Estimating labor market power from job applications*

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Abstract

A growing literature on monopsony in labor markets emphasizes idiosyncratic preferences over differentiated jobs as a key source of market power, borrowing tools from industrial organization to estimate firm-level labor supply elasticities. While promising, this discrete choice approach —when applied to job applications data— typically assumes that each job seeker applies to only one job. This assumption is at odds with observed behavior and overlooks how a wage increase affects the supply of applications not only through substitution across jobs, but also through its impact on the number of applications submitted. This paper relaxes that assumption by extending the standard framework to allow for multiple applications in a simultaneous search environment, where uncertainty about job offers induces multiple-application behavior.

1 Introduction

This is not really an introduction. I just moved some incomplete paragraphs from the model section to the introduction because I think these are things that should be discussed here.

Mirroring the case of product markets, the measurement and estimation of labor market power is usually based on the identification of wage markdowns measuring the wedge between wages and the marginal revenue product of labor that results from imperfect competition in the labor market. While a part of the literature follows a direct approach to the estimation of wage markdowns

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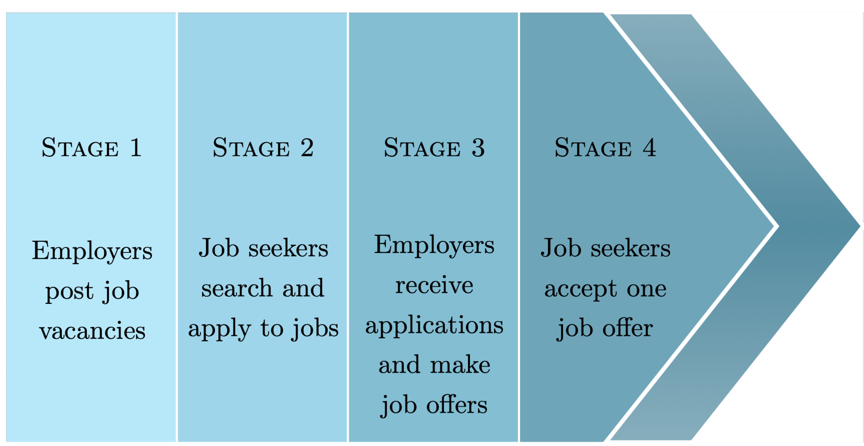
leveraging production function estimation techniques and revenue data (e.g., Brooks et al., 2021; Yeh et al., 2022; Mertens and Mottironi, 2023), this paper follows the tradition of estimating firm-level labor supply elasticities (e.g., Dal Bó et al., 2013; Azar et al., 2022; Roussille and Scuderi, 2025).[1] As discussed by Manning (2003, 2021), the key idea behind monopsony is that the labor supply curve to an individual employer is less than perfectly elastic. The markdown at the firm level is a function —typically the reciprocal— of this elasticity in a broad class of models of monopsony power (Azar and Marinescu, 2024)...

Card et al. (2018) and, more recently, Card (2022) advocate for the adoption of the industrial organization tradition of estimating discrete choice models of demand for differentiated products... incomplete paragraph

As discussed by Azar and Marinescu (2024), the setups of different articles estimating the firm-level labour supply elasticity focus on different parts of the process that determines the firm's level of employment. While it is important to note how this implies the need for a further transformation of the estimates into labour supply elasticities, of equal importance is to note that models that appropriately describe one part of the process might be inappropriate for another. INCLUDE DIAGRAM! CITE AZAR, BERRY, MARINESCU (2022)! CITE HIRSCH ET AL (2022)!... incomplete paragraph

Figure 1: Timing of recruitment process



Notes: Timeline of the recruitment process in a stylized labor market. Traditional discrete choice models that allow at most one job to be chosen are well-suited for stage 4, where workers decide among final job offers (see, e.g., Hirsch et al., 2022). However, when applied to stage 3, as in Azar et al. (2022), these models miss a key feature of the economic environment: under job-offer uncertainty and costly applications, job seekers optimally apply to multiple vacancies. Our framework captures this behavior.

2 A job differentiation model of labor supply

[Intro to model here]

[1] See Manning (2021) and Azar and Marinescu (2024) for an overview of both strands of the literature.

2.1 Risky discrete choice and the job application portfolio problem

Consider the simultaneous search setting studied by Chade and Smith (2006), where each decision maker solves a static portfolio choice problem. Job seeker i faces a finite set $\mathcal{J}$ consisting of $J \equiv |\mathcal{J}|$ job vacancy advertisements and chooses a subset $A_i \subseteq \mathcal{J}$ of vacancies to apply to. The cost of applications, $c_i(n_i)$, depends only on the number of applications, $n_i \equiv |A_i|$, where $c_i : \mathbb{N} \to \mathbb{R}_+$ is increasing and convex with $c_i(0) = 0$. Conditional on applying, the job seeker gets an offer from job j with probability $\alpha_{ij} \in (0, 1]$. Recruitment decisions are independent in the sense that the events $\{j \text{ makes an offer to } i \mid i \text{ applied to } j\}$ and $\{\ell \text{ makes an offer to } i \mid i \text{ applied to } \ell\}$ are independent for $j, \ell \in \mathcal{J}$, $j \neq \ell$. The job seeker can accept at most one offer.

In this setting, each job vacancy represents a risky option, and at most one option will be exercised. Let $j = 0$ represent the outside option, corresponding to either unemployment or the current job if employed. The ex post payoff of exercising option j is represented by Bernoulli utility function $u_i : \mathcal{J} \cup \{0\} \to \mathbb{R}$, with shorthand notation $u_{ij} = u_i(j)$. We rule out weakly dominated (by the outside option) jobs by assuming $u_{ij} \geq u_{i0}$ for al $j \in \mathcal{J}$, implying the job seeker accepts at least one offer, if any.[2] Thus, the outside option is exercised only when either every application in A_i is rejected or no applications are made ($A_i = \emptyset$). Realization of any option in the application portfolio depends on receiving an offer from that job and being rejected by every preferred job application.

Let $r_i : \mathcal{P}(\mathcal{J}) \times \{1, \ldots, J\} \to \mathcal{J}$ identify the k–th most preferred job within portfolio $A \subseteq \mathcal{J}$ by $r_i(A, k) \in A$, with shorthand notation r^A_{ik}. Here, $k \in \{1, \ldots, |A|\}$ and $\mathcal{P}(S)$ denotes the power set of set S. We assume that preferences are strict, meaning $r_i(\cdot, \cdot)$ is indeed a function (as opposed to a correspondence) and $u_{ir_i(\mathcal{J},1)} > \cdots > u_{ir_i(\mathcal{J},J)}$.[3] Each application portfolio $A \subseteq \mathcal{J}$ gives rise to a lottery over state space $\mathcal{J} \cup \{0\}$, where outcomes $j \in \mathcal{J}$ represent exercising option j —i.e., getting the job— and outcome $j = 0$ corresponds to exercising the outside option. The lottery assigns positive probability only to jobs in the application portfolio, $j \in A$, and to the outside option, $j = 0$. As discussed above, option $j \in A$ is exercised if and only if the job seeker (i) receives an offer from job j, and (ii) is rejected by every job in the portfolio that is (ex post) preferred to j. Therefore, if j is ranked in the k–th position among $m \in A$, then the probability of exercising this option is given by

$$p_i(A, j) = \alpha_{ij} \prod_{\ell=1}^{k-1} \left(1 - \alpha_{ir_i(A,\ell)}\right). \tag{1}$$

[2] Chade and Smith (2006) impose the stronger assumption that $\alpha_{ij} u_{ij} - c_i(1) > u_{i0}$ for all $j \in \mathcal{J}$, which further implies that at least one application is made. In contrast, we allow job seekers to make no applications by choosing $A_i = \emptyset$.

[3] Moreover, for fixed $A \subseteq \mathcal{J}$, $r_i(A, \cdot)$ is a bijection from $\{1, \ldots, |A|\}$ to A. This implies the existence of an inverse $r_i^{-1}(A, j)$ that returns the position of alternative $j \in A$ in the ranking of the alternatives in A.

Similarly, the probability of exercising the outside option is[4]

$$p_i(A,0) = \prod_{m\in A} (1-\alpha_{im})\,. \tag{2}$$

Let $U_i : \mathcal{P}(\mathcal{J}) \to \mathbb{R}$ represent the (ex ante) von Neumann–Morgenstern expected utility of the lottery induced by portfolio $A \subseteq \mathcal{J}$ and, without loss of generality, normalize $u_{i0} = 0$. Then, considering the cost of applications —which is incurred in any event—,

$$U_i(A) = \sum_{k=1}^{n} u_{ir_i(A,k)}\, \alpha_{ir_i(A,k)} \prod_{\ell=1}^{k-1} \left(1-\alpha_{ir_i(A,\ell)}\right) - c_i(n), \tag{3}$$

where $n = |A|$ is the size of portfolio A. The resulting utility maximization problem,

$$\max_{A\subseteq\mathcal{J}} U_i(A), \tag{4}$$

is a complex combinatorial optimization problem. In principle, it involves computation and comparison of the expected utilities from the $|\mathcal{P}(\mathcal{J})| = 2^J$ feasible application portfolios that can be chosen from $\mathcal{J}$ (including the empty set $A = \emptyset$). However, Chade and Smith (2006) exploit the downward-recursive structure of this class of portfolio choice problem to show that their marginal improvement algorithm (MIA) efficiently finds the optimal portfolio in $J(J+1)/2 = O(J^2)$ steps. The MIA is a greedy algorithm that starts by identifying the best singleton portfolio, then finds the best alternative to add to the best singleton portfolio to form the best portfolio of size two, and so on until the next best portfolio addition decreases expected utility (see Appendix A for details).

The discrete choice methods typically used in the estimation of demand for differentiated products rely on revealed (or sometimes stated) preference in the sense that the (actual or hypothetical) ex ante choice of alternative j over alternative ℓ truthfully reveals that the decision maker prefers j to ℓ ex post. This is not generally true in our simultaneous search setting. In particular, $j \in A_i$ and $\ell \notin A_i \;\not\Rightarrow\; u_{ij} > u_{i\ell}$. In a special case of this model, however, a revealed-preference structure emerges by imposing the following simplifying assumptions, which are maintained throughout the paper unless stated otherwise.

Assumption 1. Homogeneous admission probabilities: $\alpha_{ij} = \alpha_i \in (0,1), \forall j \in \mathcal{J}$.

Assumption 2. Constant marginal cost of applications: $c_i(|A|) = \gamma_i\,|A|\,, \forall A \subseteq \mathcal{J}$, where $\gamma_i > 0$.

Under Assumption 1, the model retains a sufficient degree of uncertainty to induce job seekers to make multiple applications, while the mechanism preventing preference revelation disappears as

[4] Equation (2) is a special case of Equation (1) since every inside option is preferred to the outside option and the outside option is not risky ($\alpha_{i0} \equiv 1$). It can be verified that these probabilities sum to one over $A \cup \{0\}$.

the order of the (ex ante) expected values of the risky options coincides with the ex-post preference order: $\alpha_i u_{ij} > \alpha_i u_{i\ell} \iff u_{ij} > u_{i\ell}$. Therefore, for any currently available pair j, ℓ such that $u_{ij} > u_{i\ell}$, the MIA will choose j over ℓ for the next optimal portfolio addition in any given iteration, giving portfolio choice the revealed-preference property $j \in A_i$ and $\ell \notin A_i \implies u_{ij} > u_{i\ell}$. This intuitive result follows as a corollary to Lemma 2 of Chade and Smith (2006). Further imposing Assumption 2 yields a stopping rule that determines the size of the optimal portfolio as a function of preferences and the parameters α_i and γ_i. This stopping rule follows directly from the MIA stopping rule. Proposition 1 below formalizes these insights. The resulting choice rule can be combined with an additive random utility model for the ex post utilities $\{u_{ij}\}_{j\in\mathcal{J}}$ to produce a tractable econometric model of portfolio choice.

Proposition 1. *Under Assumptions 1 and 2, the portfolio choice model (3)–(4) reduces to a two-stage choice rule comprising:*

(i) Stopping rule: Determine optimal portfolio size n_i following the rule

$$n_i = \max\left\{\left\{n \in \{1,\dots,J\} : u_{i,r_i(\mathcal{J},n)} \geq \frac{\gamma_i}{\alpha_i\left(1-\alpha_i\right)^{n-1}}\right\} \cup \{0\}\right\}. \tag{5}$$

(ii) Choice of best ex post alternatives: Conditional on optimal portfolio size n_i, choose the optimal portfolio A_i of size n_i by including the alternatives with the n_i highest ex post utilities such that

$$A_i = \left\{r_i(\mathcal{J},1),\dots,r_i(\mathcal{J},n_i)\right\}. \tag{6}$$

Proof. See Appendix B. □

It is easy to see that the two-step choice rule described in Proposition 1 can be equivalently —and more compactly— represented as a one-step rule of the form

$$j \in A_i \iff u_{ij} \geq \frac{\gamma_i}{\alpha_i(1-\alpha_i)^{r_i^{-1}(\mathcal{J},j)-1}}. \tag{7}$$

However, the sequential representation will prove useful in estimation after specifying an additive random utility model. We proceed to discuss this in more detail in Sections 2.2 and 3.2 below.

2.2 An additive random utility model for ex-post job preferences

We complete our model of the supply of applications to the firm by specifying a random utility model (ARUM hereafter) for the Bernoulli utilities $\{u_{ij}\}_{j\in\mathcal{J}}$ representing job seekers' ex-post preferences over the available vacancies. We impose a simple logit preference structure in order to keep the model tractable while cleanly illustrating the mechanisms introduced by uncertainty and the application portfolio problem discussed in Section 2.1. This approach has the advantage of yielding closed-form solutions for the relevant choice probabilities —which are generalizations of the well-known choice probability in the one-application setting—, but at the cost of imposing restrictive assumptions on preference heterogeneity and substitution patterns as a consequence of the independence of irrelevant alternatives (IIA) property. We further discuss these limitations and compare the model to single-application benchmarks in Section 2.3. Our ARUM is as follows.

A finite set $\mathcal{I}$ of job seekers, with size $I \equiv |\mathcal{I}|$, faces the portfolio choice problem described by Equations (3) and (4). The ex post utility that job seeker $i \in \mathcal{I}$ derives from working in job $j \in \mathcal{J}$ takes the additively separable form

$$u_{ij} = \delta_j + \varepsilon_{ij}, \tag{8}$$

where $\delta_j \in \mathbb{R}$ is the deterministic component, or mean utility, and ε_{ij} is a random taste shock representing the idiosyncratic component of ex-post utility. Equation (8) and Assumption 3 below comprise the core of our logit ARUM structure.[5]

Need to continue this paragraph to connect with the next one!

Assumption 3. The idiosyncratic taste shocks ε_{ij} are independent and identically distributed draws from a standard type-1 extreme value distribution, with cumulative distribution function $F_\varepsilon(x) = \exp(-\exp(-x))$ for $x \in \mathbb{R}$.

To simplify notation, let $\mathcal{J} = \{1, \ldots, J\}$ so we can use vector notation for quantities such as $\boldsymbol{\delta} = (\delta_1, \ldots, \delta_J)' \in \mathbb{R}^J$ or $\boldsymbol{\varepsilon}_i = (\varepsilon_{i1}, \ldots, \varepsilon_{iJ})' \in \mathbb{R}^J$.[6] The expected number of applications to job j in our model is given by

$$q_j(\boldsymbol{\delta}) = I \sum_{n=1}^{J} \mathscr{s}_{j|n}(\boldsymbol{\delta}) \mathscr{s}_n(\boldsymbol{\delta}), \tag{9}$$

where

$$\mathscr{s}_{j|n}(\boldsymbol{\delta}) = \sum_{k=J-n}^{J-1} \sum_{A \in \mathcal{R}_k(\mathcal{B}_j)} \frac{\exp(\delta_j)}{\sum_{\ell \in \{j\} \cup A} \exp(\delta_\ell)} + \sum_{s=1}^{|\mathcal{B}_j \setminus A|} (-1)^s \sum_{B \in \mathcal{R}_s(\mathcal{B}_j \setminus A)} \frac{\exp(\delta_j)}{\sum_{\ell \in \{j\} \cup A \cup B} \exp(\delta_\ell)} \tag{10}$$

is the probability that job j belongs to the application portfolio conditional on portfolio size, $\mathbb{P}(j \in A_i \mid n_i = n)$ for $n \in \{1, \ldots, J\}$, $\mathcal{B}_j \equiv \mathcal{J} \setminus \{j\}$ is the set of jobs excluding j, and $\mathcal{R}_k(S) =$

[5] It is possible, at the risk of reduced tractability, to derive richer, more flexible models by combining a more general structure for Equation (8) with different distributional assumptions in place of Assumption 3. Such generalizations are out of the scope of this paper and are thus left for future research. See, for example, Section 3 of Berry and Haile (2021), Chapters 2–6 of Train (2009), or Chapter 2 of Aguirregabiria (2021) for detailed discussions in the setting where only one alternative is selected.

[6] Alternatively, fix a bijection $j : \mathcal{J} \to \{1, \ldots, J\}$ such that $\boldsymbol{\delta} = (\delta_{j^{-1}(1)}, \ldots, \delta_{j^{-1}(J)})'$ is simply the permutation of $\{\delta_j\}_{j \in \mathcal{J}}$ induced by $j(\cdot)$. So far, we have left the nature of job identities $\mathcal{J}$ unspecified for clarity when defining mappings from jobs to rankings of jobs. It will be useful to work with vectors in what follows, so it is convenient to fix an ordering of $\mathcal{J}$.

$\{\sigma \subseteq S : |S| = k\}$ is the set of all size-k subsets of set S.[7] The conditional probability mass function (pmf) of portfolio size n_i —i.e., the number of applications— given admission probability α_i and marginal cost of application γ_i, $\mathbb{P}(n_i = n \mid \alpha_i, \gamma_i)$, is

$$
\begin{aligned}
\mathscr{s}_{n|\alpha,\gamma}(\boldsymbol{\delta}, \alpha_i, \gamma_i) = \sum_{s=0}^{J} \sum_{A \in \mathcal{R}_s(\mathcal{J})} \prod_{\ell \in A} \left[F_\varepsilon\big(\psi_i^{n+1}\big)^{\exp(\delta_\ell)} - F_\varepsilon\big(\psi_i^{n}\big)^{\exp(\delta_\ell)} \right] \\
\sum_{t \in \tau_n^s} \sum_{B \in \mathcal{R}_t(\mathcal{J} \setminus A)} F_\varepsilon\big(\psi_i^{n}\big)^{\sum_{p \in B} \exp(\delta_p)} \prod_{q \in \mathcal{J} \setminus (A \cup B)} \left[1 - F_\varepsilon\big(\psi_i^{n+1}\big)^{\exp(\delta_q)} \right]
\end{aligned}
\tag{11}
$$

for $n \in \{1, \ldots, J-1\}$, where $\tau_n^s = \{\max(J-n-s, 0), \ldots, \min(J-n, J-s)\}$ is a set of consecutive natural numbers, and

$$
\psi_i^n = \frac{\gamma_i}{\alpha_i(1-\alpha_i)^{n-1}}, \quad n \in \{1, \ldots, J\}
\tag{12}
$$

is shorthand for the thresholds in part (i) of Proposition 1. For the extreme cases $n = 0$ and $n = J$, the conditional pmf is

$$
\mathscr{s}_{0|\alpha,\gamma}(\boldsymbol{\delta}, \alpha_i, \gamma_i) = F_\varepsilon\big(\psi_i^1\big)^{\sum_{\ell \in \mathcal{J}} \exp(\delta_\ell)}
\tag{13}
$$

and

$$
\mathscr{s}_{J|\alpha,\gamma}(\boldsymbol{\delta}, \alpha_i, \gamma_i) = 1 - \sum_{k=1}^{J} \sum_{A \in \mathcal{R}_k(\mathcal{J})} F_\varepsilon\big(\psi_i^J\big)^{\sum_{\ell \in A} \exp(\delta_\ell)} \prod_{m \in \mathcal{J} \setminus A} \left[1 - F_\varepsilon\big(\psi_i^J\big)^{\exp(\delta_m)} \right],
\tag{14}
$$

respectively. The corresponding unconditional pmf, $\mathbb{P}(n_i = n)$ for $n \in \{0, \ldots, J\}$, is

$$
\mathscr{s}_n(\boldsymbol{\delta}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathscr{s}_{n|\alpha,\gamma}(\boldsymbol{\delta}, \alpha_i, \gamma_i) \mathrm{d}F_\alpha(\alpha_i) \mathrm{d}F_\gamma(\gamma_i).
\tag{15}
$$

See Appendix C for a full derivation.

The elasticity of the supply of applications to job $j \in \mathcal{J}$ with respect to the wage of vacancy $\ell \in \mathcal{J}$ answers the question "*If the wage offered by job ℓ increases by one percent, what is the percent increase in the number of applications to job j?*" and is given by

$$
\eta_{q_j, w_\ell} = \frac{1}{q_j(\boldsymbol{\delta})} \left[I \sum_{n=1}^{J} \frac{\partial \mathscr{s}_{j|n}(\boldsymbol{\delta})}{\partial \delta_\ell} \mathscr{s}_n(\boldsymbol{\delta}) + \mathscr{s}_{j|n}(\boldsymbol{\delta}) \frac{\partial \mathscr{s}_n(\boldsymbol{\delta})}{\partial \delta_\ell} \right] \beta.
\tag{16}
$$

Our object of interest is the own-wage elasticity of the supply of applications to the firm. This quantity answers the question "*If the firm raises the wages it offers for all its vacancies by one*

[7]Note that (i) $\mathscr{s}_{j|J}(\boldsymbol{\delta}) = 1$, consistent with the trivial fact that $\mathbb{P}(j \in A_i \mid n_i = J) = 1$; (ii) $\mathscr{s}_{j|1}(\boldsymbol{\delta}) = \exp(\delta_j) / \sum_{\ell \in \mathcal{J}} \exp(\delta_\ell)$ coincides with the well-known choice probability in the multinomial logit model; (iii) $\sum_{j \in \mathcal{J}} \mathscr{s}_{j|n}(\boldsymbol{\delta}) = n$, consistent with the conditioning event that n alternatives are chosen; and (iv) $\mathscr{s}_{j|n}(\boldsymbol{\delta})$ increases monotonically with n, consistent with the fact that, for any job seeker, the n most preferred alternatives include the $n-1$ most preferred alternatives.

percent, what is the percent increase in the total number of applications it receives?". Since the total number of applications to firm $f \in \mathcal{F}$ posting job vacancies $\mathcal{J}^f$,

$$q^f(\boldsymbol{\delta}) = \sum_{j \in \mathcal{J}^f} q_j(\boldsymbol{\delta}), \tag{17}$$

is simply the sum of the supply of applications to each of its posted vacancies, its elasticity is a weighted average of the corresponding vacancy-level elasticities:

$$\eta_{q^f, \boldsymbol{w}^f} = \frac{1}{q^f(\boldsymbol{\delta})} \sum_{\ell \in \mathcal{J}^f} \sum_{j \in \mathcal{J}^f} q_j(\boldsymbol{\delta}) \eta_{q_j, w_\ell}. \tag{18}$$

See Appendix D.1 for a derivation of the vacancy- and firm-level elasticities, and Appendix D.2 for closed-form expressions for the partial derivatives of $\mathscr{s}_{j|n}(\cdot)$ and $\mathscr{s}_n(\cdot)$ —up to integration over $F_\alpha(\cdot) \times F_\gamma(\cdot)$.

2.3 Implications and limitations

Differences with single-application models. Having developed the model and derived the implied supply of applications and own-wage elasticity, we now compare our framework to single-application benchmarks. These comparisons clarify the mechanisms driving differences in application behavior and wage elasticities.

The textbook multinomial logit (MNL) model assigns an idiosyncratic taste shock to the outside option. In contrast, our framework treats the outside option deterministically and assumes that all considered vacancies are at least as attractive as the status quo. Our treatment of the outside option is more natural in the context of job applications: rational job seekers would never consider applying to vacancies that are worse than their current position, be it a job or unemployment. To illustrate the implications of allowing multiple applications, we benchmark our model against two natural alternatives: (i) an MNL model with a deterministic, ex-post dominated outside option, and (ii) a restricted version of our model in which job seekers can submit at most one application. Comparing these models highlights how portfolio choice affects both the expected number of applications per vacancy and the implied wage elasticities.

Baseline model. To facilitate cleaner comparisons with single-application benchmarks, we focus on a simplified version of our model in which we abstract away from job-seeker heterogeneity by setting $\alpha_i = \alpha \in (0, 1)$ and $\gamma_i = \gamma > 0$ for all $i \in \mathcal{I}$. Under these degenerate distributions for the uncertainty and cost parameters, the unconditional pmf of the number of applications per job seeker in Equation (15) coincides with the conditional pmf in Equations (11), (13) and (14). In this case, the thresholds in Equation (12) simplify to

$$\psi^n = \frac{\gamma}{\alpha(1-\alpha)^{n-1}}.$$

The expected number of applications received by each job vacancy and the conditional application shares are then given by Equations (9) and (10), respectively. We use this baseline model as the point of comparison for the deterministic-outside option MNL benchmark and the restricted single-application version of our framework introduced below.

Deterministic-outside MNL benchmark. As a first benchmark, we derive an MNL model with a deterministic, ex-post dominated outside option. Intuitively, this corresponds to a setting where job seekers face no job-offer uncertainty ($\alpha = 1$) and therefore never apply to more than one job. The resulting model preserves the deterministic treatment of the outside option but shuts down the portfolio-choice mechanism entirely. Formally, we establish in Lemma 1 below that setting $\alpha = 1$ in the baseline model produces an MNL model where the outside option, with a deterministic utility normalized to 0, is chosen only when application costs are too high. Conditional on applying, the choice among the inside options is standard MNL.[8]

Lemma 1. *When $\alpha_i = 1$ and $c_i(|A|) = \gamma\,|A| > 0$ for all $i \in \mathcal{I}$, the additive random utility model of portfolio choice in Equations (3), (4) and (8) with extreme value type 1 independent and identically distributed random taste shocks $\{\varepsilon_{ij}\}_{j\in\mathcal{J}}$ collapses to a model where:*

(i) The optimal portfolio A_i is either a singleton or the empty set:

$$n_i \equiv |A_i| \in \{0, 1\}\,.$$

(ii) Job seekers choose not to apply only when applications are too costly, with probability

$$\mathcal{s}_0^{(i)}(\boldsymbol{\delta}) = F_\varepsilon\Bigg(\gamma - \ln\Bigg(\sum_{\ell\in\mathcal{J}} \exp\left(\delta_\ell\right)\Bigg)\Bigg) = \exp\Bigg(-\exp\left(-\gamma\right)\sum_{\ell\in\mathcal{J}}\exp\left(\delta_\ell\right)\Bigg).$$

(iii) Conditional on application —i.e., $n_i = 1$—, the expected share of applications to job $j \in \mathcal{J}$ takes the standard logit form

$$\mathcal{s}_{j|1}^{(i)}(\boldsymbol{\delta}) = \frac{\exp\left(\delta_j\right)}{\sum_{\ell\in\mathcal{J}}\exp\left(\delta_\ell\right)}.$$

Proof. See Appendix D.3. ☐

The expected number of applications to job j in this benchmark model is given by

Need to complete this benchmarking exercise!

[8] A model resembling the textbook MNL with random outside utility can be obtained by setting $\alpha_i = 1$ for all $i \in \mathcal{I}$, taking $F_\gamma(\cdot) = F_\varepsilon(\cdot)$, and letting $\varepsilon_{i0} \equiv \gamma_i \overset{iid}{\sim} \mathrm{EV}_1$. Since the EV_1 distribution has support $\mathbb{R}$, this requires relaxing the convexity assumption on application costs and, under a linear cost $c_i(|A|)$ as in Assumption 2, some job seekers draw negative costs. For those with $\gamma_i > 0$, applications are costly and the model collapses to a standard MNL in which each job seeker applies to at most one job: $A_i = \{j_i^*\}$, where $j_i^* = \arg\max_{j\in\mathcal{J}\cup\{0\}}\ u_{ij} - \mathbb{1}\left[j \in \mathcal{J}\right]\gamma_i = \arg\max_{j\in\mathcal{J}\cup\{0\}}\ \widetilde{u}_{ij}$, $\widetilde{u}_{ij} = \delta_j + \varepsilon_{ij}$, and $\delta_0 = 0$. However, for job seekers with $\gamma_i \leq 0$, applications are (weakly) subsidized and the optimal choice is $A_i = \mathcal{J}$, meaning they apply to all vacancies even when only one will be exercised. Thus, the textbook MNL emerges as a special case for job seekers with positive application costs, but the equivalence is only partial due to the behavior of those with $\gamma_i \leq 0$.

$$q_j^{(i)}(\boldsymbol{\delta}) = I\, s_{j|1}^{(i)}(\boldsymbol{\delta}) \left[1 - s_0^{(i)}(\boldsymbol{\delta})\right] \tag{19}$$

Single-application benchmark.

[Second benchmark model here]

Limitations.

[Discussion of limitations here]

3 An empirical application: Online job applications

[Intro section to empirical application here]

3.1 Chilean job board data

[Data description here]

3.2 Estimation strategy

[Estimation strategy here]

Full derivation of minorize-maximize algorithm in Appendix E

3.3 Empirical results

[Results and discussion here]

4 Conclusion

[Conclusion here]

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Appendix

A Marginal improvement algorithm

This appendix describes the Chade and Smith (2006) marginal improvement algorithm (MIA) within the context and notation of Section 2.1. The MIA is a greedy algorithm in the sense that it makes a locally optimal choice in each iteration. Despite its greedy nature, it converges to the global optimum, as shown by Chade and Smith (2006).

Consider the portfolio choice problem described by Equations (3) and (4). The MIA follows the following iterative procedure to find the optimal portfolio $A_i = \arg\max_{A\in\mathcal{P}(\mathcal{J})} U_i(A)$. Let $\Lambda_0 = \emptyset$. At iteration $t \in \{1, \ldots, J\}$:

- Step 1: Choose any $j_t \in \underset{j\in\mathcal{J}\setminus\Lambda_{t-1}}{\arg\max}\, U_i(\Lambda_{t-1} \cup \{j\})$.
- Step 2: Stop if $U_i(\Lambda_{t-1} \cup \{j_t\}) - U_i(\Lambda_{t-1}) < 0$.
- Step 3: Set $\Lambda_t = \Lambda_{t-1} \cup \{j_t\}$ and go to step 1 for the next iteration.

The algorithm will stop at iteration $t = \min(n_i + 1, J)$, where $n_i \equiv |A_i| \leq J$, identifying A_i.

B Proof of Proposition 1

Proof. Consider the portfolio choice problem (3)–(4). Let us start by showing that Assumption 1 implies that, conditional on $|A_i| = n$ —where $A_i = \arg\max_{A\in\mathcal{P}(\mathcal{J})} U_i(A)$—, A_i consists of the n (ex-post) best alternatives. This can be established by induction.

Consider iteration $t = 1$ of the marginal improvement algorithm (MIA) described in Appendix A. The best singleton portfolio must be the best ex post alternative since the order of expected values $\{\alpha_i u_{ij}\}_{j\in\mathcal{J}}$ coincides with the order of ex post utilities $\{u_{ij}\}_{j\in\mathcal{J}}$. Formally,

$$\begin{aligned}
\underset{j\in\mathcal{J}\setminus\Lambda_0}{\arg\max}\, U_i(\Lambda_0 \cup \{j\}) &= \underset{j\in\mathcal{J}}{\arg\max}\, U_i(\{j\}) \\
&= \underset{j\in\mathcal{J}}{\arg\max}\, \alpha_i u_{ij} - c_i(1) \\
&= \underset{j\in\mathcal{J}}{\arg\max}\, u_{ij} \\
&= \Big\{ r_i(\mathcal{J}, 1) \Big\},
\end{aligned}$$

where the first equality follows from $\Lambda_0 = \emptyset$, the second equality follows by direct evaluation of (3) at $A = \{j\}$, the third equality follows because quantities $\alpha_i > 0$ and $c_i(1)$ do not vary with j, and the last equality follows from the definition of the ranking function $r_i(\cdot, \cdot)$.

Next, consider iteration $t > 1$ and suppose that

$$\Lambda_{t-1} = \Big\{ r_i(\mathcal{J}, 1), \ldots, r_i(\mathcal{J}, t-1) \Big\}, \tag{B.1}$$

i.e., the MIA-optimal portfolio of size $t-1$ consists of the $t-1$ (ex-post) best alternatives. The induction hypothesis (B.1) implies that any alternative still available for selection by the MIA must be ranked higher —i.e., worse— than all the alternatives the MIA has already selected in previous iterations. That is, for all $j \in \mathcal{J} \setminus \Lambda_{t-1}$ and $\ell \in \Lambda_{t-1}$,[9]

$$r_i^{-1}(\mathcal{J}, j) > r_i^{-1}(\mathcal{J}, \ell). \tag{B.2}$$

Moreover, the ranking order over Λ_{t-1} must obviously coincide with the first $t-1$ positions of the ranking order over $\mathcal{J}$, i.e.,

$$r_i(\Lambda_{t-1}, k) = r_i(\mathcal{J}, k) \tag{B.3}$$

for all $k \in \{1, \ldots, t-1\}$. It follows that the MIA-optimal addition to Λ_{t-1} in iteration t must be $r_i(\mathcal{J}, t)$ since

$$\begin{aligned}
\operatorname*{arg\,max}_{j \in \mathcal{J} \setminus \Lambda_{t-1}} U_i(\Lambda_{t-1} \cup \{j\}) &= \operatorname*{arg\,max}_{j \in \mathcal{J} \setminus \{r_i(\mathcal{J},k)\}_{k=1}^{t-1}} \alpha_i \left[\sum_{k=1}^{t-1} (1-\alpha_i)^{k-1} u_{i r_i(\mathcal{J},k)} + (1-\alpha_i)^{t-1} u_{ij} \right] - c_i(t) \\
&= \operatorname*{arg\,max}_{j \in \mathcal{J} \setminus \{r_i(\mathcal{J},k)\}_{k=1}^{t-1}} u_{ij} \\
&= \Big\{ r_i(\mathcal{J}, t) \Big\},
\end{aligned}$$

where the first equality follows from (B.2)–(B.3) and direct evaluation of (3) at $\Lambda_{t-1} \cup \{j\}$ under Assumption 1, the second equality follows by discarding all (non-negative when appropriate) quantities that do not vary with j, and the last equality follows from the definition of the ranking function. Since $t > 1$ is arbitrary and we have proved the induction hypothesis holds for $t = 1$, the principle of mathematical induction establishes part (ii) of Proposition 1.

Part (i) of Proposition 1 follows directly from the stopping rule in step 2 of the MIA under Assumptions 1 and 2 by noting that, by part (ii) of the proposition, the optimal portfolio size n_i is also the position in the ranking over $\mathcal{J}$ of the last chosen alternative. This means $r_i(\mathcal{J}, n_i)$ is the last alternative the MIA picks up. Hence, the optimal portfolio contains n_i alternatives if and only if (a) the MIA does not stop in step 2 of iteration n_i, and (b) either $n_i = J$ or the MIA stops in step 2 of iteration $n_i + 1$.

[9] An equivalent statement to (B.2) is $u_{ij} < u_{i\ell}$, but the former expression highlights the order of alternatives that determines the relevant lottery whose expected utility the MIA maximizes in iteration t.

From (a), we obtain

$$
\begin{aligned}
0 &\leq U_i(\Lambda_{n_i-1} \cup \{r_i(\mathcal{J}, n_i)\}) - U_i(\Lambda_{n_i-1}) \\
&= \alpha_i \sum_{k=1}^{n_i-1} (1-\alpha_i)^{k-1} u_{ir_i(\mathcal{J},k)} + \alpha_i(1-\alpha_i)^{n_i-1} u_{ir_i(\mathcal{J},n_i)} - \gamma_i n_i \\
&\qquad - \left[\alpha_i \sum_{k=1}^{n_i-1} (1-\alpha_i)^{k-1} u_{ir_i(\mathcal{J},k)} - \gamma_i(n_i-1)\right] \\
&= \alpha_i(1-\alpha_i)^{n_i-1} u_{ir_i(\mathcal{J},n_i)} - \gamma_i,
\end{aligned}
$$

which holds if and only if

$$
u_{ir_i(\mathcal{J},n_i)} \geq \frac{\gamma_i}{\alpha_i(1-\alpha_i)^{n_i-1}}. \tag{B.4}
$$

Similarly from (b), either $n_i = J$ or

$$
\begin{aligned}
0 &> U_i(\Lambda_{n_i} \cup \{r_i(\mathcal{J}, n_i+1)\}) - U_i(\Lambda_{n_i}) \\
&= \alpha_i \sum_{k=1}^{n_i} (1-\alpha_i)^{k-1} u_{ir_i(\mathcal{J},k)} + \alpha_i(1-\alpha_i)^{n_i} u_{ir_i(\mathcal{J},n_i+1)} - \gamma_i(n_i+1) \\
&\qquad - \left[\alpha_i \sum_{k=1}^{n_i} (1-\alpha_i)^{k-1} u_{ir_i(\mathcal{J},k)} - \gamma_i n_i\right] \\
&= \alpha_i(1-\alpha_i)^{n_i} u_{ir_i(\mathcal{J},n_i+1)} - \gamma_i,
\end{aligned}
$$

which holds if and only if

$$
u_{ir_i(\mathcal{J},n_i+1)} < \frac{\gamma_i}{\alpha_i(1-\alpha_i)^{n_i}}. \tag{B.5}
$$

Finally, note that the following monotonicity properties must hold.

$$
u_{ir_i(\mathcal{J},k)} \geq \frac{\gamma_i}{\alpha_i(1-\alpha_i)^{k-1}}, \forall k \in \{1, \ldots, n_i - 1\}, \tag{B.6}
$$

$$
u_{ir_i(\mathcal{J},k)} < \frac{\gamma_i}{\alpha_i(1-\alpha_i)^{k-1}}, \forall k \in \{n_i + 2, \ldots, J\}, \tag{B.7}
$$

where $\{n_i + 2, \ldots, J\} \equiv \emptyset$ for $n_i \geq J - 1$. Suppose (B.6) does not hold, so $u_{ir_i(\mathcal{J},k)} < \gamma_i \alpha_i^{-1}(1 - \alpha_i)^{-(k-1)}$ for some $k \in \{1, \ldots, n_i - 1\}$. Then, we get the contradiction

$$
u_{ir^{\mathcal{J}}_{ik}} > u_{ir^{\mathcal{J}}_{in_i}} \geq \frac{\gamma_i}{\alpha_i(1-\alpha_i)^{n_i-1}} = (1-\alpha_i)^{k-n_i} \frac{\gamma_i}{\alpha_i(1-\alpha_i)^{k-1}} > (1-\alpha_i)^{k-n_i} u_{ir^{\mathcal{J}}_{ik}} > u_{ir^{\mathcal{J}}_{ik}}
$$

since $k < n_i$ and $\alpha_i \in (0,1) \implies (1-\alpha_i)^{k-n_i} > 1$. Similarly, suppose (B.7) does not hold, so $u_{ir_i(\mathcal{J},k)} \geq \gamma_i \alpha_i^{-1}(1-\alpha_i)^{-(k-1)}$ for some $k \in \{n_i + 2, \ldots, J\}$. Then, we get the contradiction

$$
u_{ir^{\mathcal{J}}_{ik}} < u_{ir^{\mathcal{J}}_{in_i+1}} < \frac{\gamma_i}{\alpha_i(1-\alpha_i)^{n_i}} = (1-\alpha_i)^{k-(n_i+1)} \frac{\gamma_i}{\alpha_i(1-\alpha_i)^{k-1}} \leq (1-\alpha_i)^{k-(n_i+1)} u_{ir^{\mathcal{J}}_{ik}} < u_{ir^{\mathcal{J}}_{ik}}
$$

since $k > n_i + 1$ and $\alpha_i \in (0,1) \implies (1-\alpha_i)^{k-(n_i+1)} < 1$. Together, (B.4)–(B.7) establish part (i) of Proposition 1. $\square$

C Derivation of the job applications supply function

This appendix provides a full derivation of the applications supply function, the conditional applications share function, and the probability mass function (pmf) of the number of applications in Equations (9), (10) and (15), respectively. Given a finite set of job seekers, $\mathcal{I}$ with $|\mathcal{I}| \equiv I$, facing the portfolio choice problem (3)–(4) over applications to a finite set of jobs, $\mathcal{J}$ with $|\mathcal{J}| \equiv J$, the expected number of applications to job $j \in \mathcal{J}$ is

$$\begin{aligned}
\mathbb{E}[q_j] &= \mathbb{E}\left[\sum_{i\in\mathcal{I}} \mathbb{1}\,[j \in A_i]\right] \\
&= \sum_{i\in\mathcal{I}} \mathbb{E}\big[\mathbb{1}\,[j \in A_i]\,\big] \\
&= I\,\mathbb{P}(j \in A_i) \\
&= I \sum_{n=1}^{J} \mathbb{P}(j \in A_i \mid n_i = n)\,\mathbb{P}(n_i = n)\,. \qquad \text{(C.1)}
\end{aligned}$$

Our model defines (i) a mapping $s_{j|n}(\boldsymbol{\delta})$ from $\boldsymbol{\delta}$ to $\mathbb{P}(j \in A_i \mid n_i = n)$, and (ii) a mapping $s_n(\boldsymbol{\delta})$ from $\boldsymbol{\delta}$ and the joint distribution of parameters (α_i, γ_i) to $\mathbb{P}(n_i = n)$. These mappings follow directly from parts (ii) and (i) of Proposition 1, respectively.

C.1 Conditional applications share function

Consider first the conditional (expected) applications share function $s_{j|n}(\boldsymbol{\delta})$. The probability that j belongs to the application portfolio conditional on the job seeker applying to every job is trivially $\mathbb{P}(j \in A_i \mid n_i = J) = 1$. For $n \in \{1, \dots, J-1\}$, the probability that job j belongs to the application portfolio conditional on the job seeker applying to n jobs is the probability that the ex post utility of job seeker i from job j is larger than the ex post utility from their $(n+1)$–th most preferred alternative, i.e., $\mathbb{P}(j \in A_i \mid n_i = n) = \mathbb{P}\left(u_{ij} > u_{ir_i(\mathcal{J},n+1)}\right)$. This is true since job seeker i applies to job j if and only if j is among i's $n_i = n$ most preferred alternatives. We can derive the expression for $\mathbb{P}\left(u_{ij} > u_{ir_i(\mathcal{J},n+1)}\right)$ as a function of $\boldsymbol{\delta}$ —and, obviously, of j and n, which we indicate by the subscript in $s_{j|n}(\boldsymbol{\delta})$— defined by our model by applying a well-established result from the literature on order statistics.

Let $\{u_{i(n)}\}_{n=1}^{J}$ represent the order statistics of $\{u_{ij}\}_{j\in\mathcal{J}}$ such that $u_{i(1)} < \cdots < u_{i(J)}$, and note that

$$u_{ir_i(\mathcal{J},n+1)} = u_{i(J-n)} \qquad \text{(C.2)}$$

for all $n \in \{1, \dots, J-1\}$. Similarly, let $\mathcal{B}_j \equiv \mathcal{J} \setminus \{j\}$ represent the leave-out set of available jobs excluding j, and $\{u^j_{i(n)}\}_{n=1}^{J-1}$ the order statistics of $\{u_{i\ell}\}_{\ell\in\mathcal{B}_j}$ such that $u^j_{i(1)} < \cdots < u^j_{i(J-1)}$.

Notice that "j `is among the best` n `jobs in` $\mathcal{J}$" if and only if "j `is better than the` $J-n$ `worse jobs in` $\mathcal{J}$" if and only if "j `is better than the` $J-n$ `worse jobs in` $\mathcal{B}_j$" for any $n \in \{1, \ldots, J-1\}$. The mutual independence of $\{u_{i\ell}\}_{\ell \in \mathcal{J}}$ implies that $u^j_{i(n)}$ is independent of u_{ij} for all $n \in \{1, \ldots, J-1\}$.

The *iid* assumption on $\{\varepsilon_{ij}\}_{j \in \mathcal{J}}$ implies that the ex post utilities $\{u_{ij}\}_{j \in \mathcal{J}}$ are independently but non-identically distributed with cumulative distribution function (cdf)

$$\begin{aligned} F_{u_j}(x) &\equiv \mathbb{P}\left(u_{ij} \leq x\right) \\ &= \mathbb{P}\left(\varepsilon_{ij} \leq x - \delta_j\right) \\ &= F_\varepsilon(x - \delta_j), \end{aligned} \tag{C.3}$$

where $F_\varepsilon(\cdot)$ is the marginal cdf of ε_{ij}. The cdf of the n–th order statistic $u^j_{i(n)}$ is then given by (see, e.g., David and Nagaraja, 2003, p. 96)

$$\begin{aligned} F_{u^j_{(n)}}(x) &= \sum_{k=n}^{J-1} \sum_{A \in \mathcal{R}_k(\mathcal{B}_j)} \prod_{\ell \in A} F_{u_\ell}(x) \prod_{m \in \mathcal{B}_j \setminus A} \left[1 - F_{u_m}(x)\right] \\ &= \sum_{k=n}^{J-1} \sum_{A \in \mathcal{R}_k(\mathcal{B}_j)} \prod_{\ell \in A} F_\varepsilon(x - \delta_\ell) \prod_{m \in \mathcal{B}_j \setminus A} \left[1 - F_\varepsilon(x - \delta_m)\right], \end{aligned} \tag{C.4}$$

where $\mathcal{R}_k(S) \equiv \{\sigma \subseteq S : |\sigma| = k\}$ is the set of all size-k subsets of set S —that is, all the k-combinations of S. Combining these results and leveraging the properties of the EV_1 distribution, $F_\varepsilon(x) = \exp\left(-\exp\left(-x\right)\right)$, we obtain

$$\begin{aligned} \mathscr{s}_{j|n}(\boldsymbol{\delta}) &= \mathbb{P}\left(u_{ij} > u_{ir_i(\mathcal{J},n+1)}\right) \\ &= \mathbb{P}\left(u_{ij} > u_{i(J-n)}\right) \\ &= \mathbb{P}\left(u_{ij} > u^j_{i(J-n)}\right) \\ &= \int_{-\infty}^{\infty} \mathbb{P}\left(x > u^j_{i(J-n)}\right) \mathrm{d}F_{u_j}(x) \\ &= \int_{-\infty}^{\infty} F_{u^j_{(J-n)}}(x) \mathrm{d}F_\varepsilon(x - \delta_j) \\ &= \int_{-\infty}^{\infty} \sum_{k=J-n}^{J-1} \sum_{A \in \mathcal{R}_k(\mathcal{B}_j)} \prod_{\ell \in A} F_\varepsilon(x - \delta_\ell) \prod_{m \in \mathcal{B}_j \setminus A} \left[1 - F_\varepsilon(x - \delta_m)\right] \mathrm{d}F_\varepsilon(x - \delta_j) \\ &= \int_{-\infty}^{\infty} \sum_{k=J-n}^{J-1} \sum_{A \in \mathcal{R}_k(\mathcal{B}_j)} \prod_{\ell \in A} F_\varepsilon(x - \delta_j)^{\frac{\exp(\delta_\ell)}{\exp(\delta_j)}} \prod_{m \in \mathcal{B}_j \setminus A} \left[1 - F_\varepsilon(x - \delta_j)^{\frac{\exp(\delta_m)}{\exp(\delta_j)}}\right] \mathrm{d}F_\varepsilon(x - \delta_j) \end{aligned}$$

$$= \int_0^1 \sum_{k=J-n}^{J-1} \sum_{A \in \mathcal{R}_k(\mathcal{B}_j)} \prod_{\ell \in A} u^{\frac{\exp(\delta_\ell)}{\exp(\delta_j)}} \prod_{m \in \mathcal{B}_j \setminus A} \left[1 - u^{\frac{\exp(\delta_m)}{\exp(\delta_j)}}\right] \mathrm{d}u$$

$$= \int_0^1 \sum_{k=J-n}^{J-1} \sum_{A \in \mathcal{R}_k(\mathcal{B}_j)} u^{\frac{\sum_{\ell \in A} \exp(\delta_\ell)}{\exp(\delta_j)}} \prod_{m \in \mathcal{B}_j \setminus A} \left[1 - u^{\frac{\exp(\delta_m)}{\exp(\delta_j)}}\right] \mathrm{d}u. \tag{C.5}$$

The second equality follows from (C.2). The third equality follows from equivalence of the events as discussed above. The fourth equality follows by integrating over the marginal distribution of u_{ij}. The fifth equality follows from (C.3) and the definition of the cdf of $u^j_{i(J-n)}$. The sixth equality follows from (C.4). The seventh equality follows from the fact that $F_\varepsilon(x - \ln(a)) = F_\varepsilon(x - \ln(b))^{a/b}$ for $a, b > 0$. The eigth equality follows by the change of variable $u = F_\varepsilon(x - \delta_j)$, and the last equality follows from the algebraic rules of exponentiation.

Equation (C.5) defining the conditional expected applications share function, $\mathcal{s}_{j|n}(\boldsymbol{\delta})$, is a generalization of the well-known choice probabilities of the multinomial logit model. We can easily verify that we get the standard choice probability for $n = 1$:

$$\mathcal{s}_{j|1}(\boldsymbol{\delta}) = \int_0^1 u^{\frac{\sum_{\ell \in \mathcal{J} \setminus \{j\}} \exp(\delta_\ell)}{\exp(\delta_j)}} \mathrm{d}u$$

$$= \frac{\exp(\delta_j)}{\sum_{\ell \in \mathcal{J}} \exp(\delta_\ell)},$$

since $\mathcal{R}_{J-1}(\mathcal{B}_j) = \mathcal{R}_{|\mathcal{B}_j|}(\mathcal{B}_j) = \{\mathcal{B}_j\}$ and $\mathcal{B}_j \setminus \mathcal{B}_j = \emptyset$. Note that the conditional expected shares $\mathcal{s}_{j|n}(\boldsymbol{\delta})$ satisfy the following recursive relation. We can rewrite (C.5) as

$$\mathcal{s}_{j|n}(\boldsymbol{\delta}) = \int_0^1 f_{j|n}(u, \boldsymbol{\delta}) \, \mathrm{d}u,$$

where

$$f_{j|n}(u, \boldsymbol{\delta}) = \sum_{k=J-n}^{J-1} f_j(u, \boldsymbol{\delta}, k)$$

and

$$f_j(u, \boldsymbol{\delta}, k) = \sum_{A \in \mathcal{R}_k(\mathcal{B}_j)} u^{\frac{\sum_{\ell \in A} \exp(\delta_\ell)}{\exp(\delta_j)}} \prod_{m \in \mathcal{B}_j \setminus A} \left[1 - u^{\frac{\exp(\delta_m)}{\exp(\delta_j)}}\right].$$

For $n \in \{2, \ldots, J\}$, we can recursively decompose

$$f_{j|n}(u, \boldsymbol{\delta}) = \sum_{k=J-(n-1)}^{J-1} f_j(u, \boldsymbol{\delta}, k) + \sum_{k=J-n}^{J-n} f_j(u, \boldsymbol{\delta}, k)$$

$$= f_{j|n-1}(u, \boldsymbol{\delta}) + f_j(u, \boldsymbol{\delta}, J-n)$$

$$\vdots$$

$$= f_{j|1}(u, \boldsymbol{\delta}) + \sum_{k=J-n}^{J-2} f_j(u, \boldsymbol{\delta}, k),$$

implying the recursive relations

$$\mathscr{s}_{j|n}(\boldsymbol{\delta}) = \mathscr{s}_{j|n-1}(\boldsymbol{\delta}) + \int_0^1 f_j(u, \boldsymbol{\delta}, J-n)\,\mathrm{d}u, \tag{C.6}$$

$$\mathscr{s}_{j|n}(\boldsymbol{\delta}) = \mathscr{s}_{j|1}(\boldsymbol{\delta}) + \int_0^1 \sum_{k=J-n}^{J-2} f_j(u, \boldsymbol{\delta}, k)\,\mathrm{d}u. \tag{C.7}$$

Furthermore, since $f_j(u, \boldsymbol{\delta}, J-n) \geq 0$ for $u \in [0, 1]$, Equation (C.6) establishes that the conditional share $\mathscr{s}_{j|n}(\boldsymbol{\delta})$ increases monotonically with the number of applications n. Finally, while our derivation of $\mathscr{s}_{j|n}(\boldsymbol{\delta})$ assumed $n \in \{1, \ldots, J-1\}$, it is possible to show that the resulting expression is also valid for $n = J$, integrating to $\mathscr{s}_{j|J}(\boldsymbol{\delta}) = 1$, and that $\sum_{j \in \mathcal{J}} \mathscr{s}_{j|n}(\boldsymbol{\delta}) = n$ for all $n \in \{1, \ldots, J\}$.

Given parameters $\boldsymbol{\delta}$, the integral on the right-hand side of Equation (C.5) can be accurately approximated by numerical quadrature for any $n \in \{1, \ldots, J\}$. Alternatively, we can obtain a closed-form solution by noting that

$$\prod_{m \in \mathcal{B}_j \setminus A} \left[1 - u^{\frac{\exp(\delta_m)}{\exp(\delta_j)}}\right] = 1 + \sum_{s=1}^{|\mathcal{B}_j \setminus A|} (-1)^s \sum_{B \in \mathcal{R}_s(\mathcal{B}_j \setminus A)} u^{\frac{\sum_{m \in B} \exp(\delta_m)}{\exp(\delta_j)}},$$

by standard combinatorics —e.g., by a straightforward generalization of the binomial theorem—, so (C.5) simplifies to

$$\begin{aligned}
\mathscr{s}_{j|n}(\boldsymbol{\delta}) &= \sum_{k=J-n}^{J-1} \sum_{A \in \mathcal{R}_k(\mathcal{B}_j)} \int_0^1 u^{\frac{\sum_{\ell \in A} \exp(\delta_\ell)}{\exp(\delta_j)}} \mathrm{d}u + \sum_{s=1}^{J-1-k} (-1)^s \sum_{B \in \mathcal{R}_s(\mathcal{B}_j \setminus A)} \int_0^1 u^{\frac{\sum_{\ell \in A \cup B} \exp(\delta_\ell)}{\exp(\delta_j)}} \mathrm{d}u \\
&= \sum_{k=J-n}^{J-1} \sum_{A \in \mathcal{R}_k(\mathcal{B}_j)} \frac{\exp(\delta_j)}{\sum_{\ell \in \{j\} \cup A} \exp(\delta_\ell)} + \sum_{s=1}^{J-1-k} (-1)^s \sum_{B \in \mathcal{R}_s(\mathcal{B}_j \setminus A)} \frac{\exp(\delta_j)}{\sum_{\ell \in \{j\} \cup A \cup B} \exp(\delta_\ell)},
\end{aligned} \tag{C.8}$$

where $\sum_{s=1}^{0}(\cdot) \equiv 0$ for notational consistency. Given parameter estimates $\widehat{\boldsymbol{\delta}}$, the computational burden in estimating these generalized conditional choice probabilities, either numerically or analytically, grows quickly with the number of alternatives due to the combinatorics involved.

C.2 Probability mass function of the number of applications

Consider now the conditional pmf of the number of applications conditional on the admission probability α_i and the cost of applications γ_i, $\mathscr{s}_{n|\alpha,\gamma}(\boldsymbol{\delta}, \alpha_i, \gamma_i)$. We can recover the unconditional pmf, $\mathscr{s}_n(\boldsymbol{\delta})$, by integrating the conditional pmf over the joint distribution of parameters (α_i, γ_i), which we assume to be statistically independent. We start by obtaining the conditional pmf at

$n = 0$ despite the conditioning event $n_i = 0$ not appearing explicitly Equation (C.1).[10] The job seeker does not apply to any jobs when the expected utility of the singleton portfolio comprising the best ex post alternative is negative, i.e.,

$$n_i = 0 \iff U_i(\{r_i(\mathcal{J}, 1)\}) < 0 \iff u_{i(J)} < \psi_i^1,$$

where the thresholds $\{\psi_i^n\}_{n=1}^J$ are defined as functions of (α_i, γ_i) in Equation (12). Conditional on (α_i, γ_i), the probability of this event is

$$\begin{aligned} \mathscr{s}_{0|\alpha,\gamma}(\boldsymbol{\delta}, \alpha_i, \gamma_i) &= \mathbb{P}\left(n_i = 0 \mid \alpha_i, \gamma_i\right) \\ &= F_{u_{(J)}}\left(\psi_i^1\right) \\ &= \prod_{\ell \in \mathcal{J}} F_\varepsilon\left(\psi_i^1\right)^{\exp\left(\delta_\ell\right)} \\ &= F_\varepsilon\left(\psi_i^1\right)^{\sum_{\ell \in \mathcal{J}} \exp\left(\delta_\ell\right)}. \end{aligned} \tag{C.9}$$

Similarly, for the case $n = J$, the job seeker applies to every job when even the marginal gain in expected utility from expanding the locally-optimal size $J - 1$ portfolio to include their least preferred job is non-negative, i.e.,

$$n_i = J \iff U_i(\{r_{i1}^{\mathcal{J}}, \ldots, r_{iJ}^{\mathcal{J}}\}) - U_i(\{r_{i1}^{\mathcal{J}}, \ldots, r_{iJ-1}^{\mathcal{J}}\}) \geq 0 \iff u_{i(1)} \geq \psi_i^J.$$

The conditional probability is given by

$$\begin{aligned} \mathscr{s}_{J|\alpha,\gamma}(\boldsymbol{\delta}, \alpha_i, \gamma_i) &= \mathbb{P}\left(n_i = J \mid \alpha_i, \gamma_i\right) \\ &= 1 - F_{u_{(1)}}\left(\psi_i^J\right) \\ &= 1 - \sum_{k=1}^J \sum_{A \in \mathcal{R}_k(\mathcal{J})} \prod_{\ell \in A} F_\varepsilon\left(\psi_i^J - \delta_\ell\right) \prod_{m \in \mathcal{J} \setminus A} \left[1 - F_\varepsilon\left(\psi_i^J - \delta_m\right)\right] \\ &= 1 - \sum_{k=1}^J \sum_{A \in \mathcal{R}_k(\mathcal{J})} F_\varepsilon\left(\psi_i^J\right)^{\sum_{\ell \in A} \exp\left(\delta_\ell\right)} \prod_{m \in \mathcal{J} \setminus A} \left[1 - F_\varepsilon\left(\psi_i^J\right)^{\exp\left(\delta_m\right)}\right]. \end{aligned} \tag{C.10}$$

Finally, for the interior events $n_i = n \in \{1, \ldots, J-1\}$, the stopping rule in part (i) of Proposition 1 implies that

$$n_i = n \iff u_{i(J-n+1)} \geq \psi_i^n \text{ and } u_{i(J-n)} < \psi_i^{n+1},$$

[10]The application of the law of total probability in Equation (C.1) actually requires consideration of the case $n_i = 0$, but the conditional probability $\mathbb{P}\left(j \in A_i \mid n_i = 0\right)$ is obviously zero. We include the event $n_i = 0$ for completeness, but also because it illustrates the reasoning behind the derivations for $n_i > 0$ in the simplest possible scenario.

where $\psi_i^{n+1} > \psi_i^n$. That is, the event that the job seeker applies to n jobs depends on the realization of two consecutive order statistics. Instead of explicitly integrating over the joint distribution of the order statistics of ex post utilities, we can directly derive an expression for the probability that $u_{i(J-n+1)} \geq \psi_i^n$ and $u_{i(J-n)} < \psi_i^{n+1}$ by considering the following combinatorial arguments.

To find the probability measure of the set of all realizations of the expost utilities of a job seeker such that the $(J-n)$–th and $(J-n+1)$–th order statistics satisfy $u_{i(J-n+1)} \geq \psi_i^n$ and $u_{i(J-n)} < \psi_i^{n+1}$, we can partition this set according to how many realizations lie in the interval $[\psi_i^n, \psi_i^{n+1})$. Since the resulting subsets are disjoint events, we need simply compute the sum of the probabilities of each event in the partition. Figure C.1 below depicts the configurations of the order statistics that obtain for different sets in this partition.

Let s be the number of realizations in $[\psi_i^n, \psi_i^{n+1})$. As can be seen in Panel (a), the event $s = 0$ in our partition only includes realizations of random vector $\boldsymbol{u}_i$ such that exactly $J-n$ elements lie below ψ_i^n and the remaining n elements lie above ψ_i^{n+1}. The probability of this subset can be obtained by considering all possible combinations of $J-n$ alternatives and computing the probability that the utilities of these alternatives are less than ψ_i^n and the utilities of the remaining alternatives are larger than ψ_i^{n+1}, i.e.,

$$\sum_{B \in \mathcal{R}_{J-n}(\mathcal{J})} \prod_{p \in B} F_{u_p}(\psi_i^n) \prod_{q \in \mathcal{J} \setminus B} \left[1 - F_{u_q}(\psi_i^{n+1})\right].$$

Panel (b) of Figure C.1 illustrates the element of the partition where $s = J$. This case includes all realizations of $\boldsymbol{u}_i$ such that every element lies in $[\psi_i^n, \psi_i^{n+1})$. The probability of this subset is simply the probability that the utility of every alternative lies in $[\psi_i^n, \psi_i^{n+1})$ since there is only one combination of size J from $\mathcal{J}$ —i.e., $\mathcal{R}_J(\mathcal{J}) = \mathcal{R}_{|\mathcal{J}|}(\mathcal{J}) = \{\mathcal{J}\}$. The corresponding expression is

$$\prod_{\ell \in \mathcal{J}} \left[F_{u_\ell}(\psi_i^{n+1}) - F_{u_\ell}(\psi_i^n)\right].$$

Finally, for the cases $s \in \{1, \ldots, J-1\}$ depicted in panel (c), let $u_{i(r)}$ represent the smallest order statistic that lies in $[\psi_i^n, \psi_i^{n+1})$. Note that $u_{i(J-n)} < \psi_i^{n+1}$ implies the largest order statistic in $[\psi_i^n, \psi_i^{n+1})$ is at least the $(J-n)$–th, while $u_{i(J-n+1)} \geq \psi_i^n$ implies the smallest order statistic in $[\psi_i^n, \psi_i^{n+1})$ is at most the $(J-n+1)$–th. Therefore, r must satisfy $r + s - 1 \geq J - n$ and $r \leq J - n + 1$. Since the number of elements of $\boldsymbol{u}_i$ that lie in $(-\infty, \psi_i^n)$ is $r-1$ and there are only $J-s$ elements that lie outside $[\psi_i^n, \psi_i^{n+1})$, the probability of the s–th subset in the partition can be obtained by (i) considering all combinations of size s of the J alternatives, $A \in \mathcal{R}_s(\mathcal{J})$, (ii) considering all the combinations of size $t \in \{\max(J-n-s, 0), \ldots, \min(J-n, J-s)\}$ of the remaining $J-s$ alternatives, $B \in \mathcal{R}_t(\mathcal{J} \setminus A)$, and (iii) computing the probability that the utilities of the alternatives in A lie between ψ_i^n and ψ_i^{n+1}, the utilities of the alternatives in B lie below ψ_i^n, and the remaining alternatives in $\mathcal{J} \setminus (A \cup B)$ have utilities larger than ψ_i^{n+1}. The corresponding expression is

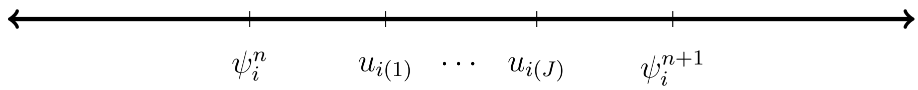
$$\sum_{A\in\mathcal{R}_s(\mathcal{J})}\prod_{\ell\in A}\left[F_{u_\ell}(\psi_i^{n+1})-F_{u_\ell}(\psi_i^{n})\right]\sum_{t\in\tau_n^s}\sum_{B\in\mathcal{R}_t(\mathcal{J}\setminus A)}\prod_{p\in B}F_{u_p}(\psi_i^{n})\prod_{q\in\mathcal{J}\setminus(A\cup B)}\left[1-F_{u_q}(\psi_i^{n+1})\right],$$

where $\tau_n^s\equiv\{\max{(J-n-s,0)},\ldots,\min{(J-n,J-s)}\}$. Summing over all values of s, we obtain

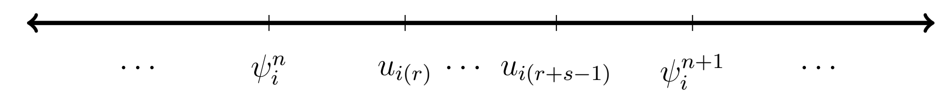
$$\begin{aligned}
s_{n|\alpha,\gamma}(\boldsymbol{\delta},\alpha_i,\gamma_i) &= \mathbb{P}\left(n_i=n\mid\alpha_i,\gamma_i\right)\\
&=\sum_{s=0}^{J}\sum_{A\in\mathcal{R}_s(\mathcal{J})}\prod_{\ell\in A}\left[F_{u_\ell}\big(\psi_i^{n+1}\big)-F_{u_\ell}\big(\psi_i^{n}\big)\right]\\
&\qquad\sum_{t\in\tau_n^s}\sum_{B\in\mathcal{R}_t(\mathcal{J}\setminus A)}\prod_{p\in B}F_{u_p}\big(\psi_i^{n}\big)\prod_{q\in\mathcal{J}\setminus(A\cup B)}\Big[1-F_{u_q}\big(\psi_i^{n+1}\big)\Big]\\
&=\sum_{s=0}^{J}\sum_{A\in\mathcal{R}_s(\mathcal{J})}\prod_{\ell\in A}\Big[F_\varepsilon\big(\psi_i^{n+1}\big)^{\exp{(\delta_\ell)}}-F_\varepsilon\big(\psi_i^{n}\big)^{\exp{(\delta_\ell)}}\Big]\\
&\qquad\sum_{t\in\tau_n^s}\sum_{B\in\mathcal{R}_t(\mathcal{J}\setminus A)}F_\varepsilon\big(\psi_i^{n}\big)^{\sum_{p\in B}\exp{(\delta_p)}}\prod_{q\in\mathcal{J}\setminus(A\cup B)}\Big[1-F_\varepsilon\big(\psi_i^{n+1}\big)^{\exp{(\delta_q)}}\Big].
\end{aligned}\tag{C.11}$$

Figure C.1: Realizations of the order statistics consistent with n applications

(a) No realizations of u_{ij} lie between ψ_i^n and ψ_i^{n+1}



(b) All realizations of u_{ij} lie between ψ_i^n and ψ_i^{n+1}



(c) $s \in \{1, \ldots, J-1\}$ realizations of u_{ij} lie between ψ_i^n and ψ_i^{n+1}

Notes: This figure depicts the realizations of the order statistics of utilities u_{ij} that are consistent with the job seeker applying to n jobs according to the stopping rule in part (i) of Proposition 1. That is, $u_{i(J-n)} < \psi_i^{n+1}$ and $u_{i(J-n+1)} \geq \psi_i^n$ for $n \in \{1, \ldots, J-1\}$. The thresholds ψ_i^n and ψ_i^{n+1} are defined in Equation (12). Cases are indexed by the number of realizations of u_{ij} in the interval $\left[\psi_i^n, \psi_i^{n+1}\right)$, $s \in \{0, \ldots, J\}$. The case $s = 0$ in Panel (a) is equivalent to exactly $J - n$ realizations of u_{ij} below ψ_i^n and exactly n above ψ_i^{n+1}. The case $s = J$ in Panel (b) is equivalent to exactly J realizations of u_{ij} between ψ_i^n and ψ_i^{n+1}. For the cases $s \in \{1, \ldots, J-1\}$ in Panel (c), r must satisfy $r \leq J - n + 1$ so that $u_{i(J-n+1)} \geq \psi_i^n$, and $r + s - 1 \geq J - n$ so $u_{i(J-n)} < \psi_i^{n+1}$, where $u_{i(r)}$ is the smallest order statistic that lies between ψ_i^n and ψ_i^{n+1}. Then, we have at least $\max(J - n - s, 0)$ and at most $\min(J - n, J - s)$ realizations below ψ_i^n, with the remaining realizations above ψ_i^{n+1}.

D Other proofs and derivations

D.1 The wage elasticity of the job applications supply

The elasticity of the applications supply to job $j \in \mathcal{J}$ with respect to the wage of job $\ell \in \mathcal{J}$ is

$$\begin{aligned}
\eta_{q_j,w_\ell} &= \frac{\partial \ln\big(q_j(\boldsymbol{\delta})\big)}{\partial \ln(w_\ell)} \\
&= \frac{1}{q_j(\boldsymbol{\delta})} \frac{\partial q_j(\boldsymbol{\delta})}{\partial \ln(w_\ell)} \\
&= \frac{1}{q_j(\boldsymbol{\delta})} \frac{\partial q_j(\boldsymbol{\delta})}{\partial \delta_\ell} \frac{\partial \delta_\ell}{\partial \ln(w_\ell)} \\
&= \frac{1}{q_j(\boldsymbol{\delta})} \left[I \sum_{n=1}^{J} \frac{\partial s_{j|n}(\boldsymbol{\delta})}{\partial \delta_\ell} s_n(\boldsymbol{\delta}) + s_{j|n}(\boldsymbol{\delta}) \frac{\partial s_n(\boldsymbol{\delta})}{\partial \delta_\ell} \right] \beta,
\end{aligned} \tag{D.1}$$

where the last equality follows from partially differentiating Equation (9) with respect to δ_ℓ and Equation (8) —for job ℓ— with respect to $\ln(w_\ell)$.

The elasticity of the aggregate supply of applications at the firm level is

$$\begin{aligned}
\eta_{q^f,w_\ell} &= \frac{\partial \ln\big(q^f(\boldsymbol{\delta})\big)}{\partial \ln(w_\ell)} \\
&= \frac{1}{q^f(\boldsymbol{\delta})} \frac{\partial q^f(\boldsymbol{\delta})}{\partial \ln(w_\ell)} \\
&= \frac{1}{q^f(\boldsymbol{\delta})} \sum_{j \in \mathcal{J}^f} \frac{\partial q_j(\boldsymbol{\delta})}{\partial \ln(w_\ell)} \\
&= \frac{1}{q^f(\boldsymbol{\delta})} \sum_{j \in \mathcal{J}^f} q_j(\boldsymbol{\delta})\, \eta_{q_j,w_\ell},
\end{aligned} \tag{D.2}$$

where the third equality follows from (17), and the last equality follows from the definition of the vacancy-level elasticity.

Finally, the elasticity of the firm-level supply of applications with respect to a simultaneous increase of the wages the firm offers for all its vacancies, $\boldsymbol{w}^f = \{w_\ell\}_{\ell \in \mathcal{J}^f}$, is given by

$$\eta_{q^f,\boldsymbol{w}^f} = \frac{1}{q^f(\boldsymbol{\delta})} \sum_{\ell \in \mathcal{J}^f} \sum_{j \in \mathcal{J}^f} q_j(\boldsymbol{\delta})\, \eta_{q_j,w_\ell}. \tag{D.3}$$

D.2 Closed-form derivatives

We can obtain closed-form solutions for the partial derivatives of the conditional share $\mathscr{s}_{j|n}(\boldsymbol{\delta})$ and the conditional pmf $\mathscr{s}_{n|\alpha,\gamma}(\boldsymbol{\delta}, \alpha_i, \gamma_i)$ with respect to δ_ℓ for $n \in \{1, \ldots, J\}$. The partial derivative of the unconditional pmf, $\mathscr{s}_n(\boldsymbol{\delta})$, with respect to δ_ℓ is then obtained by integrating the partial of the conditional pmf over $F_\alpha(\cdot) \times F_\gamma(\cdot)$:

$$\begin{aligned}\frac{\partial \mathscr{s}_n(\boldsymbol{\delta})}{\partial \delta_\ell} &= \frac{\partial}{\partial \delta_\ell} \int\int \mathscr{s}_{n|\alpha,\gamma}(\boldsymbol{\delta}, \alpha_i, \gamma_i) \mathrm{d}F_\alpha(\alpha_i) \mathrm{d}F_\gamma(\gamma_i) \\ &= \int\int \frac{\partial \mathscr{s}_{n|\alpha,\gamma}(\boldsymbol{\delta}, \alpha_i, \gamma_i)}{\partial \delta_\ell} \mathrm{d}F_\alpha(\alpha_i) \mathrm{d}F_\gamma(\gamma_i), \end{aligned} \tag{D.4}$$

where the second equality follows since $\mathscr{s}_{n|\alpha,\gamma}(\boldsymbol{\delta}, \alpha_i, \gamma_i)$ is continuously differentiable in δ_ℓ and the supports of F_α and F_γ do not depend on δ_ℓ.[11]

To find the partial derivatives of $\mathscr{s}_{j|n}(\boldsymbol{\delta})$, fix $(j, n) \in \mathcal{J} \times \{1, \ldots, J\}$ and let

$$E_\ell(S) = \frac{\exp(\delta_\ell)}{\sum_{k \in \{j\} \cup S} \exp(\delta_k)}, \tag{D.5}$$

for $\ell \in \mathcal{J}$ and $S \subseteq \mathcal{B}_j$. Note that the expression on the right-hand side of Equation (10) is a finite sum of terms —some with a negative sign— of the form $E_j(S)$ for different subsets S of the choice set that do not contain j. Each such term has partial derivative with respect to δ_ℓ

$$\frac{\partial E_j(S)}{\partial \delta_\ell} = \begin{cases} E_j(S)\big[1 - E_j(S)\big] & \text{if } \ell = j \\ \\ -\mathbb{1}\,[\ell \in S]\, E_j(S) E_\ell(S) & \text{otherwise} \end{cases}. \tag{D.6}$$

Thus,

$$\begin{aligned}\frac{\partial \mathscr{s}_{j|n}(\boldsymbol{\delta})}{\partial \delta_j} &= \sum_{k=J-n}^{J-1} \sum_{A \in \mathcal{R}_k(\mathcal{B}_j)} \frac{\partial E_j(A)}{\partial \delta_j} + \sum_{s=1}^{|\mathcal{B}_j \setminus A|} (-1)^s \sum_{B \in \mathcal{R}_s(\mathcal{B}_j \setminus A)} \frac{\partial E_j(A \cup B)}{\partial \delta_j} \\ &= \sum_{k=J-n}^{J-1} \sum_{A \in \mathcal{R}_k(\mathcal{B}_j)} E_j(A)\big[1 - E_j(A)\big] + \sum_{s=1}^{|\mathcal{B}_j \setminus A|} (-1)^s \sum_{B \in \mathcal{R}_s(\mathcal{B}_j \setminus A)} E_j(A \cup B)\big[1 - E_j(A \cup B)\big] \end{aligned} \tag{D.7}$$

and, similarly for $\ell \neq j$,

$$\frac{\partial \mathscr{s}_{j|n}(\boldsymbol{\delta})}{\partial \delta_\ell} = \sum_{k=J-n}^{J-1} \sum_{A \in \mathcal{R}_k(\mathcal{B}_j)} \frac{\partial E_j(A)}{\partial \delta_\ell} + \sum_{s=1}^{|\mathcal{B}_j \setminus A|} (-1)^s \sum_{B \in \mathcal{R}_s(\mathcal{B}_j \setminus A)} \frac{\partial E_j(A \cup B)}{\partial \delta_\ell}$$

[11] The function $\mathscr{s}_{n|\alpha,\gamma}(\boldsymbol{\delta}, \alpha_i, \gamma_i)$ is a finite sum of products of terms of the form $F_\varepsilon(\psi_i^{n+1})^{\exp(\delta_k)} - F_\varepsilon(\psi_i^n)^{\exp(\delta_k)}$, $F_\varepsilon(\psi_i^n)^{\exp(\delta_k)}$, or $\big[1 - F_\varepsilon(\psi_i^{n+1})^{\exp(\delta_k)}\big]$. Each of these factors is uniformly bounded since $F_\varepsilon(x) \in [0, 1]$ for all $x \in \mathbb{R}$ and the thresholds $\{\psi_i^n\}_{n=1}^J$ do not depend on $\boldsymbol{\delta}$.

$$= -\sum_{k=J-n}^{J-1} \sum_{A \in \mathcal{R}_k(\mathcal{B}_j)} \Bigg[\mathbb{1}\left[\ell \in A\right] E_j(A) E_\ell(A)$$

$$+ \sum_{s=1}^{|\mathcal{B}_j \setminus A|} (-1)^s \sum_{B \in \mathcal{R}_s(\mathcal{B}_j \setminus A)} \mathbb{1}\left[\ell \in A \cup B\right] E_j(A \cup B) E_\ell(A \cup B) \Bigg]. \tag{D.8}$$

To find the partial derivative of $\mathcal{s}_{n|\alpha,\gamma}(\boldsymbol{\delta}, \alpha_i, \gamma_i)$ with respect to δ_ℓ for $n \in \{1, \dots, J-1\}$, rewrite Equation (11) as

$$\mathcal{s}_{n|\alpha,\gamma}(\boldsymbol{\delta}, \alpha_i, \gamma_i) = \sum_{s=0}^{J} \sum_{A \in \mathcal{R}_s(\mathcal{J})} \sum_{t \in \tau_n^s} \sum_{B \in \mathcal{R}_t(\mathcal{J} \setminus A)} \left[\left(\prod_{k \in A} a_k \right) \left(\prod_{p \in B} b_p \right) \left(\prod_{q \in \mathcal{J} \setminus (A \cup B)} c_q \right) \right], \tag{D.9}$$

where $a_k = (F_{n+1})^{\exp(\delta_k)} - (F_n)^{\exp(\delta_k)}$, $b_k = (F_n)^{\exp(\delta_k)}$, $c_k = 1 - (F_{n+1})^{\exp(\delta_k)}$, and $F_k = F_\varepsilon(\psi_i^k)$ for $k \in \{1, \dots, J\}$. Note that the expression on the right-hand side of (D.9) is a finite sum of products of terms of the form a_k, b_p, or c_q for k, p, and q in different, mutually exclusive subsets of $\mathcal{J}$. Since ℓ belongs to only one of these subsets, the chain rule yields

$$\frac{\partial \mathcal{s}_{n|\alpha,\gamma}(\boldsymbol{\delta}, \alpha_i, \gamma_i)}{\partial \delta_\ell} = \sum_{s=0}^{J} \sum_{A \in \mathcal{R}_s(\mathcal{J})} \sum_{t \in \tau_n^s} \sum_{B \in \mathcal{R}_t(\mathcal{J} \setminus A)} \Bigg[\mathbb{1}\left[\ell \in A\right] \frac{\partial a_\ell}{\partial \delta_\ell} \left(\prod_{k \in A \setminus \{\ell\}} a_k \right) \left(\prod_{p \in B} b_p \right) \left(\prod_{q \in \mathcal{J} \setminus (A \cup B)} c_q \right)$$

$$+ \mathbb{1}\left[\ell \in B\right] \frac{\partial b_\ell}{\partial \delta_\ell} \left(\prod_{k \in A} a_k \right) \left(\prod_{p \in B \setminus \{\ell\}} b_p \right) \left(\prod_{q \in \mathcal{J} \setminus (A \cup B)} c_q \right)$$

$$+ \mathbb{1}\left[\ell \notin A \cup B\right] \frac{\partial c_\ell}{\partial \delta_\ell} \left(\prod_{k \in A} a_k \right) \left(\prod_{p \in B} b_p \right) \left(\prod_{q \in \mathcal{J} \setminus (A \cup B \cup \{\ell\})} c_q \right) \Bigg]$$

$$= \sum_{s=0}^{J} \sum_{A \in \mathcal{R}_s(\mathcal{J})} \sum_{t \in \tau_n^s} \sum_{B \in \mathcal{R}_t(\mathcal{J} \setminus A)} \Bigg[\left(\prod_{k \in A} a_k \right) \left(\prod_{p \in B} b_p \right) \left(\prod_{q \in \mathcal{J} \setminus (A \cup B)} c_q \right)$$

$$\left(\mathbb{1}\left[\ell \in A\right] \frac{1}{a_\ell} \frac{\partial a_\ell}{\partial \delta_\ell} + \mathbb{1}\left[\ell \in B\right] \frac{1}{b_\ell} \frac{\partial b_\ell}{\partial \delta_\ell} + \mathbb{1}\left[\ell \notin A \cup B\right] \frac{1}{c_\ell} \frac{\partial c_\ell}{\partial \delta_\ell} \right) \Bigg], \tag{D.10}$$

where $\frac{\partial a_\ell}{\partial \delta_\ell} = \exp(\delta_\ell) \left[(F_{n+1})^{\exp(\delta_\ell)} \ln(F_{n+1}) - (F_n)^{\exp(\delta_\ell)} \ln(F_n) \right]$, $\frac{\partial b_\ell}{\partial \delta_\ell} = \exp(\delta_\ell)(F_n)^{\exp(\delta_\ell)} \ln(F_n)$, and $\frac{\partial c_\ell}{\partial \delta_\ell} = -\exp(\delta_\ell)(F_{n+1})^{\exp(\delta_\ell)} \ln(F_{n+1})$.

D.3 Proof of Lemma 1

Remark. The following proof makes use of the properties of the EV_1 distribution and the ARUM structure discussed in Appendix C, which we omit here to avoid repetition.

Proof. Start by noting how Equation (3) changes when $\alpha_i = 1$. In this case, the job seeker faces no uncertainty regarding her ability to exercise any option in the application portfolio —i.e., getting the job—, but the constraint that only one can be exercised binds. Given any nonempty application portfolio $A \neq \emptyset$, only the most ex-post preferred option in the portfolio, $r_i(A,1)$, will be exercised. Thus, the von Neumann–Morgenstern utility from nonempty portfolio $A \subseteq \mathcal{J}$ is

$$U_i(A) = u_{ir_i(A,1)} - c_i(|A|). \tag{D.11}$$

For an empty portfolio, expected utility simply coincides with the ex-post Bernoulli utility of the outside option:

$$U_i(\emptyset) = -c_i(0) = 0 = u_{i0}. \tag{D.12}$$

Now, let $\gamma > 0$, set $c_i(|A|) = \gamma\,|A|$, and note that

$$\begin{aligned} U_i(A) &= u_{ir_i(A,1)} - \gamma\,|A| \\ &\leq u_{ir_i(A,1)} - \gamma \\ &= U_i\big(\,\{r_i(A,1)\}\,\big) \end{aligned}$$

for any nonempty $A \subseteq \mathcal{J}$ since $|A| \in \{1,\ldots,J\}$. Therefore, conditional on applying, the optimal portfolio is a singleton. Accounting for the case $A_i = \emptyset$, we conclude $A_i \in \{0,1\}$, establishing part (i) of Lemma 1.

Next, to prove part (ii), consider the non-application margin. Notice that, conditional on applying, the optimal portfolio is the singleton containing the best ex-post alternative:

$$\operatorname*{arg\,max}_{A \in \{\sigma \subseteq \mathcal{J} : |\sigma| > 0\}} U_i(A) = \{r_i(\mathcal{J},1)\}\,.$$

Not applying —i.e., choosing the outside option— is optimal if and only if the marginal cost of applications exceeds the highest ex-post utility among the inside alternatives:

$$A_i = \emptyset \iff U_i\big(\,\{r_i(\mathcal{J},1)\}\,\big) < U_i(\emptyset) \iff u_{ir_i(\mathcal{J},1)} - \gamma < 0.$$

This event has probability

$$\begin{aligned} \mathbb{P}\left(\max_{\ell \in \mathcal{J}} u_{i\ell} < \gamma\right) &= F_\varepsilon\left(\gamma - \ln\left(\sum_{\ell \in \mathcal{J}} \exp\left(\delta_\ell\right)\right)\right) \\ &= \exp\left(-\exp\left(-\gamma\right)\sum_{\ell \in \mathcal{J}} \exp\left(\delta_\ell\right)\right), \end{aligned}$$

establishing part (ii) of Lemma 1.

Finally, for any $j \in \mathcal{J}$, note that

$$\mathbb{P}\left(A_i = \{j\} \mid A_i \neq \emptyset\right) = \mathbb{P}\left(\max_{\ell \in \mathcal{J}} u_{i\ell} \leq u_{ij}\right)$$

$$= \frac{\exp(\delta_j)}{\sum_{\ell \in \mathcal{J}} \exp(\delta_\ell)},$$

which establishes part (iii) of Lemma 1. $\square$

E Minorize-maximize algorithm

This appendix closely follows Appendix D of Roussille and Scuderi (2025). The likelihood contribution of job seeker i can be written as

$$\begin{aligned}
f_i(\boldsymbol{\delta} \mid \mathcal{A}_i) &= \mathbb{P}\left(\bigcap_{j \in A_i, \ell \in \overline{A}_i} \left\{\delta_j + \varepsilon_{ij} > \delta_\ell + \varepsilon_{i\ell}\right\}\right) \\
&= \mathbb{P}\left(\bigcap_{j \in A_i} \left\{\delta_j + \varepsilon_{ij} > \max_{\ell \in \overline{A}_i} \delta_\ell + \varepsilon_{i\ell}\right\}\right) \\
&= \int_{-\infty}^{\infty} \left(\prod_{j \in A_i} 1 - F_\varepsilon(x - \delta_j)\right) \mathrm{d}F_\varepsilon\left(x - \ln\left(\sum_{\ell \in \overline{A}_i} \exp(\delta_\ell)\right)\right) \\
&= \int_{-\infty}^{\infty} \left(\prod_{j \in A_i} 1 - F_\varepsilon\left(x - \ln\left(\sum_{\ell \in \overline{A}_i} \exp(\delta_\ell)\right)\right)^{\frac{\exp\left(\delta_j\right)}{\sum_{\ell \in \overline{A}_i} \exp(\delta_\ell)}}\right) \mathrm{d}F_\varepsilon\left(x - \ln\left(\sum_{\ell \in \overline{A}_i} \exp(\delta_\ell)\right)\right) \\
&= \int_0^1 \left(\prod_{j \in A_i} 1 - u^{\frac{\exp(\delta_j)}{\sum_{\ell \in \overline{A}_i} \exp(\delta_\ell)}}\right) \mathrm{d}u \\
&= \int_0^1 \left(\prod_{j \in A_i} 1 - z^{\exp(\delta_j)}\right) \left(\sum_{\ell \in \overline{A}_i} \exp(\delta_\ell)\right) z^{\sum_{\ell \in \overline{A}_i} \exp(\delta_\ell) - 1} \mathrm{d}z,
\end{aligned} \tag{E.1}$$

where $\mathcal{A}_i = \left\{A_i, \overline{A}_i\right\}$ is job seeker i's partition of the choice set into chosen and unchosen alternatives and $F_\varepsilon(x) = \exp(-\exp(-x))$ is the cdf of the EV_1 distribution. The second equality follows from the equivalence of the corresponding events, the third equality follows from the assumption of independent observations and the fact that $\{\varepsilon_{ij}\}_{j \in \mathcal{J}} \overset{iid}{\sim} \mathrm{EV}_1 \implies \mathbb{P}\left(\max_{\ell \in \overline{A}_i} \delta_\ell + \varepsilon_{i\ell} \leq x\right) = F_\varepsilon\left(x - \ln\left(\sum_{\ell \in \overline{A}_i} \exp(\delta_\ell)\right)\right)$, the fourth equality uses the fact that $F_\varepsilon(x - \ln(a)) = F_\varepsilon(x - \ln(b))^{a/b}$ for $a, b > 0$, the fifth equality applies the change of variable $u = F_\varepsilon\left(x - \ln\left(\sum_{\ell \in \overline{A}_i} \exp(\delta_\ell)\right)\right)$, and the last equality makes the change of variable $z = u^{1/\sum_{\ell \in \overline{A}_i} \exp(\delta_\ell)}$. Numerical evaluation of the resulting integral allows us to avoid iterating over all the permutations of the application portfolio A_i to break ties, which becomes an increasingly demanding computational task as the number of alternatives grows.

Given the *iid* assumption, the log-likelihood function takes the form

$$\ell\left(\boldsymbol{\delta} \mid \{\mathcal{A}_i\}_{i\in\mathcal{I}}\right) = \sum_{i\in\mathcal{I}} \ln\left(f_i\left(\boldsymbol{\delta} \mid \mathcal{A}_i\right)\right),$$

which could be directly maximized using the expression in Equation (E.1).[12] Instead, we gain some computational speed by implementing a minorize-maximize (MM) algorithm based on monotonically increasing a suitable surrogate function satisfying an ascent property that guarantees monotonic increases of the objective function.[13]

Let $\boldsymbol{\delta}^{(n)}$ represent the current iterate in our MM algorithm. A *minorizing function* of the real-valued function $\ell(\boldsymbol{\delta})$ at the point $\boldsymbol{\delta}^{(n)}$ is any function $g\left(\boldsymbol{\delta} \mid \boldsymbol{\delta}^{(n)}\right)$ satisfying

$$g\left(\boldsymbol{\delta} \mid \boldsymbol{\delta}^{(n)}\right) \leq \ell(\boldsymbol{\delta}), \ \forall \boldsymbol{\delta}$$

$$g\left(\boldsymbol{\delta}^{(n)} \mid \boldsymbol{\delta}^{(n)}\right) = \ell\left(\boldsymbol{\delta}^{(n)}\right).$$

Note that if our iterative procedure is such that $g\left(\boldsymbol{\delta}^{(n+1)} \mid \boldsymbol{\delta}^{(n)}\right) \geq g\left(\boldsymbol{\delta}^{(n)} \mid \boldsymbol{\delta}^{(n)}\right)$ —i.e., each iteration (weakly) increases the corresponding surrogate minorizing function—, then

$$\begin{aligned}
\ell\left(\boldsymbol{\delta}^{(n+1)}\right) &\geq g\left(\boldsymbol{\delta}^{(n+1)} \mid \boldsymbol{\delta}^{(n)}\right) \\
&\geq g\left(\boldsymbol{\delta}^{(n)} \mid \boldsymbol{\delta}^{(n)}\right) \\
&= \ell\left(\boldsymbol{\delta}^{(n)}\right),
\end{aligned}$$

where the first inequality follows from the definition of $g\left(\cdot \mid \boldsymbol{\delta}^{(n)}\right)$ as a minorizing function of $\ell(\cdot)$ at $\boldsymbol{\delta}^{(n)}$, the second inequality is our assumption, and the equality follows again from the definition of a minorizing function. This ascent property of minorizing functions guarantees that MM algorithms force the objective function uphill.

MM algorithms typically construct a suitable surrogate minorizing function at the current iterate and then maximize it to obtain the next iterate, i.e.,

$$\boldsymbol{\delta}^{(n+1)} = \arg\max_{\boldsymbol{\delta}} \ g\left(\boldsymbol{\delta} \mid \boldsymbol{\delta}^{(n)}\right),$$

leading to significant computational efficiency gains when the surrogate is easy to maximize. However, the ascent property only requires *increasing* the surrogate function, as shown above. Consequently, we follow Roussille and Scuderi (2025) in replacing full maximization in the 'maximization' step with a single gradient ascent update.

[12] For notational simplicity, we hereafter suppress the dependence of the likelihood function on the data $\{\mathcal{A}_i\}_{i\in\mathcal{I}}$.
[13] See Wu and Lange (2010) for an introduction to MM algorithms.

To construct our minorizing surrogate of the log-likelihood function at $\boldsymbol{\delta}^{(n)}$, we start by defining

$$\rho_i\left(\delta_j \mid \boldsymbol{\delta}^{(n)}\right) = \frac{\exp\left(\delta_j\right)}{\sum_{\ell \in \overline{A}_i} \exp\left(\delta_\ell^{(n)}\right)},$$

$$\varphi_i\left(\boldsymbol{\delta}, z \mid \boldsymbol{\delta}^{(n)}\right) = \left(\prod_{j \in A_i} 1 - z^{\rho_i\left(\delta_j \mid \boldsymbol{\delta}^{(n)}\right)}\right) \sum_{\ell \in \overline{A}_i} \rho_i\left(\delta_\ell \mid \boldsymbol{\delta}^{(n)}\right) z^{\sum_{\ell \in \overline{A}_i} \rho_i\left(\delta_\ell \mid \boldsymbol{\delta}^{(n)}\right) - 1},$$

$$\pi_i\left(z \mid \boldsymbol{\delta}^{(n)}\right) = \frac{\varphi_i\left(\boldsymbol{\delta}^{(n)}, z \mid \boldsymbol{\delta}^{(n)}\right)}{\int_0^1 \varphi_i\left(\boldsymbol{\delta}^{(n)}, x \mid \boldsymbol{\delta}^{(n)}\right) \mathrm{d}x},$$

and noting that

$$\frac{f_i\left(\boldsymbol{\delta}\right)}{f_i\left(\boldsymbol{\delta}^{(n)}\right)} = \int_0^1 \frac{\varphi_i\left(\boldsymbol{\delta}, z \mid \boldsymbol{\delta}^{(n)}\right)}{\varphi_i\left(\boldsymbol{\delta}^{(n)}, z \mid \boldsymbol{\delta}^{(n)}\right)} \pi_i\left(z \mid \boldsymbol{\delta}^{(n)}\right) \mathrm{d}z,$$

which follows from the fact that $f_i\left(\boldsymbol{\delta} + \alpha \boldsymbol{\iota}\right) = f_i\left(\boldsymbol{\delta}\right) \ \forall \alpha \in \mathbb{R}$ and choosing $\alpha = -\ln\left(\sum_{\ell \in \overline{A}_i} \exp\left(\delta_\ell^{(n)}\right)\right)$, where $\boldsymbol{\iota}$ is a vector of ones. Since $\pi_i\left(z \mid \boldsymbol{\delta}^{(n)}\right) \geq 0$ and $\int_0^1 \pi_i\left(z \mid \boldsymbol{\delta}^{(n)}\right) \mathrm{d}z = 1$, applying Jensen's inequality yields

$$\ln\left(\int_0^1 \frac{\varphi_i\left(\boldsymbol{\delta}, z \mid \boldsymbol{\delta}^{(n)}\right)}{\varphi_i\left(\boldsymbol{\delta}^{(n)}, z \mid \boldsymbol{\delta}^{(n)}\right)} \pi_i\left(z \mid \boldsymbol{\delta}^{(n)}\right) \mathrm{d}z\right) \geq \int_0^1 \ln\left(\frac{\varphi_i\left(\boldsymbol{\delta}, z \mid \boldsymbol{\delta}^{(n)}\right)}{\varphi_i\left(\boldsymbol{\delta}^{(n)}, z \mid \boldsymbol{\delta}^{(n)}\right)}\right) \pi_i\left(z \mid \boldsymbol{\delta}^{(n)}\right) \mathrm{d}z$$

$$\iff \ell_i\left(\boldsymbol{\delta}\right) \geq \ell_i\left(\boldsymbol{\delta}^{(n)}\right) + \int_0^1 \ln\left(\frac{\varphi_i\left(\boldsymbol{\delta}, z \mid \boldsymbol{\delta}^{(n)}\right)}{\varphi_i\left(\boldsymbol{\delta}^{(n)}, z \mid \boldsymbol{\delta}^{(n)}\right)}\right) \pi_i\left(z \mid \boldsymbol{\delta}^{(n)}\right) \mathrm{d}z, \tag{E.2}$$

where $\ell_i\left(\boldsymbol{\delta}\right) = \ln\left(f_i\left(\boldsymbol{\delta}\right)\right)$ is the log-likelihood contribution of observation i. We obtain our first minorization of this log-likelihood contribution by defining

$$H_{\pi i}^{(n)} = -\int_0^1 \ln\left(\pi_i\left(z \mid \boldsymbol{\delta}^{(n)}\right)\right) \pi_i\left(z \mid \boldsymbol{\delta}^{(n)}\right) \mathrm{d}z$$

and rewriting (E.2) as

$$\ell_i\left(\boldsymbol{\delta}\right) \geq H_{\pi i}^{(n)} + \int_0^1 \ln\left(\varphi_i\left(\boldsymbol{\delta}, z \mid \boldsymbol{\delta}\right)^{(n)}\right) \pi_i\left(z \mid \boldsymbol{\delta}^{(n)}\right) \mathrm{d}z, \tag{E.3}$$

which holds with equality at $\boldsymbol{\delta} = \boldsymbol{\delta}^{(n)}$. We can improve on this minorization to obtain a surrogate function that is separable in $\boldsymbol{\delta}$ by noting that

$$\ln\left(\varphi_i\left(\boldsymbol{\delta}, z \mid \boldsymbol{\delta}\right)^{(n)}\right) = \sum_{j \in A_i} \ln\left(1 - z^{\rho_i\left(\delta_j \mid \boldsymbol{\delta}^{(n)}\right)}\right) + \ln\left(\sum_{\ell \in \overline{A}_i} \rho_i\left(\delta_\ell \mid \boldsymbol{\delta}^{(n)}\right)\right) + \left(\sum_{\ell \in \overline{A}_i} \rho_i\left(\delta_\ell \mid \boldsymbol{\delta}^{(n)}\right) - 1\right) \ln\left(z\right) \tag{E.4}$$

and

$$\begin{aligned}
\ln\left(\sum_{\ell\in\overline{A}_i}\rho_i\left(\delta_\ell \mid \boldsymbol{\delta}^{(n)}\right)\right) &= \ln\left(\sum_{\ell\in\overline{A}_i}\frac{\exp\left(\delta_\ell\right)}{\exp\left(\delta_\ell^{(n)}\right)}\rho_i\left(\delta_\ell^{(n)} \mid \boldsymbol{\delta}^{(n)}\right)\right) \\
&\geq \sum_{\ell\in\overline{A}_i}\ln\left(\frac{\exp\left(\delta_\ell\right)}{\exp\left(\delta_\ell^{(n)}\right)}\right)\rho_i\left(\delta_\ell^{(n)} \mid \boldsymbol{\delta}^{(n)}\right)
\end{aligned}$$

$$\iff \ln\left(\sum_{\ell\in\overline{A}_i}\exp\left(\delta_\ell\right)\right) \geq \sum_{\ell\in\overline{A}_i}\delta_\ell\,\rho\left(\delta_\ell^{(n)} \mid \boldsymbol{\delta}^{(n)}\right) + H_{\rho i}^{(n)}, \tag{E.5}$$

where $H_{\rho i}^{(n)} = -\sum_{\ell\in\overline{A}_i}\rho_i\left(\delta_\ell^{(n)} \mid \boldsymbol{\delta}^{(n)}\right)\ln\left(\rho_i\left(\delta_\ell^{(n)} \mid \boldsymbol{\delta}^{(n)}\right)\right)$ and the inequality follows from Jensen's inequality since $\rho_i\left(\delta_\ell^{(n)} \mid \boldsymbol{\delta}^{(n)}\right) \geq 0$ and $\sum_{\ell\in\overline{A}_i}\rho_i\left(\delta_\ell^{(n)} \mid \boldsymbol{\delta}^{(n)}\right) = 1$. Notice that (E.5) holds with equality at $\boldsymbol{\delta} = \boldsymbol{\delta}^{(n)}$. Finally, combining with (E.3) and (E.4) yields

$$\ell_i\left(\boldsymbol{\delta}\right) \geq H_i^{(n)} + g_i\left(\boldsymbol{\delta} \mid \boldsymbol{\delta}^{(n)}\right), \tag{E.6}$$

where

$$\begin{aligned}
g_i\left(\boldsymbol{\delta} \mid \boldsymbol{\delta}^{(n)}\right) = \int_0^1 \sum_{j\in A_i}\ln\left(1 - z^{\rho_i\left(\delta_j \mid \boldsymbol{\delta}^{(n)}\right)}\right)\pi_i\left(z \mid \boldsymbol{\delta}^{(n)}\right)\mathrm{d}z + \sum_{\ell\in\overline{A}_i}\delta_\ell\,\rho_i\left(\delta_\ell^{(n)} \mid \boldsymbol{\delta}^{(n)}\right) \\
+ \sum_{\ell\in\overline{A}_i}\rho_i\left(\delta_\ell \mid \boldsymbol{\delta}^{(n)}\right)\int_0^1 \ln\left(z\right)\pi_i\left(z \mid \boldsymbol{\delta}^{(n)}\right)\mathrm{d}z,
\end{aligned}$$

$$H_i^{(n)} = H_{\pi i}^{(n)} + H_{\rho i}^{(n)} - \int_0^1 \ln\left(z\right)\pi_i\left(z \mid \boldsymbol{\delta}^{(n)}\right)\mathrm{d}z,$$

and (E.6) holds with equality at $\boldsymbol{\delta} = \boldsymbol{\delta}^{(n)}$. Thus, the log-likelihood function $\ell\left(\boldsymbol{\delta}\right) = \sum_{i\in\mathcal{I}}\ell_i\left(\boldsymbol{\delta}\right)$ is minorized at $\boldsymbol{\delta}^{(n)}$ by the surrogate function

$$g\left(\boldsymbol{\delta} \mid \boldsymbol{\delta}^{(n)}\right) = H^{(n)} + \sum_{i\in\mathcal{I}} g_i\left(\boldsymbol{\delta} \mid \boldsymbol{\delta}^{(n)}\right), \tag{E.7}$$

where $H^{(n)} = \sum_{i\in\mathcal{I}} H_i^{(n)}$.

In its n^{th} iteration, our MM algorithm looks for $\boldsymbol{\delta}^{(n+1)}$ such that $g\left(\boldsymbol{\delta}^{(n+1)} \mid \boldsymbol{\delta}^{(n)}\right) \geq g\left(\boldsymbol{\delta}^{(n)} \mid \boldsymbol{\delta}^{(n)}\right)$, producing an increase in the log-likelihood function by the ascent property. Notice that increasing $\sum_{i\in\mathcal{I}} g_i\left(\boldsymbol{\delta} \mid \boldsymbol{\delta}^{(n)}\right)$ is sufficient to obtain an increase in $g\left(\boldsymbol{\delta} \mid \boldsymbol{\delta}^{(n)}\right)$ since $H^{(n)}$ is constant in $\boldsymbol{\delta}$. The Newton-Raphson update for maximization of $\sum_{i\in\mathcal{I}} g_i\left(\boldsymbol{\delta} \mid \boldsymbol{\delta}^{(n)}\right)$ is given by

$$\boldsymbol{\delta}^{(n+1)} = \boldsymbol{\delta}^{(n)} + \left(-\sum_{i\in\mathcal{I}}\frac{\partial^2 g_i\left(\boldsymbol{\delta} \mid \boldsymbol{\delta}^{(n)}\right)}{\partial\boldsymbol{\delta}\partial\boldsymbol{\delta}'}\right)^{-1}\left(\sum_{i\in\mathcal{I}}\frac{\partial g_i\left(\boldsymbol{\delta} \mid \boldsymbol{\delta}^{(n)}\right)}{\partial\boldsymbol{\delta}}\right)\Bigg|_{\boldsymbol{\delta}=\boldsymbol{\delta}^{(n)}}$$

and, as mentioned above, we use only one such gradient ascent update in each iteration to obtain an increase in the objective function. The fact that $g_i\left(\boldsymbol{\delta} \mid \boldsymbol{\delta}^{(n)}\right)$ has a diagonal Hessian greatly simplifies computation of this update. The j^{th} entry of its gradient and the j^{th} diagonal element of its Hessian are respectively given by

$$\left.\frac{\partial g_i\left(\boldsymbol{\delta} \mid \boldsymbol{\delta}^{(n)}\right)}{\partial \delta_j}\right|_{\boldsymbol{\delta}=\boldsymbol{\delta}^{(n)}} = \mathbb{1}\left[j \in A_i\right] \left.\left(-\rho_i\left(\delta_j \mid \boldsymbol{\delta}^{(n)}\right) \int_0^1 \frac{z^{\rho_i\left(\delta_j \mid \boldsymbol{\delta}^{(n)}\right)}}{1 - z^{\rho_i\left(\delta_j \mid \boldsymbol{\delta}^{(n)}\right)}} \ln(z) \pi_i\left(z \mid \boldsymbol{\delta}^{(n)}\right) \mathrm{d}z\right)\right|_{\boldsymbol{\delta}=\boldsymbol{\delta}^{(n)}}$$

$$+ \mathbb{1}\left[j \in \overline{A}_i\right] \left.\left(\rho_i\left(\delta_j^{(n)} \mid \boldsymbol{\delta}^{(n)}\right) + \rho_i\left(\delta_j \mid \boldsymbol{\delta}^{(n)}\right) \int_0^1 \ln(z) \pi_i\left(z \mid \boldsymbol{\delta}^{(n)}\right) \mathrm{d}z\right)\right|_{\boldsymbol{\delta}=\boldsymbol{\delta}^{(n)}},$$

$$\left.\frac{\partial^2 g_i\left(\boldsymbol{\delta} \mid \boldsymbol{\delta}^{(n)}\right)}{\partial \delta_j^2}\right|_{\boldsymbol{\delta}=\boldsymbol{\delta}^{(n)}} = \mathbb{1}\left[j \in A_i\right] \Bigg(-\rho_i\left(\delta_j \mid \boldsymbol{\delta}^{(n)}\right) \int_0^1 \frac{z^{\rho_i\left(\delta_j \mid \boldsymbol{\delta}^{(n)}\right)}}{1 - z^{\rho_i\left(\delta_j \mid \boldsymbol{\delta}^{(n)}\right)}} \ln(z) \pi_i\left(z \mid \boldsymbol{\delta}^{(n)}\right) \mathrm{d}z$$

$$\left. - \rho_i\left(\delta_j \mid \boldsymbol{\delta}^{(n)}\right)^2 \int_0^1 \frac{z^{\rho_i\left(\delta_j \mid \boldsymbol{\delta}^{(n)}\right)}}{\left[1 - z^{\rho_i\left(\delta_j \mid \boldsymbol{\delta}^{(n)}\right)}\right]^2} \ln(z)^2 \pi_i\left(z \mid \boldsymbol{\delta}^{(n)}\right) \mathrm{d}z \Bigg)\right|_{\boldsymbol{\delta}=\boldsymbol{\delta}^{(n)}}$$

$$+ \mathbb{1}\left[j \in \overline{A}_i\right] \left.\left(\rho_i\left(\delta_j \mid \boldsymbol{\delta}^{(n)}\right) \int_0^1 \ln(z) \pi_i\left(z \mid \boldsymbol{\delta}^{(n)}\right) \mathrm{d}z\right)\right|_{\boldsymbol{\delta}=\boldsymbol{\delta}^{(n)}}.$$

Therefore, since the Hessian is diagonal, the gradient ascent update for the j^{th} component of $\boldsymbol{\delta}^{(n)}$ takes the form

$$\delta_j^{(n+1)} = \delta_j^{(n)} + \frac{\sum_{i \in \mathcal{I}} \rho_{ij}^{(n)} \kappa_{ij}^{(n)}}{\sum_{i \in \mathcal{I}} \rho_{ij}^{(n)} \lambda_{ij}^{(n)}}, \tag{E.8}$$

where

$$\kappa_{ij}^{(n)} = \begin{cases} -\int_0^1 \frac{z^{\rho_{ij}^{(n)}}}{1 - z^{\rho_{ij}^{(n)}}} \ln(z) \pi_i^{(n)}(z) \, \mathrm{d}z & \text{if } j \in A_i \\ \\ 1 + \int_0^1 \ln(z) \pi_i^{(n)}(z) \, \mathrm{d}z & \text{if } j \in \overline{A}_i \end{cases},$$

$$\lambda_{ij}^{(n)} = \begin{cases} \int_0^1 \frac{z^{\rho_{ij}^{(n)}}}{1 - z^{\rho_{ij}^{(n)}}} \ln(z) \pi_i^{(n)}(z) \, \mathrm{d}z + \rho_{ij}^{(n)} \int_0^1 \frac{z^{\rho_{ij}^{(n)}}}{\left(1 - z^{\rho_{ij}^{(n)}}\right)^2} \ln(z)^2 \pi_i^{(n)}(z) \, \mathrm{d}z & \text{if } j \in A_i \\ \\ -\int_0^1 \ln(z) \pi_i^{(n)}(z) \, \mathrm{d}z & \text{if } j \in \overline{A}_i \end{cases},$$

$\rho_{ij}^{(n)} = \rho_i\left(\delta_j^{(n)} \mid \boldsymbol{\delta}^{(n)}\right)$, $\pi_i^{(n)}(z) = \pi_i\left(z \mid \boldsymbol{\delta}^{(n)}\right)$, and all the integrals involved can be approximated by numerical quadrature. Finally, since the level of $\boldsymbol{\delta}$ is not identified, we impose the normalizations $\left|\left|\boldsymbol{\delta}^{(0)}\right|\right| = 1$ and $\sum_{j \in \mathcal{J}} \exp\left(\delta_j^{(N+1)}\right) = 1$ for the initial ($n = 0$) and terminal ($n = N + 1$) values, respectively.

F Data description

This appendix...

This is old and informal stuff... will need rewriting.

F.1 Job advertisements

Number of vacancies: A small number of job ads —6 out of 1,137,965, mapping to 54 of 39,480,855 applications— report zero vacancies being offered in the raw data. We treat the number of vacancies as missing for these observations.

Hours of work: Combined the two part-time categories into one (the distinction was only nominal).

Ad availability (dates and duration): Some of the reported publication and expiry dates and durations of job ads are nonsensical, leading to many application dates falling out of the reported ad availability period. We redefine job ad availability periods as spells of clustered applications according to the following procedure.

1. Define the maximum length (in days) of an application cluster, $\overline{\tau}$. If two consecutive applications to a given ad are more than $\overline{\tau}$ days apart, they belong to different application clusters. We set $\overline{\tau} = 120$ days.

2. For each job ad j identified by the unique ad ID in the raw data, let $\boldsymbol{d}_j^1 = (d_{j1}^1, \ldots, d_{jT_j}^1)'$ be a column vector containing the T_j numerical dates in which ad j received at least one application in ascending order, where numerical values are assigned to dates following, e.g., Stata's convention.

3. Assign all dates d_{jt}^1 such that $d_{jt}^1 - d_{j1}^1 + 1 \leq \overline{\tau}$ to the first application-cluster spell.

4. If all application dates fall within $\overline{\tau}$ dates of the first application date, stop —and job ad j has only one application-cluster spell. Otherwise, if the first application date more than $\overline{\tau}$ days apart from the first application date is the $\bar{t}_1^{\text{th}}$ one —i.e., $d_{jt}^1 - d_{j_1}^1 + 1 \leq \overline{\tau} \; \forall \, t \leq \bar{t}_1 - 1$ and $d_{jt}^1 - d_{j_1}^1 + 1 > \overline{\tau} \; \forall \, t \geq \bar{t}_1$—, repeat the previous steps for Let $\boldsymbol{d}_j^2 = (d_{j1}^2, \ldots, d_{j(T_j - \bar{t}_1 + 1)}^2)' = (d_{j\bar{t}_1}^1, \ldots, d_{jT_j}^1)'$.

The algorithm stops at a n^{th} iteration when $\dim(\boldsymbol{d}_j^n) > 0$ and $\dim(\boldsymbol{d}_j^{n+1}) = 0$, producing n distinct application-cluster spells with durations of at most $\overline{\tau}$ days. We define a new ad ID that maps to unique combinations of the original ad ID (j) and application-cluster spell (s).

Finally, we exploit the information contained in the original publication and expiry dates reported in the raw data by imputing the corresponding ad-availability spell publication and expiry dates as folows. Let $\boldsymbol{d}_{js} = (d_{js1}, \dots, d_{jsT_{js}})'$ be a column vector containing the applications dates corresponding to application-cluster spell s of ad j in ascending order. We impute the publication date of ad j's s^{th} availability spell, $\underline{t}_{js}$, as ad j's originally reported publication date, $\underline{d}_j$, if the first application occurred at most Δ days after. Otherwise, we use the first application date of the application-cluster spell. That is, we define

$$\underline{t}_{js} = \begin{cases} \underline{d}_j & \text{if} \quad d_{js1} \in \left[\underline{d}_j, \, \underline{d}_j + \Delta\right] \\ \\ d_{js1} & \text{otherwise} \end{cases} .$$

Similarly, if the corresponding application-cluster spell contains more than one application date, we impute the expiry date of availability spell (j, s), $\overline{t}_{js}$, as the originally reported expiry date of ad j in the raw data, $\overline{d}_j$, if the last application occurred at most Δ days before, and the last application date otherwise. In the special case of unit duration —i.e., $T_{js} = 1$—

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