Measurements of Ω_M and Ω_Λ from magnitude-redshift data of 60 Type Ia Supernovae

Pallav Chanda

Project Supervisor: Dr. Rajib Saha

20 May 2018 - 15 July 2018

Abstract

Understanding Supernova Type I (Ia, Ib, Ic), and Type II, while focusing on properties of Type Ia Supernovas, we report the measurements of the mass density, Ω_M, and cosmological-constant energy density, Ω_Λ, of the universe based on 60 Type Ia supernovae, 42 of which, at redshifts between 0.18 and 0.83, were discovered by the Supernova Cosmology Project and the rest 18, at redshifts below 0.1, by Cal*á*n Tololo Supernova Survey. The corrected magnitude-redshift data has been directly taken for the following analysis from the research paper titled "Measurements of Ω and Λ from 42 High-Redshift Supernovae" published on 8 Dec 1998. For a flat ($\Omega_M + \Omega_\Lambda = 1$) cosmology we find $\Omega_M^{flat} = 0.275^{+0.090}_{-0.080}$ (1σ statistical). The data indicate results that are against a $\Lambda = 0$ flat cosmology, the simplest universe model. An open, $\Lambda = 0$ cosmology also does not fit the data well: the results show the presence of a cosmological constant that is non-zero and positive, with a confidence of P($\Lambda > 0$) = 99.8%. We discuss the Results that we obtained through the analysis and draw conclusions based upon theoretical and experimental observations.

Contents

1 Introduction **1**

1.1 Supernovae and their types 2

1.1.1 Type II Supernova 2

1.1.2 Type I Supernova 3

1.2 Proper Distance and Comoving Distance 3

1.3 Scale Factor of the Universe 4

1.4 Hubble's Law 5

1.5 Redshift 5

1.6 Luminosity Distance 6

1.7 Friedmann Models 7

2 Basic Data and Procedures **8**

2.1 Deriving the Relevant Equations 8

2.2 Data 13

2.3 Procedures 16

2.3.1 Calculating the value of the integral 16

2.3.2 Fitting $\mathcal{M}$ 16

2.3.3 Calculating D_L and $m(z)$ for each z 17

2.3.4 Fitting Ω_M and Ω_Λ 17

3 Results **19**

4 Conclusions and Discussions **22**

Chapter 1

Introduction

It has been recognized that luminous events like supernovae, since researchers started studying them, might be used as standard candles for cosmological measurements. If an absolute distance scale or magnitude scale is established, they could be used to measure the Hubble constant at closer distances, while at higher redshifts, they could be analysed to give the deceleration parameter of the universe. Measuring the Hubble constant first became possible in the 1980s, when Type Ia supernovae (SNe Ia) of more homogeneous subclass were identified.

We try to determine values of the mass density, Ω_M, and cosmological-constant energy density, Ω_Λ, of the universe based on Type Ia supernovae. This is achieved by using data for apparent magnitude of luminosity $m(z)$ with respect to corresponding z values and using various integration approximation algorithms and fitting techniques to determine Ω_M and Ω_Λ for a Flat Universe Model and a General Universe Model. The magnitude-redshift data has been obtained from the Supernova Cosmology Project (high-redshift data) and the Calán Tololo Supernova Survey (low-redshift data) while for the following analysis, the data corrected for magnitude-redshift - which is free of errors due to Lightcurve width-luminosity relation of SNe Ia, Cross-filter K Correction from observed band to restframe band, Reddening, Malmquist Bias and other Luminosity Biases, etc. - has been directly taken for the following analysis from the research paper titled "Measurements of Ω and Λ from 42 High-Redshift Supernovae" published on 8 Dec 1998.[1]

The analysis and programming needs knowledge of myriad equations and theory. The following introductory depiction of a few important concepts and theory have helped me approach and tap into the Cosmological relations and inch closer to our goal in this analysis.

1.1 Supernovae and their types

A Supernova is the explosion of a star that causes it to suddenly increase greatly in brightness and this catastrophic explosion ejects most of its mass.

1.1.1 Type II Supernova

Have hydrogen lines in its spectra that are made by explosion of a very large star. Hydrogen lines come from outer hydrogen rich layer.

The Central Regions of a supergiant star can be roughly classified as shown in the below diagram.

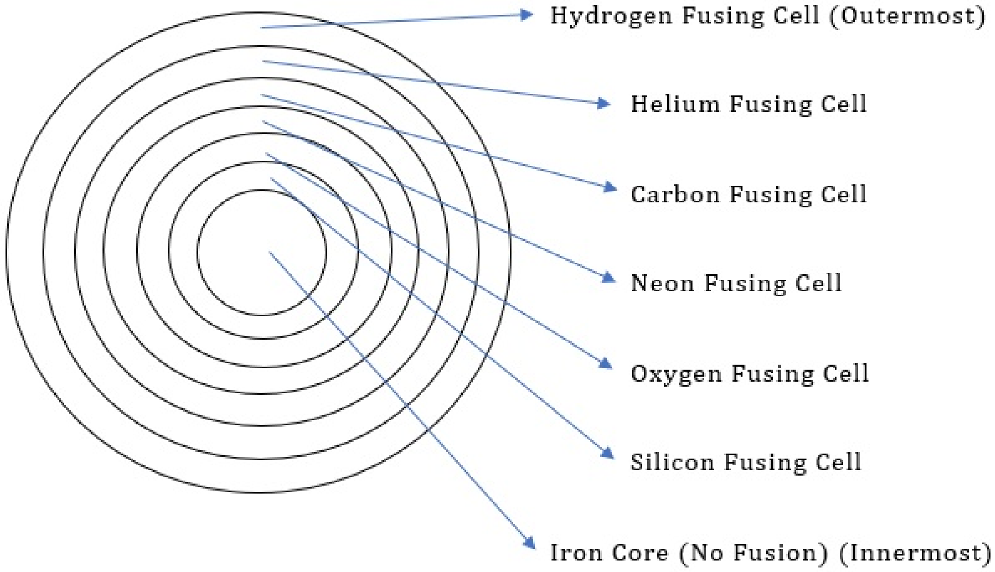

Figure 1.1: Central Regions of a supergiant star

1.1.2 Type I Supernova

No hydrogen lines in spectra.

Type Ia Supernova

Formed due to collapse of a white dwarf star.

White Dwarf is a remnant of a star that wasn't massive enough to force ignition of carbon fusion for energy. All stable white dwarfs must be smaller than the Chandrasekhar limit (approx. 1.4 solar masses).

Type Ia Supernovae blaze with equal brightness at their peaks and hence can be used to measure Cosmic Distances.

Type Ib Supernova

Due to collapse of short-lived big stars. *No hydrogen lines, but Helium lines present* in spectra.

Type Ic Supernova

Due to collapse of short-lived big stars. *No hydrogen or helium lines, but some other element lines present* in spectra.

1.2 Proper Distance and Comoving Distance

Proper distance indicates the distance of a distant object from an observer at a specific moment of time *and it changes over time* because of the expansion of the universe and the space within it.

Comoving distance *does not change over time* due to the expansion of the universe or space. (Though it changes due to other, local factors such as the motion of a galaxy in a cluster, etc.) It only factors out the expansion of the universe. [4]

At present time, Proper distance and Comoving distance are defined, deliberately, to be equal to each other.

At other times,

$$Comoving\ distance = \frac{Proper\ distance}{Scale\ factor\ at\ that\ time}$$

Comoving distance between an observer and a distant light-emitting object is given by,

$$\chi = \int_{t_e}^{t} c\frac{dt'}{a(t')} \tag{1.1}$$

$$\begin{aligned} where,\quad & c = speed\ of\ light \\ & t_e = time\ of\ emission\ of\ photons\ detected\ by\ observer \\ & t = present\ time \end{aligned}$$

1.3 Scale Factor of the Universe

As stated earlier, the universe and the space within it expands. This relative expansion of the universe is indicated abstractly by parametrizing with a dimensionless quantity a, which is known as the scale factor or scientifically as the cosmic scale factor. It gives the relation of the proper distance (which changes over time) between a pair of objects, at any arbitrary time t to their distance at some reference time t_0 as follows:

$$d(t) = a(t)d_0 \tag{1.2}$$

$$\begin{aligned} where,\quad & d(t)\ is\ the\ proper\ distance\ at\ epoch\ t \\ & d_0\ is\ the\ distance\ at\ reference\ time\ t_0 \\ & a(t)\ is\ the\ scale\ factor. \end{aligned}$$

Thus, by definition, $a(t_0) \equiv 1$ The scale factor is dimensionless. t is counted from the birth of the universe and t_0 is set to the present age of the universe, $13.799 \pm 0.021\ Gyr$. This gives the current value of a as $a(t_0)$ or 1.

1.4 Hubble's Law

Hubble's law states that the redshifts in the spectra of distant galaxies (and hence their speeds of recession) are proportional to their distance.[4] This, when coupled with the Doppler Shift Relations, we get the following equation:

$$v = H_0 d \tag{1.3}$$

$$\begin{aligned} where,\quad & v\ is\ recessional\ velocity\ of\ object \\ & d\ is\ proper\ distance\ of\ object\ from\ observer \\ & H_0\ is\ the\ Hubble\ Constant\ or\ Hubble\ Parameter \\ & \quad at\ present\ time. \end{aligned}$$

Thus, if we take time into consideration, Hubble's law takes the form:

$$\dot{d}(t) = H(t)d(t) \tag{1.4}$$

$$\begin{aligned} where,\quad & d\ is\ proper\ distance\ of\ object\ from\ observer\ at\ time\ t \\ & H(t)\ is\ Hubble\ Parameter\ at\ time\ t. \end{aligned}$$

Substituting the definition of $d(t)$ in this equation, we achieve,

$$\begin{aligned} \dot{a}(t)d_0 &= H(t)a(t)d_0 && (1.5) \\ \therefore\quad \dot{a}(t) &= H(t)a(t) && (1.6) \\ \Rightarrow\quad H(t) &= \frac{\dot{a}(t)}{a(t)} && (1.7) \end{aligned}$$

Equation (1.7) gives the definition of the Hubble Parameter, $H(t)$.

1.5 Redshift

Cosmological redshift is the shifting of the entire spectrum of light towards the red color region, i.e. high wavelength region, because of the expansion of the universe and space and the corresponding lengthening of the wavelength of any light that passes. It occurs, generally, when light emitted from a sufficiently distant light source (generally more than a few million light years away) is observed by an observer. It is the convention to refer to this change in wavelength using a dimensionless quantity called z.

$$\frac{a(t_0)}{a(t)} = \frac{1}{a(t)} = 1 + z \tag{1.8}$$

1.6 Luminosity Distance

The luminosity distance d_L is defined so as to satisfy the relation:

$$F_{obs} = \frac{L}{4\pi d_L^2} \tag{1.9}$$

where, F_{obs} is the observed flux from an astronomical source and L is its absolute luminosity. We define flux as the energy that passes per unit time through a unit area (so that the energy per unit time, or the power, collected by a telescope of area A is FA); and luminosity as the total power (energy per unit time) emitted by the source at all wavelengths.

Luminosity distance is a measure of the distance of an object from the observer determined only on the basis of the apparent brigntness, or technically luminosity, of the object as seen by the observer. Thus, it doesn't give the exact measure of actual physical distance from the observer, but a good measure of the apparent distance of a farther object (read stars and galaxies), and hence, very important in astronomical calculations.

The expression for Flux when taking the scale factor, $a(t)$, due to the expansion of universe and the redshift, z, into consideration is as follows [3]

$$F_{bol} = \frac{L_{bol}}{4\pi r_1^2 a^2(t_0)(1+z)^2} \tag{1.10}$$

where, L_{bol} is the absolute *bolometric* luminosity of the distant object, F_{bol} is the corresponding apparent *bolometric* luminosity of the distant object, r_1 is the coordinate radius of the sphere along which the radiation emmitted by the distant object is distributed uniformly, z is the cosmological redshift and $a(t)$ is the scale factor.

Comparing equations (1.9) and (1.10),

$$d_L^2 = r_1^2 a^2(t_0)(1+z)^2 \tag{1.11}$$

$$\Rightarrow d_L = \sqrt{r_1^2 a^2(t_0)(1+z)^2} \tag{1.12}$$

On logarithmic scale of magnitudes, Equation (1.10) becomes [3]

$$m_{bol} - M_{bol} = 5\log d_L + 25 \tag{1.13}$$

where, m_{bol} is apparent bolometric magnitude and M_{bol} is the absolute bolometric magnitude of a standard candle.

1.7 Friedmann Models

Friedmann hypothesizes a Model Universe which is formed by a big bang followed by expansion, and later contraction and an eventual big crunch that causes it to collapse. This model assumes a closed model of the universe but Friedmann has also proposed similar solutions for an open model of the universe (which expands infinitely) and a flat model of the universe (in which expansion continues infinitely but gradually approaches a rate of zero).[8] Friedmann also proposes equations to describe the expansion and contraction of the Universe.

The expansion of space in the homogeneous and the isotropic models of the universe are indicated by the Friedmann equations taking general relativity into consideration. The Friedmann Equations are as follows: [3]

First Friedmann Equation:

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} \tag{1.14}$$

Second Friedmann Equation:

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + \frac{3p}{c^2}) \tag{1.15}$$

where, ρ is the density of the Universe and p is the pressure. *Parameter* k is constant throughout a particular solution, but may vary from one solution to another.

Chapter 2

Basic Data and Procedures

2.1 Deriving the Relevant Equations

Using the First Friedmann Equation (1.14):

$$\begin{aligned}
H^2 &= \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} && (2.1)\\
\therefore 1 + \frac{kc^2}{H^2a^2} &= \frac{8\pi G}{3H^2}\rho && (2.2)\\
\therefore 1 + \frac{kc^2}{H^2a^2} &= \frac{\rho}{\frac{3H^2}{8\pi G}} && (2.3)
\end{aligned}$$

The term $\frac{3H^2}{8\pi G}$ is defined as Critical Density, depicted as follows

$$\rho_c \equiv \frac{3H^2}{8\pi G} \tag{2.4}$$

Critical density is the density at which the Universe would stop expanding only after an infinite time.

The Total Density Parameter $\Omega(t)$ is defined as the ratio of the average density of matter and energy taken together, in the Universe, to the critical density. Also, it is the sum of individual density parameters for Non-relativistic Matter, Radiation and Dark Energy.[7] Thus,

$$\begin{aligned}
\Omega(t) &\equiv \frac{\rho}{\rho_c} && (2.5)\\
\Omega(t) &= \Omega_M(t) + \Omega_R(t) + \Omega_\Lambda(t) && (2.6)\\
\rho &= \rho_M + \rho_R + \rho_\Lambda && (2.7)
\end{aligned}$$

$$\begin{aligned}
\Rightarrow \Omega_M(t) &= \frac{\rho_M}{\rho_c} && (2.8)\\
\Rightarrow \Omega_R(t) &= \frac{\rho_R}{\rho_c} && (2.9)\\
\Rightarrow \Omega_\Lambda(t) &= \frac{\rho_\Lambda}{\rho_c} && (2.10)
\end{aligned}$$

where, subscript M, R and Λ represent Non-relativistic Matter, Radiation and Dark Energy respectively.

The universe is mainly consisting of dust, i.e. pressureless fluid. Hence, the Continuity Equation for fluid in differential form can be applied for the Universe by considering it as a fluid. Also, each of the components - Non-relativistic Matter, Radiation and Dark Energy - can be individually considered as a fluid and under equilibrium. Thus, using the Continuity Equation,

$$\begin{aligned}
\frac{\partial \rho}{\partial t} + 3H(\rho + \frac{p}{c^2}) &= 0 && (2.11)\\
where, \qquad p &= \omega \rho c^2 && (2.12)
\end{aligned}$$

where,
$\omega = 0$ for Non-relativistic Matter
$\omega = \frac{1}{3}$ for Radiation
$\omega = -1$ for Dark Energy

Solving the partial differential,

$$\begin{aligned}
log_e \rho &= -3(1+\omega) log_e a + A && (2.13)\\
\therefore \quad \rho &= A a^{-3(1+\omega)} && (2.14)\\
\Rightarrow \rho_M &= A a^{-3} && (2.15)\\
\Rightarrow \rho_R &= A a^{-4} && (2.16)\\
\Rightarrow \rho_\Lambda &= A a^0 = A && (2.17)
\end{aligned}$$

As seen here, the energy density of Dark Energy of the Universe is constant. Thus, the individual density parameters for Non-relativistic Matter, Radiation and Dark Energy were written:

$$\begin{aligned}
\Omega_M(t) &= \frac{8\pi G A}{3} \frac{1}{\dot{a}^2 a} && (2.18)\\
\Omega_R(t) &= \frac{8\pi G A}{3} \frac{1}{\dot{a}^2 a^2} && (2.19)\\
\Omega_\Lambda(t) &= \frac{8\pi G A}{3} \frac{a^2}{\dot{a}^2} && (2.20)
\end{aligned}$$

From Equation (2.8)

$$\begin{aligned}
H^2(t)\Omega_M(t) &= \frac{\rho_M(t)}{\rho_c} && (2.21)\\
\therefore H^2(t_0)\Omega_M(t_0) &= \frac{\rho_M(t_0)}{\rho_c} \qquad at\ t = t_0 && (2.22)\\
\therefore \qquad \Omega_M(t) &= \frac{H^2(t_0)}{H^2(t)}\,\frac{\rho_M(t)}{\rho_M(t_0)}\Omega_M(t_0) && (2.23)\\
\Rightarrow \qquad \Omega_M(t) &= \frac{H^2(t_0)}{H^2(t)}\,\frac{1}{a^3(t)}\Omega_M(t_0) && (2.24)
\end{aligned}$$

Similarly,

$$\begin{aligned}
\Omega_R(t) &= \frac{H^2(t_0)}{H^2(t)}\,\frac{1}{a^4(t)}\Omega_R(t_0) && (2.25)\\
\Omega_\Lambda(t) &= \frac{H^2(t_0)}{H^2(t)}\Omega_\Lambda(t_0) && (2.26)
\end{aligned}$$

From Equations (2.3), (2.4) and (2.5)

$$\begin{aligned}
\Omega(t) &= 1 + \frac{kc^2}{H^2a^2} && (2.27)\\
\therefore \qquad -\frac{kc^2}{a^2} &= H^2(1 - \Omega(t)) && (2.28)\\
\Rightarrow \qquad -\frac{kc^2}{a^2(t_0)} &= H^2(t_0)(1 - \Omega(t_0)) \qquad at\ t = t_0 && (2.29)\\
\Rightarrow \qquad -\frac{kc^2}{a^2(t)} &= \frac{H^2(t_0)(1 - \Omega(t_0))}{a^2(t)} && (2.30)
\end{aligned}$$

From Equations (2.6), (2.24), (2.25) and (2.26)

$$\begin{aligned}
H^2(t)\Omega(t) &= H^2(t_0)(\frac{1}{a^3(t)}\Omega_M(t_0) + \frac{1}{a^4(t)}\Omega_R(t_0) + \Omega_\Lambda(t_0)) && (2.31)\\
\therefore H^2(t) + \frac{kc^2}{a^2(t)} &= H_0^2(\frac{1}{a^3(t)}\Omega_{M,0} + \frac{1}{a^4(t)}\Omega_{R,0} + \Omega_{\Lambda,0}) && (2.32)\\
\Rightarrow \qquad H^2(t) &= H_0^2(\frac{1}{a^3}\Omega_{M,0} + \frac{1}{a^4}\Omega_{R,0} + \Omega_{\Lambda,0} + \frac{1}{a^2}(1 - \Omega_0)) && (2.33)
\end{aligned}$$

But,

$$\frac{a(t_0)}{a(t)} = \frac{1}{a(t)} = 1 + z \qquad (2.34)$$

Also, Energy densities due to Non-relativistic Matter and Dark Energy are far more dominant in the Universe than due to Radiation. Thus, neglecting the Radiation Energy Density Parameter and using Equation (2.34): [2]

$$\begin{aligned}
\Rightarrow H^2(z) &= H_0^2((1+z)^3\Omega_{M,0} + \Omega_{\Lambda,0} + (1+z)^2(1-\Omega_0)) && (2.35)\\
\Rightarrow \Omega(t) &= \Omega_M(t) + \Omega_\Lambda(t) \qquad from\ (2.6) && (2.36)\\
\therefore \quad \Omega_0 &= \Omega_{M,0} + \Omega_{\Lambda,0} \qquad at\ t = t_0 && (2.37)\\
\therefore 1 - \Omega_0 &= 1 - \Omega_{M,0} - \Omega_{\Lambda,0} && (2.38)\\
\Rightarrow H^2(z) &= H_0^2((1+z)^2(1+\Omega_{M,0}\ z) - z(2+z)\Omega_{\Lambda,0}) && (2.39)\\
\Rightarrow H(z) &= H_0\sqrt{(1+z)^2(1+\Omega_{M,0}\ z) - z(2+z)\Omega_{\Lambda,0}} && (2.40)
\end{aligned}$$

The comoving coordinate distance r is related to the comoving distance χ as: [4]

$$\chi = \int_t^{t_0} c\frac{dt'}{a(t')} = \int_0^r \frac{dr}{\sqrt{1-\kappa r^2}} \tag{2.41}$$

From Equation (2.34)

$$\begin{aligned}
\frac{dz}{dt} &= \frac{-\dot{a}}{a^2} && (2.42)\\
\therefore \qquad dt &= \frac{-a}{H}dz && (2.43)\\
\Rightarrow \int_t^{t_0} c\frac{dt'}{a(t')} &= c\int_0^z \frac{dz'}{H(z')} && (2.44)
\end{aligned}$$

$$\int_0^r \frac{dr}{\sqrt{1-\kappa r^2}} = \frac{c}{H_0}\int_0^z \frac{dz'}{\sqrt{(1+z')^2(1+\Omega_{M,0}\ z') - z'(2+z')\Omega_{\Lambda,0}}} \tag{2.45}$$

Comoving coordinate distance r can be replaced by Lumonisity distance d_L while working with lumonisities and brightnesses of distant objects. Hence, the required expression can be found by replacing r by d_L in Equation (2.45). Solving (2.45), the relation for d_L is obtained: [2]

$$d_L(z;\Omega_M,\Omega_\Lambda,H_0)$$

$$= \frac{c(1+z)}{H_0\sqrt{|\kappa|}}S(\sqrt{|\kappa|}\int_0^z [(1+z')^2(1+\Omega_{M,0}z') - z'(2+z')\Omega_{\Lambda,0}]^{-\frac{1}{2}}dz') \tag{2.46}$$

where, for $\Omega_M + \Omega_\Lambda > 1$, $S(x)$ is defined as $\sin(x)$ and $\kappa = 1 - \Omega_M - \Omega_\Lambda$;
for $\Omega_M + \Omega_\Lambda < 1$, $S(x) = \sinh(x)$ and $\kappa = 1 - \Omega_M - \Omega_\Lambda$;
and for $\Omega_M + \Omega_\Lambda = 1$, $S(x) = x$ and $\kappa = 1$.

From Equation (1.13):

$$\begin{aligned}
m(z) &= M + 5\log d_L(z; \Omega_M, \Omega_\Lambda, H_0) + 25 && (2.47)\\
m(z) &= M + 5\log D_L(z; \Omega_M, \Omega_\Lambda) - 5\log H_0 + 25 && (2.48)\\
m(z) &= \mathcal{M} + 5\log D_L(z; \Omega_M, \Omega_\Lambda) && (2.49)\\
\text{where,} \quad & \mathcal{M} = M - 5\log H_0 + 25 && (2.50)\\
& \log D_L \equiv H_0 d_L
\end{aligned}$$

2.2 Data

Sr. No.	SN (1)	z (2)	m_B^{corr} (3)	$\sigma_{m_B^{corr}}$ (4)	Notes (5)
1	1990O	0.030	16.26	0.20	
2	1990af	0.050	17.63	0.18	
3	1992P	0.026	16.08	0.24	
4	1992ae	0.075	18.43	0.20	
5	1992ag	0.026	16.28	0.20	
6	1992al	0.014	14.47	0.23	
7	1992aq	0.101	19.16	0.23	
8	1992bc	0.020	15.18	0.20	
9	1992bg	0.036	16.66	0.21	
10	1992bh	0.045	17.61	0.19	
11	1992bl	0.043	17.19	0.18	
12	1992bo	0.018	15.61	0.21	B
13	1992bp	0.079	18.27	0.18	
14	1992br	0.088	19.28	0.18	B
15	1992bs	0.063	18.24	0.18	
16	1993B	0.071	18.33	0.20	
17	1993O	0.052	17.54	0.18	
18	1993ag	0.050	17.69	0.20	

Table 2.1: CalÁn Tololo Supernova Survey SNe Ia Data [1]

Col 1. IAU Name assigned to Calán Tololo supernova.[1]
Col 2. Redshift of supernova or host galaxy in Local Group restframe.[1]
Col 3. Stretch–luminosity corrected B-band peak magnitude.[1]
Col 4. Total uncertainty in corrected B-band peak magnitude. This includes uncertainties due to the intrinsic luminosity dispersion of SNe Ia of 0.17 mag, 10% of the Galactic extinction correction, 0.01 mag for K–corrections, 300 $km\ s^{-1}$ to account for peculiar velocities, in addition to propagated uncertainties from the lightcurve fits.[1]
Col 5. Fits from which given supernova was excluded.[1]

Sr. No.	SN (1)	z (2)	$m_B^{effective}$ (3)	$\sigma_{m_B^{effective}}$ (4)	Notes (5)
1	1992bi	0.458	23.11	0.46	
2	1994F	0.354	22.38	0.33	
3	1994G	0.425	22.13	0.49	
4	1994G	0.374	21.72	0.22	B
5	1994al	0.420	22.55	0.25	
6	1994am	0.372	22.26	0.20	
7	1994an	0.378	22.58	0.37	
8	1995aq	0.453	23.17	0.25	
9	1995ar	0.465	23.33	0.30	
10	1995as	0.498	23.71	0.25	
11	1995at	0.655	23.27	0.21	
12	1995aw	0.400	22.36	0.19	
13	1995ax	0.615	23.19	0.25	
14	1995ay	0.480	22.96	0.24	
15	1995az	0.450	22.51	0.23	
16	1995ba	0.388	22.65	0.20	
17	1996cf	0.570	23.27	0.22	
18	1996cg	0.490	23.10	0.20	C
19	1996ci	0.495	22.83	0.19	
20	1996ck	0.656	23.57	0.28	
21	1996cl	0.828	24.65	0.54	
22	1996cm	0.450	23.17	0.23	
23	1996cn	0.430	23.13	0.22	C
24	1997F	0.580	23.46	0.23	
25	1997G	0.763	24.47	0.53	
26	1997H	0.526	23.15	0.20	
27	1997I	0.172	20.17	0.18	
28	1997J	0.619	23.80	0.28	
29	1997K	0.592	24.42	0.37	
30	1997L	0.550	23.51	0.25	
31	1997N	0.180	20.43	0.17	
32	1997O	0.374	23.52	0.24	B
33	1997P	0.472	23.11	0.19	
34	1997Q	0.430	22.57	0.18	
35	1997R	0.657	23.83	0.23	
36	1997S	0.612	23.69	0.21	
37	1997ac	0.320	21.86	0.18	
38	1997af	0.579	23.48	0.22	

Sr. No.	SN (1)	z (2)	$m_B^{effective}$ (3)	$\sigma_{m_B^{effective}}$ (4)	Notes (5)
39	1997ai	0.450	22.83	0.30	
40	1997aj	0.581	23.09	0.22	
41	1997am	0.416	22.57	0.20	
42	1997ap	0.830	24.32	0.22	

Table 2.2: Supernova Cosmology Project (SCP) SNe Ia Data [1]

Col 1. IAU Name assigned to SCP supernova.[1]
Col 2. Geocentric redshift of supernova or host galaxy.[1]
Col 3. Stretch-luminosity corrected effective B-band peak magnitude: $m_B^{effective} \equiv m_X^{peak} + \alpha(s-1) - K_{BX} - A_X$ [1]
Col 4. Total uncertainty in corrected B-band peak magnitude. This includes uncertainties due to the intrinsic luminosity dispersion of SNe Ia of 0.17 mag, 10% of the Galactic extinction correction, 0.01 mag for K-corrections, 300 $km\ s^{-1}$ to account for peculiar velocities, in addition to propagated uncertainties from the lightcurve fits.[1]
Col 5. Fits from which given supernova was excluded.[1]

2.3 Procedures

2.3.1 Calculating the value of the integral

The value of the definite integral $\int_0^z [(1+z')^2(1+\Omega_{M,0}z') - z'(2+z')\Omega_{\Lambda,0}]^{-\frac{1}{2}} dz'$ cannot be obtained by using standard integrals and hence the process of Numerical Integration has been used to estimate the value of the integral correctly upto 16 decimal places, the error which comes later is due to Computational Errors (like truncation, etc.). Composite Simpson's $\frac{1}{3}^{rd}$ rule of Numerical Integration has been used.

Composite Simpson's $\frac{1}{3}^{rd}$ rule states: [5]

$$\int_a^b f(x)dx = \frac{h}{3}[f(a) + 4\sum_{i=2,4,6,...}^{N} f(x_i) + 2\sum_{i=3,5,7,...}^{N-1} f(x_i) + f(b)]$$

N is selected as an even number and is the number of smaller parts into which the interval is broken into.
$h = \frac{b-a}{N}$.
x_i $(1 < i \leq N)$ are the beginning points of the other sub-intervals intervals.
Error in the value is given by:

$$error = \frac{h^4}{180}(b-a)max_{\xi\epsilon[a,b]}[\frac{d^4}{dx^4}f(\xi)]$$

For the definite integral $\int_0^z [(1+z')^2(1+\Omega_{M,0}z') - z'(2+z')\Omega_{\Lambda,0}]^{-\frac{1}{2}} dz'$, for some assumed Ω_M and Ω_Λ, the interval 0 to z, for each z in Table 2.1 and Table 2.2, has been broken into $N = 1000$ equal parts and Composite Simpson's $\frac{1}{3}^{rd}$ rule has been used. The error, when represented scientifically, is in the order of 10^{-22}, and hence the value of the integral so obtained is very accurate for this analysis.

2.3.2 Fitting $\mathcal{M}$

With a set of apparent magnitude and corresponding redshift measurements ($m(z)$) for high-redshift supernovae, and a similar set of measurements for low-redshift supernovae, as in Table 2.2 and Table 2.1 respectively, value of $\mathcal{M}$ could be determined for each z. Fitting the low-redshift and high-redshift measurements, from Table 2.1 and Table 2.2, simultaneously to Equation (2.49), leaving $\mathcal{M}$ free as a fitting parameter, would also be a good alternative to the previous tedious method. [?]

2.3.3 Calculating D_L and $m(z)$ for each z

For each z in Table 2.1 and Table 2.2, and for some assumed Ω_M and Ω_Λ, $D_L = H_0 d_L$ was calculated by formulating as per the equation

$$D_L(z; \Omega_M, \Omega_\Lambda) = \frac{c(1+z)}{\sqrt{|\kappa|}} S(\sqrt{|\kappa|} \int_0^z [(1+z')^2(1+\Omega_{M,0}z') - z'(2+z')\Omega_{\Lambda,0}]^{-\frac{1}{2}} dz')$$

where, for $\Omega_M + \Omega_\Lambda > 1$, $S(x)$ is defined as $\sin(x)$ and $\kappa = 1 - \Omega_M - \Omega_\Lambda$;
for $\Omega_M + \Omega_\Lambda < 1$, $S(x) = \sinh(x)$ and $\kappa = 1 - \Omega_M - \Omega_\Lambda$;
and for $\Omega_M + \Omega_\Lambda = 1$, $S(x) = x$ and $\kappa = 1$.

The integral for each z has been calculated using steps mentioned in Section 2.3.1. Thus, $D_L(z)$ was calculated, for each z, by first assuming a flat universe model ($\Omega_M + \Omega_\Lambda = 1$) and then for a generalized model (with, parameter $\kappa \neq 1$ and $\Omega_M + \Omega_\Lambda \neq 1$).

Thus, for each z in Table 2.1 and Table 2.2, and for some assumed Ω_M and Ω_Λ, $m(z)$ was calculated using the fit value of $\mathcal{M}$ and $\log D_L(z)$.

2.3.4 Fitting Ω_M and Ω_Λ

Flat Universe Model

For fitting, Random Gaussian Number Generator `gasdev(long *idum)` [6] was used to generate random numbers with a gaussian distribution ($\sigma = 0.2$) for Ω_M; and Ω_Λ was set as $(1 - \Omega_M)$.

Metropolis Hastings Algorithm based on Markov Chain Monte Carlo (MCMC) was used to minimize the χ^2 obtained according to the Levenberg Marquardt Algorithm.

$$\chi^2 = \sum_{i=1}^{n} \frac{(m_{i,obs} - m_{i,theory})^2}{\sigma_i^2} \tag{2.51}$$

Aim is to minimize the χ^2. Thus, for a particular generated value of Ω_M (and thus Ω_Λ), $m(z)$ was calculated using the processes mentioned in Sections 2.3.1, 2.3.2 and 2.3.3 for each z in Table 2.2 and Table 2.1, these were the m_{theory} values. Corresponding χ^2 have been calculated from these values using data for $m(z)_{obs}$ and $\sigma_{m(z)}$ and thus the corresponding values of Ω_M and Ω_Λ have been accepted or rejected based on the Metropolis Hastings Algorithm to match the probability distribution $e^{-\chi^2}$. The next value of Ω_M was then generated using `gasdev(long *idum)` [6] by taking the previous accepted value as the mean for the gaussian distribution. This process was repeated for 100000 iterations. Thus, the best fit values of Ω_M^{flat} and Ω_Λ^{flat} were obtained which were the values with the highest frequency among all accepted values.

General Model

Here, the restriction $\Omega_M + \Omega_\Lambda = 1$ disappears. Because of this, both Ω_M and Ω_Λ were generated using Random Gaussian Number Generator `gasdev(long *idum)` [6]. The same Fitting procedure as in the above Flat Model case has been used, except the fact that unlike the earlier case with only one fitting parameter Ω_M, here, two fitting parameters - Ω_M and Ω_Λ - are simultaneously fit, i.e. simultaneously generated, simultaneously accepted or rejected by the Metropolis Hastings Algorithm, and simultaneously next values for the two have been generated using `gasdev(long *idum)` [6] by taking the previous accepted value of each parameter as the mean for the gaussian distribution for the next generation of values for each parameter. Thus, the best fit values of Ω_M and Ω_Λ were obtained, which were the values with the highest frequency among all accepted values.

Chapter 3

Results

Fit	N	χ^2	DOF	Ω_M^{flat}	$P(\Omega_\Lambda > 0)$	Best Fit Ω_M, Ω_Λ	Fit Description
Inclusive	*Fits*						
A	60	101.558	56	$0.300^{+0.090}_{-0.080}$	0.9987	$0.875, 1.425$	All SNe
B	56	60.992	52	$0.275^{+0.095}_{-0.085}$	0.9991	$0.825, 1.425$	Fit A, but excluding 2 residual outliers and 2 stretch outliers
Primary	*Fit*						
C	**54**	**56.875**	**50**	$\mathbf{0.275^{+0.090}_{-0.080}}$	**0.9982**	**0.725, 1.325**	**Fit B, but also excluding 2 likely reddened**

Table 3.1: Fit Results

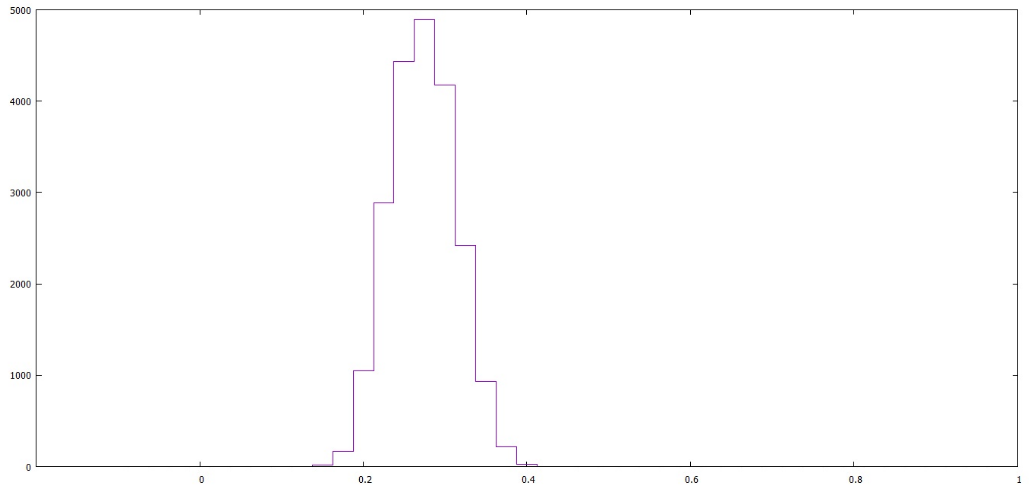

Figure 3.1: **Fit C**: Ω_M^{flat} Histogram for Flat Model of Universe ($\Omega_M+\Omega_\Lambda=1$)

X-axis: Ω_M^{flat}, Y-axis: Frequency

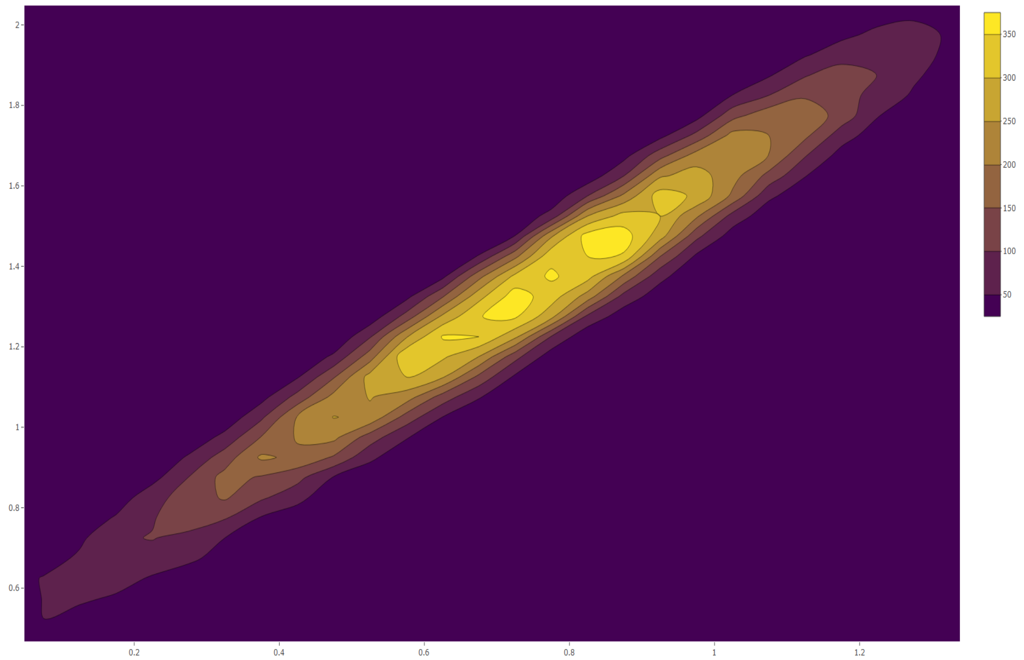

Figure 3.2: **Fit C**: Contour plot for General Model of Universe ($\Omega_M+\Omega_\Lambda\neq 1$)

X-axis: Ω_M, Y-axis: Ω_Λ

From Table 3.1, it is clear that the results of Fits A, B, and C are quite close to each other within experimental and observational error range, so we can conclude that our measurement is correct, bounding and surdy with respect to the choice of the supernova subsets. The fits with the least subjective selection of the data are the Inclusive Fits A and B. They already indicate the main cosmological results from the datasets. However, to make our results sturdy and fitting with respect to host-galaxy reddening, Fit C is used as primary fit.

For Fit C, we find $\Omega_M^{flat} = 0.275$ in a flat universe. While, for a more general model, we find, $\Omega_M = 0.725$ and $\Omega_\Lambda = 1.325$. Cosmologies with $\Omega_\Lambda \leq 0$ are a poor fit to the data at the 99.8% confidence level of Ω_Λ being greater than zero. The χ^2 per degrees of freedom for Fit C, $\chi_v^2 = 1.137$ also indicates that the fit model is a reasonable and robust description of the data. The errors stated in the Table 3.1 are as per the standard 1σ statistical error.

Chapter 4

Conclusions and Discussions

The data indicate results that are inconsistent with the $\Lambda = 0$, flat universe model that has been the theoretically favored cosmology for researchers and scientists alike. If the universe is spatially flat, according to the simplest theories, then the supernova data analysis in this project imply that there is a significant, and positive cosmological constant. This is an implication of the fact that the bestfit values of Ω_M and Ω_Λ when added lead to a value greater than 1 or unity. Thus, either the universe may be flat, or there may be little or no cosmological constant, but the data are not consistent with both possibilities being implied and present simultaneously. This is the most unambiguous result of the current dataset. Also, this could imply that the universe is spacially closed, i.e. it has a positive curvature (as $\Omega_{total} > 1$ for bestfit values). [3]

The contour plot also suggests that the cosmological constant is a significant constituent of the energy density of the universe. The bestfit value and corresponding contours within which they lie indicates that the energy density in the cosmological constant is ~ 0.55 more than that in the form of mass energy density. If the universe has a dominant energy contribution, as seen from our analysis, from a cosmological constant, we can conclude that the value of the cosmological constant is comparable to the current mass-energy density. We also conlude that the universe is expanding and will continue to expand because of the dominant cosmological constant (or recently known as Dark Energy) in the universe. As the universe expands, the matter energy density falls, while the cosmological constant remains unchanged. This could further lead to a very large part of the total energy in the universe being contributed from the cosmological constant as time proceeds further in cosmological scales.

Bibliography

[1] S. Perlmutter and G. Aldering and G. Goldhaber and R. A. Knop and P. Nugent and P. G. Castro and S. Deustua and S. Fabbro and A. Goobar and D. E. Groom and I. M. Hook and A. G. Kim and M. Y. Kim and J. C. Lee and N. J. Nunes and R. Pain and C. R. Pennypacker and R. Quimby and C. Lidman and R. S. Ellis and M. Irwin and R. G. McMahon and P. Ruiz-Lapuente and N. Walton and B. Schaefer and B. J. Boyle and A. V. Filippenko and T. Matheson and A. S. Fruchter and N. Panagia and H. J. M. Newberg and W. J. Couch and The Supernova Cosmology Project. *Measurements of* Ω *and* Λ *from 42 High-Redshift Supernovae*. The Astrophysical Journal, 517:2, 1999. `http://stacks.iop.org/0004-637X/517/i=2/a=565`

[2] S. Perlmutter and S. Gabi and G. Goldhaber and A. Goobar and D. E. Groom and I. M. Hook and A. G. Kim and M. Y. Kim and J. C. Lee and R. Pain and C. R. Pennypacker and I. A. Small and R. S. Ellis and R. G. McMahon and B. J. Boyle and P. S. Bunclark and D. Carter and M. J. Irwin and K. Glazebrook and H. J. M. Newberg and A. V. Filippenko and T. Matheson and M. Dopita and W. J. Couch. *Measurements of the Cosmological Parameters* Ω *and* Λ *from the First Seven Supernovae at* $z \geq 0.35$. The Astrophysical Journal, 483:2, 1997. `http://stacks.iop.org/0004-637X/483/i=2/a=565`

[3] Narlikar, J.V. *An Introduction to Cosmology* `ISBN 9780521793766, lCCN 2001035268`. Cambridge University Press, 2002.

[4] Theuns, Tom. *Physical Cosmology*. Springer-Verlege, Berlin, 1994.

[5] Hoffman, Joe D and Frankel, Steven. *Numerical methods for engineers and scientists*. CRC press, 2001.

[6] Press, William H and Teukolsky, Saul A and Vetterling, William T and Flannery, Brian P. *Numerical recipes in C* `ISBN 0-521-43108-5`. Cambridge University Press, 1988.

[7] Filipe B. Abdalla and Kathleen Lonsdale Building G.22. *Observational Cosmology: The Friedman equations 2.* `http://zuserver2.star.ucl.ac.uk/~hiranya/PHAS3136/PHAS3136/PHAS3136.html`

[8] The Editors of Encyclopaedia Britannica. *Friedmann universe.* ENCYCLOPÆDIA BRITANNICA `https://www.britannica.com/science/Friedmann-universe`