Statistical Computing

Lecture 13: Image Recognition Based on SVD

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Classification of handwritten digits

Read images in R

```
## The image matrix for training sample. 256x1707
azip <- read.table("azip.dat")

## The true digits given in the training sample. lenght = 1
dzip <- as.numeric(read.table("dzip.dat"))

## The testing image matrix. 256x2007
testzip <- read.table("testzip.dat")

## The true digits for the testing sample. length = 2007
dtest <- read.table("dtest.dat")

## Display the image
i <- 120
image(matrix(azip[, i], ncol = 16), col = gray(255:0/255))
```

The naive method

The naive method is to check the distance from each test image to the mean of training image.

```
## The mean of training sample of a single digit
digits <- 0:9  # The possible digits in the US postal code
img.mean <- matrix(0, 256, length(digits))

for (i in digits) {
    idx <- (i == dzip)  # the location indicator for the i
    imgi <- azip[, idx, drop = FALSE]
    imgi.mean <- rowMeans(imgi)
    img.mean[, i + 1] <- imgi.mean
}

## Plot the mean image
par(mfrow = c(2, 5))
for (i in 1:10) {
    image(matrix(img.mean[, i], ncol = 16)[, 16:1], col = g
}
```

The naive method

- Now it is the time to check the testing sample to the mean of the training sample. We pick the first five testing digits.
- We find the first, third and the fifth are rather easy to classify by eyeballs. But the second and fourth ones are particular difficult.

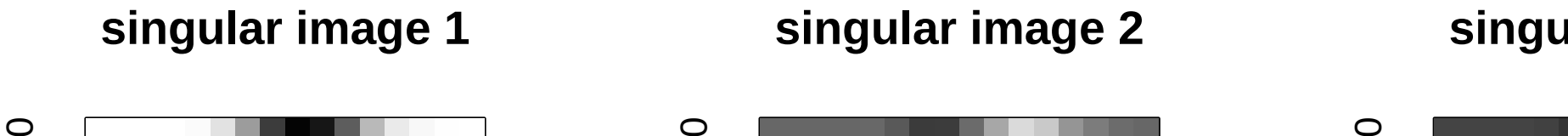
```
## Sketch a distance function to compute the Euclidean
## distance between two matrices in row wise.
rdist <- function(X, Y) {
    dim.X <- dim(X)
    dim.Y <- dim(Y)
    sum.X <- matrix(rowSums(X^2), dim.X[1], dim.Y[1])
    sum.Y <- matrix(rowSums(Y^2), dim.X[1], dim.Y[1], byrow
    dist0 <- sum.X + sum.Y - 2 * tcrossprod(X, Y)
    out <- sqrt(dist0)
    return(out)
}
```

The SVD method

- We pick the digit 9 as an example in this method and plot the first ten singular image from the SVD decomposition.
- We first use four bases, which yields the correct specification as follows We also tries to classify other digits which gives robust results. But when we increase more basis function, there comes the risk of overfitting.
- It maybe not a good idea to use all the bases but one can always pick up the bases according to the first kth largest eigen values.

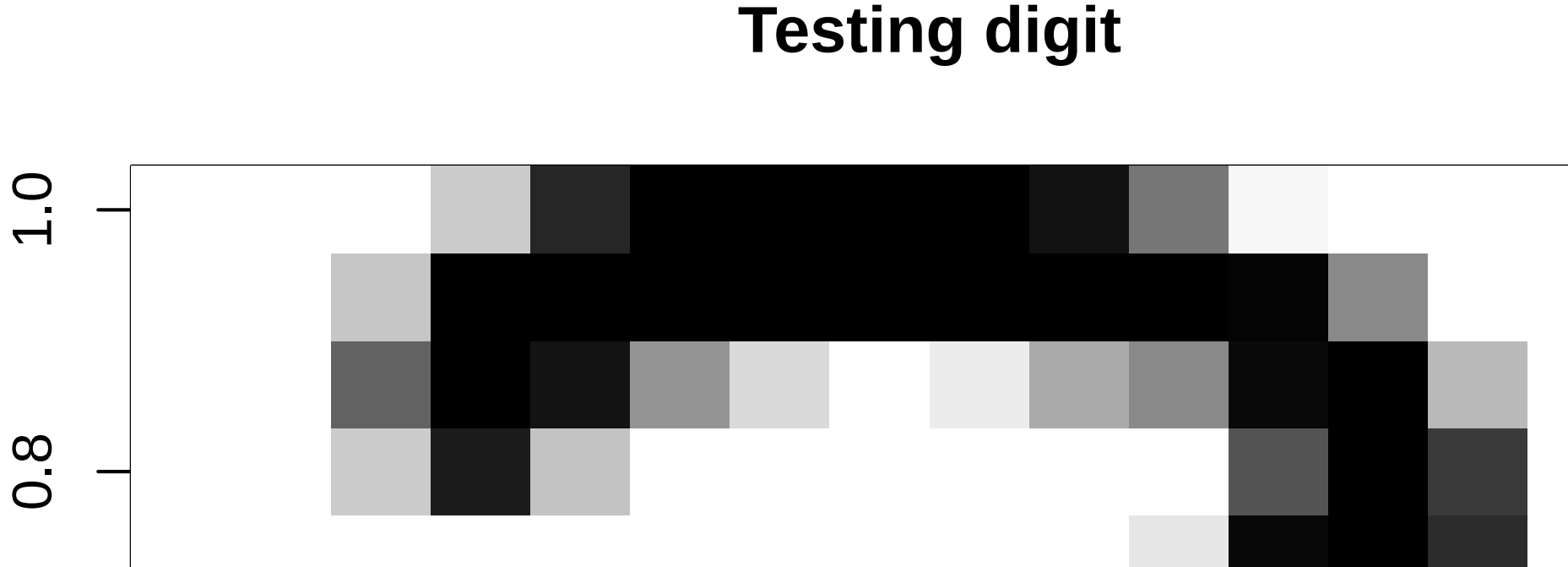
The SVD method

```
## Compute the singular matrix of a single digit in the
## training sample
digit <- 9

## Subtract the matrix for that digit
img.mat <- azip[, digit == dzip, drop = FALSE]
img.matSVD <- svd(img.mat)

## Plot the singular matrix under different basis.
par(mfrow = c(2, 5))
for (i in 1:10) {
    image(matrix(img.matSVD$u[, i], 16)[, 16:1], col = gray
        main = paste("singular image ", i, sep = ""))
}
```



Testing based on SVD

```
## Do the least square method with different basis and find
## the minimal residuals.

## The testing digit matrix
test.idx <- 3
image(matrix(testzip[, test.idx], 16)[, 16:1], col = gray(2
    main = paste("Testing digit"))
```



The SVD method

- We will find out when we overfit (see the plot of classification success as a function of the number of basis vectors.)
- To see this, we loop over all testing observations and number of bases from 1 to 88, and then count the correct specification numbers.