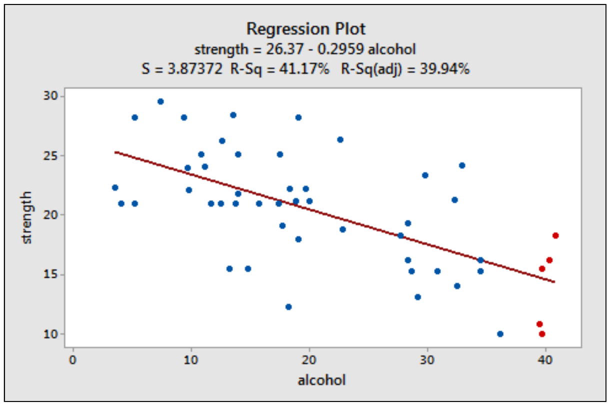
4.2 - Residuals vs. Fits Plot

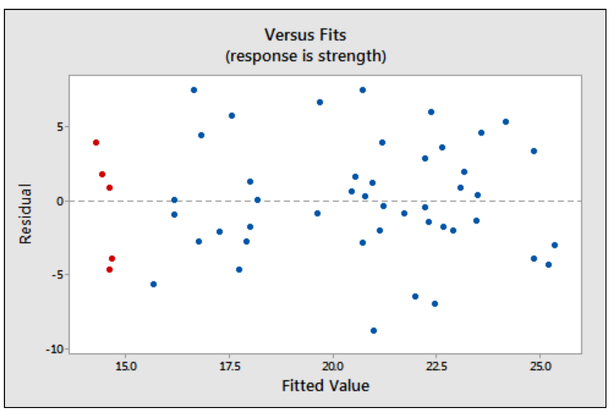
When conducting a residual analysis, a "**residuals versus fits plot**" is the most frequently created plot. It is a scatter plot of residuals on the *y* axis and fitted values (estimated responses) on the *x* axis. The plot is used to detect non-linearity, unequal error variances, and outliers.

Let's look at an example to see what a "well-behaved" residual plot looks like. Some researchers (Urbano-Marquez, *et al.*, 1989) were interested in determining whether or not alcohol consumption was linearly related to muscle strength. The researchers measured the total lifetime consumption of alcohol (x) on a random sample of $n = 50$ alcoholic men. They also measured the strength (y) of the deltoid muscle in each person's nondominant arm. A fitted line plot of the resulting data, (alcoholarm.txt [1]), looks like:



The plot suggests that there is a decreasing linear relationship between alcohol and arm strength. It also suggests that there are no unusual data points in the data set. And, it illustrates that the variation around the estimated regression line is constant suggesting that the assumption of equal error variances is reasonable.

Here's what the corresponding **residuals versus fits plot** looks like for the data set's simple linear regression model with arm strength as the response and level of alcohol consumption as the predictor:



Note that, as defined, the residuals appear on the *y* axis and the fitted values appear on the *x* axis. You should be able to look back at the scatter plot of the data and see how the data points there correspond to the data points in the residual versus fits plot here. In case you're having trouble with doing that, look at the five data points in the original scatter plot that appear in red. Note that the predicted response (fitted value) of these men (whose alcohol consumption is around 40) is about 14. Also, note the pattern in which the five data points deviate from the estimated regression line.

Now look at how and where these five data points appear in the residuals versus fits plot. Their fitted value is about 14 and their deviation from the residual = 0 line shares the same pattern as their deviation from the estimated regression line. Do you see the connection? Any data point that falls directly on the estimated regression line has a residual of 0. Therefore, the residual = 0 line corresponds to the estimated regression line.

This plot is a classical example of a well-behaved residuals vs. fits plot. Here are the characteristics of a well-behaved residual vs. fits plot and what they suggest about the appropriateness of the simple linear regression model:

- The residuals "bounce randomly" around the 0 line. This suggests that the assumption that the relationship is linear is reasonable.
- The residuals roughly form a "horizontal band" around the 0 line. This suggests that the variances of the error terms are equal.
- No one residual "stands out" from the basic random pattern of residuals. This suggests that there are no outliers.

In general, you want your residual vs. fits plots to look something like the above plot. Don't forget though that interpreting these plots is subjective. My experience has been that students learning residual analysis for the first time tend to over-interpret these plots, looking at every twist and turn as something potentially troublesome. You'll especially want to be careful about putting too much weight on residual vs. fits plots based on small data sets. Sometimes the data sets are just too small to make interpretation of a residuals vs. fits plot worthwhile. Don't worry! You will learn — with practice — how to "read" these plots.

PRACTICE PROBLEMS: Residual analysis - the basic idea

The least squares estimate from fitting a line to the data points in residuals.txt [2] are $b_0 = 6$ and $b_1 = 3$. (You can check this claim, of course).

1. Copy the data into, say, columns C1 and C2 of a Minitab worksheet.
2. Using the least squares estimates, create a new column that contains the predicted values, $\hat{y}_i$, for each x_i — you can use Minitab's calculator to do this. Select Calc >> Calculator... In the box labeled "Store result in variable", specify the new column, say C3, where you want the predicted values to appear. In the box labeled Expression, type 6+3*C1. Select OK. The predicted values, $\hat{y}_i$, should appear in column C3. You might want to label this column "fitted." You might also convince yourself that you indeed calculated the predicted values by checking one of the calculations by hand.
3. Now, create a new column, say C4, that contains the residual values — again use Minitab's calculator to do this. Select Calc >> Calculator... In the box labeled "Store result in variable", specify the new column, say C4, where you want the residuals to appear. In the box labeled Expression, type C2-C3. Select OK. The residuals, e_i, should appear in column C4. You might want to label this column "resid." You might also convince yourself that you indeed calculated the residuals by checking one of the calculations by hand.
4. Create a "residuals versus fits" plot, that is, a scatter plot with the residuals (e_i) on the vertical axis and the fitted values ($\hat{y}_i$) on the horizontal axis.(See Minitab Help Section - Creating a basic scatter plot [3]). Around what horizontal line (residuals = ??) do the residuals "bounce randomly?" What does this horizontal line represent?

(CHECK YOUR ANSWER)

Source URL: https://onlinecourses.science.psu.edu/stat501/node/277

Links:
[1] https://onlinecourses.science.psu.edu/stat501/sites/onlinecourses.science.psu.edu.stat501/files/data/alcoholarm.txt
[2] https://onlinecourses.science.psu.edu/stat501/sites/onlinecourses.science.psu.edu.stat501/files/data/residuals.txt
[3] https://onlinecourses.science.psu.edu/stat501/node/115