

*iQ3R Mathematics — Geometry*¹

*Lesson 6 Module 1*²

August 30, 2021

This sixth lesson is a bit of a departure from the New York curriculum. Instead of starting into Topic B, we find it best to review the material in Topic A and to tie together some concepts we left unaddressed in Topic A. Students will still practice using the construction tools and techniques introduced in lessons 1 through 5, but here we will close the gaps in some of the reasoning that was used to assert some of the claims made in those lessons.

Student View: Slide 1

Overview

IN THIS LESSON you will bring together all the ideas you learned in lessons 1 through 5. This will complete this unit on constructions. Specifically the aim of this lesson is to

1. Review basic constructions: you will also learn to find perpendiculars through points on lines; parallels through points not on lines.
2. Discuss postulates vs. assumptions.
3. Introduce measurement concepts: how to measure distance between various geometric objects.
4. Introduce you to symmetry.
5. Help you see “why” the concurrences we saw in lessons 4 and 5 occur.

¹ Typically in the United States grade 9 students take either Algebra 1 or Geometry depending on the school and the performance of the student in 8th grade. Generally, Geometry is taken by students who are on a more accelerated track.

² Adapted for individualized instruction from the EngageNY curriculum for New York State public and private instructors.

Overview

Lesson 6 Supported Standards

Common Core Focus Standards

G-CO.A.1

Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

G-CO.D.12

Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

G-CO.D.13

Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Student View: Slide 2

*Review with Applications**Basic Constructions*

You now know¹ three basic constructions using the drawing tools

- Given a line segment, find the perpendicular bisector to the line segment.
- Given an angle, find the angle bisector of that angle.
- Given an angle, copy the angle along another segment or ray.

Using these basic constructions, you now are able to

- Construct an equilateral triangle
- Construct a regular hexagon
- Find the perpendicular to a given line which contains a specified point not on the line.
- Construct the incenter and circumcenter of a triangle as well as the corresponding circles.

CHECK FOR UNDERSTANDING: Given a triangle $\triangle ABC$ draw

1. The circumcircle
2. The incircle

CHALLENGE: When will the circumcenter of a triangle be on the exterior of the triangle? When will it be on one of the sides of a triangle?

*Review with Applications**Basic Constructions*

1. Students have seen these procedures and have been using them over the last five lessons so the procedures will not be reproduced here. Be prepared to verbally review them if necessary.

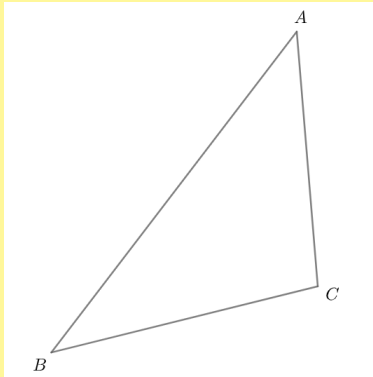
Do we want to have some form of digital handout they can reference?

Lesson Design Specifications (Please Read)
Guided Practice:

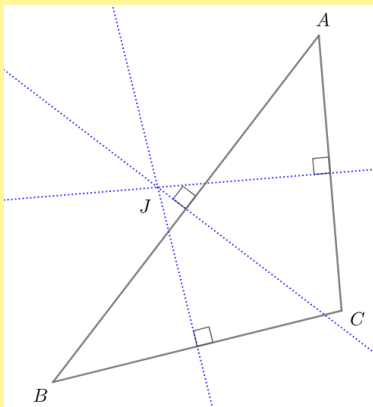
At this point student and teacher will do one of the following:

- Navigate to Geogebra on a shared screen **OR**
- Launch an app that opens up a shared window for interactive geometry **OR**
- Share a whiteboard with appropriate drawing tools.

Instructor will now use the following sequence on first a scalene then an obtuse triangle. We show only the case for the obtuse.

CONSTRUCTION SEQUENCE

STEP 1

Find the perpendicular bisectors of the segments of $\triangle ABC$. Label their point of concurrency¹ J .


STEP 2

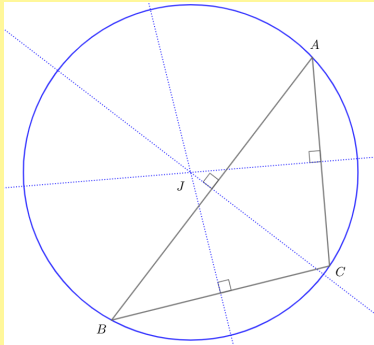
The result should look something like the figure shown². Ask, "What radius should you use for the circle centered at J ?" *Answer:* Any of the equal distances AJ , BJ or CJ will do.

Now draw circle J with the appropriate radius.

1. Ask, "Do we really need to find all three perpendicular bisectors to draw the circumcircle?"
2. Note that we have not included **arrowheads** in the picture. When the line extends to the border of the graphic, that also is intended to represent a line. When we want to emphasize that a portion of a line is being viewed, we will do that by keeping the entire structure within the viewing window and using the arrowheads where the segment terminates.

Lesson Design Specifications (Please Read)

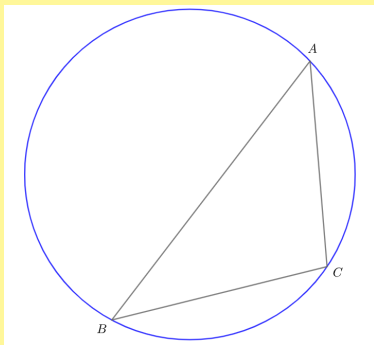
Guided Practice: Continued



STEP 3

The result should look something like the figure shown. Now clear away the auxillary lines and miscellaneous label J .

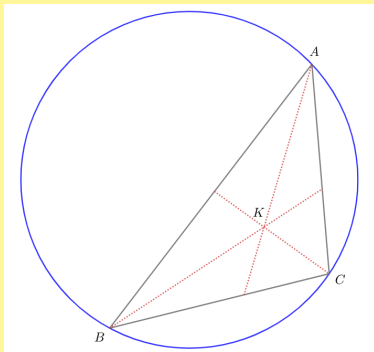
Leave the circum-circle in view.



STEP 4

Now draw the bisectors of each of the angles of triangle $\triangle ABC$.

Label the incenter as K .



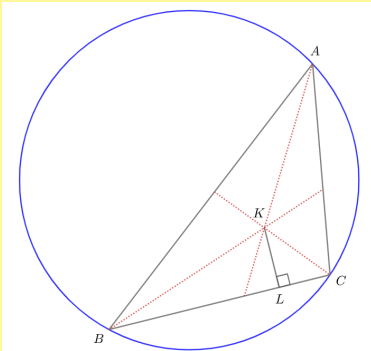
STEP 5

The result should appear somewhat like what you see here in this figure.

Next, draw a perpendicular segment from K to side BC and label its intersection with BC as L .

Lesson Design Specifications (Please Read)

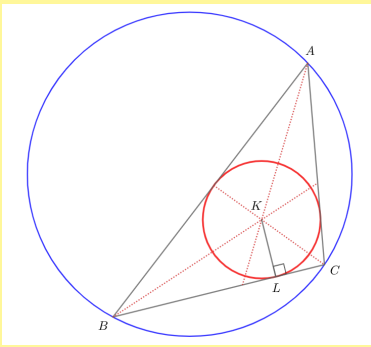
Guided Practice: Continued



STEP 6

The result should appear somewhat like what you see here in this figure.

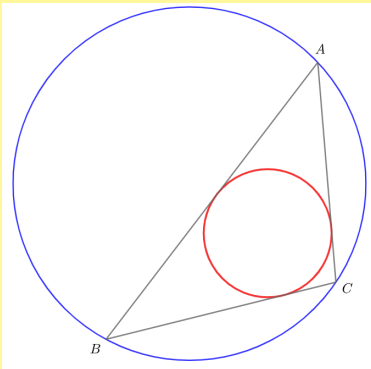
Finally, draw circle K centered at K with radius KL .



STEP 7

The result should appear somewhat like what you see here in this figure.

Now eliminate all the superfluous lines and labels.



The final figure with a triangle showing BOTH the incircle and the circumcircle.

Now we return to the main instructional sequence.

Student View: Slide 3

*Review with Applications**Extending the Results*

Previously: For given line k and point P not on k we saw how to use our basic constructions to find the line perpendicular to k through P . We now make a definition/notation which we will stay with for the course.

DEFINITION $k^\perp(P)$: Given line k and point P (either on or off of k), we represent *the unique* line which is perpendicular to k and contains P by the notation¹ $k^\perp(P)$.

DEFINITION $k^\parallel(P)$: Given a line k and a point P (again, either on or off of k), we represent the unique line which is parallel to k through P by the notation $k^\parallel(P)$. If a point P is on k we take $k^\parallel(P)$ to simply be k .

KNOWLEDGE POINT: Using the basic constructions you now can

1. Construct $k^\perp(P)$, given line k and point P in any case — whether P is on k or not.
2. Construct $k^\parallel(P)$ for any given line k and point P .

Let's work out the details of (1) first.

*Review with Applications**Extending the Results*

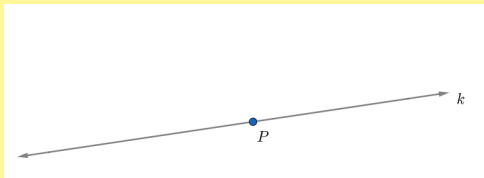
1. The function notation is intentional. For each line k in the plane k^\perp defines a function from the set of points in the plane to the set of lines in the plane. This function has interesting properties on its own.

Lesson Design Specifications (Please Read)

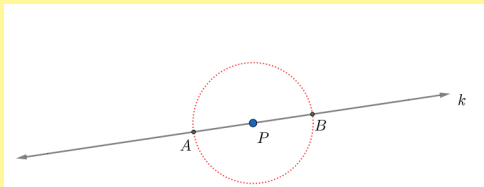
Guided Practice 2: Construction of $k^\perp(P)$; case P is on k .

At this point student and teacher will do one of the following:

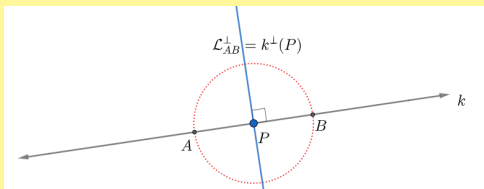
- Navigate to Geogebra on a shared screen **OR**
- Launch an app that opens up a shared window for interactive geometry **OR**
- Share a whiteboard with appropriate drawing tools.



STEP 1: Fix line k with selection of arbitrary point P on k . Draw a circle with center at P and label intersections as A and B .



STEP 2: Now find \mathcal{L}_{AB}^\perp — the perpendicular bisector of segment AB .



STEP 3: Done. We have done what we set out to do. We have found $k^\perp(P)$ — the line perpendicular to k through the point P *located on k* .

We now return to the main sequence of instructional slides.

Student View: Slide 4

*Review with Applications**Construction of $k^\perp(P)$.*

NOTATION: If a line k is specified with selected point P on k , we will represent the line perpendicular to k containing the point P by the symbol $k^\perp(P)$ — pronounced “k perp of P”.

KNOWLEDGE POINT: Construction of $k^\perp(P)$ from line k and point P on line k .

- Step 1: Locate P on line k .
- Step 2: Draw circle P centered at P with any positive radius r . Since $r > 0$ circle P meets k in exactly two points which we label A and B .
- Step 3: Draw \mathcal{L}_{AB}^\perp — the perpendicular bisector of segment AB .
- Observation: $\mathcal{L}_{AB}^\perp = k^\perp(P)$.

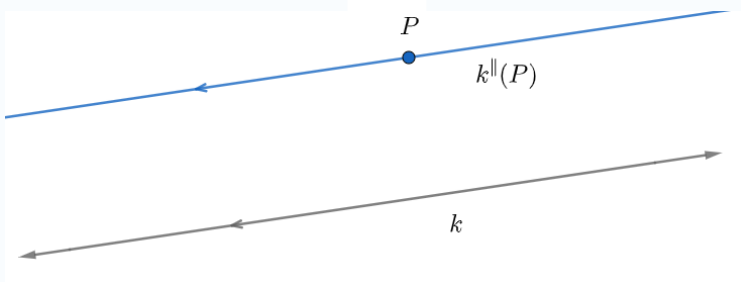
We now address the second problem we considered.

*Review with Applications**Construction of $k^\perp(P)$.*

Student View: Slide 5

*Review with Applications**Construction of $k^{\parallel}(P)$*

IMPORTANT PROBLEM: Given line k and point P , off k , find a line which is both parallel to k and contains P . Recall that we use the notation¹ $k^{\parallel}(P)$ to represent this line: **The line *parallel* to line k containing the point P which is off k .** This is illustrated in the figure below.



We use the arrows² along the lines for emphasis that these lines are parallel.

Let's see how this construction works 🧐

*Review with Applications**Construction of k^{\parallel}_P*

1. Be sure to emphasize that the words go with the symbol. When they see the symbol $k^{\parallel}(P)$ they should be thinking "the line parallel to line k through the point P ." We use the symbol so we don't have to keep writing the cumbersome phrase over and over again, but we **MUST** remember that the symbol is encoding the language of geometry.
2. We mean the arrows around the "center" of the figure. Recall our convention: lines are either represented as in the figure by k using arrows on the ends of the segment, or as in the figure by $k^{\parallel}(P)$ which simply runs from one side of the viewing window to the other.

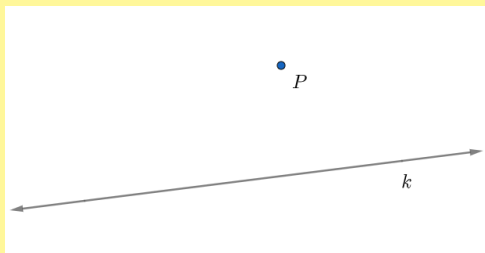
Lesson Design Specifications (Please Read)

Guided Practice:

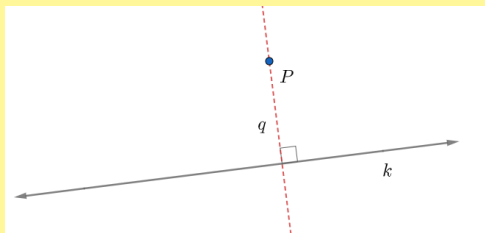
At this point student and teacher will do one of the following:

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- Launch an app that opens up a shared window for interactive geometry **OR**
- Share a whiteboard with appropriate drawing tools.

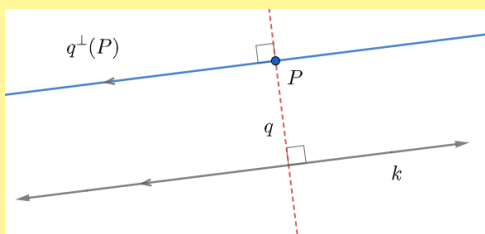
Instructor will now guide the student through the steps required to construct $k^{\parallel}(P)$.



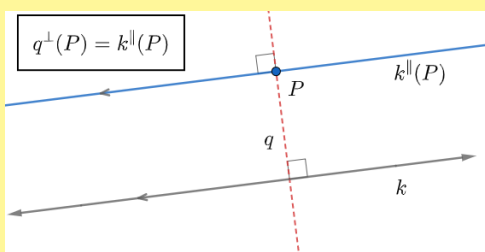
STEP 1: Start with a given line k and point not¹ on k which we have labeled P .² Using the ideas from lesson 4, construct the perpendicular to k containing point P . Call this q . (Observe that $q = k^{\perp}(P)$.)



STEP 2: You should get a figure similar to the one shown here at the left. Now have the student construct $q^{\perp}(P)$ — the line perpendicular to q through P .



STEP 3: That is it! Amazing! Actually $q^{\perp}(P)$ is what you are looking for. We emphasize this in the next figure.



ALL DONE! QED³ 🧐

1. What would happen if we tried to pick point P on line k ?
2. Again, stress that what you label the lines and points does not matter. Notice that when we label lines other than by using notation like \overleftrightarrow{AB} , we use either lower case letters or the more elaborate \mathcal{L} .
3. QED — *quod erat demonstrandum* — Latin for “what was to be demonstrated”

Student View: Slide 6

*Review with Applications**Construction of $k^{\parallel}(P)$*

KNOWLEDGE POINT: If a line k and a point P not on¹ k are specified, then we use the following procedure to find $k^{\parallel}(P)$ — the line parallel to k through the point P .

- Step 1: Draw line $k^{\perp}(P)$ containing point P .
- Step 2: Draw the perpendicular through P to line $k^{\perp}(P)$.
- Step 3: Verify that in fact the following beautiful equation² holds!

$$k^{\parallel}(P) = (k^{\perp}(P))^{\perp}(P)$$

OBSERVATION: This equation of line descriptors is an extremely concise way of saying that, "Two lines perpendicular to the same line are parallel to each other."

CHECK FOR UNDERSTANDING: Using our notation³, what should be the result of finding $(k^{\parallel}(P))^{\perp}(P)$?

*Review with Applications**Construction of $k^{\parallel}(P)$*

1. Recall that if P is on K then $k^{\parallel}(P)$ is just k .
2. This is truly an instance of where the choice of effective notation really does convey more information than expected. The language here that goes with this little equation is, "The perp *through* P on the perp *from* P to k is the *parallel* to k through P ."
3. Focus on the language. The expression says "find the perpendicular through P , to the line *parallel to* k which contains P . This must be the perpendicular through P to k . So the answer is simply $k^{\perp}(P)$."

Student View: Slide 7

*Concept Development**Postulates****Parallel Postulate:***

Given a point P and line k not containing P there exists *exactly one* line containing P which is parallel to k .

Assumptions in Geometry are actually called “postulates”.

- An assumption can be easily changed.
- A postulate¹ is something that is taken to be true and is not easily changed.
- This “parallel postulate” changed the world of geometry!
- Since it is an assumption and not a “fact”, people wondered if it was a necessary condition² of the assumed “Euclidean Assumptions.”
- This postulate actually characterizes what we call “flat space” and it is not necessarily true for all “spaces.”³

*Concept Development**Postulates*

1. Postulates are the foundation of the theory of geometry. We can change the parallel postulate to read that there are infinitely many lines parallel to k . When we do that we change from “flat” geometry to non-Euclidean geometry.
2. It is not a necessary condition of the assumptions of Euclidean Geometry. This is in fact the starting point for non-Euclidean geometries. For centuries, mathematicians tried to “prove” the parallel postulate as a consequence of other assumed “facts” and were miserably thwarted. Now we know that this postulate is actually what characterizes the Euclidean “flat” space.
3. The physical universe is generally considered to be a 4-dimensional space-time manifold that is “curved” due to the presence of matter which is equivalent to gravity according to Einstein.

Student View: Slide 8

Concept Development

Postulates

The Ruler Postulate:

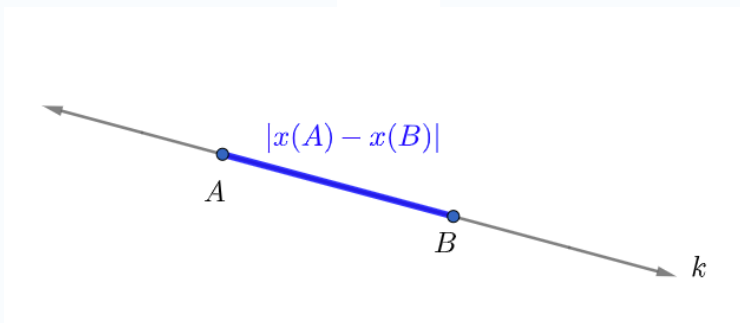
Let point O and P be chosen on line k . Let the real number system, \mathbb{R} , represent measurements in a fixed system of units.¹ Then there exists a one to one² correspondence $x : k \rightarrow \mathbb{R}$, such that $x(O) = 0$, $x(P) = 1$, and for all points A and B on k ,

$$|x(A) - x(B)| = AB.$$

Moreover, given any real number ζ ,³ there exists exactly one point Z on k , such that $x(Z) = \zeta$.

The mapping x is called a *coordinate system* on k . It is also referred to as a *coordinationization* of k .

So the content of the ruler postulate is that any line can be coordinatized. Coordinationization of a line is far from unique. Indeed, if x and y are both (linear)⁴ coordinate systems on k , a previous challenge problem showed that $y = mx + b$ for some constants m , and b .



Concept Development

Postulates

1. For example, it could be units of time, money, length and so on. The idea is that scalars are not just numbers when it comes to measurement in geometry. We measure lengths, or intervals, in units in some fixed system. Be sure to emphasize this. In geometry when we say, for example $AB = 2$, we most always mean 2 of "something" — unless you are working in a highly specialized dimensionless framework which is way beyond the scope of this course.
2. One-to-one means that if $x(A) = x(B)$, then $A = B$. The mapping is also surjective (onto). This is the content of the second part of the postulate.
3. This is the Greek letter "zeta".
4. Curvilinear coordinate systems are also possible: a standard coordinate system on the "postive" axis is given by $x = \log t$. This is the system in which earthquake magnitudes are measured.

Student View: Slide 9

Concept Development

Point Plotting Theorem

From the Ruler Postulate we immediately get the important

Point Plotting Theorem: Let \overrightarrow{OQ} be any given ray. Let $r > 0$ be given in a specified unit system for measurements on \overrightarrow{OQ} . Then there exists a unique point P on \overrightarrow{OQ} such that $OP = r$.

We will be studying the idea of proof very soon. Here we will demonstrate a little bit about what is meant by “proof”. A proof in mathematics is simply a reasoned argument based on accepted assumptions.

What have we to assume here? **The Ruler Postulate:** From this we get a coordinate system “ x ” on line OQ with $x(O) = 0$ and $x(Q) = 1$. Hence \overrightarrow{OQ} is the set of points with “positive” coordinates in the “ x ” system.¹

Take $\zeta = r$ in the second part of the Ruler Postulate. The postulate assures us the existence of a point $Z = P$ with $x(P) = r$. Since

$$OP = |x(P) - x(O)| = |x(P) - 0| = |x(P)| = x(P) = r,$$

we have demonstrated what we desired to show².

CHECK FOR UNDERSTANDING: Will x composed with $t \mapsto t^2$ still give us these results? ³

Concept Development

Point Plotting Theorem

1. A coordinatization is exactly that ... it is a system for assigning numbers to points and conversely. Emphasize that this is entirely up to the experimenter. But once a coordinatization has been selected, all measurements must be agreed on within that coordinate system. Be very careful if you have to “change coordinates”. Make sure to keep track of the bookkeeping if you do that. This is especially important in Physics.
2. Well, we have really only shown “existence” ... can you explain why uniqueness also follows? (Answer: If P' is any other point on ray OQ with $OP' = r$, then $x(P) = x(P') = r$, and by injectivity of x we have that $P = P'$.)
3. Yes! Since $t \mapsto t^2$ is one to one on the positive axis, take $\zeta = \sqrt{r}$ and apply the Ruler Postulate.

Student View: Slide 10

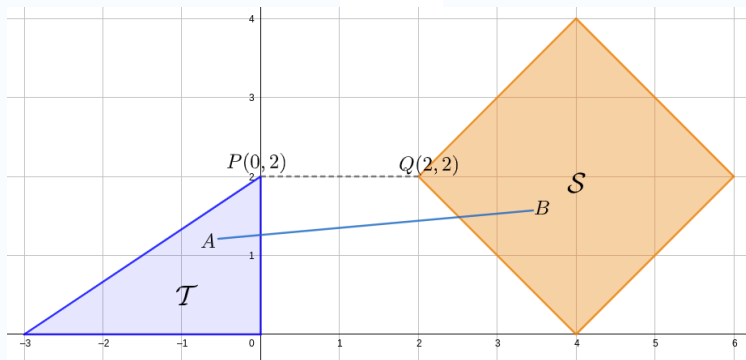
Concept Development: Measurement

Distance

KNOWLEDGE POINT: *Distance Between Points* Let a system of units be selected for measurements of distance between points on ANY line. If x is a coordinate system on line AB then the distance between A and B is $|x(A) - x(B)|$.

We define distance between “extended objects”¹ in terms of the distance between points.

EXAMPLE: What is the distance between the triangle and the square in the figure below?



Since $PQ = 2$ and every other segment \overline{AB} with A in T and B in S satisfies $AB > PQ = 2$, we say that the distance between S and T is simply $PQ = 2$.

DEFINITION: *Distance Between Sets of Points*

Let T and S be sets of points in the plane (or space). We define the *distance between* T and S , denoted $\text{dist}(T, S)$ to be the *minimum*² of all lengths AB where A lies in T and B lies in S .

Concept Development: Measurement

Distance

1. Extended objects are objects which are composed of possibly more than one point. The idea here is to use our notion of distance between points to develop the concept of distance between objects.
2. In actuality, we should have

$$\text{dist}(T, S) = \inf\{AB \mid A \in T, B \in S\}$$

The definition given here works fine when the sets are compact. It is certainly true for simple plane figures like polygons. This is only a course in elementary geometry. So this definition will do well for us. Interested and thoughtful students will do well to consider the shortcomings of this definition. For example, the min does not exist between the point $(2, 4)$ and the *open* unit disc.

Student View: Slide 11

*Concept Development: Measurement**Distance*

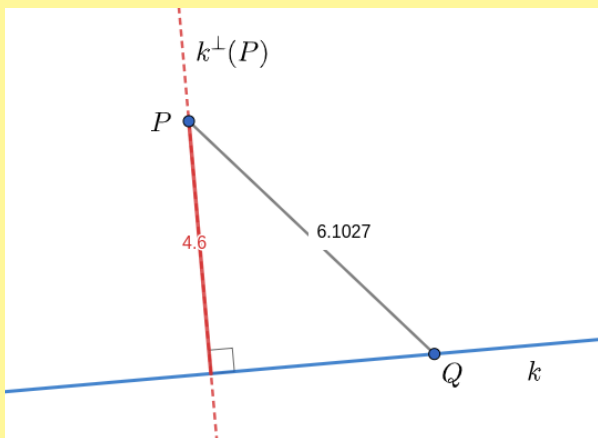
CHECK FOR UNDERSTANDING: Use our generalized definition of *Distance Between Sets of Points* to verify that the right definition of distance from a point to a line is measured along a perpendicular¹.

Lesson Design Specifications (Please Read)*Guided Practice:*

At this point student and teacher will do one of the following:

- Navigate to Geogebra on a shared screen **OR**
- Launch an app that opens up a shared window for interactive geometry **OR**
- Share a whiteboard with appropriate drawing tools.

Here the Instructor will guide the student to use the following app² which allows the student to move point Q along k and see the distance PQ continuously update. They will see that PQ takes its “min” exactly when Q takes on the location of $k^\perp(P) \cap k$.

*Concept Development: Measurement**Distance*

1. Let P be a point selected off of a given line k . You will want to help the student identify the two sets $T = \{P\}$ and $S = k$. The exercise consists in showing that given any arbitrary point Q on k the distance PQ is minimized when Q is on $k^\perp(P) \cap k$.
2. This app is very easy to use. Students will see at once that the only way to get the smallest value of PQ is to select that point Q both on k and on the perpendicular $k^\perp(P)$.

Student View: Slide 12

*Concept Development**Symmetry*

DEFINITION: **Reflection in a Line:** Let line k in the plane be given. A **reflection** in k is a function R_k from the plane to itself which has the following properties:

1. For all P not on k , $\overline{R_k(P)P} \perp k$.
2. For all P on k , $R_k(P) = P$.
3. k is the perpendicular bisector of $\overline{R_k(P)P}$ for all points P not on k .

*Concept Development**Symmetry*

1. This says that segments formed from images and their pre-images are always perpendicular to the line of reflection.
2. This says that the line of reflection is fixed under the transformation.
3. This says that k contains the mid-points of each of the segments $\overline{R_k(P)P}$.

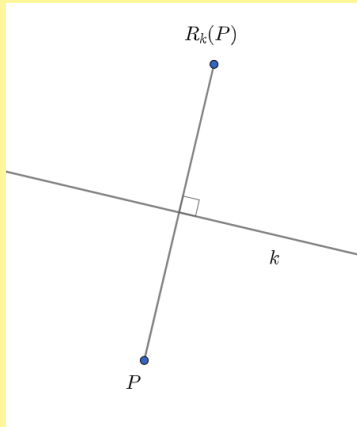
Lesson Design Specifications (Please Read)*Guided Practice:*

At this point student and teacher will do one of the following:

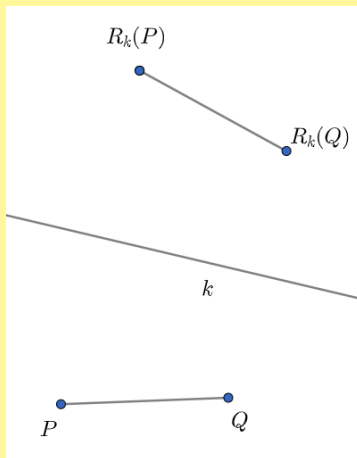
- Navigate to Geogebra on a shared screen **OR**
- Launch an app that opens up a shared window for interactive geometry **OR**
- Share a whiteboard with appropriate drawing tools.

This sequence of demonstrations will help our students understand the idea of “reflection in a line” as a rigid transformation of the plane. This is a huge concept coming up. We get a brief glimpse of it here.

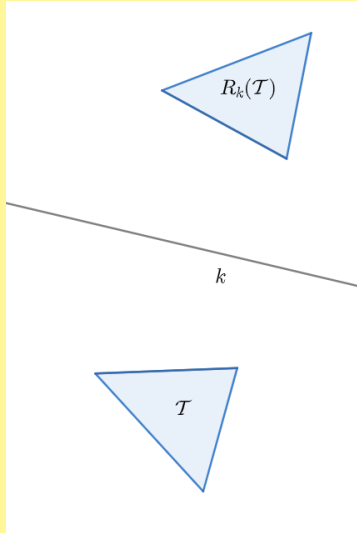
1. Reflection of point P in line k .



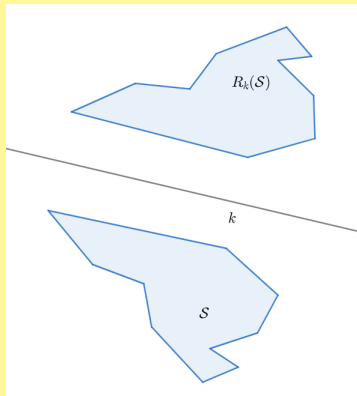
2. Reflection of a segment PQ in line k .



3. Reflection of a triangle in line k .



4. Reflection of a more complicated figure about line k .



We now return to the main instruction sequence.

Student View: Slide 13

(HUGE¹) KNOWLEDGE POINT: A figure \mathcal{K} is *symmetric* with respect to a line k if $R_k(\mathcal{K}) = \mathcal{K}$.

SYMMETRIES OF IMPORTANT FIGURES:

1. Let Q be a square and let d be a given diagonal. Then Q is symmetric about d .
2. Let d be any diameter of any circle. The circle is symmetric about d .
3. Let \overline{AB} be any given segment. Then \overline{AB} is symmetric with respect to its perpendicular bisector \mathcal{L}_{AB}^\perp .²
4. Let angle $\angle ABC$ be given. Let \overrightarrow{BK} be the angle bisector of $\angle ABC$. Then $\angle ABC$ is symmetric with respect to \overrightarrow{BK} .³

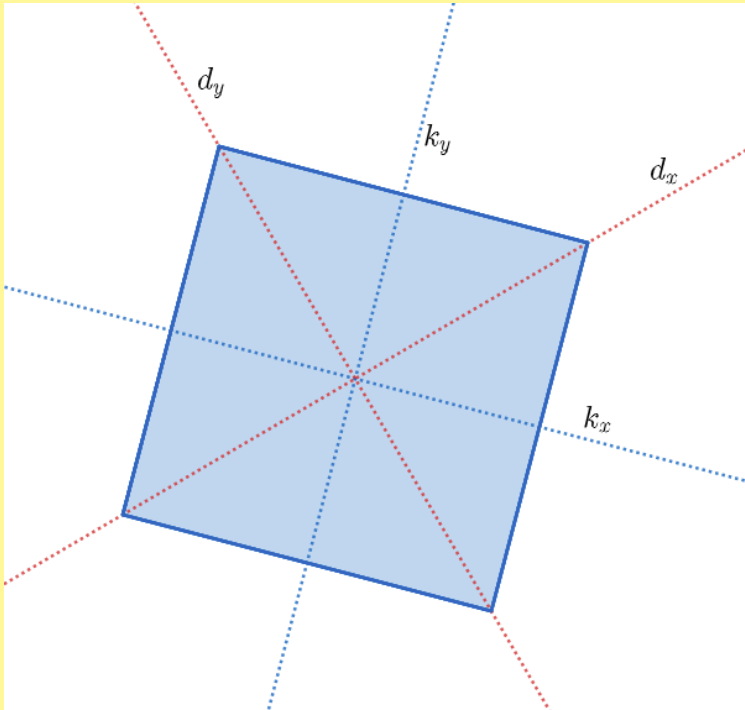
Lesson Design Specifications (Please Read)

Guided Practice: Demonstrations of Symmetries

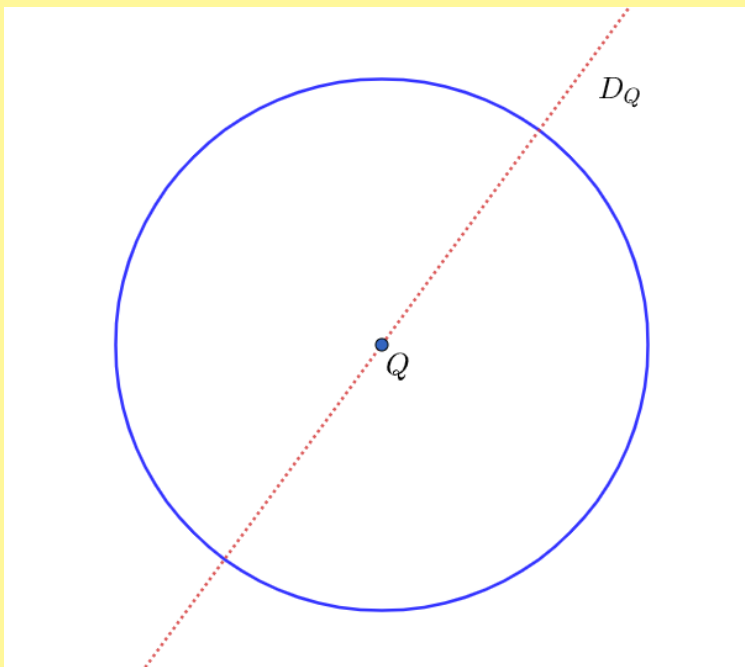
At this point student and teacher will do one of the following:

- Navigate to Geogebra on a shared screen **OR**
 - Launch an app that opens up a shared window for interactive geometry **OR**
 - Share a whiteboard with appropriate drawing tools.
1. Use the reflection tool to show that the square is invariant under reflection symmetries in each of the lines k_x , k_y , d_x , and d_y .

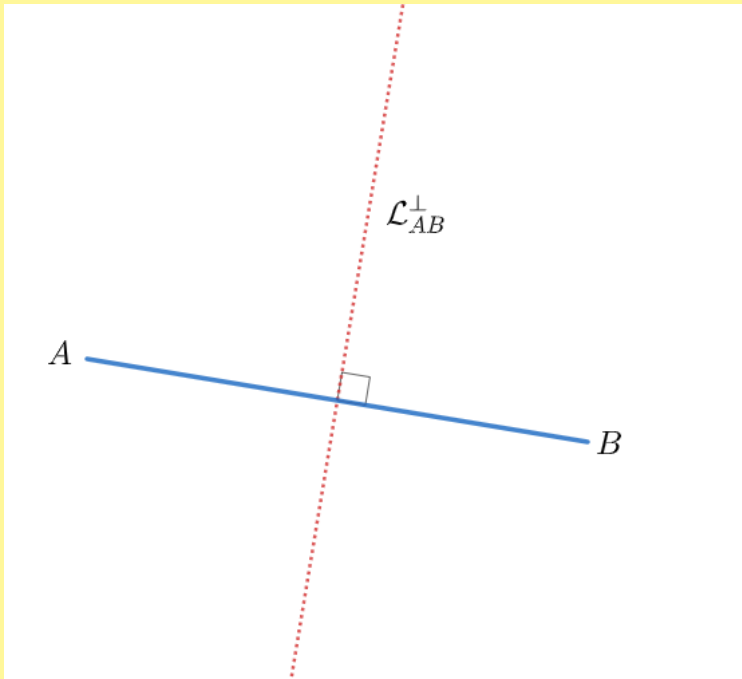
1. It cannot be overstated how important this concept is. The idea of symmetry in mathematics is ubiquitous. Later we will investigate rotational symmetry and symmetry via reflection in points. Figures in the plane can be described in large measure by their symmetry groups. For example, a circle is characterized by rotational invariance about its center through every angle, and reflection invariance about every line containing its center. It is also properties of symmetry that inform our ideas of probability; since a coin is “symmetric” about its axes of rotation, we should expect a 50:50 chance of getting heads or tails.
2. It is this fact that allows us to conclude WHY the perpendicular bisectors of the sides of a triangle are concurrent.
3. It is this fact that allows us to conclude WHY the angle bisectors of a triangle are concurrent.



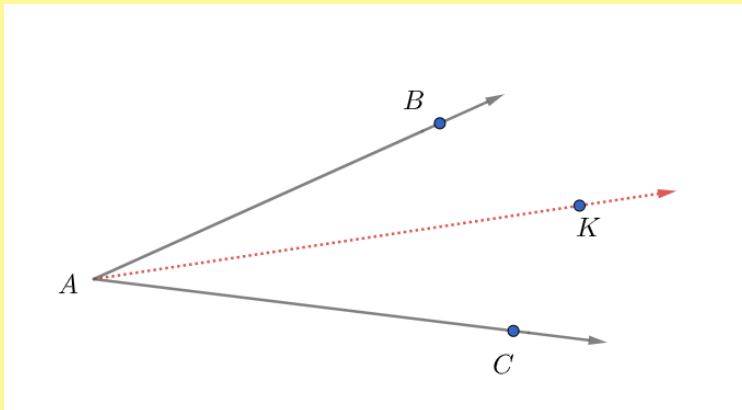
2. Use the reflection tool to show that a circle is reflection invariant through any line containing a diameter. Any line D_Q containing the center of a circle Q contains its diameter as shown.



3. Use the reflection tool to show that a segment \overline{AB} is reflection invariant about its perpendicular bisector \mathcal{L}_{AB}^{\perp} .



4. Use the reflection tool to show that an angle $\angle BAC$ is reflection symmetric about its angle bisector \overrightarrow{AK} .

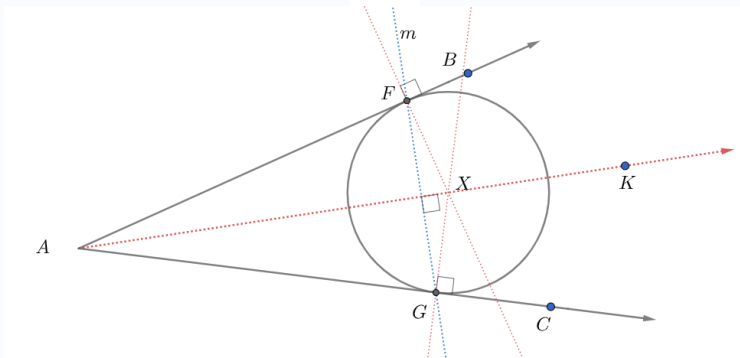


We now return to the main instruction sequence.

Student View: Slide 14

Let angle $\angle BAC$ be given with angle bisector \overrightarrow{AK} . Let Q be any point on \overrightarrow{AK} . Then Q is equidistant¹ from the sides of $\angle BAC$.

PROOF: Let m be any perpendicular through \overrightarrow{AK} and say m intersects $\angle BAC$ at points F and G . Then \overrightarrow{AK} is contained in the perpendicular bisector of \overline{FG} by symmetry. Therefore, given any point on \overrightarrow{AK} , say X , $XF = XG$. Now take X suitably chosen on ray AK to establish the result.²

*Application**Angle Bisector Theorem*

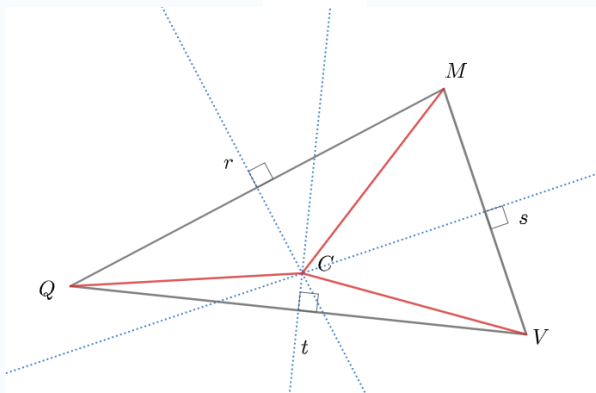
1. It must be emphasized that measurement of distance is along lines perpendicular to the sides of the angle. Any line perpendicular to the angle bisector is in fact "bisected" by it, but that is not the way distance is measured. Make sure to make this point clear.
2. If time permits, extend this discussion to account for why the angle bisectors of a triangle must be concurrent.

Student View: Slide 15

CHECK FOR UNDERSTANDING: How does the concept of symmetry help us understand WHY:

1. The perpendicular bisectors of the sides of a triangle are concurrent.
2. The angle bisectors of the angles of a triangle are concurrent.

EXPLANATION OF 1.



GIVEN $\triangle QMV$ with circumcenter C on lines r, s as shown.

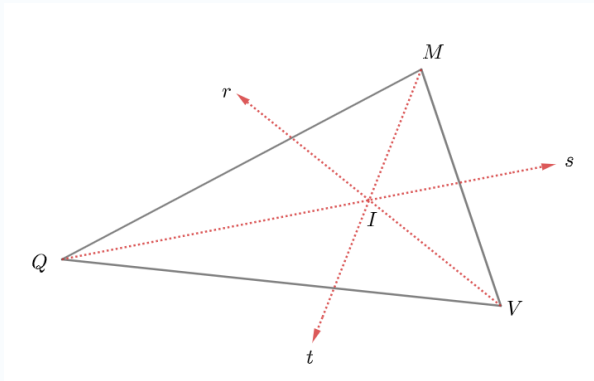
We need to explain WHY C also lies on t . Here is why. r is the perpendicular bisector of segment QM and C is on the intersection of r and s . Therefore $CQ = CM$. But C is also on s , the perpendicular bisector of segment MV , hence $CM = CV$ and it follows therefore that $CQ = CV$ since these are both equal to CM . Since $CQ = CV$, it follows that C must be equidistant from Q and V which means that C is on t , the perpendicular bisector of \overline{QV} . It follows that r, s, t must be concurrent at C .

Applications

Explaining Concurrencies

1. In this first discussion, note that the distances are computed from the circumcenter to the vertices of the triangle. Since the bisectors of the sides of the triangle are lines of symmetry, their incidence point forms three isosceles triangles with respect to the vertices of the ambient figure. This is the heart of the reasoning here.
2. In the second discussion the emphasis is on the distance between the alleged point of concurrence and the sides of the angles. Note that in both discussions, critical dependence is made on the concept of measure. In both of these discussions we are preparing students to be able to fashion arguments for themselves using reasoning skills based on previously established facts and assumed principles.

EXPLANATION OF 2.



GIVEN $\triangle QMV$ with incenter I on angle bisectors r, s as shown. We need to explain WHY I also lies on angle bisector t . Here is why. Since I lies on the angle bisector r ,

$$\text{dist}(I, \overline{MV}) = \text{dist}(I, \overline{QV}).$$

Now, since I also lies on the angle bisector s ,

$$\text{dist}(I, \overline{QM}) = \text{dist}(I, \overline{QV}).$$

Hence

$$\text{dist}(I, \overline{QM}) = \text{dist}(I, \overline{MV}).$$

And this means that I must lie on the angle bisector of $\angle QMV$ which of course is t .

Student View: Slide 16

We have done what we set out to do. 🧐

In this lesson we have covered a lot of material!

- We reviewed our basic constructions and learned more about constructing perpendicular and parallel lines.
- We introduced the formal term “postulate” that we will continue to use throughout this course.
- We discussed in detail the meaning of measurement.
- We introduced the concept of symmetry and its role in geometry.
- We concluded with clear explanations of why the concurrencies we saw in lessons 4 and 5 must be the case.

In our next lesson we will continue to apply these ideas and discuss further conclusions we can draw regarding angles, lines and structures associated with them.

Concluding Thoughts