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Title of research report:

Evaluating the effects of different weighting methods for cryptocurrency portfolios in terms of reducing risk by measuring average percentage change and volatility.

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Evaluating the effects of different weighting methods for cryptocurrency portfolios in terms of reducing risk by measuring average percentage change and volatility.

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Abstract

In light of the recent negative interest rates propositions, it is important to find out whether it is viable to invest money in cryptocurrencies as a method of storage. This study aims to obtain a cryptocurrency portfolio that would offer the least amount of risk to its investor through the examination of 4 portfolio strategies: equal weights, variance, correlation, and covariance. From a risk-based perspective, we evaluate the portfolios solely on their aptitude to minimise risk, and, thus, we follow an approach which avoids hyper-parameter tuning to obtain the optimal risk-based portfolio model. Based on our percent change and volatility measurements, the Variance portfolio was most effective in reducing risk, while the correlation portfolio, was least effective. Regarding the use of cryptocurrencies as a method of storage, we found that 3% of cryptocurrencies in a portfolio of stocks is optimal.

Key words:

Key terminology essential to our study is outlined below. Firstly, a cryptocurrency or crypto is a system of electronic money that is used for buying and selling online without the need of a central bank. Cryptocurrencies¹ are volatile in comparison to other assets, and volatility² is the measure of a security's stability. It is calculated as the standard deviation from a certain continuously compounded return over a given period of time. In our study, we use statistical models to compare the volatilities of different cryptocurrencies, a large part of which is examining the correlation³ - the degree of association between two variables. To reduce the effect of volatility, we apply diversification by investing in different cryptocurrencies.

Acknowledgments

We would like to thank our supervisor Mr. Fieldhouse for his support and encouragement. This is our first time writing a full-scale research paper, so his feedback and the time he spent proofreading our research paper were invaluable. We are also grateful for him stepping up to be our supervisor in this competition, especially since our research paper concerns such a niche area of economics.

¹ Oxford University Press - <https://www.lexico.com/definition/cryptocurrency>

² Farlex Financial Dictionary - <https://financial-dictionary.thefreedictionary.com/volatilities>

³ Collins Dictionary of Economics - <https://www.collinsdictionary.com/us/dictionary/english/correlation>

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Introduction

As the world enters the 21st century, the effectiveness of traditional financial systems is being reassessed. The first cryptocurrency emerged when Satoshi Nakamoto invented the Bitcoin in 2009, a decentralised digital currency, with a fixed money supply, that enables peer-to-peer transactions to take place without the need of intermediaries. The Bitcoin⁴ rose steadily despite fluctuations in 2014 and 2015; yet, it was not until 2017 that it surpassed \$1,000. 2017 saw Bitcoin lead an unprecedented and the longest rally by cryptocurrencies to date. Its price soared from \$5,000 in October 2017, peaking at \$19,783 on December 17th 2017. However, five days later, it experienced a plunge of 45%, that of which continued throughout 2018. Having lost more than 80% of its market value since the December spike, Bitcoin stood at less than \$3,500 in November 2018; other cryptocurrencies followed suit. According to critics, it was akin to the Tulipmania of 17th century Holland⁵ - a bursting financial bubble. On the other hand, some see the sharp decline as a natural response to the increase; unlike the Dollar or Euro, having a fixed money supply allowed the Bitcoin to refind its price equilibrium rapidly. We elucidate this by comparing the price change of the Euro (EUR) exchanged with the US Dollar (USD) to the price change of the cryptocurrency Bitcoin (BTC) exchanged with the US Dollar (USD) in the form of two graphs shown overleaf: EUR to USD and BTC to USD. Contrasting them, we see that the exchange rate between two national currencies EUR-USD is less volatile than the BTC-USD. This can be explained by the difference in nature of the two types of currencies, with cryptocurrencies being decentralised and the national currencies often being intervened by their respective governments. Regardless, it is wise to question the contribution and volatile performance of this new device as technology continues to advance, and cryptocurrency gains traction with more people.

⁴ CoinDesk - <https://www.coindesk.com/price/bitcoin>

⁵ Qtd. in The New York Times - www.nytimes.com/2019/04/23/technology/bitcoin-tulip-mania-internet.html

Figure 1: EUR to USD⁶

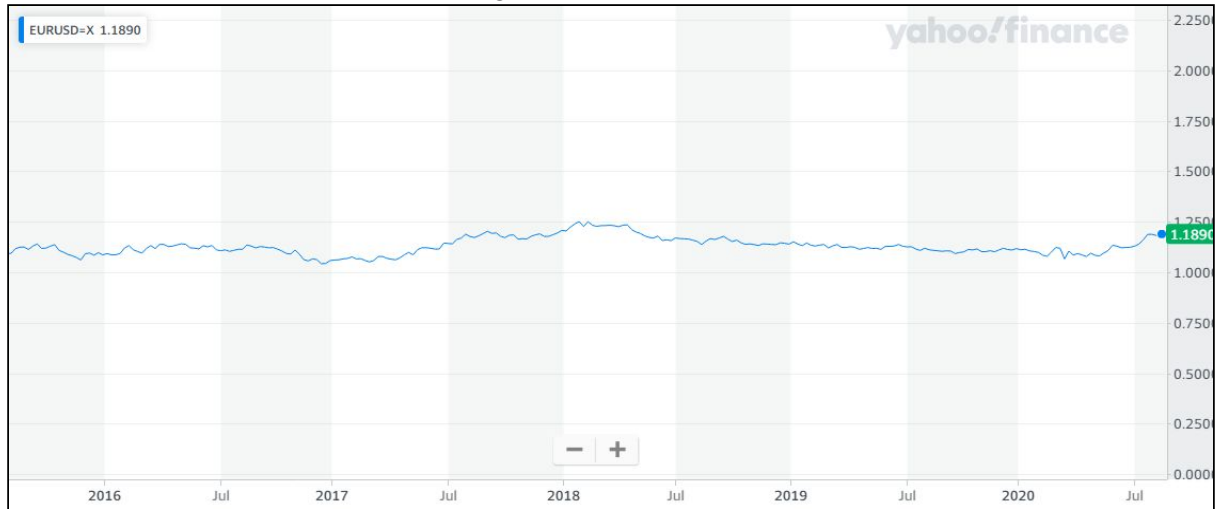


Figure 2: BTC to USD⁷



In 2008 the financial crisis struck, causing halts to major global economies, skyrocketing unemployment rates and tamping down stock values; once a dominant financial institution Lehman Brothers went bankrupt. The aftermath of this recession brought light to the instability of our modern banking systems, and one year later, the first ever digital currency - Bitcoin - was launched. Although there seems to be strong links between the financial crisis and this groundbreaking invention, whether one did cause the other is unclear. Nevertheless, as Professor David Yermack of New York University put it during an interview with Marketwatch, the “timing of its launch could only have helped attract users.”⁸ Following suit, more than 2000 cryptocurrencies emerged over the past decade. Investments in

⁶ Yahoo Finance EUR-USD - <https://finance.yahoo.com/quote/EURUSD=X?p=EURUSD=X&.tsrc=fin-srch>

⁷ Yahoo Finance BTC-USD - <https://finance.yahoo.com/quote/BTC-USD>

⁸ Qtd. in MarketWatch - www.marketwatch.com/story

them have made people affluent, but, in other cases, have wiped off their fortunes. This was especially evident after the cryptocurrency crash of 2018, which reminded cryptocurrency holders of the 2008 financial crisis and amplified the potential pitfalls of investing in digital peer-to-peer currency. And this is also our primary reason to carry out this research. In the volatile settings of cryptocurrencies, we understand investors' need to reduce risk, and, hence, this is why we place heavy emphasis on exploring a multitude of cryptocurrency portfolio arrangements that would minimise the effects of a bear market. Here is a relist of the cryptocurrency portfolios that we examine in this study: equal weights, variance, correlation, and covariance. It is important to note that expected returns are not weighted in our final evaluation of these portfolios, as our risk-based methods may lead to possible errors when estimating expected returns, that of which would harm the accuracy of our results.

Although there has not been a significant amount of research made on the topic of cryptocurrency portfolios, there are abundant studies on portfolio optimisations for traditional assets. Some of these findings prove useful and relatable to our study, offering us insight into the topic of portfolio management; we will outline them in the section below. To restate the research question of this study, we are examining different portfolio strategies to obtain a cryptocurrency portfolio that would offer the least amount of risk to its investor. Our research would make a contribution to the literature of this field of study. This is because we base our study on four common portfolio strategies; however, we factor in diversification, which means that we run two different tests per strategy - one portfolio test with 4 cryptocurrencies and one with 20 cryptocurrencies. This step is to ensure that we cover the different factors that may affect the risk-reducing aptitude of a cryptocurrency portfolio, and using 4 common portfolio strategies allow our study to have a greater usage and contribution to ordinary cryptocurrency investors. Next, to further expand the scope of our study, we add the most risk-reducing cryptocurrency portfolio we found to a standard portfolio of ETF (Exchange-traded fund) to see whether the incorporation of cryptocurrencies that way would decrease the overall risk of the mixed portfolio.

Literature review

Cryptocurrencies are now well regarded by investors as an alternative source of income to traditional assets such as stocks and bonds despite its relatively short history of existence. Although there is little literature on cryptocurrency-only portfolios, there exists an abundance of research on the general topic of portfolio management that prove useful for cryptocurrency research such as the publications of different portfolio strategies that we trial in our study. Below is a summary table of the strategies that we employ, including their author(s) and year of publication.

Table 1: Literature review

Equally-weighted	I. Simonson (1990) ⁹
Variance (Minimum Variance)	Markowitz (1952) ¹⁰
Correlation	Pearson (1844)
Covariance	W.Härdle, L.Simar (2003) ¹¹
The inclusion of cryptocurrencies to a traditional asset portfolio	Chuen et al. (2017) ¹²
Bitcoin and its effect on portfolio diversification and risk-adjusted returns in an optimised portfolio	Eisl et al. (2015)

However, we cannot expect identical results from cryptocurrency portfolios as with stock portfolios due to the highly correlated cryptocurrency market¹³, namely altcoins with Bitcoin. This could be explained by their relatively small size of the market cap, which suggests that prices of cryptocurrencies are only established by a few exchanges, those of which are connected to arbitrage

⁹ Journal of Marketing Research

¹⁰ The Journal of Finance

¹¹ Applied Multivariate Statistical Analysis

¹² The Journal of Alternative investment

¹³ Ref Figure 16 pg.35 - Cryptocurrency return correlation heat map

bots that encourage high frequency trading. In addition to this, the BTC/USD is often used to calculate the exchange prices of other cryptocurrencies. Consequently, any changes in the market of Bitcoin (and stable coins like USDT) will be followed by other cryptocurrencies. For example, a fall in the price of Bitcoin would spark a price decline of other cryptocurrencies.¹⁴ Another cause for this high correlation lies within the decision making of headstrong asset investors who have similar trading motives; when one cryptocurrency changes, they react as if this change would also occur to other cryptocurrencies, thereby contributing to the positive correlation between cryptocurrencies.

A portfolio with assets that are highly correlated alludes to a high return with all assets advancing in the same direction. However, this also increases the chance of a large loss. To counter this uncertainty, traditional stock investors tend to add stocks with negative correlations to their existing portfolio of assets, acting as a counterbalance. While this technique may be effective to utilise for traditional stocks, diversification in a cryptocurrency-only portfolio is often less promising, even though it is possible to mitigate idiosyncratic risk that way. Another alternative would be to short, though we set the constraint of long-only as there are very few central institutions/platforms to regulate a short.

As for previous literature on cryptocurrencies, the majority focuses on the Bitcoin (BTC) and its overall performance, especially when integrating it to asset portfolios such as *Chuen et al. (2017)*, who explored the inclusion of cryptocurrencies to a traditional portfolio consisting of assets such as gold and S&P 500. *Eisl et al. (2015)* investigated whether the presence of Bitcoin in an optimised portfolio would improve portfolio diversification and risk-adjusted returns. Our study, on the other hand, focuses on cryptocurrency-only portfolios, and its contributions to the literature of cryptocurrencies are indicated in the Introduction.

¹⁴ Qtd. in Cointelegraph

Methodology

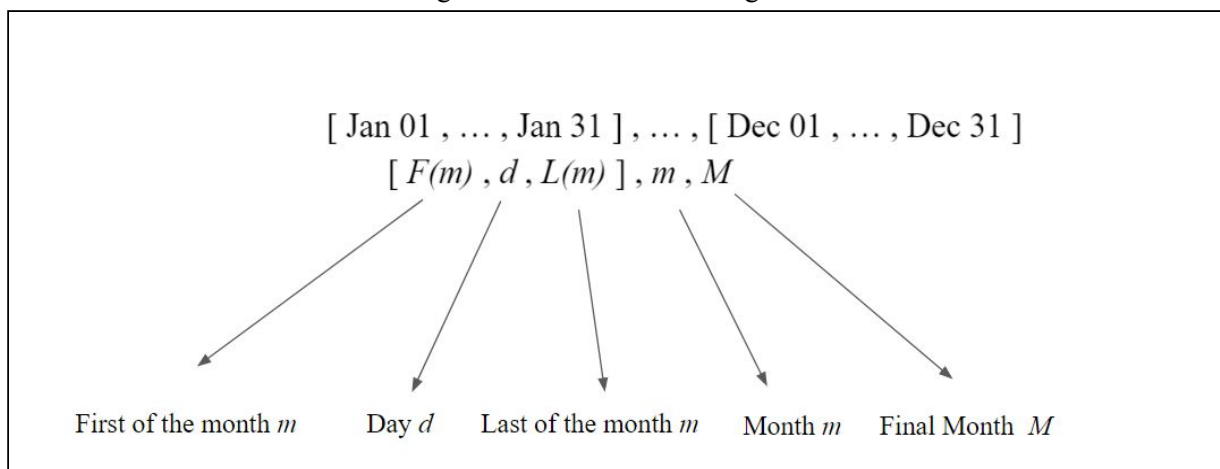
In the methodology we present the following elements, including the application of Table 1 formulae:

- Unique time variables' structure,
- General formulae which are used multiple times in different method sections,
- Method 1 - Control,
- Method 2 - Variance,
- Method 3 - Correlation,
- Method 4 - Covariance,
- Complete tabulation of all variables utilised.

Time Variables:

It is important to note that cryptocurrencies are traded daily, unlike stocks and bonds which are not traded on weekends. We avoid using rolling 30 day windows, as it does not suit the purpose of reassessing our portfolio only once at the beginning of each month. By contrast, we program it so that the complete previous month's data is selected. We accurately portray this method mathematically by using special time variables.

Figure 3: Time variables diagram



$$\therefore F(m) = L(m-1) + 1 \quad \text{eg. Feb 01} = \text{Jan 31} + 1$$

$$\therefore \text{Length of month } l_m = L(m) - F(m) + 1 \quad \text{eg. } l_{(2)} = \text{Feb 28} - \text{Feb 01} + 1 = 28$$

General Formulae:

Variables	Indexes
$Q_{i,m}$ = Quantity of i th cryptocurrency and month m $w_{i,m}$ = Weight for i th crypto and month m , where $\sum_{j=1}^N w_{jm} = 1$ $V_{L(m-1)}$ = Portfolio value for last day of previous month m $P_{i,d}$ = Adj Close Price of i th cryptocurrency at day d $P_{i,F(m)}$ = Adj Close Price of i th cryptocurrency for first of month m	N = Number of cryptos M = Number of months $\forall i = \text{crypto } 1, \dots, \text{crypto } N$ $\forall m = \text{month } 1, \dots, \text{month } M$ $\forall d = \text{day } F(m), \dots, L(m)$

$$Q_{i,m} = \frac{w_{i,m} \times V_{L(m-1)}}{P_{i,F(m)}}$$

$$\text{eg. } Q_{1,1} = \frac{0.25 \times 100,000}{5,000} = 5 \text{ BTC at month } 1$$

Equations	Variables
Return: $R_{i,d} = \frac{P_{i,d} - P_{i,d-1}}{P_{i,d-1}}$	$R_{i,d}$ = Return for i th crypto and day d $\forall d = \text{day } F(m), \dots, L(m)$ $d-1$ = previous day
Return mean: $\mu_{i,m} = \frac{\sum_{d=F(m)}^{L(m)} R_{i,d}}{l_m}$	$\mu_{i,m}$ = Return mean for i th crypto and month m $F(m)$ = First day of month m $L(m)$ = Last day of month m l_m = Length of month m

Method 1 - Control Method:

The control method uses the naive diversification strategy $1/N$, also known as the equal-weight strategy¹⁵. This method allocates the same quantity of funds for each cryptocurrency regardless of any factor in the portfolio. An equally-weighted portfolio is often employed by asset investors to increase diversification and thereby reduce risk; in other words, it is in congruence with the aim of the other strategies we test, which is to reduce risk through diversification. Therefore, this strategy is often used as a benchmark to compare the results of other methods due to its neutral nature.

Formulae:

Equations	Variables
<p>Equal weights:</p> $w_{i,m} = \frac{1}{N} \quad \text{eg. } w_{i,m} = \frac{1}{4} = 0.25$	<p>$w_{i,m}$ = Weight for ith crypto and month m</p> <p>N = Number of cryptos</p>
<p>Substitution - Quantity:</p> $\therefore Q_{i,m} = \frac{V_{L(m-1)}}{N \times P_{i,F(m)}}$	<p>$V_{L(m-1)}$ = Portfolio value for last day of previous month m</p> <p>$P_{i,F(m)}$ = Adj Close Price of ith cryptocurrency for first of month m</p>

¹⁵ Ref. Table 1: Literature review

Method 2 - Variance Method:

Variance denotes how much the data points in a set are away from its mean¹⁶. The Variance formula¹⁷ is found by obtaining the sum of the squared deviations of every data point and dividing the said sum by the total number of data points in the set.

Formulae:

Equations	Variables
<p>Sample Return Variance:</p> $\sigma^2_{i,m} = \frac{\sum_{d=F(m)}^{L(m)} [R_{i,d} - \mu_{i,m}]^2}{l_m - 1}$	<p>$\sigma^2_{i,m}$ = Variance for <i>i</i>th crypto based on month <i>m</i>'s returns.</p> <p>$R_{i,d}$ = Return for <i>i</i>th crypto on day <i>d</i></p> <p>$\mu_{i,m}$ = Return mean for <i>i</i>th crypto of month <i>m</i></p> <p>l_m = Length of month <i>m</i></p>
<p>Weight for inverse to variance:</p> $w_{im} = \frac{1/\sigma^2_{i,m-1}}{\sum_{j=1}^N 1/\sigma^2_{j,m-1}}$ <p>eg. $\frac{\frac{1}{\sigma^2_{1,1}}}{\frac{1}{\sigma^2_{1,1}} + \dots + \frac{1}{\sigma^2_{4,1}}}$</p>	<p>$\sigma^2_{i,m-1}$ = Variance for <i>i</i>th crypto and previous month <i>m</i> calculated based on previous month <i>m</i>'s returns.</p> <p>N = Number of cryptos</p>

¹⁶ Educba - www.educba.com/pearson-correlation-coefficient-formula/

¹⁷ Ref. Table 1: Literature review

Method 3 - Correlation Methods

The method we chose to calculate correlation is "Pearson's"¹⁸, which is widely used and also known as the product-moment correlation coefficient (PMCC). Correlation signifies the strength of the linear relationship between the two variables denoted by "r". The range of correlation coefficient is from -1 to 1. 1 represents a perfect linear relationship between the two variables: when one increases the other increases by the same proportion in the same direction. Conversely, -1 represents a perfect negative linear relationship between the two variables: when one increases the other decreases by the same proportion in the opposite direction. If the correlation coefficient is 0, this represents that there is no linear relationship between the two variables¹⁹.

Formulae:

Equations	Variables
<p>Pearson's correlation coefficient equation:</p> $\sigma_{i,m} = \sqrt{\sigma_{i,m}^2}$ $\rho_{i,j,m} = \frac{\sum_{d=F(m)}^{L(m)} \{[R_{i,d} - \mu_{i,m}] \times [R_{j,d} - \mu_{j,m}]\}}{\sigma_{i,m} \times \sigma_{j,m}}$	<p>$\forall i = \text{crypto } 1, \dots, \text{crypto } N$</p> <p>$\forall j = \text{crypto } 1, \dots, \text{crypto } N$</p> <p>$\sigma_{i,m}$ = standard deviation</p> <p>$\rho_{i,j,m}$ = Pearson's correlation coefficient for (i,j,m)</p> <p>$\mu_{i,m}$ = Return mean for ith crypto and month m</p>

With $\rho_{i,j,m}$ correlation coefficient, we try three methods given that negative values will be changed to 0, as we are not able to short (long-only):

- Inversing correlation matrix,
- Applying $1/\rho_{i,j,m}$,
- Applying $1/|\rho_{i,j,m}|$ - Most successful.

¹⁸ Ref. Table 1: Literature review

¹⁹ Educba - www.educba.com/variance-formula/

A. Inversing correlation matrix:

Equation	Variables
$w_{i,m} = \frac{\omega_{i,m-1}^*}{\sum_{j=1}^N \omega_{j,m-1}^*}$	<p>Let: ϕ = unit column vector of size N</p> <p>Let: Matrix Δ_m have entries (row i, column j) equal to $\rho_{ij,m}$</p> <p>Let: Matrix Ω_m equal the inverse of matrix Δ_m</p> <p>Let: Vector $\omega_m = \Omega_m \cdot \phi$</p> <p>Let: Entry $\omega_{i,m} =$ entry i of ω_m</p> <p>Let: Entry $\omega_{i,m}^* = \max(0, \omega_{i,m})$</p>

B. Applying 1 / correlation:

Equation	Variables
$w_{i,m} = \frac{\lambda_{i,m-1}^*}{\sum_{j=1}^N \lambda_{j,m-1}^*}$	<p>ϕ = unit column vector of size N</p> <p>Let: $f(x) = \frac{1}{x}$</p> <p>Let: Matrix Λ_m have entries (row i, column j) equal to $f(\rho_{ij,m})$</p> <p>Let: Vector $\lambda_m = \Lambda_m \cdot \phi$</p> <p>Let: Entry $\lambda_{i,m} =$ entry i of λ_m</p> <p>Let: Entry $\lambda_{i,m}^* = \max(0, \lambda_{i,m})$</p>

C. Applying $1/|\text{correlation}|$:

Equation	Variables
$w_{i,m} = \frac{\theta_{i,m-1}}{\sum_{j=1}^N \theta_{j,m-1}}$	<p>ϕ = unit column vector of size N</p> <p>Let: $g(x) = \frac{1}{ x }$</p> <p>Let: Matrix Θ_m have entries (row i, column j) equal to $g(\rho_{ij,m})$</p> <p>Let: Vector $\theta_m = \Theta_m \cdot \phi$</p> <p>Let: Entry $\theta_{i,m}$ = entry i of θ_m</p>

Method Analysis:

Method	+ Advantages	- Disadvantages
A. Inversing correlation matrix	Very effective in certain situations, as negative weights indicate shorting and this improves diversification.	When using long-only the negative weights are rendered ineffective.
B. Applying $1/\rho_{ij,m}$	Creative method which led to a better method C which was successful.	Not very orthodox and was the least effective method out of the three.
C. Applying $1/ \rho_{ij,m} $ - Most successful	In this scenario this method utilised all the data, something method A and B failed to do.	Not very orthodox either, maybe a better approach can be devised to tackle the issue at hand.

Method 4 - Covariance Method:

Covariance signifies the linear relationship of two variables, not to be confused with correlation which determines both the magnitude and direction. The range for covariance is from negative to positive infinity.²⁰

Formulae:

Equation	Variables
<p>Sample covariance equation²¹:</p> $\Gamma_{i,j,m} = \frac{\sum_{d=F(m)}^{L(m)} [(R_{i,d} - \mu_{i,m}) \times (R_{j,d} - \mu_{j,m})]}{l_m - 1}$	<p>$\Gamma_{i,j,m}$ = covariance factor for i, j, m</p> <p>l_m = Length of month m</p>

With $\Gamma_{i,j,m}$ correlation coefficient, we try three methods given that negative values will be changed to 0, as we are not able to short:

- I. Inversing covariance matrix,
- II. $1/\Gamma_{i,j,m}$,
- III. $1/|\Gamma_{i,j,m}|$ - Most successful.

The formulae to apply these three methods are the same as those presented in the correlation section.

Instead of using the $\rho_{i,j,m}$ as entries we use $\Gamma_{i,j,m}$. This is possible because the two

aforementioned items function similarly by having the same indexes i, j , and m .

²⁰ Educba - www.educba.com/covariance-formula/

²¹ Ref. Table 1: Literature review

Table 2: Variables Tabulation

Latin	Definitions	Greek	Definitions
d	Index $\forall d : \text{day } F(m), \dots, L(m)$	$\Gamma_{i,j,m}$	Covariance factor for i, j, m
$F(m)$	First day of the month m	Δ_m	Matrix with entries (row i , column j) equal to $\rho_{i,j,m}$ at month m
$f(x)$	Function $(x): \frac{1}{x}$	Θ_m	Matrix with entries (row i , column j) equal to $g(\rho_{i,j,m})$ at month m
$g(x)$	Function $(x): \frac{1}{ x }$	θ_m	Method 3.C. Correlation matrix $\Theta_m \times$ unit column vector ϕ
i	Index $\forall i : \text{crypto } 1, \dots, \text{crypto } N$	$\theta_{i,m}$	Entry i of method 3.C. correlation factors θ_m
j	Index $\forall j : \text{crypto } 1, \dots, \text{crypto } N$	Λ_m	Matrix that has entries (row i , column j) equal to $f(\rho_{i,j,m})$ at month m
$L(m)$	Last day of the month m	λ_m	Method 3.B. Correlation matrix $\Lambda_m \times$ unit column vector ϕ
l_m	Length of the month m	$\lambda_{i,m}$	Entry i of method 3.B. correlation factors λ_m
M	Number of months for data analysis	$\lambda_{i,m}^*$	$\text{Max}(0, \lambda_{i,m})$
m	Index $\forall m : \text{month } 1, \dots, \text{month } M$	$\mu_{i,m}$	Return mean for i th crypto and month m
N	Number of cryptos	$\rho_{i,j,m}$	Pearson's correlation coefficient for (i, j, m)
$P_{i,d}$	Adj Close Price of i th crypto at day d	$\sigma_{i,m}^2$	Variance for i th crypto based on month m 's returns.
$P_{i,F(m)}$	Adj Close Price of i th crypto for first day of month m	ϕ	Unit column vector of size N
$Q_{i,m}$	Quantity of i th crypto at month m	Ω_m	Inverse of correlation matrix Δ_m
$R_{i,d}$	Return for i th crypto at day d	ω_m	Inverse correlation matrix $\Omega_m \times$ unit column vector ϕ
$V_{L(m)-1}$	Portfolio value at last day of previous month m	$\omega_{i,m}$	Entry i of inverse correlation factors ω_m
$w_{i,m}$	Weight for i th crypto and month m , where $\sum_{i=1}^N w_{im} = 1$	$\omega_{i,m}^*$	$\text{max}(0, \omega_{i,m})$

Data Analysis

In the data analysis we review the following elements:

- Application overview,
- Key correlations,
- Method 1 (Control) - Equally Weighted - Small and Large Portfolio,
- Method 2 - Inverse to variance - Small and Large Portfolio,
- Method 3 - Correlation - Small and Large Portfolio,
- Method 4 - Covariance - Small and Large Portfolio.

Application overview:

For this section, we use monthly average percentage change and monthly volatility measures to evaluate each cryptocurrency portfolio's aptitude of reducing risk. Firstly, we set up a Python environment which includes scraping 20 Yahoo Finance cryptocurrency adjusted close price datasets in addition to their respective market caps. Secondly, in the environment we created using Jupyter Notebook, we simulate each type of portfolio according to their mathematical definitions²². The simulation tests each portfolio for the max range of historical data, from Sep 2017 to Aug 2020 for a total of 34 months, and calculates the average monthly portfolio percentage change. We run each portfolio strategy twice for two varying portfolio sizes - 4 types of cryptocurrencies and 20 types, all with a market cap higher than 200M USD. This allows us to take into account any effect there may be with investing in a larger portfolio. The last step in the process is the visualisation, where we graph the portfolio success over time. Furthermore, we supplement our analysis by simulating yet another portfolio, combining different proportions of our best cryptocurrency portfolio with the ETF to see if this would reduce overall risk.

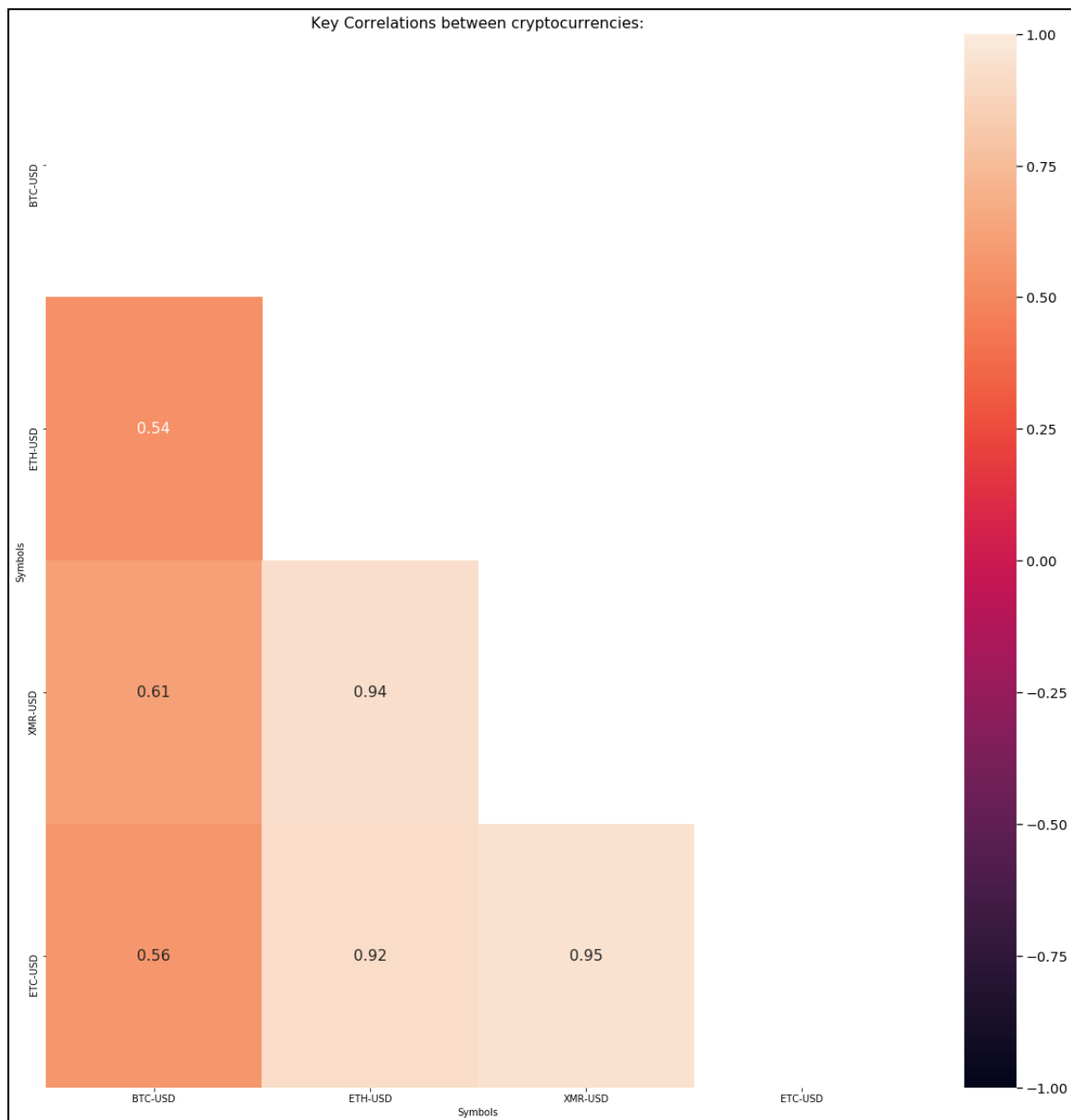
²² Ref Methodology

Key Correlations:

Due to the fact that cryptocurrencies are highly correlated to one another, we commence our analysis by providing an overview of their correlations with reference to two different heatmaps overleaf. This is because we test two portfolio sizes for each strategy. The colour gradient allows us to visualise the correlations between each of the intersecting labels, creating a matrix. The beige colour represents a strong positive linear correlation and amber colour moderate positive linear correlation. This is important to note as the correlations between cryptocurrencies invested in a portfolio affect the overall diversification of the portfolio; for example, high correlation within a portfolio reduces its diversity. The author of *Journal of Investment Strategies*²³, who introduced the maximum diversity portfolio strategy, viewed diversity as the “the ratio of the weighted average of the volatilities of assets to the volatility of the portfolio of the same assets.” This said, diversifying the portfolio is widely regarded as a risk management strategy of reducing the overall risk of a portfolio, as investing in different cryptos would reduce the negative effects experienced in an unstable market. Therefore, diversifying our tested portfolio with different strategies would ultimately lead us to an optimal strategy for reducing risk, but first, it is important to examine the overarching factor that affects diversification in all strategies - the correlations between cryptocurrencies.

²³ Choueifaty et al.

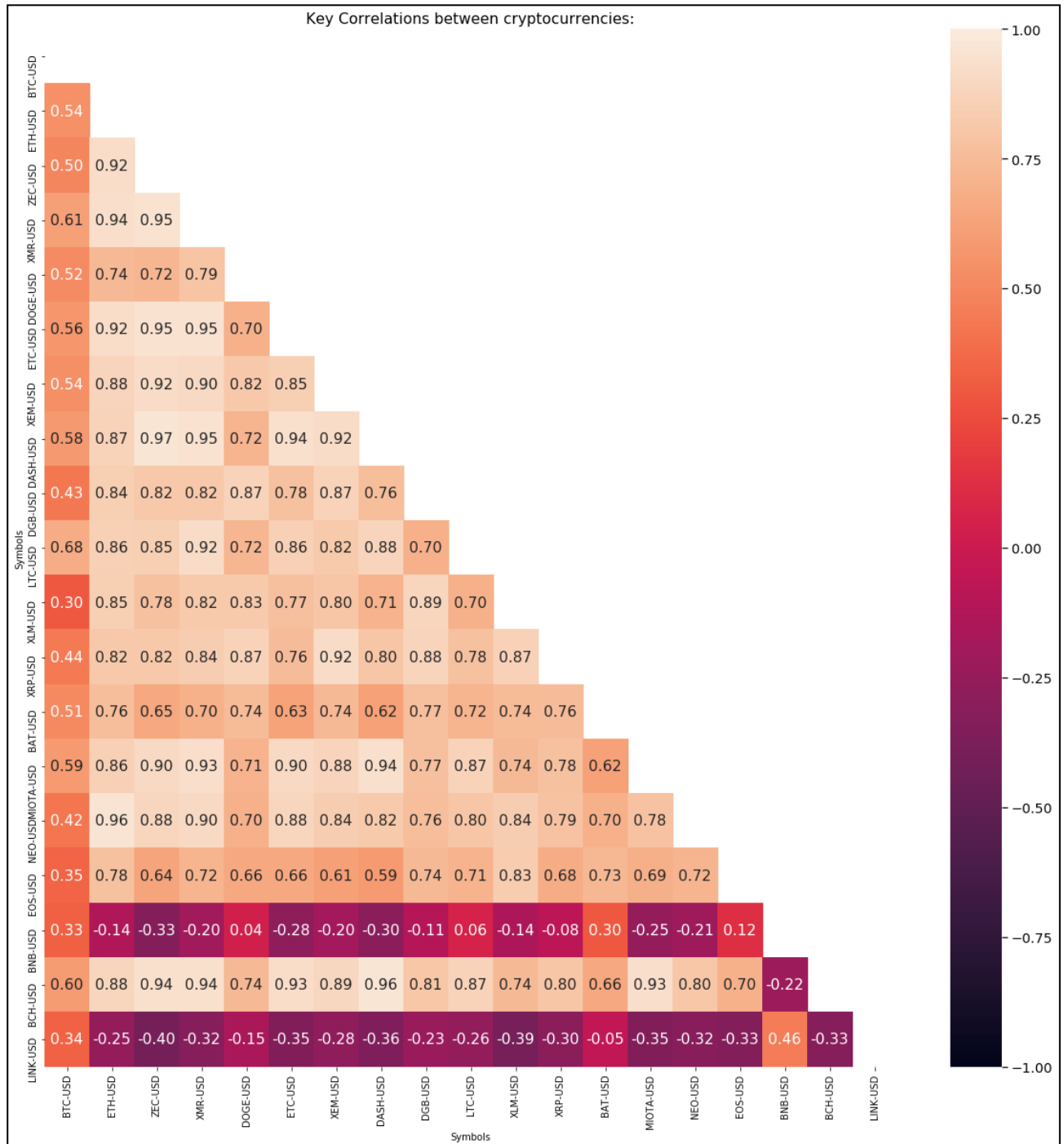
Figure 4: Small Portfolio Adjusted Close Correlations



The first heatmap above shows the correlation of the four cryptocurrencies invested as part of the smaller portfolio testing: they are 'BTC-USD', 'ETH-USD', 'XMR-USD' and 'ETC-USD' respectively. Overall, all cryptocurrencies above have a positive linear correlation with one another; there is no negative correlation. In addition to this, from the visualisation we observe that the first column of cryptocurrencies - namely the correlation of 'BTC-USD' with 'ETH-USD', 'XMR-USD' and 'ETC-USD' - have a medium correlation of 0.55, 0.61, and 0.56, in respective order, in comparison to

the strong positive linear correlation shown by the lighter cells of the second and third column. Most notably, there is a 0.95 positive correlation between 'ETC-USD' and 'XMR-USD'.

Figure 5: Large Portfolio Adjusted Close Correlations



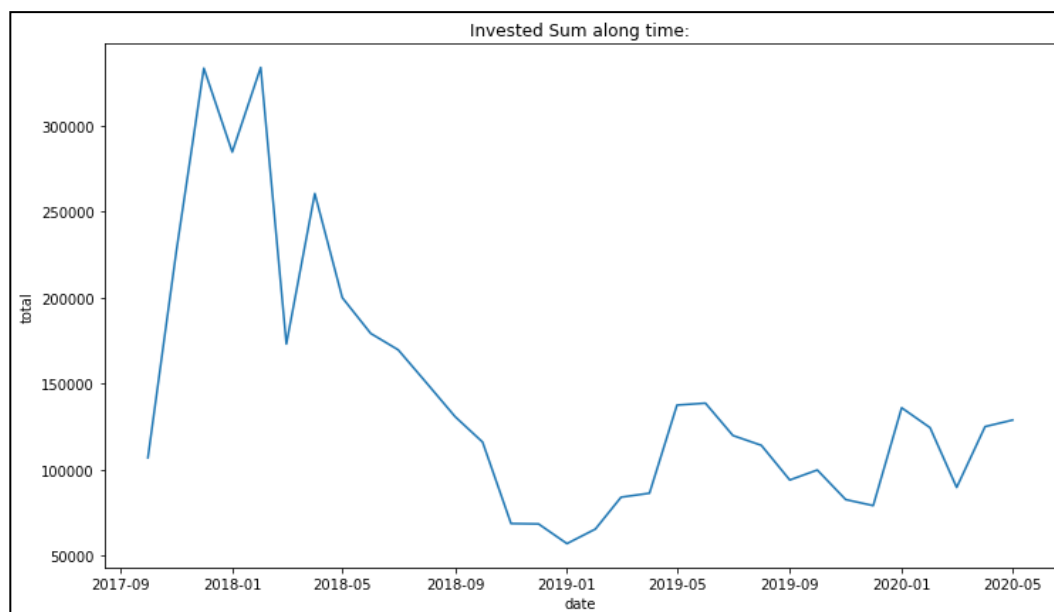
The second heat map above shows the correlation of the twenty cryptocurrencies invested as part of the larger portfolio testing: they are 'BTC-USD', 'ETH-USD', 'ZEC-USD', 'XMR-USD', 'DOGE-USD',

'ETC-USD', 'XEM-USD', 'DASH-USD', 'DGB-USD', 'LTC-USD', 'XLM-USD', 'XRP-USD', 'BAT-USD', 'MIOTA-USD', 'NEO-USD', 'EOS-USD', 'BNB-USD', 'BCH-USD', 'OMG-USD', and 'LINK-USD'. The majority of the cryptocurrencies above have a positive linear correlation with one another; however, 'BNB-USD' and 'LINK-USD' have a negative positive correlation with nearly all other cryptocurrencies but themselves. This positive linear correlation between the 'BNB-USD' and 'LINK-USD' signifies the similarity between the two cryptocurrencies, while their negative correlations with the other cryptocurrencies illustrates the dissimilarity. From this assumption, we view and classify the cryptocurrencies on this correlation heatmap as two separate groups based on the polarity of their linear correlations. A notable outlier to this observation is the positive linear correlation between 'BTC-USD' (Bitcoin) and the cryptocurrencies of both groups, as well as Bitcoin itself having the lowest overall correlations. This could suggest that Bitcoin is like a bridge between them, having properties from both groups.

Method 1 (control): Equally-weighted portfolio

Equally-weighted small portfolio:

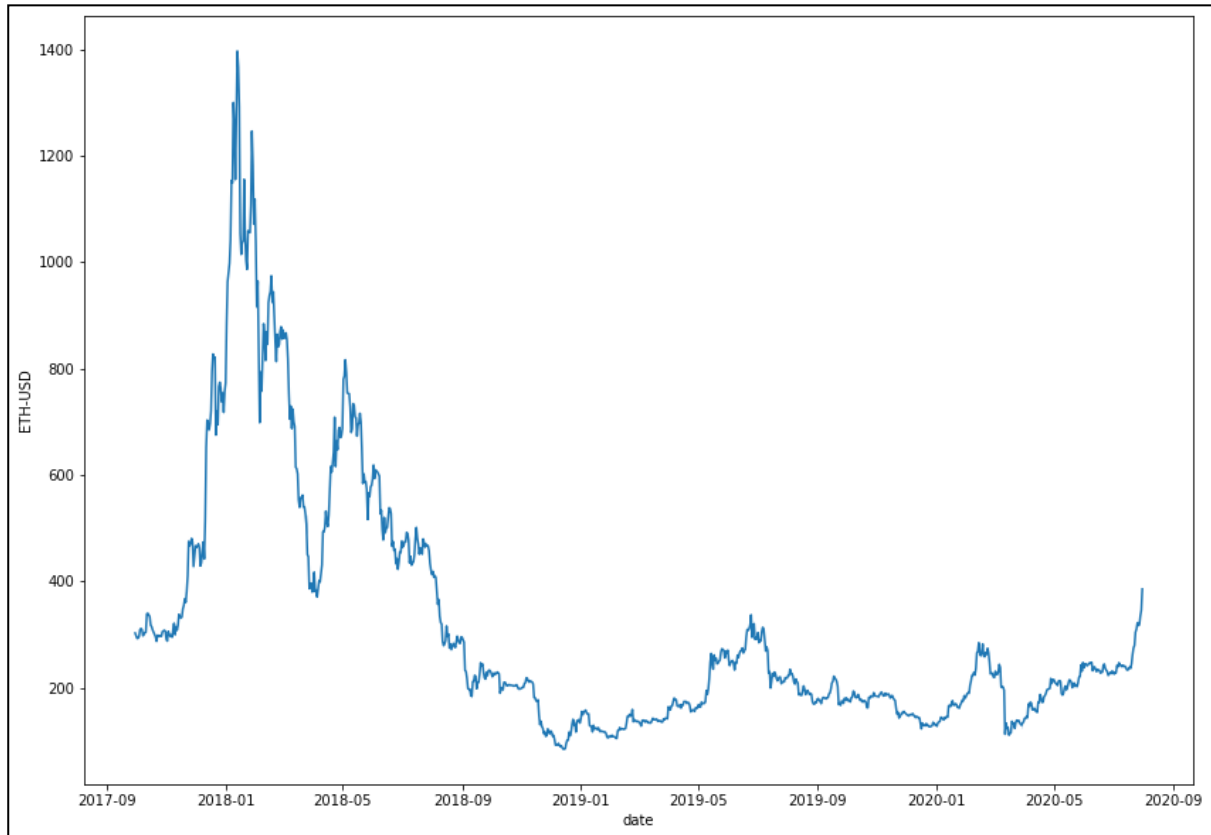
Figure 6: Equally-weighted small portfolio along time



The average percentage change for the above graph = 22.31 % , volatility = 32.79%

22.31% is relatively high for a portfolio. One of the main points to note from this graph is the resemblance to the Ethereum graph below:

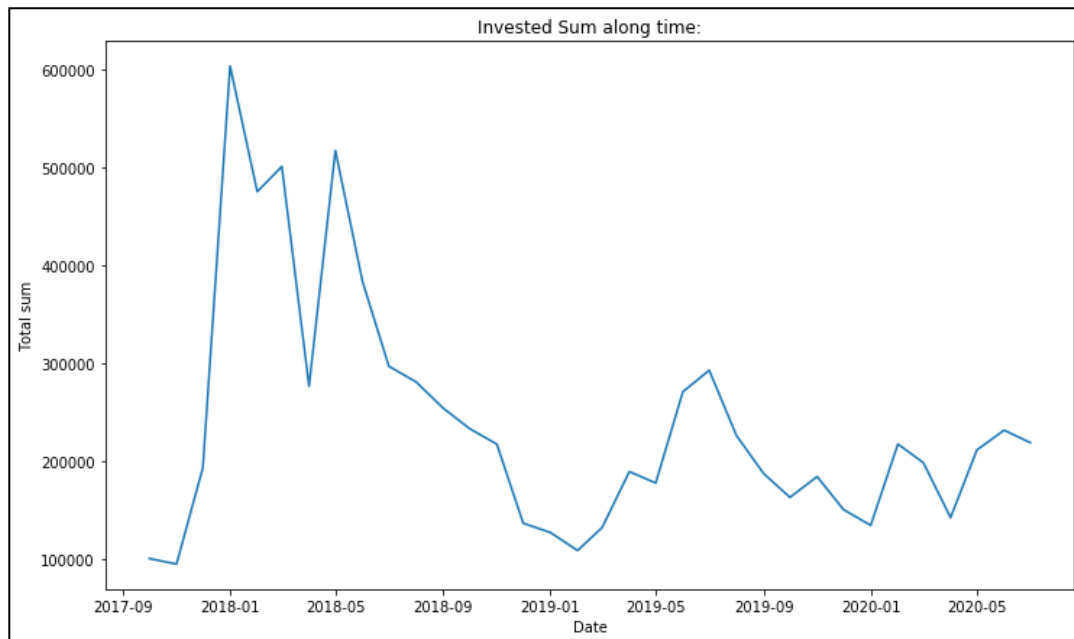
Figure 7: ETH - USD along time



We can see that the two graphs have a very similar trend, most notably the peak in January 2018 and the trough in January 2019. This is because $\frac{1}{4}$ of the portfolio is based on ETH, which is also highly correlated with other cryptocurrencies.

Equally-weighted large portfolio:

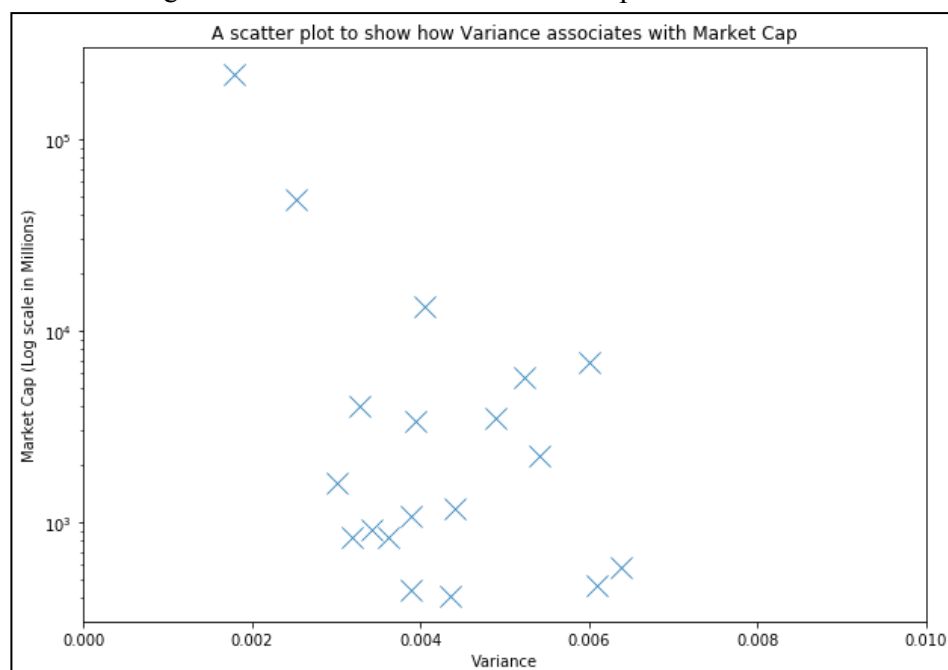
Figure 8: Equally-weighted large portfolio along time



The average percentage change for the above graph = 30.58 %, volatility = 32.79%

Here, we can see that the percentage change is 37.06% higher than the small portfolio. We believe this could be because the low market capitalisation cryptocurrencies are more variant, hence increasing the average percentage change.

Figure 9: How variance and market cap are associated



There is -0.52 of correlation between the two factors, which shows that there is some linear relationship between market cap and variance. This validates our previously mentioned hypothesis that the larger portfolio is susceptible to higher variances by using lower market cap cryptocurrencies.

Method 2: Inverse to variance

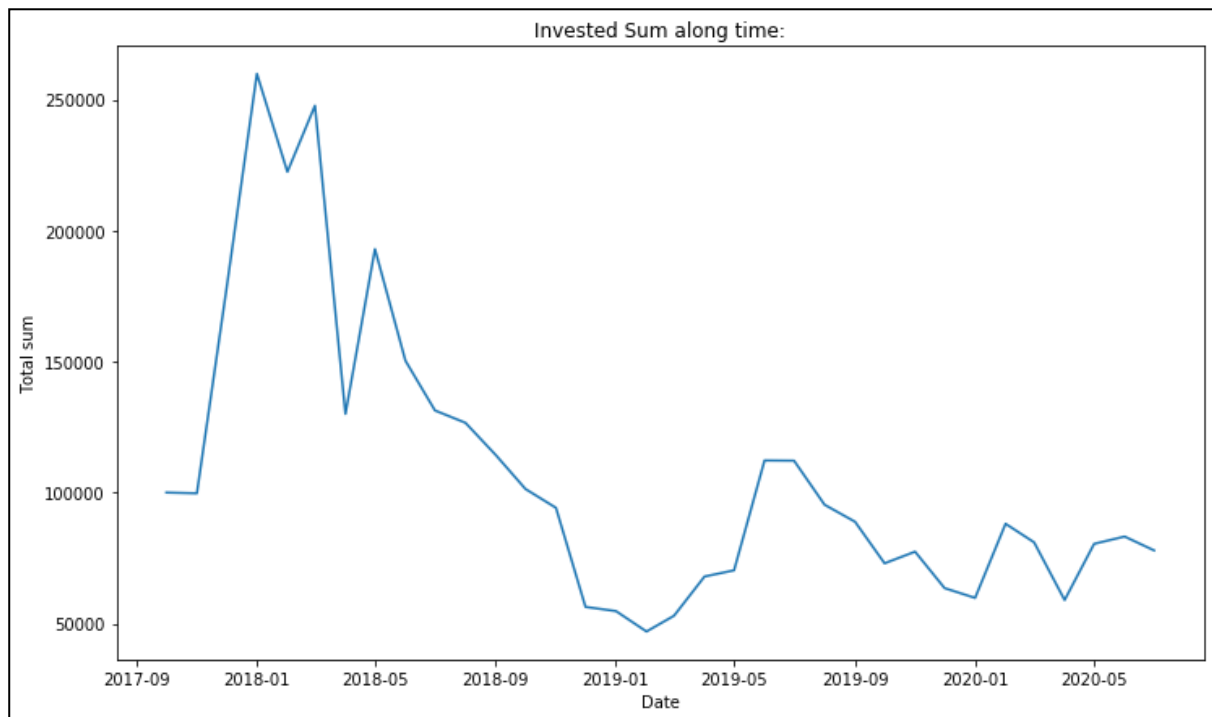
The table below shows the monthly variance of each cryptocurrency. Row 0 is the variance at the start of the data collection, September of 2017, and Row 33 being the variance at the end of the data collection. The variance of each cryptocurrency is different to one another. But the variance of all 4 cryptocurrencies generally rise and fall at the same time, following a similar trend but with different values of variance.

Table 3: Table of monthly variances

Months	vBTC-USD	vETH-USD	vXMR-USD	vETC-USD
0	0.00130759	0.000877848	0.000891313	0.00053025
1	0.00206084	0.00232119	0.0063037	0.0150027
2	0.0075072	0.00573187	0.00971586	0.00779168
...
31	0.00137702	0.00159129	0.0013056	0.00183572
32	0.000381122	0.000532542	0.000488399	0.000510846
33	0.000534414	0.00111803	0.000648924	0.000930994

Inverse to variance small portfolio:

Figure 10: Inverse to variance small portfolio along time

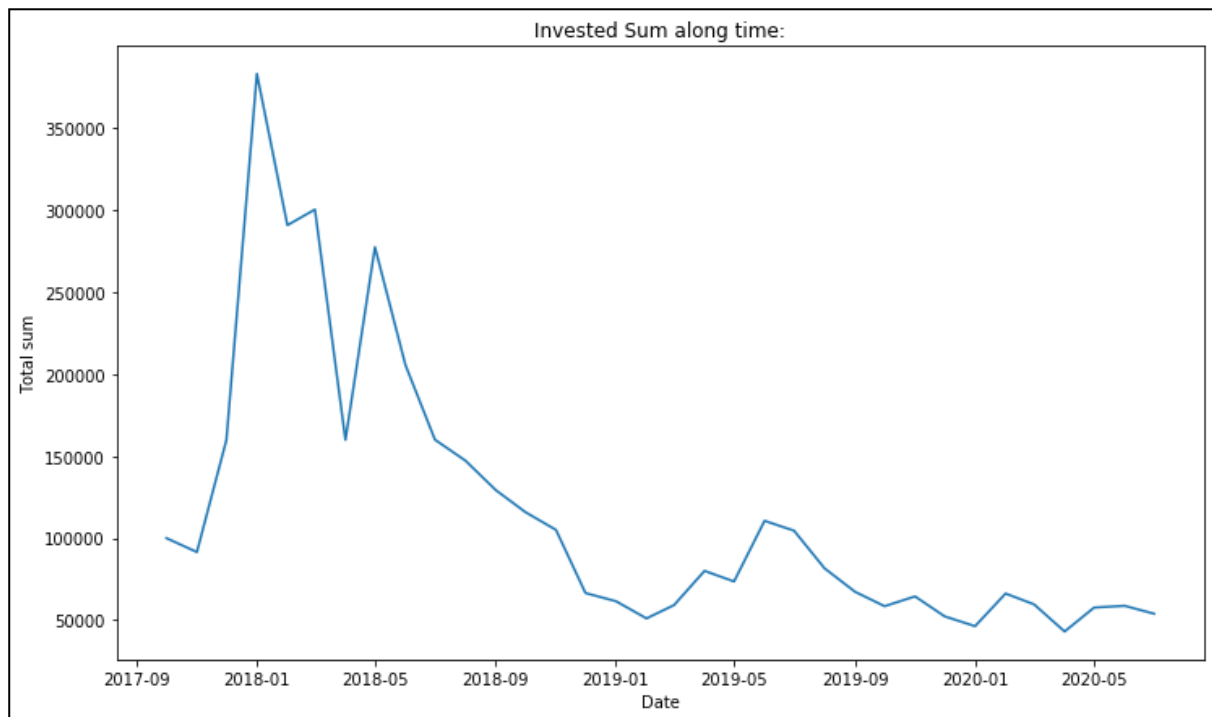


The average percentage change for the above graph = 20.38 % , volatility = 28.29%

The average percentage change in the graph is substantially lower than the control method, this means that the method successfully reduced the risk by 8.65%. This is a good sign because the inverse to variance method should definitely be superior to equal weights, due to the fact that it takes into account variance, a factor which maps an aspect of risk itself.

Inverse to variance large portfolio:

Figure 11: Inverse to variance large portfolio along time



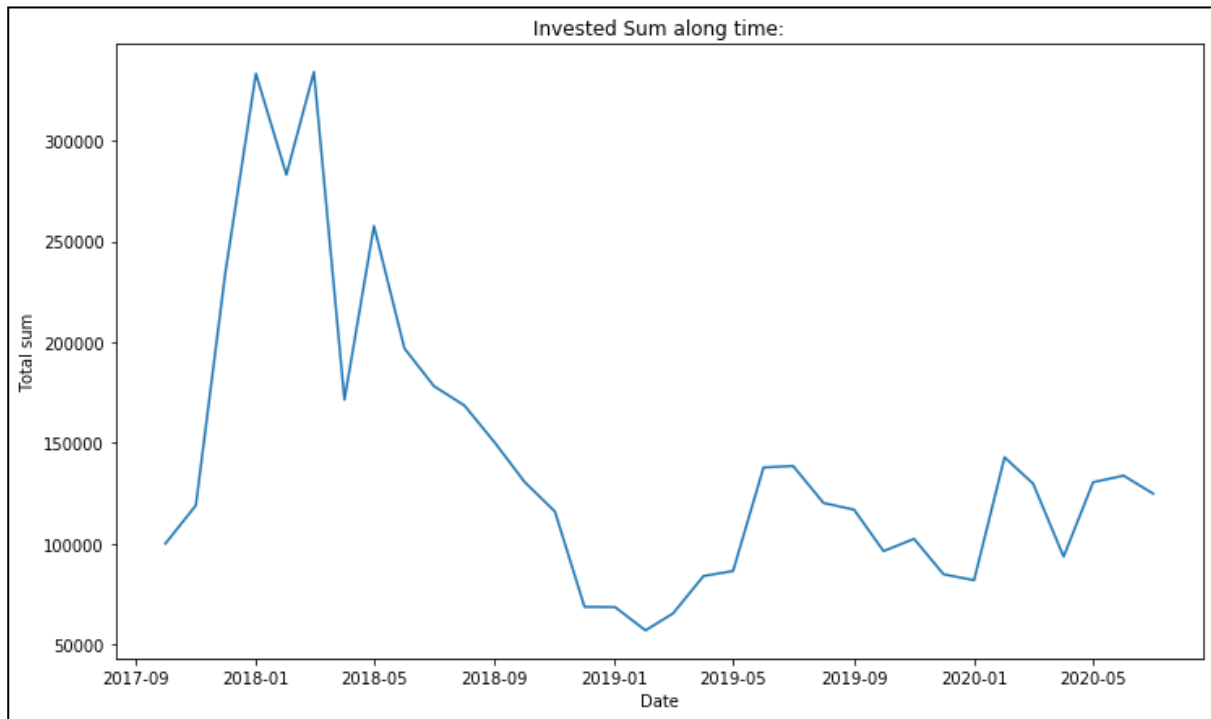
The average percentage change for the above graph = 25.81 %, volatility = 37.38%

This method had an even stronger impact on the larger portfolio, reducing its average percentage change by 15.60%. Also, we can note that this larger portfolio has a larger percentage change than the smaller portfolio. In addition, although this strategy greatly reduces risk it also seems to make a net loss on both occasions.

Method 3: Correlation

Correlation small portfolio:

Figure 12: Correlation small portfolio along time

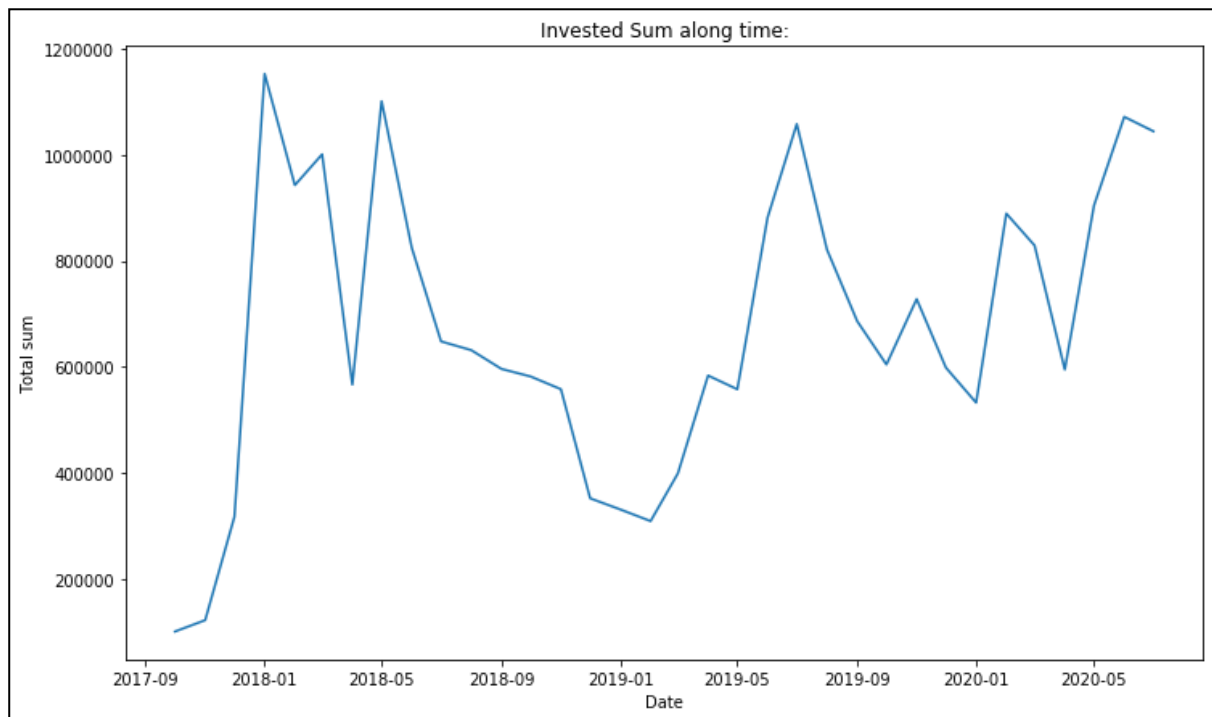


The average percentage change for the above graph = 23.91 %, volatility = 36.07%

This method performed worse than the equal weights portfolio. This is due to the sub-optimal methodology of applying Pearson's correlation coefficient. In traditional settings, inverting the matrix and using the negative values as short to maximise diversification is effective. As previously mentioned before, this setting forces us to create a new method which is not effective.

Correlation large portfolio:

Figure 13: Correlation large portfolio along time



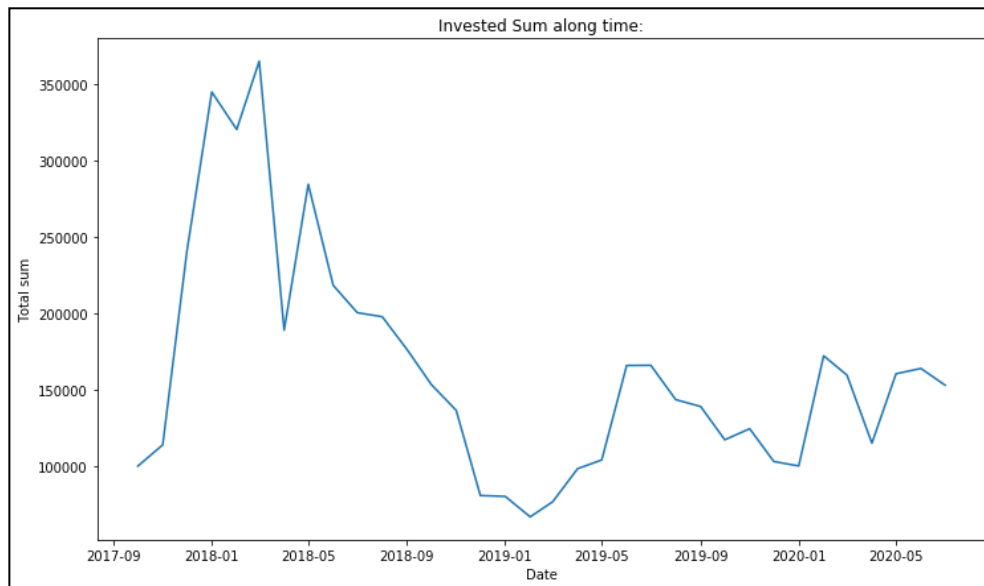
The average percentage change for the above graph = 33.69 %, volatility = 57.86%

Return on Investment (ROI) = over 10x

The method had a relatively strong impact on the larger portfolio, increasing the average percentage change by 10.17%. This is not ideal because the correlation method should be better than equal weights, as it takes into account the Pearson's correlation formula. Despite this, what is interesting is that the Return on Investment (ROI) for this method is over 10 times, coming from the original 100,000 'invested'. Despite not reducing risk at all but, rather, increasing risk, this strategy has strongly rebounded during both bear markets (2018-05 and 2020-05) compared to the variance strategy, which has reduced risk but has made losses on both occasions. Nevertheless, we believe that the historical sample sizes are not big enough to draw a definite conclusion that the correlation strategy is better than the variance strategy in terms of profitability.

Method 4: Covariance

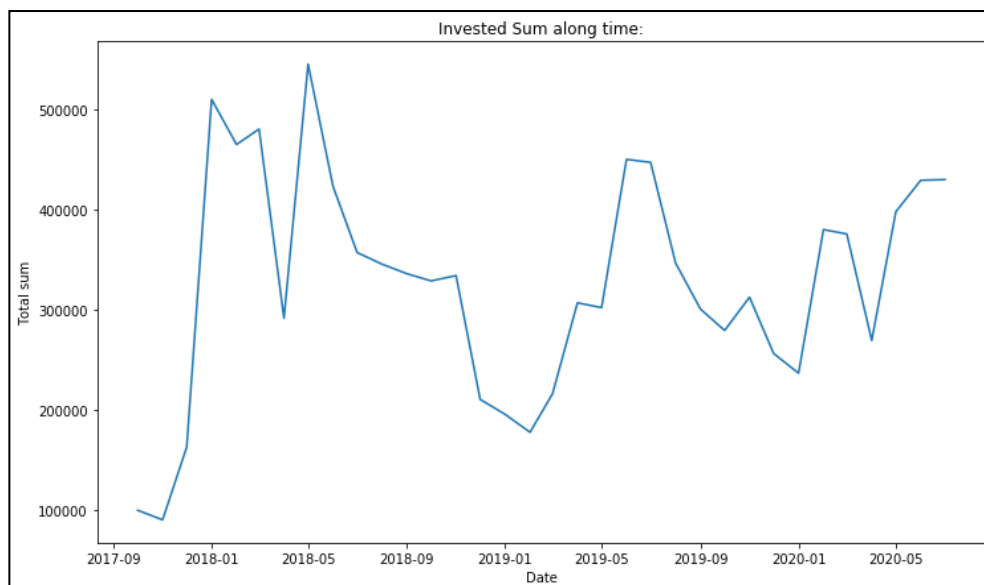
Figure 14: Covariance small portfolio along time



The average percentage change for the above graph = 22.31 %

Same result as the control, the proof that this is a coincidence is that the large portfolio differed.

Figure 15: Covariance large portfolio along time



The average percentage change for the above graph = 25.81 % volatility = 47.44 %

The 12.43% decrease and 4x ROI are successful.

Conclusion

Our conclusion covers the following elements:

- Final results,
- Research question outcome,
- Possible reasons for research question outcome,
- Unexpected findings,
- Final summary.

Final Results:

Table 4: Final results table

Percentage change between portfolio value m-1 to m for different methods (%)					
Type	Control	Variance	Covariance	Correlation	Comments
Percent - Small	22.31	20.38	22.31	23.91	This percentage change is relatively high compared to general stocks
Percent - Large	30.58	25.81	26.78	33.69	
Volatility - Small	32.79	28.29	32.79	36.07	$Volatility / \sqrt{30} \approx 7.3\%$ daily Vol.
Volatility - Large	50.06	37.83	47.44	57.86	
Difference between control and other methods (%)					
Type		Variance	Covariance	Correlation	Comments
Percent - Small		-8.65%	0.00%	7.17%	These methods seem to beat the control by more with a large portfolio.
Percent - Large		-15.60%	-12.43%	10.17%	
Volatility- Small		-13.72%	0.00%	10.00%	Variance method is the most effective in all domains.
Volatility- Large		-24.43%	-5.23%	15.58%	

The results above were obtained without hyperparameter tuning, such as not testing for time periods different than a month of variance calculations or large rolling windows, not testing for more portfolio sizes, and not using alternative variance and covariance techniques such as shrinkage. We choose not

to explore the aforementioned themes, as we value understanding the overview of the different general strategies. This is because it is futile to apply a highly technical strategy when a much more elegant strategy is available. In other words, the aim of this paper is breadth before depth.

Research question outcome:

This study successfully evaluates the effects of different weighting methods for cryptocurrency portfolios in terms of reducing risk by measuring average percentage change and volatility. By means of virtual portfolio management simulation based on historical data from September 2017 - August 2020, we tracked four weighting strategies: equal weights (naive diversification) by $1/N$, minimum variance, inverse correlation, and minimum covariance, we found that variance was the most successful.

Possible reasons for research question outcome:

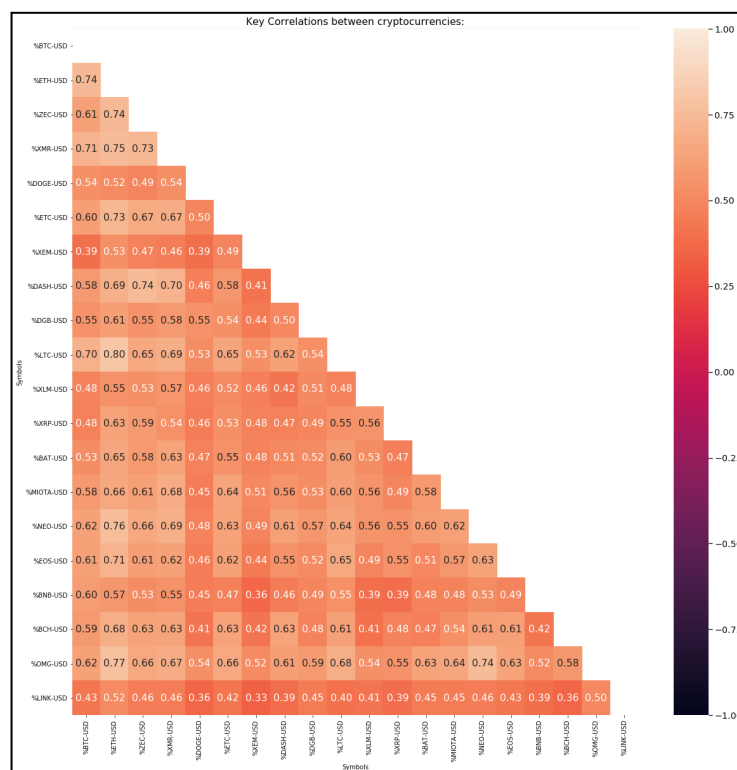
From our study, the most successful model in reducing risk is the variance model, followed by covariance, control, and correlation. The Variance portfolio strategy is the most ideal strategy, as there is a volatility decrease of 13.72% and 24.43% from our control method (equal weights). We know that the reasons for the ranking are too complex for us explain with certainty; however, we speculate:

- Having a portfolio mostly consisting of positive correlations decreases the success of diversification with traditional methods.
- If we use shorting as a method of diversification, it would be highly impactful in these situations where there are mostly positive correlations. Theoretically, this is applied by inverting the correlation coefficient matrix and using the negative values as short options. However, we do not implement this strategy as we set a long-only constraint.
- Because covariance is partially a function of correlation, it is also negatively affected by the decreased success of correlation.

Unexpected findings:

1. We find that diversification by increasing the number of cryptocurrencies through the testing of larger portfolios is not necessarily optimal. One of the main explanations we attained is that smaller cryptocurrencies, in terms of market capitalisation, have a tendency to be more variant (-.52 correlation), as seen in Figure 9 pg.26.
2. There is a complete positive inter-correlation between cryptocurrency returns, and this is evident from the heatmap below.

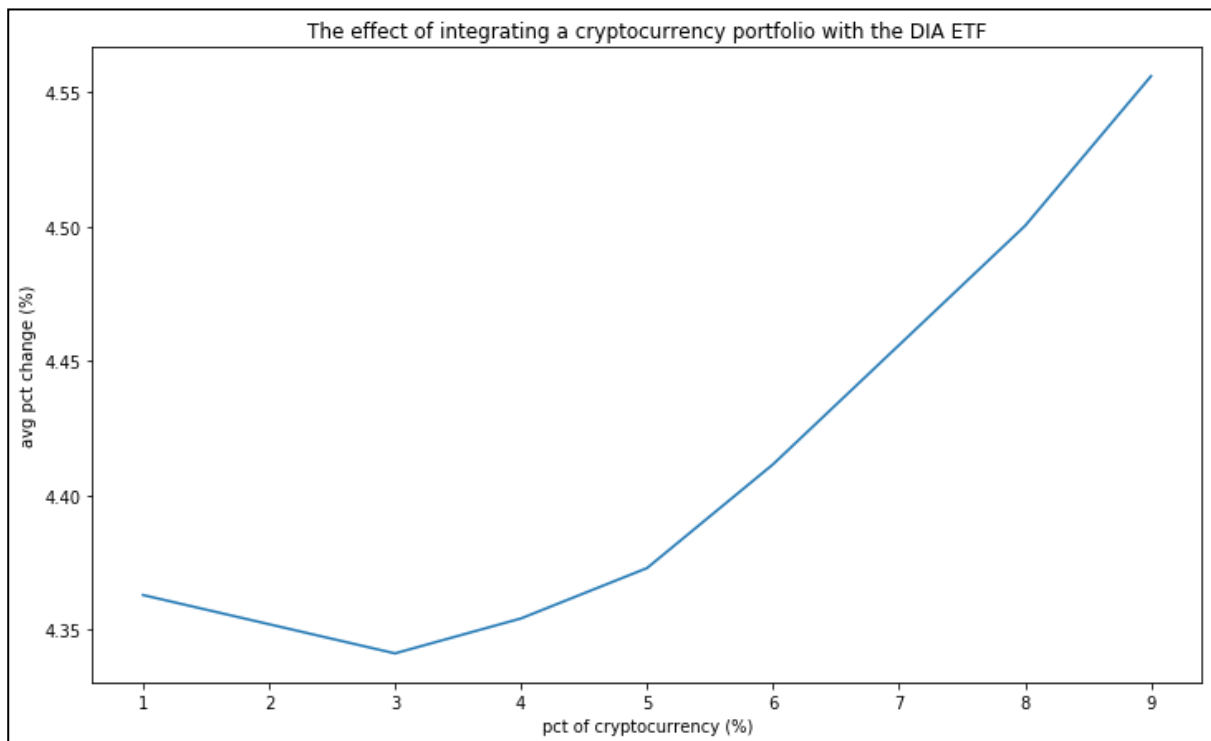
Figure 16: Large portfolio daily return correlations



This high inter-correlation seems to be a factor that hinders the performance of industry-standard portfolio diversification strategies. A future prospect would be to adapt or invent a new strategy fit for these particular properties, including point one.

3. We combine the independently calculated variance cryptocurrency strategy with the Dow Jones Industrial Average ETF (DIA) at varying ratios. Below is a graph of such a combination.

Figure 17: Integrating a preset cryptocurrency portfolio with an ETF at varying percentages



From this graph we see that this combination process is most successful in terms of reducing risk at 3%. At the scale of Russell 3000 market cap of 2018, 30T USD²⁴, three percent would be the equivalent of 900B USD, which is close to 2.5 times the current total cryptocurrency market cap.²⁵ Overall this is important, as it serves as evidence that not all cryptocurrency investors are necessarily speculators, but, rather may be, risk diversifying investors. In our case we pre-calculate the weights and merge with the ETF, when a more ideal approach would be to consider the ETF during the weight calculation process; hence, 3% serves as a rough estimate for how much people looking to store their money should invest in cryptos.

²⁴ Nasdaq articles - <https://www.nasdaq.com/articles/us-stock-market-now-worth-30-trillion-2018-01-18>

²⁵ Trading view - <https://www.tradingview.com/markets/cryptocurrencies/global-charts/>

Final summary:

To conclude, our paper fully addresses the research question by establishing the most successful strategy as variance, which reduces the average small portfolio percentage change by 8.65%, volatility by 13.72% and for the large portfolio by 15.60%, volatility by 24.43% for the large portfolio. The importance of such findings stems from the fact that cryptocurrencies are becoming ever more available and popular (as seen in Figure 18), thus creating the need to develop a safe strategy, independent of speculation. In addition to a safe strategy, this paper also estimates the point at which cryptocurrencies integrated in general portfolios reduce risk at 3%. This also leads to the suggestion that the current cryptocurrency market is 40% of what it could be as of submission date.

Figure 18: Graph to show the increasing total volume of cryptocurrencies trade over time²⁶



²⁶ TradingView - <https://www.tradingview.com/markets/cryptocurrencies/global-charts/>

Bibliography

cryptocurrency. (n.d.) *Oxford University Press*. Accessed on 15 August 2020 from Lexico, <https://www.lexico.com/definition/cryptocurrency>.

volatility. (n.d.) *Farlex Financial Dictionary*, published in 2012. Accessed on 15 August 2020 from The Free Dictionary, <https://financial-dictionary.thefreedictionary.com/volatilities>.

correlation. (n.d.) *Collins Dictionary of Economics*, 4th ed.. (2005). Accessed on August 15 2020 from The Free Dictionary, <https://financial-dictionary.thefreedictionary.com/correlation>.

Wendorf, Marcia. "Explore the Similarities between Bitcoin and Tulip Mania." *Interesting Engineering*, Interesting Engineering, 6 June 2019. Accessed on 15 August 2020 from www.interestingengineering.com/what-do-bitcoin-and-tulip-mania-have-in-common.

"Bitcoin", *CoinDesk*. Accessed on 16 August 2020 from <https://www.coindesk.com/price/bitcoin>.

Popper, Nathaniel. "After the Bust, Are Bitcoins More Like Tulip Mania or the Internet?" *The New York Times*, The New York Times, 23 Apr. 2019. Accessed on 8 August 2020 from www.nytimes.com/2019/04/23/technology/bitcoin-tulip-mania-internet.html.

Hankin, Aaron. "Bitcoin Wasn't a Response to the Financial Crisis, Says NYU Professor." *MarketWatch*, MarketWatch, 15 Sept. 2018. Accessed on 16 August 2020 from www.marketwatch.com/story/bitcoin-wasnt-a-response-to-the-financial-crisis-says-nyu-professor-2018-09-13.

"EUR-USD", *Yahoo Finance*. Accessed on 16 August 2020 from <https://finance.yahoo.com/quote/EURUSD=X?p=EURUSD=X&tsrc=fin-srch>.

"BTC-USD", *Yahoo Finance*. Accessed on 16 August 2020 from <https://finance.yahoo.com/quote/BTC-USD>.

Simonson, Itamar. "The Effect of Purchase Quantity and Timing on Variety-Seeking Behavior." *Journal of Marketing Research*, vol. 27, no. 2, 1990, p. 150., doi:10.2307/3172842.

Markowitz, Harry. "Portfolio Selection." *The Journal of Finance*, vol. 7, no. 1, 1952, p. 77–91. *JSTOR*, www.jstor.org/stable/2975974.

Härdle, Wolfgang, and Léopold Simar. "Multivariate Distributions." *Applied Multivariate Statistical Analysis*, 2003, pp. 119–154., doi:10.1007/978-3-662-05802-2_4.

Thompson, Patrick. "How To Diversify Away Risk In A Crypto Portfolio: Correlation And Variance." *Cointelegraph*, Cointelegraph, 3 May 2018, Accessed on 9 August 2020 from www.cointelegraph.com/news/how-to-diversify-away-risk-in-a-crypto-portfolio-correlation-and-variance.

Chuen, David Lee Kuo, et al. "Cryptocurrency: A New Investment Opportunity?" *The Journal of Alternative Investments*, vol. 20, no. 3, 2017, pp. 16–40., doi:10.3905/jai.2018.20.3.016.

Eisl, Alexander, et al. "Caveat Emptor: Does Bitcoin Improve Portfolio Diversification?" *SSRN Electronic Journal*, 2015, doi:10.2139/ssrn.2408997.

"Pearson Correlation Coefficient Formula: Examples & Calculator." *EDUCBA*, 17 Dec. 2019. Accessed on 10 August 2020 from www.educba.com/pearson-correlation-coefficient-formula/.

"Variance Formula: Calculation (Examples with Excel Template)." *EDUCBA*, 6 Sept. 2019. Accessed on 10 August 2020 from www.educba.com/variance-formula/.

"Covariance Formula: Examples: How To Calculate Correlation?" *EDUCBA*, 21 Apr. 2020. Accessed on 10 August 2020 from www.educba.com/covariance-formula/.

Choueifaty, Yves, et al. "Properties of the Most Diversified Portfolio." *SSRN Electronic Journal*, 2011, doi:10.2139/ssrn.1895459.

Publisher Barron's. "The U.S. Stock Market Is Now Worth \$30 Trillion." *Nasdaq*. Accessed on 15 August 2020 from www.nasdaq.com/articles/us-stock-market-now-worth-30-trillion-2018-01-18.

"Crypto Market Cap and Dominance Charts." *TradingView*. Accessed on 16 August 2020 from www.tradingview.com/markets/cryptocurrencies/global-charts/.

Higgins, Stan. "From \$900 to \$20,000: Bitcoin's Historic 2017 Price Run Revisited." *CoinDesk*, CoinDesk, 30 Dec. 2017, Accessed on 15 August 2020 from www.coindesk.com/900-20000-bitcoins-historic-2017-price-run-revisited.