}\ln(\sigma_s)}\right)\right);$$

The following relations hold:

$$(1 - E_T)F_o(x) + E_T F_u(x) = F(x);$$

$$F_o(r_f, x'_{50}, x) = 0.5\,\frac{\operatorname{eceI}(b, z'_{50}, z)}{\operatorname{erfc}(a \cdot z'_{50})};$$

$$F_u(r_f, x'_{50}, x) = 0.5\,\frac{1 + \operatorname{erf}(z) + \widehat{r_f} \cdot \operatorname{eeI}(b, z'_{50}, z)}{1 + \widehat{r_f} \cdot \operatorname{erf}(a \cdot z'_{50})}$$

where

$$a = \frac{\ln(\sigma_s)}{\sqrt{\ln^2(\sigma_g) + \ln^2(\sigma_s)}}; \quad b = \frac{\ln(\sigma_g)}{\ln(\sigma_s)}; \quad \widehat{r_f} = \frac{1 - r_f}{1 + r_f}$$

$$z = \frac{\ln(x) - \ln(x_g)}{\sqrt{2}\ln(\sigma_g)}; \quad z'_{50} = \frac{\ln(x_g) - \ln(x'_{50})}{\sqrt{2}\ln(\sigma_s)}$$

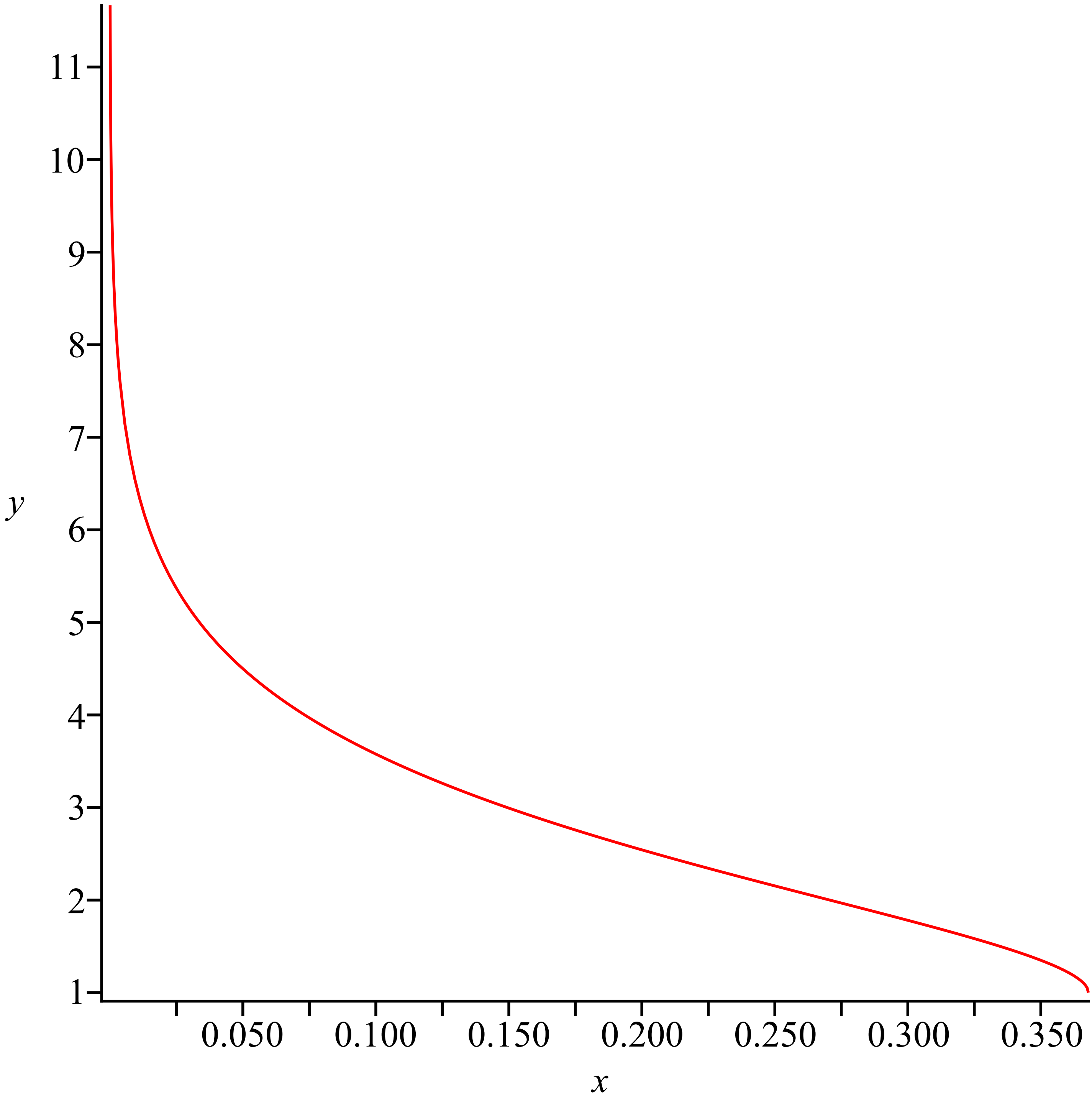

FIGURE 2. $y = \mathrm{expL}_{+}^{\infty}(x)$

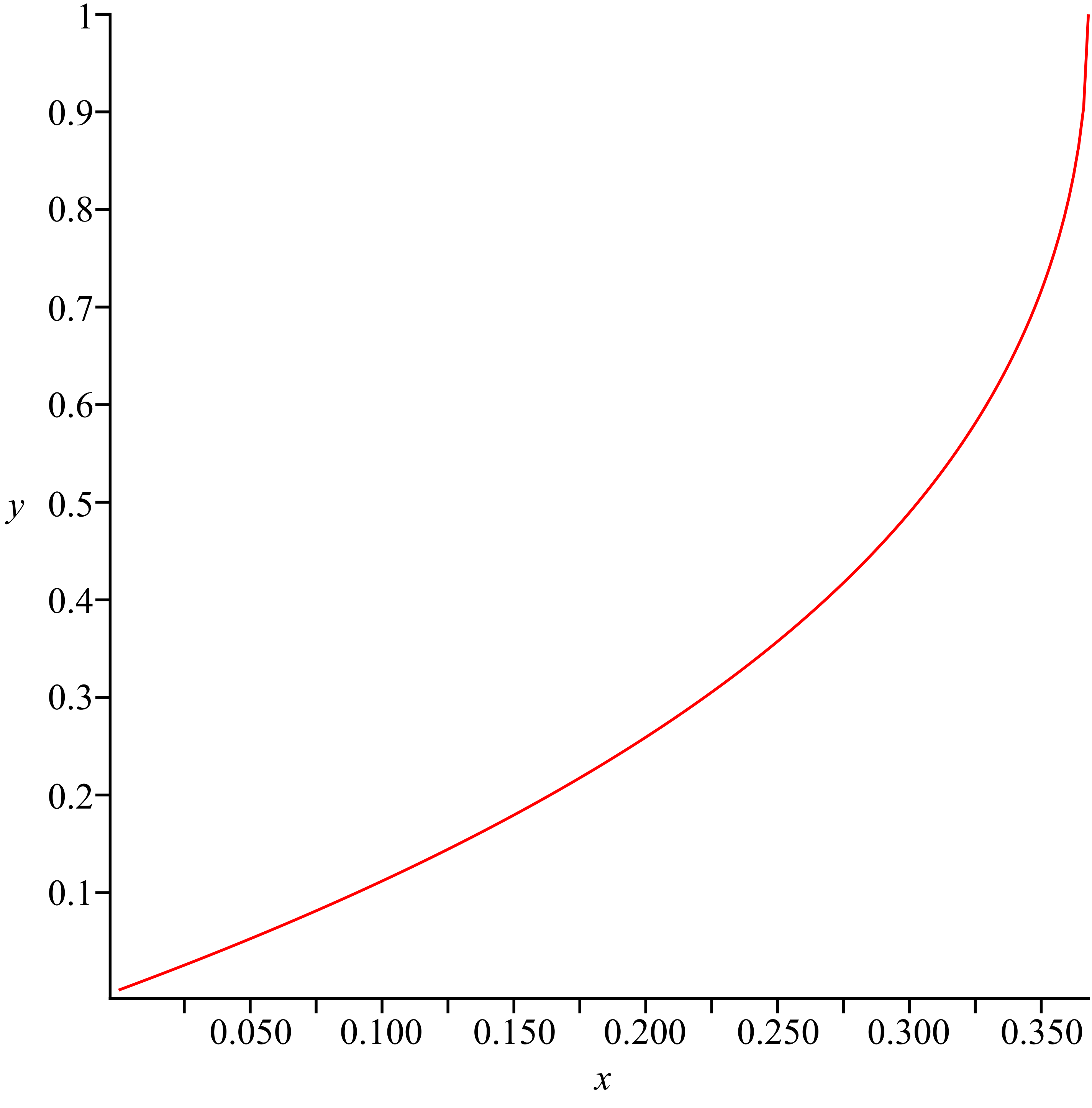

FIGURE 3. $y = \mathrm{expL}_{+}^{0}(x)$

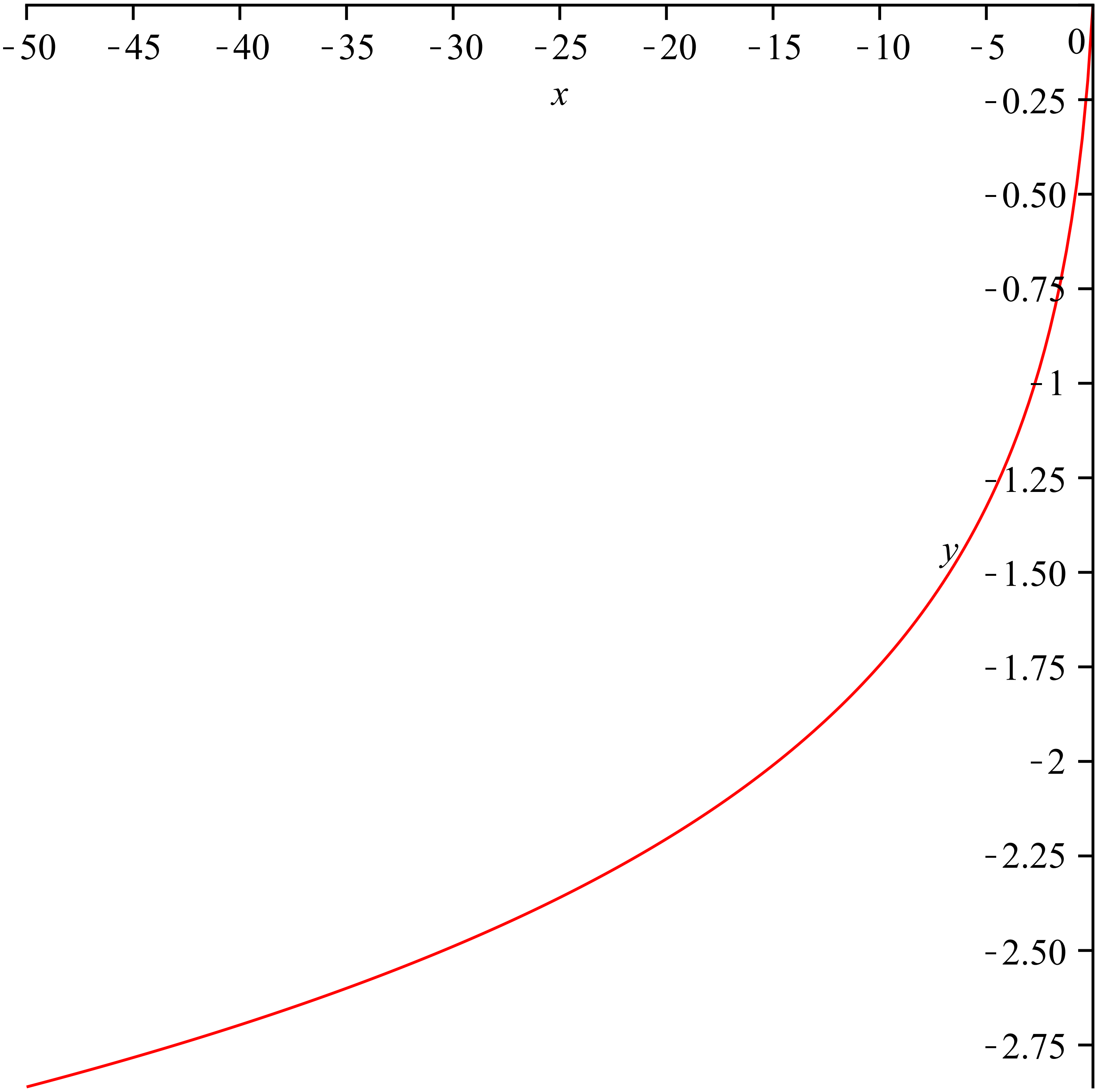

FIGURE 4. $y = \mathrm{expL\_}(x)$

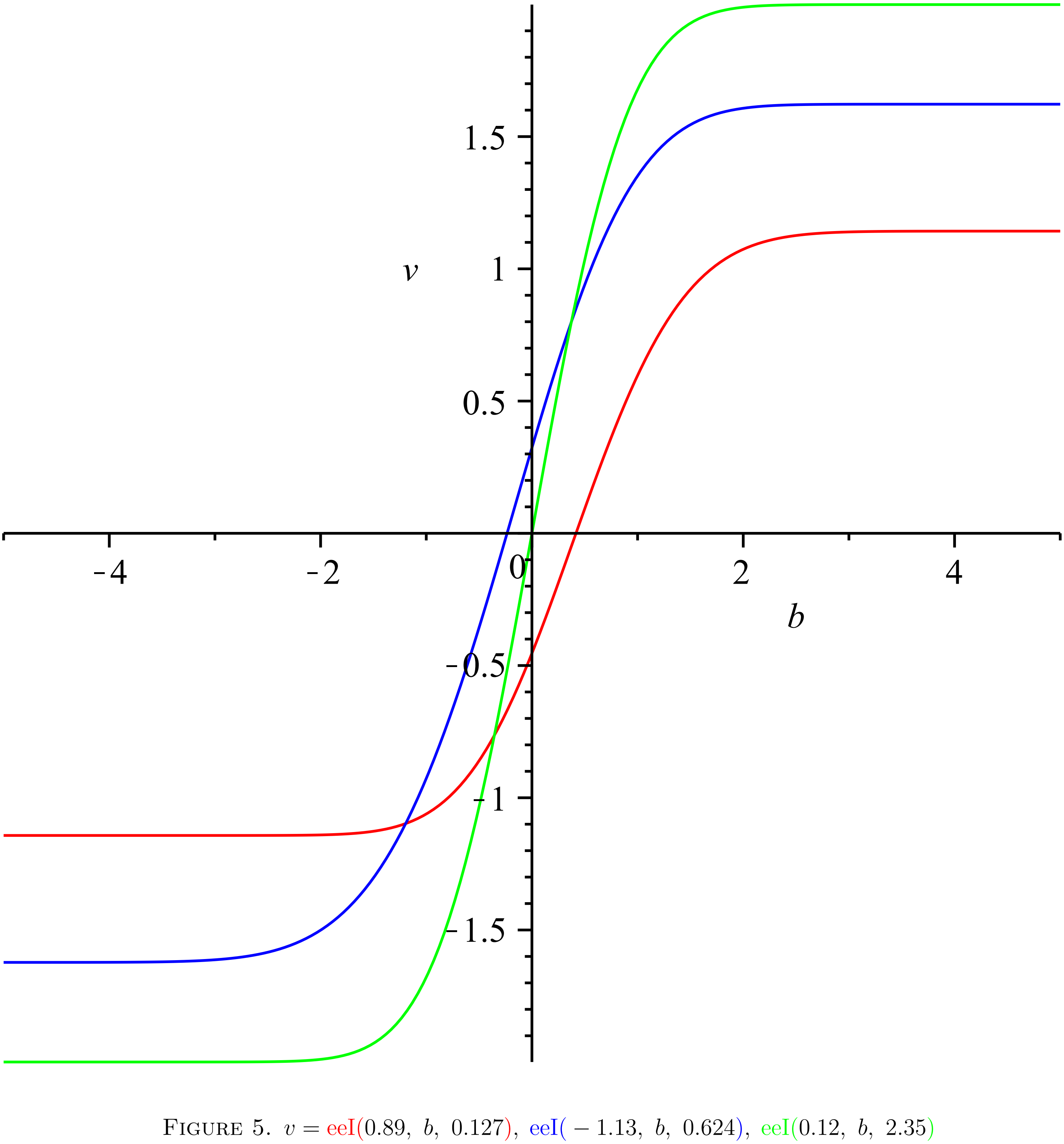

FIGURE 5. $v = \mathrm{eeI}(0.89,\ b,\ 0.127),\ \mathrm{eeI}(-1.13,\ b,\ 0.624),\ \mathrm{eeI}(0.12,\ b,\ 2.35)$

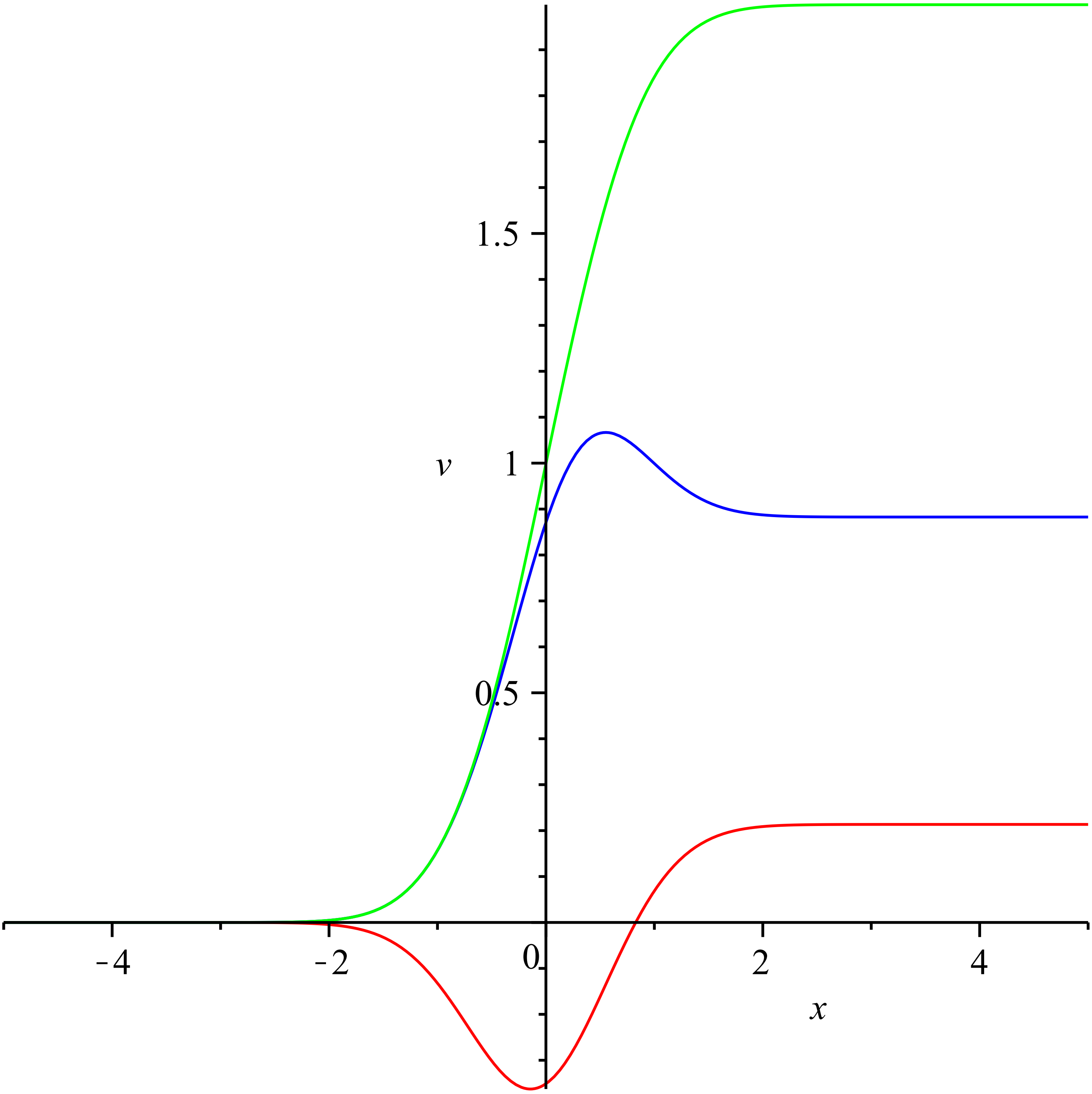

FIGURE 6. $v = \mathrm{eeI}(0.89,\ 0.127,\ x),\ \mathrm{eeI}(-1.13,\ 0.624,\ x),\ \mathrm{eeI}(0.12,\ 2.35,\ x)$