Gödel, Turing, and the Untimely Demise of Hilbert’s Problems

David Hilbert was a considerably prominent mathematician in his own time, as well as after, and he was rightly so, because he contributed a great deal to the mathematics community. This community, during this time period, was not distinct and separate from the philosophical community. At the International Congress of Mathematicians in Paris in 1900, Hilbert presented his desire for more of a separation from the philosophy area by presenting twenty-three problems that defined the direction of the math community in the 20th century. Although all of the aptly named Hilbert Problems were important and helpful to the definition and understanding of the new age of mathematics, one of these problems has a considerable amount to do with the topic at hand. This problem is known as the Halting problem. It asks if there is a way of knowing whether a program or proof will halt or no, without evaluating the program or proof in question. Also, Hilbert greatly desired to make a formal system of axioms within mathematics that are complete and consistent. He wanted to formalize mathematics and separate it from philosophy completely in the process. His way of thinking gave way to formalism, which is still believed by many today, who think mathematics can be defined as complete and that what can be derived is able to be derived by the axioms presented within the system of mathematics itself.

Gödel also made a significant impact on the mathematics in the 20th century. Gödel had not been born when Hilbert had given his first set of problems, but when he was 25, in 1931, he incompleteness theorems, one of which disproved one of Hilbert’s problems. His most famous incompleteness theorem, which is the one that disproved Hilbert’s problem, proved that if a system is complete, then it cannot be consistent, and vice-versa. Hilbert’s problem, since it was asking for a complete and consistent system, was, therefore, impossible. This only applies to systems powerful enough to count natural numbers, but that was what Hilbert’s problem desired. Furthermore, Gödel also proved that there are true facts about natural numbers that cannot be proven with axioms of that system. He proved this all using recursive logic, where a variable was defined within itself. Before all of that, Gödel published a completeness theorem as his dissertation that established the completeness of first-order predicate calculus.

As if the impossibility of this Hilbert problem was not enough, Alan Turing, using ideas from Gödel’s incompleteness theorem, proved the Halting problem impossible as well. Alan Turing was a considerably successful mathematician and cryptanalyst, with some chemistry on the side. As for his contribution to chemistry, as well as biology, he wrote a paper regarding the chemical basis of morphogenesis, which is how organisms dictate how to shape themselves, as well as how to predict oscillating chemical reactions. As a cryptanalyst, he worked in England during the Second World War and decrypted the supposedly “un-decryptable” enigma machines with which the Germans communicated, whose code changed upon each sent character. He created a number of bombes that could decrypt the code, but he had the arduous task of deciding when to use the information and when not to so that the Germans did not know that their code had been broken.

In relation to the Halting problem, though, Turing first though of the Turing machine, which is a theoretical machine of infinite tape separated into tape with a method of printing on the tape and moving from square to square. This machine is capable of computing any possible mathematical computation representable by an algorithm. This machine was taken to the next level with the idea of a Universal Turing machine which could compute anything computable. This original Turing machine, though disproves the halting problem because it is impossible to tell whether a Turing machine would halt. Therefore, the Halting problem has no solution.